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Towards Meaningful Information Processing: A unifying representation for Peirce's sign types

Abstract An open problem in AI is the definition of meaningful information processing. That human interpretation and information processing by current computers can be different is well illustrated by Searle's famous Chinese room argument thought experiment. In this paper we suggest that an answer to the above open problem of AI can be given by introducing a model of information processing which is embedded in a Peircean theory of (meaningful) signs. Peirce's sign theory, that he systematically derived from his concept of a category, is seen by many as a theory of the knowable (the types of distinctions that can be signified by signs). We show that our model of information processing has the potential for representing three types of relation that are analogous to Peirce's three classifications of sign, consisting of 10, 28, and 66 elements.

Keywords: information, human processing, Searle, sign theory, Peirce, process model

1 Introduction

In his famous Chinese room argument thought experiment (CRA), Searle has shown that computations by current computers can be qualitatively less meaningful than human interpretation. We may ask: Is it possible to model human interpretation as a process? In this paper we suggest that the answer can be positive. Following the assumption that interpretation is related to a goal hence a process, we show that on the basis of an analysis of stimulus–reaction phenomena a model of human interpretation can be defined as a process (learning is beyond our current scope). By associating the events of that process with Peircean sign aspects we establish a relation between our model and Peirce's theory of signs and interpretation. It is by virtue of *this* relation that we can posit our process as a model of meaningful information processing. Our

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process oriented approach may be interesting from a semiotic perspective as well. Indeed, by offering an informational analysis to our model we are able to show its potential for a representation of three types of relation, consisting of 10 ('meaningful'), 28 ('syntactic'), and 66 ('semantic') classes, which are analogous to Peirce's three classifications of sign.

The possibility of a process model of human interpretation may enable a paradigmatic change in human–computer interfacing. As Peirce's sign theory is seen by many as a theory of the knowable (the types of distinctions that can be signified by signs), our model may facilitate the introduction of a novel approach in knowledge representation, natural language modeling, and problem specification. The possibility of a common representation for Peirce's different classifications may open new perspectives in sign theoretical research as well (Weiss & Burks, 1945), (Farias & Queiroz, 2003), (Burch, 2011), (Sanders, 1970).

The model suggested in this paper, and the relation between the used representation and Peirce's sign aspects has been introduced earlier in (Sarbo, Farkas, & van Breemen, 2011), amongst others. The goal of this paper is to show that our model is able to represent the more complex concepts of Peircean sign theory as well.

In past research, we experimentally tested that our theory is in line with human interpretation. To this end we developed models of information processing in different knowledge domains, such as the domain of natural language, 'naive' logic, and 'naive' reasoning. The test results showed that concepts generated by these models may be meaningful from a human perspective, as well.

Our theory is remotely related to Situation Calculus (McCarthy & Hayes, 1969). In our model, situations are represented by percepts (cf. sets of perceived qualities), fluents by elements of a percept, actions by relations between qualities (cf. bindings). Formulae used in action preconditions are restricted to Boolean expressions.

Note that ‘meaningful information’ (Vitanyi, 2006) and ‘meaningful information processing’ are not the same. The first is related to Shannon information² and a distinction between useful regularity and redundancy (Adriaans, 2009), the second to information as stimulus and interpretation by the mind. Maybe the negligence of this difference explains to some extent the blind spot which information scientists have had in their approach to the importance of embedding information processing in a theory of signs. Curiously, in information theory the term meaningful may refer to a formal property, but also to a property of (human) processing. An example of the second is the Turing Test, which assumes the existence of an observer, capable of (re)cognizing intelligent hence meaningful communication. Notably the same concept is involved in Shannon’s concept of information as event probability, as well. Whereas the concept of probability is usually formally defined, not so the concept of an event which is related to conceptualization (how can we know that an event has occurred) hence to meaningfulness from an interpreter’s perspective. As knowledge may arise from (meaningful) interpretation through generalization, limitations caused by a lack of semiotic embedding may characterize traditional, formal knowledge representation too.

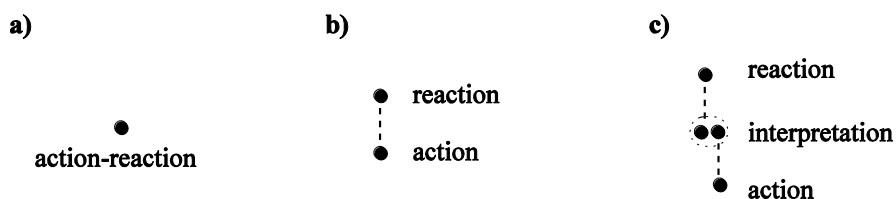
The structure of this paper is the following. In Sect. 2, we introduce a model of information processing on the basis of an analysis of action–reaction phenomena. This is followed by an analysis of our model, from a ‘syntactic’, and a ‘semantic’ point of view, in Sect. 3–5. In Sect. 6 we elaborate on the relation between our model and Peirce’s different classifications of sign. We close the paper with a summary.

²Following (Adriaans et al., 2010), Shannon information can be used to find an optimal compression for a sequence of messages. The related concept, Kolmogorov complexity can be used to define an ‘optimal’ probability distribution for a binary string: the universal distribution. Kolmogorov complexity and Shannon information seem to be dual notions: the shortest code for a binary string in the sense of Kolmogorov complexity is its optimal code in the sense of Shannon information using the universal distribution.

2 Action–reaction phenomena

We assume that the goal of human interpretation is the generation of a response to (external) stimuli.³ On a physical level, stimuli appear as forces. A theory of forces can be found in Newton’s work (Newton, 1999/1687). In his 3rd law of motion he postulates: “For every action there is an equal and opposite reaction”. An illustrative example is a nail, hit by a hammer. Following Newton’s 2nd law, the applied force accelerates the nail much harder, by virtue of its smaller mass, than the reaction force accelerates the hammer. In the end, the nail may get deeply driven into the underlying piece of timber, while the hammer only gets slightly bounced back in the opposite direction. Newton’s 3rd law, in combination with the 2nd law, not only predicts the reaction force, but, in specific cases, also the possible consequences of the action, such as the driving of the nail. Remarkably, in Newton’s world, there are only action–reaction phenomena. This may explain why in everyday life the term reaction ambiguously denotes a force as well as the effects triggered by that force.

From the point of view of information processing, Newton’s model of action–reaction phenomena may be conceived as too narrow: by knowing the applied force and the existing mass, everything else can be computed. Being neutral to the direction of time, this *monadic* model lacks the notion of a development, for instance, from action to reaction. See Fig. 1(a).



³The theory of this section is based on (Sarbo et al., 2011).

Figure 1: A monadic (a), dyadic (b), and triadic (c) concept of action–reaction phenomena. A dashed line is used to express a dependency between events. A pair of nodes that can be merged into a single node is indicated by a dotted circle.

If we are interested in the way in which reactions arise from actions, we may introduce the hypothesis that, in everyday phenomena, action and reaction are related to each other according to a relation of dependency. Assuming, for the moment, causal dependency, the result is a *dyadic* concept of action–reaction phenomena. See Fig. 1(b).

We may further refine our model by assuming that reactions arise from actions through an act of interpretation. Indeed, even at the physical level, the ability of objects to show different reactions to different actions may be seen as their potential for *interpreting* actions by ‘(re)cognizing’ them and ‘generating’ a reaction. Just as, in principle, physical objects involved in action–reaction phenomena must be independent (otherwise their co-occurrence cannot appear as a phenomenon), interpretation assumes the existence of knowledge about those objects and the possible consequence of their co-occurrence. For example, in our running example, the nail may be said to interpret the appearing force, by ‘(re)cognizing’ its measure and ‘generating’ a counter force, as well as a diametral piercing movement. In sum, through the introduction of the concept of interpretation in our *triadic* model, the relation between action and reaction can be split into two relations: one between action and corresponding knowledge (recognition) and another between that knowledge and a corresponding reaction (generation), see Fig. 1(c).

With the assumption of an involved dependency between action and reaction, the concept of development comes into sight.

2.1 Knowledge through internal processing

In Newton's world, action and reaction are unambiguously related to each other. Objects occurring in the Newtonian world simply do not have the potential of changing their 'reaction strategy'. The 'knowledge' in the node 'interpretation' in Fig. 1(c) remains the same. What happens if we extend our focus and include in our model objects that do have the potential to acquire knowledge?

Obviously, Newton's 3rd law holds for such entities as well. However, the potential to generate more complex reactions enables the introduction of more refined models of action–reaction phenomena. Through memorization, the interpreting system obtains information about occurring action–reaction events that may prove useful in later interactions. If the interpreting system has the potential to observe itself and to memorize its observations,⁴ it will be able to distinguish information about external actions from the possible consequences those actions can bring about to the interpreting system. The latter kind of information may be called the system's knowledge about itself. Through abstraction and generalization, the interpreting system may introduce concepts that can favorably be used to predict the consequences that appearing external qualities may have. This potential of the interpreting system to predict future events assumes an ability to cope with modalities other than the mechanical one, such as wave-type qualities, for instance, the observation of light rays in visual action–reaction phenomena: if we see the qualities of a hammer moving in our direction, we may step away, or shout, in order to prevent certain unfortunate mechanical effects.

2.2 Two modes of operation

Arguably, Newton's theory considers action–reaction phenomena from the point of view of an external observer (*external-view*). If we are interested in the question of how interpretation may capitalize on memorized information for the

⁴ An *observation* is defined as an event interpretation of an interaction.

generation of reactions, we must switch perspective and analyze action–reaction phenomena from the stance of the interpreting system itself (*internal-view*). Following this stance, we suggest that the interpreting system occurring in some *state* is in interaction with the external force (quality).⁵ This external force appears as an *effect* in the interpreting system. The qualities of this state and effect will be called the input state and effect qualities or, briefly, input qualities. The relation between *this* state and effect is the ground for the reaction generated by the observer.

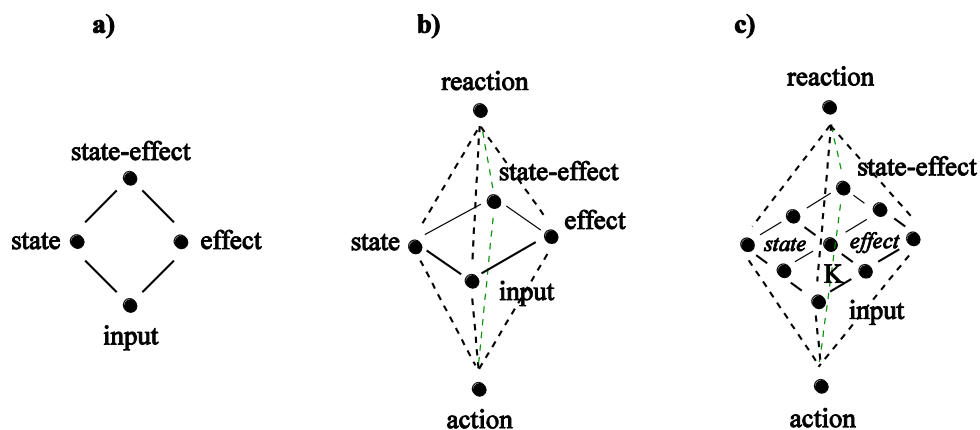


Figure 2: Informational relation underlying interpretation (a), matching mode operation (b), and analysis mode operation (c). A continuous line is used to represent a flow of emerging information, not just a dependency. ‘K’ stands for the observer’s knowledge

The aforementioned interaction is stored by the interpreting system in a collection of (unanalyzed) ‘input’, ‘state’, ‘effect’ and (interrelated) ‘state–effect’ qualities. These ‘storing events’, which come down to establishing relations, are all triggered by the external force (‘action’). Interpretation can be successful only if these events consistently *match* memorized information. This informational relation underlying the generation of reactions is depicted in Fig.2(a). A schematic model of the corresponding ‘*matching*’ mode operation of

⁵ External-view representation is possible also in this case. By considering the external force to be a representation of a phenomenon, the observer may be able to derive hypotheses about the qualities of that phenomenon.

information processing is depicted in Fig. 2(b). For example, the 'state' can be defined by the mass and the velocity of the nail and the hammer, the 'effect' by the change in their velocities.

Note that all 'storing events' included in Fig. 2(b) are internal. They are related to the occurring action and reaction, which are external. The latter relations are expressed by means of dashed lines, connecting the nodes 'action' and 'reaction' via the nodes 'input', 'state', 'effect' and 'state-effect' relation, refining our model in the node 'interpretation', in Fig. 1(c).

We assume that, through generalization, perceived data obtained in past experiences may be transformed into knowledge. In the case of indeterminate input qualities the generation of a state–effect relation may require an *analysis* of all possible matches, e.g., in a cyclic fashion,⁶ as well as the selection of a solution, on the basis of some strategy. As part of this analysis, different interpretations of the input state (cf. 'state') and effect (cf. 'effect') as well as their relation (cf. 'state-effect') can be generated internally by means of the system's knowledge about corresponding state and effect qualities.

Following the above considerations, a model of action–reaction phenomena can be derived as follows. By considering the input state and effect qualities to be external (cf. effect), in relation to the system's knowledge which is internal (cf. state), the interaction between the state and effect qualities on the one hand, and the system's knowledge on the other can be modeled by two instances of a 'matching' mode process. As the observer's knowledge is shared by those process instances, it can be represented by a single node ('K'). In Fig. 2(c) the two sub-processes (represented by a pair of structures consisting of four nodes) are marked by the labels *state* and *effect*. Note that the above 'internal' refinement of our model (in the nodes 'state' and 'effect', in Fig. 2(b)) does not affect the dependency between 'action' and 'reaction' (cf. dashed

⁶We return to this point in Sect. 5.4.

lines).⁷ In Fig. 2(c), the nodes between ‘action’ and ‘reaction’ represent the nine types of relation involved in action–reaction phenomena.

By the introduction of analysis and selection in the interpretation process, the possibility of anticipatory responses and habit formation are added to the mechanistic Newtonian model.

3 Informational analysis

What information is necessary for the nine types of relation of our model of action–reaction phenomena? Following the analysis presented in the previous section, the interpreting system must have information about:

1. Potential relational properties of qualities involved in an input interaction (in order to be able to perceive the input as a co-occurrence of a state and effect)
2. Actual relational properties of qualities involved in an observed change (in order to be able to interpret co-occurring state and effect qualities as ‘constituents’ of a relation)
3. Properties of relations involved in an observed phenomenon (in order to be able to interpret the input as a relation between a perceived state and effect).

For instance, that a nail and a hammer have kinetic properties, which are a potential for a relation (1), that a nail can be in relation with a hammer by resisting its effect, and a hammer can be in relation with a nail by affecting its state (2), and that a nail and a hammer having certain kinetic properties, may establish a relation between a diametral force and counter force (3).

⁷In Fig. 2(c), dashed lines connect the nodes ‘action’ and ‘reaction’ with the nodes ‘state’ and ‘effect’ comprised by the two sub-processes, respectively. The labels of the latter two nodes are omitted in the diagram.

How can we know about the three types of information? The Newtonian world can be defined by a set of action–reaction phenomena, which implies that *only* action-reaction phenomena can be observed hence, from an informational stance, only relations (3) can be experienced. Actual relational properties (2) must be derived from perceived relations (3), and potential properties (1) from those actual relational properties (2). The other way round, potential properties (1) must underly actual relational properties (2), and those the properties of perceived relations (3). For instance, from the experience of a reaction (3), we may derive the existence of a relation between a state and effect (2), and from that relation, a co-occurrence of certain kinetic qualities (1).

Information about qualities may enable an interpretation of co-occurring qualities as a relation, involved in the observed phenomenon. An example is a co-occurrence of a nail and a hammer, interpreted as ‘nail-driving’. Note that interpretation requires knowledge about possible co-occurrences of qualities that can be experienced as a phenomenon, hence are meaningful.

The granularity of information may depend on the resolution by the interpreting system. In this paper we assume finite resolution and the existence of a mapping of qualities, which are a continuum, to discrete values called *qualia*.⁸ Below, we use the terms quality and qualia interchangeably.

Postulating the existence of three types of information enables action–reaction phenomena to be interpreted as a process, combining qualities involved in an interaction into a relation.

3.1 Categories and types of information

In this section we will settle on the idea that the three types of information may coincide with the three categories of information involved in phenomena. According to Peirce, phenomena can be distinguished in three categories, that he consequently called firstness, secondness, and thirdness:

⁸ Following (Harnad, 1987) we assume qualia to be ‘internal’ representations of ‘external’ qualities. Qualia is plural for quale.

The first is that whose being is simply in itself, not referring to anything nor lying behind anything. The second is that which is what it is by force of something to which it is second. The third is that which is what it is owing to things between which it mediates and which it brings into relation to each other (CP: 2.356)

An example of firstness are qualities in themselves, e.g., nail, hammer. An example of secondness are (ad-hoc) relations between qualities, e.g., a co-occurrence of a nail and a hammer. An example of thirdness are habitual (meaningful) relations of qualities, e.g., a co-occurrence of a nail and a hammer (known) as 'nail-driving'. Another example, this time in the context of the CRA, is the perception of an input string as a quality (cf. firstness), as a sequence of words in some language (cf. secondness), and a sentence in a familiar language (cf. thirdness). In the first case, the reaction can be a copying of the input (cf. impression), in the second case, the generation of a syntactic parsing, in the third case, the establishing of a meaningful interpretation.

The three categories are related to each other according to a relation of dependency: categories of a higher ordinal number *involve* a lower order category. A distinguishing property of the Peircean categorical schema is that thirdness can only be experienced, firstness may only appear through secondness, and secondness only through thirdness.⁹ This *subservience* relation of the three categories implies that categories of a lower ordinal number *evolve* to hence *need* a higher order category. The two relations share the property that they are *reflexive* and *transitive*. Below we refer to the two types of dependency between categories by the relations 'involve' (' \geq ') and 'evolve' (' \leq ').

⁹ Note the difference with Aristotle's categories, that are the possible kind of things that can be the subject or the predicate of a proposition.

We suggest that the three types of information of qualities, introduced in the previous section, are analogous to the three classes of a categorical definition of information involved in phenomena, as mentioned above:

- 1. data:** information involved in a quality in itself, such as its potential for a relation (cf. firstness)
- 2. connection:** information involved in a relation of a quality with another quality, such as its actual relational properties (cf. secondness)
- 3. relation:** information involved in a quality, establishing a relation, such as its habitual properties (cf. thirdness).

By considering the dependency between the Peircean categories to be an expression of a *development*, the three classes of information can be interpreted as *stages* of an informational process, generating a relation (cf. reaction) involved in the input action. We assume that information of a higher class *involves* information of a lower class, and the other way around, information of a lower class *needs* information of a higher class it is evolving to. For instance, $1 \leq 2$: a potential relational property may become operational only through an actual relation; and $3 \geq 2$: an arising relation involves an actual relation between qualia. An example, in the context of the CRA, are the relations involved in the interpreting system's potential for generating a copy of the input (1), for analyzing it syntactically (2), and in a meaningful fashion (3).

By establishing a link between categories and types of information, which are relations, we open the way for a relational interpretation of action–reaction phenomena.

4 Relational interpretation

From a categorical perspective, qualities involved in an interaction must be independent (cf. firstness). The reaction, which arises through interpretation, must involve a qualitative change (cf. thirdness). From this we may conclude that each one of the interacting qualities, and reaction, which too is a quality, must be independent.

An example of qualities involved in an interaction are a nail and hammer; an example of a qualitative change is the arising force and counter force. Another example, this time in the domain of wave-type phenomena, are interacting wave signals (cf. qualities), and an arising standing wave (cf. a qualitative change).

From an informational stance, an interaction can be characterized by a pair of qualities (q1, q2), the reaction by the quality of the arising force and counter force (q3). As information involved in each one of the 3 qualities can be distinguished in 3 classes (cf. stages), we suggest that information involved in action–reaction phenomena can be represented by $3*3*3=27$ possible relations or combinations of information stages; the arising phenomenon itself by the combination corresponding to the ‘arising’ relation (which can be any one of those possible relations). As an observation is always related to some perspective by the observer, the ‘arising’ relation must involve information about a selection from the 27 combinations. By virtue of this information, the ‘arising’ relation must be different from those possible relations.

In sum, we suggest that action–reaction phenomena can be characterized by $27+1=28$ types of relation. According to our model, those relations must be involved in the interaction between q1 and q2. Information about the reaction, involved in q3, may enable a novel interpretation of the existing relations, but no introduction of relations independent from q1 and q2 (the existence of such relations would imply that representation by the observer may not be truthful).

4.1 Internal-view representation

Although from the point of view of interaction, the observer can be identical to one of the interacting qualities, from the stance of interpretation, the observer may represent the input interaction from the perspective of any one of the two input qualities. The two perspectives of interpretation are: q1 is affecting q2, and

q2 is affecting q1, that we will call effect-, and state-view, respectively.¹⁰ By virtue of the isomorphism between the two perspectives, their relations can be merged into a single representation. We will return to this point in Sect. 5, in which we incorporate in our model a representation of q3, as well.

Below, we elaborate on an effect-view interpretation (q1 is affecting q2). Conform this perspective, the two qualia are interpreted differently: q1 as an expression of the input effect, q2 as an expression of a relation of the input state with the input effect. The three classes of relational information involved in q2 can be defined as follows (see Fig. 3):

(1) independent: an expression of the relation of q2 with q1, as a co-occurrence of independent qualia (cf. firstness)

(2) co-existing: an expression of the relation of q2 with q1, as a co-existence of interrelated qualia (cf. secondness)

(3) corresponding: an expression of the relation of q2 with q1, as a correspondence of qualia (cf. thirdness).

Following the dependency between the categories (see Sect. 3.1), the above classes are related to one another: $(1) \leq (2) \leq (3)$ and $(3) \geq (2) \geq (1)$ where ' \leq ' and ' \geq ' are used as polymorphic operators.

As q2 and q1 are commonly interpreted from the perspective of q1, their information can be merged into a single representation. The three up-right diagonals depicted in this diagram represent q1 from the perspective of firstness, secondness, and thirdness, according to q2, that we call the three representations of q1. The terminology used in Fig. 3, as well as below, complies with the terminology used in (Sarbo et al., 2011).

(1) First representation of q1 according to q2

neutral=independent*data: an expression of the input qualia as neutral relational properties or a potential for a relation

¹⁰ In syntactic language phenomena, related notions are active and passive voice.

passive=independent*connection: an expression of the input qualia as passive relational properties enabling an actual relation

active=independent*relation: an expression of the input qualia as active relational properties involved in a habitual relation

An example are independent qualia of an interaction appearing as data (cf. neutral); the potential of q1 for a relation with an independent other quality, represented by q2 (cf. passive); and the habitual properties of a relation involved in q1, in combination with a type of qualia represented by q2 (cf. active).

(2) Second representation of q1 according to q2

constituent=co-existing*data: an expression of the input qualia as constituents of a potential relation

modifier=co-existing*connection: an expression of the input qualia as passive constituents enabling an actual relation

predicate=co-existing*relation: an expression of the input qualia as interrelated active constituents involved in a habitual relation

An example, in syntactic language phenomena, are co-existent words of an input string, appearing as data (cf. constituent); syntactic modifiers, connecting to their arguments (cf. modifier); and verb phrases, predicating the subject of the sentence (cf. predicate).

(3) Third representation of q1 according to q2

abstract=corresponding*data: an expression of the input qualia as abstract properties involved in a potential relation

structure=corresponding*connection: an expression of the input qualia as interrelated properties involved in an actual relation

binding=corresponding*relation: an expression of the input qualia as an amalgamation of properties involved in a habitual relation.

An example, this time in wave-type phenomena, are corresponding signals of a wave phenomenon, abstracted as longitudinal or transversal wave-forms (cf. abstract); a combination of wave signals into an interference or superposition (cf.

structure); and a relation of wave signals, appearing as a standing wave, characterized by an arising new wave pattern (cf. binding).

By combining information from q2 and q1 into a single representation, we lay the ground for an informational analysis of our model of action–reaction phenomena.

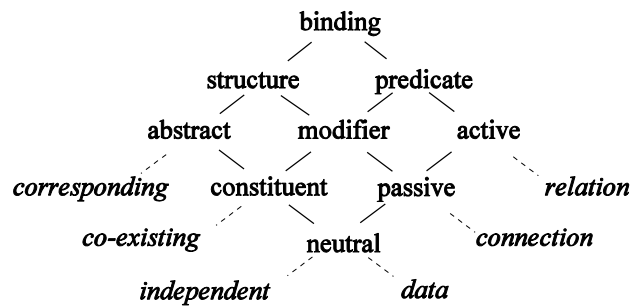


Figure 3: Combined state and effect information in effect-view interpretation. The three up-right diagonals illustrate the three representations of q1 (according to q2)

5 Information structures

The goal of this section is to show that the 27+1 combinations of information stages can be represented by relations enabled by q2 and q1. Our model is categorically inspired. We show that those relations, as well as the operations generating them, exhibit the properties of the three categories. Following this line of thought we assume that, from an informational stance, action–reaction phenomena must involve ternary relations between three qualities (cf. thirdness), binary relations between a pair of qualities (cf. secondness), and unary relations of qualities to themselves (cf. firstness). As, in effect-view, the observed phenomenon is represented by information involved in q1, in the relation between q2 and q1, and in the relation between q3, q2 and q1, we assume that the 27+1 combinations of information stages is represented by *unary* relations of q1 to itself, *binary* relations between q2 and q1, and *ternary* relations between all three qualities. As q3 is merging q2 and q1 into a single

quality, we assume that q_3 can be represented by merging information involved in q_2 and q_1 .

We suggest that unary, binary, and ternary relations between qualities can be represented by relations between stages from a single, a pair of, and all three representations of q_1 , respectively. By virtue of their categorical foundation, relations between information stages must respect the dependency between categories. We call this the condition of *categorical dependency*. For example, a secondness category information stage can be in an ‘involve’ relation (\geq) with firstness and secondness category information stage(s) only. Below we begin with an analysis of unary and binary relations. We return to ternary relations in Sect. 5.1.3.

5.1 Syntactic structures

In order to introduce an informational analysis of our representation, we embed the set of stages of a quality into the ordered set of integers $S=\{1,2,3\}$. For $i\in\{1,2\}$, we define: $q_i=\langle S,\leq \rangle$. We represent the set of relations enabled by q_2 and q_1 by the operation lattice multiplication (Davey & Priestley,1990). In this section, we restrict ourselves to an analysis of the relations represented by the Hasse-diagram of the arising ordering,¹¹ that we refer to by the symbol ‘ q_2*q_1 ’. We allow ‘ \leq ’ to designate the relations ‘involve’ and ‘evolve’, ambiguously. We call an element of the set underlying q_2*q_1 a stage.¹² A stage of q_2*q_1 can be referred to by its coordinates ‘(i)j’, designating stage ‘j’ of q_1 , from the ‘ith’ perspective, according to q_2 . For instance, (3)1 designates stage 1 of q_1 , from the perspective of 3rdness, according to q_2 . See also Fig. 4. Elements of q_2*q_1 , that are relations, can be referred to by a pair of coordinates, separated by a hyphen symbol, for instance, (3)1-(3)2. Reference to a set of relations can be abbreviated. For example, (3)1-(3)2-(2)2 is short for $\{(3)1-(3)2, (3)2-(2)2\}$.

¹¹The Hasse diagram of a lattice L is a representation of a minimal relation generating L by means of a closure operation.

¹² Note the ambiguous use of this term.

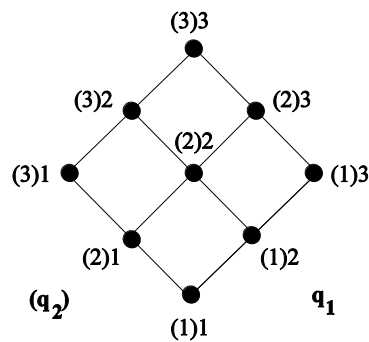


Figure 4: A representation of $q_2 * q_1$. Stages of $q_2 * q_1$ are labelled by a pair of integers, representing the category of a stage of q_2 (in parentheses) and a stage of q_1 . Stages of q_2 are used as (relational) representations of a stage of q_1 . For example, (3)1 designates stage 1 of q_1 , from a 3rdness perspective according to q_2 . This convention is used in later diagrams as well.

In order to achieve our goal set in the beginning of Sect. 5, we derive decompositions of $q_2 * q_1$. In the case of unary relations (cf. firstness) this boils down to decompositions of q_1 (Birkhoff & Bartee, 1970). In our categorically inspired analysis we restrict ourselves to decompositions that are homogeneous hence can be characterized by a single type or category. We define a homogeneous decomposition of a relation R , as follows: $R = \cup_{i=1,k} (r_i \cong r)$, for $r \subseteq R$, $k \geq 1$, and $r_i \cap r_j = \emptyset$ for $i \neq j$. The equivalence class r is also called a unit relation. By virtue of the two orderings (cf. ‘involve’ and ‘evolve’), r has two versions. We assume that a single version of r is defined by one of the order relations only. In the diagrams of this section, relations are represented by undirected edges (information about ordering is omitted).

In our analysis we always begin with the smallest unit relation of a kind (unary, binary, and ternary), from which we develop new unit relations (and decompositions) by means of three functions on relations:¹³ (1) identity, (2) composition, (3) recursion. Identity (1) enables a definition of instances of r in R . Composition (2) is a function on instances of r . By virtue of the existence of the

¹³ We specify our functions by means of examples (a formal definition is omitted).

two order relations, this operation has two versions, combination and complementation, enabling a composition of instances of r , having identical and different orderings, respectively. Finally, recursion (3) enables an introduction of a new interpretation of a relation. This function has three versions: transitive closure, encapsulation, and merging. Encapsulation enables a subset of R to be interpreted as an element of an ordering. A pair of such elements are considered to be in relation if they include a shared element of R . Merging introduces a conceptually new interpretation for R .

Note the categorical nature of the three functions above. Identity operates on a singleton relation (cf. firstness), composition on a pair of subsets of a relation (cf. secondness), recursion on subsets underlying a new concept (cf. thirdness). Also note the analogy between these functions, and the functions used by a mathematical theory of categories (Barr & Wells, 1990).

We call a homogeneous decomposition of R a *conceptualization*, an instance of r an *information structure*. In our analysis, trivial decompositions ($r \in R$) are omitted. As the structures revealed by our analysis do not represent information involved in q_3 , they can be called *syntactic* structures.

An example, in geometry, are the different decompositions of a quadrangle, defined by a relation between four points and edges. By considering a point and a pair of edges which it is incident to be a single relation (cf. identity), the quadrangle can be recursively conceptualized (cf. encapsulation) as a relation between four angles (cf. merging). By combining the relations representing a pair of complementary angles into a single relation (cf. complementation), the quadrangle can be conceptualized as a couple of pairs of angles (Sarbo, 1996).

5.1.1 Unary structures

The smallest unary unit relation is defined by the relation of an information stage of q_1 . See Fig. 5(a). The corresponding unary decomposition of q_1 is a trivial one. A more interesting unit relation can be found through composition and transitive closure. See Fig. 5(b). A homogeneous decomposition of the three representations of q_1 into three unary information structures is depicted in

Fig. 5(c). An application of both versions of the above unit relation do not enable new conceptualizations, different from the existing ones. Unary information structures satisfy the condition of categorical dependency, trivially.

The number of unary information structures is: **3**.

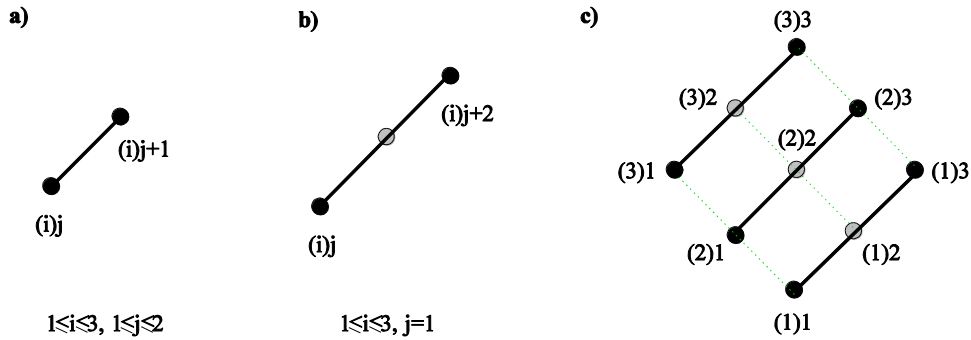


Figure 5: Unit relations enabling a trivial (a), and a non-trivial unary decomposition of q_1 (b). The three unary information structures (c) generated by unit relation (b). Bullets of a lighter grey shade stand for stages of q_1 that are closed. Dotted lines are used for an expression of the three representations of q_1 , as firstness category relations involved in $q_2 * q_1$.

5.1.2 Binary structures

The smallest binary unit relation is defined by the pair of relations of an information stage of $q_2 * q_1$. The two versions of this relation are displayed in Fig. 6(a) and Fig. 6(b). An instance of the first version is $(2)1-(1)1-(1)2$. A homogeneous decomposition of $q_2 * q_1$ into binary information structures is depicted in Fig. 7(a), by small ‘triangles’.¹⁴ Instances of the two versions of the unit relation, ordered by ‘evolve’ (\leq) and ‘involve’ (\geq), are represented by ‘triangles’ pointing downward and upward, respectively. Through composition (cf. combination) and transitive closure, $q_2 * q_1$ can be conceptualized as a set of large ‘triangles’. See Fig. 7(b).

¹⁴ For reasons, explained later in Sect. 5.1.3, binary information structures are represented by triangles.

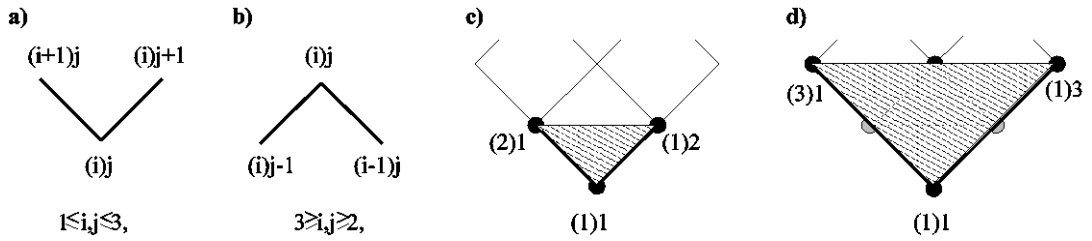


Figure 6: The two versions of the used binary unit relation, ordered by ‘evolve’ (a) and ‘involve’ (b). A sample binary information structure (c), and the closure of a composition of binary information structures (d). A filling pattern is used for an illustration of binary structures as ‘triangles’

By combining a pair of small ‘triangles’ into a single relation (cf. complementation), we may conceptualize $q_2 * q_1$ as a set of small ‘quadrangles’. See Fig. 8(a). An example is the complementation of $(2)1-(1)1-(1)2$ by $(2)1-(2)2-(1)2$, obtaining $(2)1-(2)2-(1)2-(1)1$. Considering small ‘quadrangles’ to be a single element (cf. encapsulation), enables $q_2 * q_1$ to be recursively conceptualized as a large ‘quadrangle’ (cf. merging). See Fig. 8(b). Binary information structures satisfy the condition of categorical dependency, trivially. For example, in Fig. 6(c), $(1)1 \leq (2)1$ and $(1)1 \leq (1)2$, as $(1) \leq (2)$ and $1 \leq 1$, and $(1) \leq (1)$ and $1 \leq 2$, respectively.

The number of binary information structures is: $(8+2)+(4+1) = 15$.

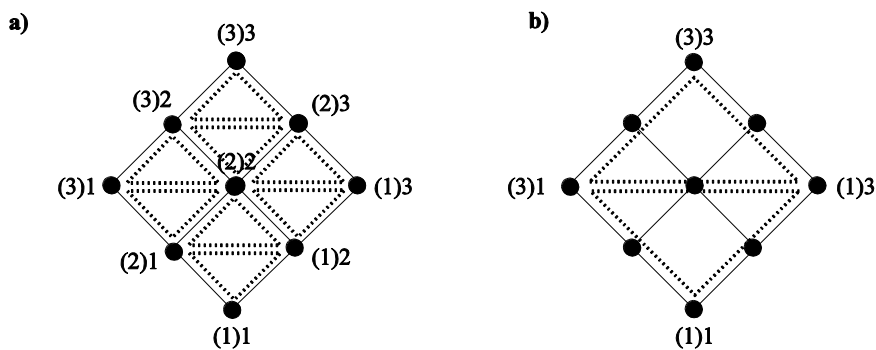


Figure 7: A conceptualization of $q_2 * q_1$ as small (a) and large ‘triangles’ (b)

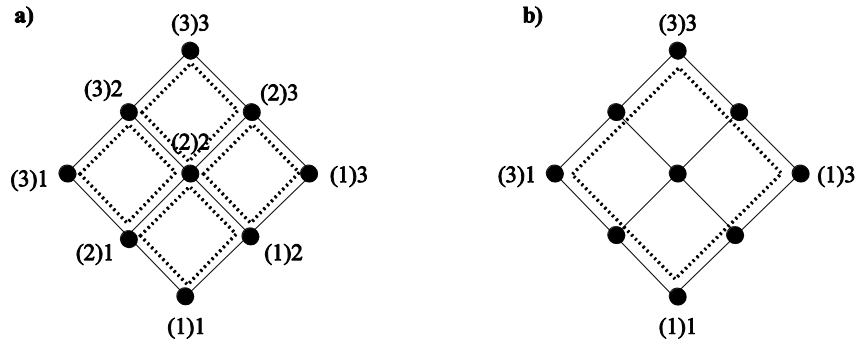


Figure 8: A conceptualization of $q_2 \cdot q_1$ as small (a) and large ‘quadrangles’ (b). Edges representing relations arising through closure, are omitted (e.g., in (a), $(1)1$ - $(2)2$, and in (b), $(1)1$ - $(3)3$)

5.1.3 Ternary structures

Ternary relations are relations between q_3 , q_2 , and q_1 . From an analytical perspective, q_3 (cf. reaction) must involve information about q_2 and q_1 , as well as about itself. However, binary relations enabled by $q_2 \cdot q_1$ may represent information involved in q_2 and q_1 only. We may capture the above completeness of q_3 , by defining ternary relations to include information from all three representations of q_1 (according to q_2). As a result, we may define the smallest ternary unit relation to be a relation between a single information stage from each one of the three representations of q_1 . An instance of this unit relation is defined by the information stages $(3)1$, $(2)1$, $(1)2$.

We represent ternary information structures by triples. To this end, we assume the existence of an order preserving mapping between the three categories and the three positions of a triple. This way we ensure that, in a triple, the condition of categorical dependency for stages from q_2 is respected. In addition we require that the above condition is respected by the stages of q_1 , occurring in the three positions of a triple. We use the convention that a triple ‘ (i,j,k) ’ stands for the ternary relation $(3)_i$ - $(2)_j$ - $(1)_k$, for $1 \leq i, j, k \leq 3$. For example, $(1,2,2)$ is representing $(3)1$ - $(2)2$ - $(1)2$. This relation satisfies the condition of

categorical dependency, as $1 \leq 2 \leq 2$. An example of a triple which does not satisfy that condition is (2,1,1).

A homogeneous decomposition of $q_2 * q_1$ into ternary information structures is defined by the set: $\{(1,1,1), (1,1,2), (1,2,2), (2,2,2), (1,1,3), (1,2,3), (2,2,3), (1,3,3), (2,3,3), (3,3,3)\}$, which is a strict lexicographic ordering. See Fig. 9. In this diagram, stages occurring in a ternary relation are connected by edges (cf. relations between information stages). An application of both versions of the above unit relation, as well as the use of the operations, composition and recursion,¹⁵ enable no new decompositions. Note, in Fig. 9, the existence of 'horizontal' edges, e.g., (2)1-(1)2, which explains our earlier representation of binary information structures by means of triangles, in Fig. 7.

The number of ternary information structures is: **10**.

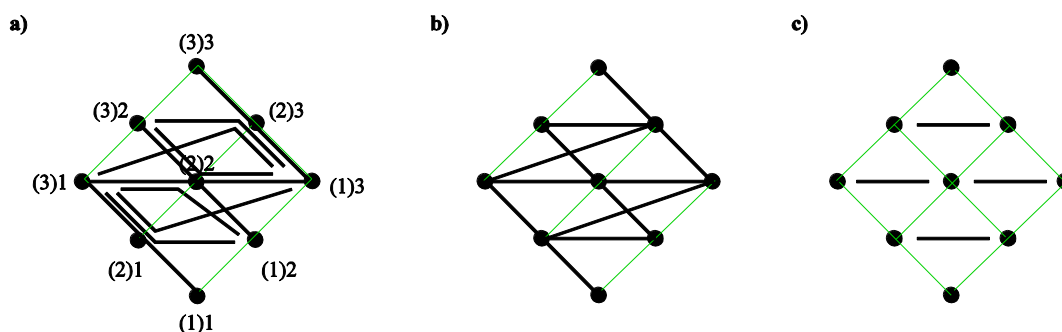


Figure 9: Ternary information structures depicted by edges between stages occurring in the three positions of a triple (a). The involved set of relations (b) and 'horizontal' dependencies (c)

Following the analysis of this section we conclude that the number of unary, binary, and ternary information structures involved in $q_2 * q_1$ is: $(3+15+10)=$ **28**. Note the categorical nature of the three types of structures. Unary information structures are independent (cf. firstness), binary information structures are

¹⁵ Note that the number of values that can be assigned to a position of a triple is 3, which is a prime number hence not decomposable.

related to one another, but their relation is not an ordering (cf. secondness), ternary information structures are related and their relation defines an induced ordering (cf. thirdness). Also note that, from a formal perspective, the set of decompositions of $q_2 * q_1$ introduced in this section is not complete. Decompositions that are not mentioned are not categorically founded hence are not considered to be representations of action–reaction phenomena.

The relations introduced in this section are an expression of a co-existence of information stages. As, from an analytical point of view, action–reaction phenomena merge the qualities of an interaction into a single quality (cf. reaction), we are interested in relations expressing a combination of information involved in qualities of a phenomenon. An analysis of such relations, represented by $q_2 * q_1$, is the subject of the next section.

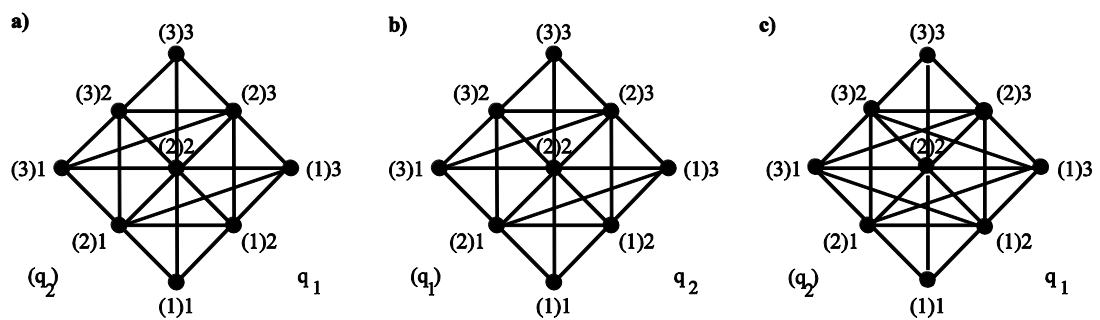


Figure 10: Relations in effect- (a) and state-view interpretation (b), and the set of relations obtained through merging (c)

5.2 Semantic structures

The set of relations generated by the two perspectives of interpretation, q_1 is affecting q_2 (effect-view) and q_2 is affecting q_1 (state-view), are isomorphic. See Fig. 10(a) and Fig. 10(b). Although in a single observation, the observer may develop one of the two interpretations only, we assume that through *memorization*, relations from the two perspectives can be merged into a single representation by the observer (this requires that one of the two representations is reflected). The arising full set of relations is depicted in Fig. 10(c). Note, in this

diagram, the existence of an edge between (1)1 and (3)3, and the absence¹⁶ of one between (3)1 and (1)3. See also Fig. 8.

The potential of the observer to consider phenomena to be interactions between some state and effect, is independent from its potential to merge information from different interpretations, through memorization. We assume that, in a single observation, the observer may develop *one* of the two possible views of interpretation in the presence of memory information too. How can we comply with this condition of our model?

By analyzing the relations, displayed in Fig. 10(c), we may observe that there are two relations, (2)1-(3)2 and (1)2-(2)3, which are an expression of a combined growth of categorical information involved in q2 and q1 hence can be used for distinguishing between two perspectives of interpretation. As (2)1-(3)2 is an expression of an increase of information of the state (q2), from secondness to thirdness (this is opposed to an increase of information of the effect (q1), from firstness to secondness, which is less meaningful), this relation can be associated with state-view interpretation. For symmetry reasons, (1)2-(2)3, expressing an increase of information of the effect, can be associated with effect-view interpretation.¹⁷ In sum, we assume that 'state-view' interpretation is characterized by the presence of (2)1-(3)2 and the absence of (1)2-(2)3; effect-view interpretation by the presence (1)2-(2)3 of and the absence of (2)1-(3)2. See Fig. 11.

By re-introducing the two views of interpretation, the possibility of an analysis of relations representing a combination of information from different perspectives comes into sight.

¹⁶This is a consequence of the 'V'-shape relations underlying small 'triangles' (cf. Sect. 5.1.2).

¹⁷(1)1-(2)2, (2)2-(3)3, and (1)1-(3)3 do not represent an increase of categorical information specific for q2, or q1; (2)1-(1)3, (3)1-(2)3, etc., do not represent a combined growth of categorical information of q2 and q1.

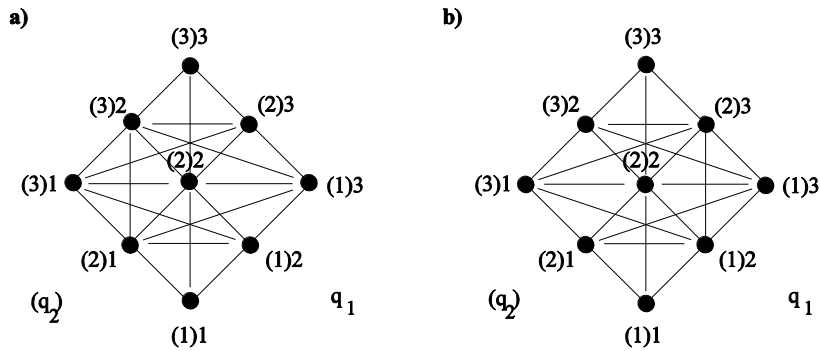


Figure 11: Relations in state- (a) and effect-view interpretation (b)

5.3 Octahedron representation

In order to expose our relations as an expression of information combination, we offer a transformation to our representation, depicted in Fig. 11. We illustrate this for state-view interpretation, shown in Fig. 11(a). Relations enabling a combination of information from different perspectives of q_1 must be relations between stages that are categorically different¹⁸ (relations between information stages that are different in one of their q_2 , or q_1 coordinates only, have been considered already by our analysis). These are the ‘crossing’ relations, (3)2-(1)3, (3)1-(1)2, (2)1-(1)3, (3)1-(2)3, and the ‘vertical’ relations, (2)1-(3)2, (1)1-(2)2, (2)2-(3)3 and (1)1-(3)3. For example, in (3)2-(1)3, (3) and (1), as well as 2 and 3, are categorically different. An example of a relation that does not satisfy our needs is (3)2-(2)2.

An essential element of our transformation is a combination of information from a pair of stages of a relation into a single stage. As a result, the edge connecting the two stages can be removed. We illustrate our transformation for (3)1-(1)2 and (3)2-(1)3. Through merging them into (1)2 and (1)3, respectively, information shared by the relations (3)1-(2)2 and (3)2-(2)2 may get lost (cf. information ‘coordinated’ by (2)2). In order to keep the represented information

¹⁸A combination of information from different interpretations is meaningful for binary and ternary relations only.

invariant, we introduce an identical copy of (2)2, as shown in Fig. 12(a).¹⁹ As (1)1 and (3)3 are in relation with (2)2, the relations (1)1-(2)2 and (2)2-(3)3 can be replaced by (1)1-(2)2 and (2)2-(3)3. A stepwise transformation of the relations in Fig. 11(a) is illustrated by Fig. 12(b)-(d). A transformation of the relations in Fig. 11(b) is depicted in Fig. 13.

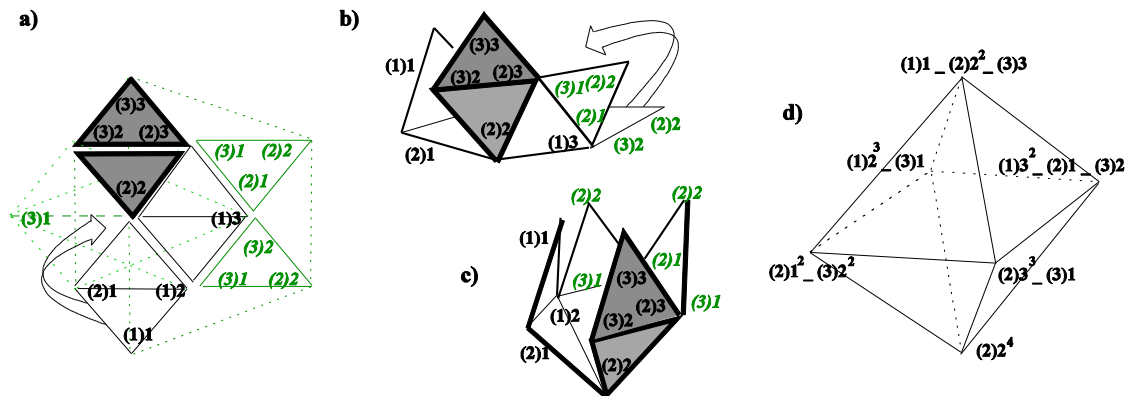
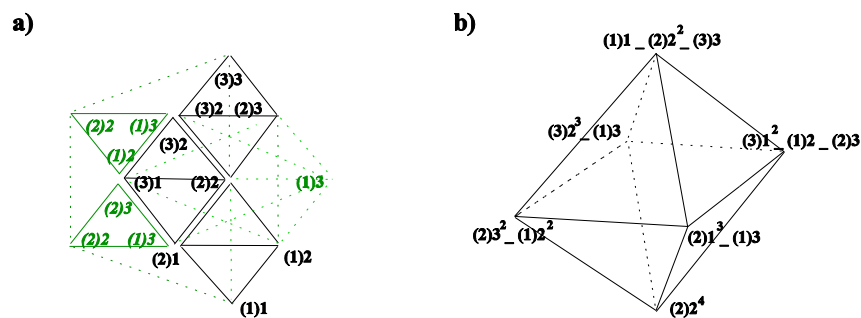


Figure 12: A transformation of relations in state-view interpretation. In (a)–(c), stages and edges involved in merging are given in italics, and by dotted lines, respectively. In (d), vertices are labelled by an expression on stages; combination of information is designated by ‘_’. The number of instances of a stage can be given by a superscript, for example, (2)3³_(3)1 designates a combination of information from 3 instances of (2)3 and a single instance of (3)1. Grey shade and dotted lines are used for visualization purposes:



¹⁹ A copy is required as coordination may refer to all information represented by (2)2. Note that information is idempotent for addition. Stages involved in information combination are given in italics, e.g., (2)2.

Figure 13: A transformation of relations in effect-view interpretation (a) into an octahedron (b)

5.4 Cyclic information processing

The category of information obtained through combination can be distinguished in two types: ‘and’- and ‘or’-type. For example, firstness type information, designated by ‘1’, is ‘and’-type in (1)1, by virtue of occurrences of ‘1’ in both coordinates; and ‘or’-type in (1)2-(3)1, because of occurrences of ‘1’ in different coordinates. This way, vertices of the octahedron can be labelled by their categorical type of information. We elaborate this for ‘effect-view’ representation, depicted in Fig. 13. In our presentation below, superscripts of a stage are omitted (cf. idempotence of information). We represent the categorical type of information of a vertex by an expression, in which ‘and’-, and ‘or’-type occurrences of a category are separated by a ‘*’, and ‘+’ symbol, respectively.

(1)1_(2)2_(3)3:	1*2*3
(2)1_(1)3:	1
(2)3_(1)2:	2
(3)2_(1)3:	3
(3)1_(1)2_(2)3:	1+2+3
(2)2_(2)2_(2)2_(2)2:	2*2*2*2

In the case of indeterminate input, the interpreting system may have to consider all possible interpretations, one by one, in a cyclic fashion. Cyclic operation can be modelled by mapping the vertices of the octahedron to the nodes of our Newtonian model of action–reaction phenomena. The vertex labelled by 2*2*2*2 can be mapped to the node ‘action’, by virtue of the meaning of a pointer involved in secondness (cf. 2) and the potential of the node ‘action’ for a presentation of qualia of a (next) input phenomenon. The vertex labelled by 1*2*3 can be mapped to the node ‘reaction’, by virtue of the

completeness of thirdness²⁰ and the potential of the node 'reaction' to combine information from all three qualities. The other four vertices can be mapped to the four nodes of our Newtonian model, as well as to stages of $q_2 \cdot q_1$, as follows. See also Fig. 14.

1: **state**, (3)1; abstract input state, representing the potential for a relation with any effect (cf. firstness)

2: **state-effect**, (3)3; binding, between the input state and effect, representing a relation (cf. secondness)

3: **effect**, (1)3; abstract input effect, representing the potential for a habitual relation with a type of input state which is involved (cf. thirdness)

1+2+3: **input**, (1)1; input state and effect, representing a collection of qualia (cf. relational potential) offered for interpretation.

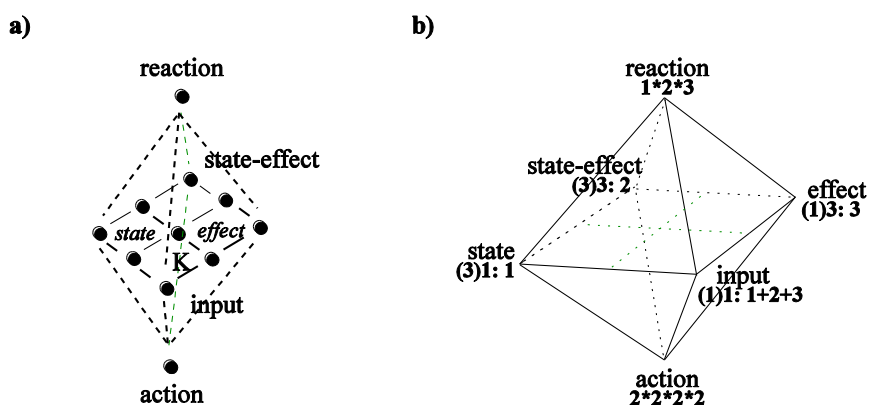


Figure 14: A recap of our Newtonian model of interpretation (a), cf. Fig. 2(c), and a cyclic process interpretation of the octahedron (b), cf. Fig. 13 (which is clockwise rotated by 90°). In (b), a vertex can be labelled by a stage and a categorical type of information separated by a colon symbol

By transforming our relations into an octahedron and through making use of the mapping above, the cutting plane of the octahedron can be interpreted as an

²⁰Thirdness involves secondness and firstness.

instance of 'matching' mode operation. If the generated relations do not enable this mode of operation, the interpreting system may switch to 'analysis' mode, in order to generate other relations, by means of the interpreting system's knowledge, eventually enabling a matching of the (augmented) input qualia.

5.5 Relations re-presented

Our mapping, displayed in Fig. 14(b), reveals the possibility of the cutting plane of the next cycle to be 'primed' by information from the previous cycle. It also reveals the potential of this structure to be interpreted as an instance of $q_2 * q_1$. Following our analysis, in Sect. 5, $q_2 * q_1$ enables a representation of 28 syntactic structures. According to our Newtonian model, those structures must be in relation with the nodes, 'action' and 'reaction'. From these premises we conclude that, through considering relations to be expressions of information combination, the octahedron structure can be interpreted as a representation of $2 * 28$ structures. By virtue of the potential of the nodes, 'action' and 'reaction', for a representation of properties of the interpreting system itself, those structures can be called *semantic* structures.

In Sect. 2 we have shown that our Newtonian model can be interpreted as a representation of relations between the three qualities involved in action–reaction phenomena. The nodes 'action', and 'reaction', are an expression of this relation as a firstness, and a thirdness, respectively; the cutting plane as a representation of the same relation as a secondness. By virtue of the dependency between the cutting plane, and the nodes 'action', and 'reaction', the two sets of 28 semantic structures (cf. $2 * 28$) can be said to represent a relation between firstness and secondness ('action'–cutting plane), and between secondness and thirdness (cutting plane–'reaction'), respectively.

Earlier we represented ternary relations by syntactic structures (cf. triples), in which a position is filled by information from a single perspective of q_1 (cf. firstness). Later we introduced a representation of ternary relations by semantic structures, in which a position is filled by information from a pair of perspectives

of q1 (cf. secondness). Eventually we define a third type of (*triadic*) ternary relation, in which a position is defined by information from all three perspectives of q1. We assume that triadic ('meaningful') ternary information structures are in relation with the nodes 'action' and 'reaction', both. As a result we may extend the set of semantic structures by 10 triadic structures, one of which representing the 'arising' relation generated by the interpreting system.

The number of semantic structures is: $2 \cdot 28 + 10 = 66$.

way of re- presentation \ type of relation	unary	binary	ternary	category- relation
1. 28 syntactic	1st-type	1st-type	1st-type	relation in itself
2. 2*28 semantic		2nd-type	2nd-type	relation--`action', relation--`reaction'
3. 10 triadic			3rd-type	`action'--relation--`reaction'

Figure 15: An overview of the types of relation used by our model

In summary, relational information is represented by our model in three ways (see Fig. 15) by 28 syntactic structures (information from a single representation of q1), 2*28 semantic structures (information from a pair of representations of q1), and 10 triadic structures (information from all three representations of q1). To this end, we introduced three types of relation (unary, binary, ternary), and within that classification we made a distinction between three types of representation. Relations of the first type represent information stages of a category in themselves (cf. unary, binary, ternary relations); relations of the second type are an expression of a combination of information from a pair of categories (cf. binary and ternary relations); relations of the third type represent a merging of information from all three categories (cf. ternary relations). Note that unary relations have a single type, binary relations two types, and ternary relations have three types.

An interesting property of $q1*q1$ is that its number of elements is 12, enabling a definition of 66 pairs (cf. binary relations).²¹ This may be considered to be an expression of the potential of $q1*q2$ for a *formal* representation of action–reaction phenomena.

A representation of *meaningful* interpretation is beyond our possibilities. In line with Peircean semiotics, we assume that interpretation²² involves an irreducible triadic relation between the three qualities of action–reaction phenomena. This may explain why triadic (‘meaningful’) ternary relations have been introduced as a subset of semantic relations.

6 Peircean sign relations

The goal of this section is to show that our model of action–reaction phenomena is congruent with Peirce’s theory of signs. To this end, we generalize the concept of an interaction in the concept of a sign, and present action–reaction phenomena as sign interpretation processes.

6.1 Categories, signs, and sign aspects

Fundamental notions in Peircean theory are his three categories of phenomena: firstness, secondness, and thirdness. An example of the three categories, in the domain of signs, is the quality of pain (firstness), the relation of pain with its object, e.g., toothache (secondness), and the relation between pain and its interpreting thought by some agent, e.g., “call a dentist” (thirdness).²³

By analyzing his concept of a sign along categorical lines, Peirce concluded that all signs must involve nine sign aspects. Note that sign aspects are different from signs. They may be characterized as potential signs, that are becoming a sign. This difference between signs and sign aspects can be illustrated by the phenomenon of apparent motion perception. In this phenomenon, a series of

²¹ $\binom{12}{2}=66$

²²From a semiotic perspective, interpretation is always meaningful.

²³The relation of the sign with its interpretant involves the relation of the sign with its object.

steady pictures (cf. sign aspect) are presented. Although each picture can be meaningful in itself, combined they can be interpreted as parameters in the experience of the entire series of pictures as motion (cf. sign).

The nine sign aspects can be introduced by a triadic (sub-)classification of the sign's relations. We call the category of a sub-class a *categorical aspect*, the relation between sub-classes of a triad a *categorical relation*. Below, we introduce a sign aspectual characterization of the sign's primary relations: the relation of the sign to itself, to its object, and its interpretant. As a consequence of the dynamical nature of successions of sign aspects (cf. steady pictures), we illustrate them by means of signs (cf. motion). We designate categories by integers, e.g., firstness by '1', and categorical aspects by integers in parentheses, e.g. the aspect of firstness by '(1)'.

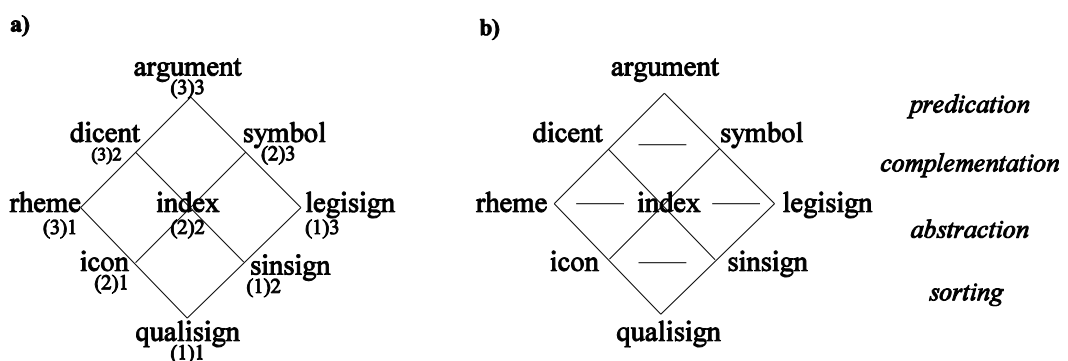


Figure 16: A Peircean classification of sign aspects (a) and its process interpretation (b). In (a), the categorical aspect and category of a sign aspect is designated by a pair of integers, e.g., (2)1. In (b), neighboring sign aspects are connected by a horizontal edge. The types of interpretation events are depicted on the right-hand side of this diagram, in italics

A classification of sign aspects in triads:

- 1 The relation of the sign to itself: (1) qualisign, (2) sinsign, (3) legisign. E.g., the perception of pain as (1) a quality, (2) an actual event occurring now, and (3) a habitual feeling.
- 2 The relation of the sign to its object: (1) iconical, (2) indexical, (3) symbolic. E.g., pain interpreted as (1) a measure of a quality, (2) a pointer pointing to the teeth, and (3) a conventional concept such as a toothache.
- 3 The relation of the sign to its interpretant: (1) rhematic, (2) dicentric or propositional, (3) argumentative. E.g., the perception of pain as (1) an abstract concept (cf. 'pain'), (2) an actual existence of 'pain' in the teeth, and (3) a premise that we should call a dentist.

The first triad in the above classification refers to the sign in itself hence to a category; the second and third to a relation involved in the sign hence to a categorical aspect. On the basis of their category, Peirce's nine sign aspects can be arranged in a partial order (Walther, 1979), (Bense, 1976). See Fig. 16(a). References to sign aspects can be given by means of adjectival and nominal phrases that we use interchangeably.²⁴

The isomorphism between the partial order displayed in Fig. 16(b), and the cutting plane of the octahedron depicted in Fig. 14 (b), enables the definition of a mapping from our informational concepts (cf. Fig. 3) to Peircean sign aspects. For example, 'neutral' (qualities in themselves) can be mapped to the sign aspect qualisign, 'constituent' (co-existent qualities) to the sign aspect icon,²⁵ 'abstraction' (abstract qualia) to the sign aspect rheme.

6.2 Process model

In earlier research (Sarbo et al., 2011), we have shown that Peirce's classification of sign aspects can be interpreted as a process of interactions between

²⁴In Peircean theory, the kinds of references are used differently. An adjectival reference emphasizes the service rendered by the sign aspect, for example, *iconic*. This is opposed to a nominal, which is a reference to a sign, by a pars pro toto use of an aspect, e.g., *icon* (Van Breemen, pers. comm., 2012).

²⁵Co-existence involves constituency hence (iconic) resemblance to the input as a whole.

neighboring sign aspects. This is depicted in Fig. 16(b), in which neighboring sign aspects are connected by a horizontal edge.

The input of this process is defined by the input qualia, interpreted as an expression of the qualisign sign aspect.²⁶ Through *sorting* the input into a collection of state, and effect qualia, the input qualia are interpreted as an expression of the icon and sinsign sign aspects, respectively. By separating the two types of qualia from one another, in *abstraction*, the input qualia are interpreted as an expression of the rheme and legisign sign aspects. In *complementation*, the abstract state and effect is augmented with indexical information by the interpreting system, enabling an interpretation of the input qualia as an expression of the dicent and symbol sign aspects. By merging the last two representations, in *predication*, the input qualia are interpreted as an expression of the argument sign aspect.

In the next section we delve into an analysis of Peirce's theory of sign relations that themselves are signs, not just sign aspects. Our goal is to reveal the potential of our model for an aspectual representation of these more complex notions of the sign as well.

6.3 The sign as an object

Arguably the most important property of a process view of sign interpretation is its potential for considering the sign to be an object. The possibility of this perspective complies with Peirce's theory of interpretation, as it is pointed out by van Breemen (van Breemen, 2012). Peirce writes (Peirce, 1992, 1998):²⁷

It seems best to regard a sign as a determination of a quasi-mind; for if we regard it as an outward object, and as addressing itself to a human mind, that mind must first apprehend it as an object in itself, and only after that

²⁶ This, and all other expressions by this process are representations of sign aspects involved in the input interaction.

²⁷ Vol. 2, p. 391, 1906.

consider it in its significance; and the like must happen if the sign addresses itself to any quasi-mind. It must begin by forming a determination of that quasi-mind, and nothing will be lost by regarding that determination as the sign.

Peirce classified the significative effect of a sign in three types, which he called immediate, dynamic, and final or normal interpretant. His classification indicates a development in interpretation as a process (cf. levels), from immediate, through dynamic, to final. In this section we suggest that the above levels may apply to the apprehension of the sign as an object ('sign-object'), as well. As, from an analytical perspective, the relations involved in triadic signs must be a result of interpretation, we must be able to characterize those relations from the perspective of the three levels of interpretation.

Following our assumption of the sign to be an object, the firstness of the relation of the sign to itself may be associated with the 'immediate' representation (1) of the sign (S), (2) the sign of the immediate object (IO) and immediate interpretant (II), and (3) the sign of the dynamic object (DO), dynamic interpretant (DI), and final or normal interpretant (NI).²⁸ The secondness of representation of the relation of the sign to its object may be associated with the more developed, 'dynamic' representations by the interpreting system: by the relation between the sign and its immediate object (cf. S–DO)²⁹ and between the sign and its immediate (cf. S–DI) and dynamic interpretant (cf. S–NI), following information augmentation of the IO, II, and DI, respectively. Finally, the thirdness of the relation of the sign to its object and interpretant may be associated with the relation between the most developed, 'final' representation of the sign, object, and interpretant (cf. S–NI–DO), following a (final) information augmentation of the DO and DI.

²⁸The normal interpretant (NI) is the tendency of the final interpretant towards its limit. In this paper the two types of interpretant are used interchangeably.

²⁹Following the above process view of interpretation, S–DO is a representation of the IO, following information augmentation mediated by S.

An example of firstness sign relations is the phenomenon toothache, signified by unsorted qualia of a sensation of pain (S). Via interpretation we may generate a thought sign, e.g., 'toothache' (II), as a response on the perceived input qualia. By sorting the input in a form and event, we may conclude that the perceived qualia stand for pain in the dental area (IO). Our first or immediate interpretant may trigger more responses such as a motor reaction or an interpreting thought (DI), by means of knowledge by the interpreting system (observer). Examples are 'toothache', 'call a dentist', 'make an appointment', 'take an analgesic', etc., representing increasingly more developed responses (DIs) on the input sign. The tendency of these responses (NI) can be paraphrased by the interpretant 'stay calm, do what is necessary, e.g. take medication, consult a dentist'.

We suggest that the above interpretation of the Peircean sign relations (Peirce, 1865–1909) can be used for the introduction of an aspectual representation of those relations. In the next section we show that Peirce's sign relations can be mapped to categorical relations between sign aspects, as well as to relations involved by the octahedron structure, including its process interpretation.

6.4 A representation of sign relations

The 10 relations of the sign, that are themselves sign, are: S, IO, DO, II, DI, NI, S–DO, S–DI, S–NI, S–NI–DO. An interpretation of these, more complex notions of the sign requires cyclic processing. To this end we make use of a result from (Sarbo et al., 2011), proving that processing of a single sign, and a series of signs can be modeled in an isomorphic fashion. The cycles enabled by a process, that has three types, first or initial, intermediate, and final, can be associated with the three levels of interpretation, immediate, dynamic, and final, respectively. Below we revisit our process model, depicted in Fig. 16(b), in order to reveal its potential for an aspectual representation of the 10 relations of the sign. We refer

to sign aspects by means of their Peircean term, for example, the qualisign sign aspect by the term 'qualisign'.

We begin our analysis with the first event in sign processing, which is the definition of the input for processing. As a consequence of the possibility of the sign to be an object, the representation of the input by the qualisign sign aspect (S) must be an expression of the immediate object of interpretation (IO), as well. See Fig. 17.

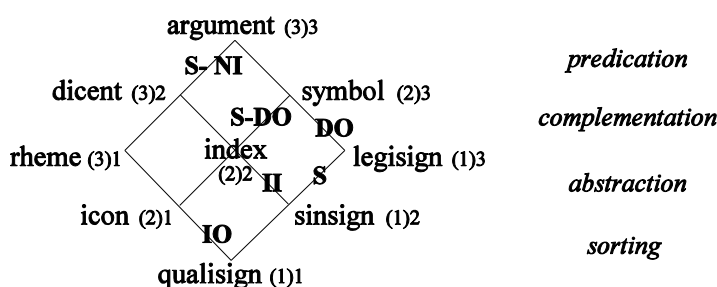


Figure 17: Sign relations mapped to categorical relations between sign aspects, and to relations between stages of $q_2 \cdot q_1$. For instance, IO can be mapped to a relation between rheme, icon, and qualisign, and to the relation $(3)1-(2)1-(1)1$.

In *sorting*, the representation of a relation between the input state and effect, by the sinsign sign aspect (S), is also an expression of the event involved in the immediate interpretant (II). An expression of the above relation, this time from the perspective of constituency, by the icon sign aspect, amounts to a more developed expression of the sign-object (IO), as well as a representation of the initial value of information augmentation (S-DO). Finally, the representation of the above relation by the index sign aspect, as a pointer to complementary information about the input state and effect, is also an expression of a development in the process of information augmentation, both as a value (S-DO), and an event (II).

In *abstraction*, the input sign is represented by the legisign sign aspect, which is also an expression of the initial value in the process of information

augmentation of the habitual relation involved in the input sign (DO). A representation of the input by the rheme sign aspect, is an expression of the sign-object as an abstract state (IO). This state being the subject of interpretation, the rheme is also an expression of the initial value of that process (S–NI).

In *complementation*, the representation of the sign-object is augmented from immediate to dynamical. This is witnessed by a representation of the sign's object (cf. legisign; DO) and interpretant (cf. rheme; S–NI). If the current processing cycle is a final one, the above representation of the interpretant can be used as a representation of the final or normal interpretant (S–NI); otherwise, more cycle(s) may be required hence that representation must be an expression of the dynamic interpretant (S–DI).

In *predication*, augmented representations of the sign-object (S–NI, DO) are combined into a single representation. This is expressed as the final input representation (S–NI–DO), if the current cycle is a final one. If it is not, the above representation is offered as input for further processing by the next cycle (S–DI).³⁰

A mapping of the remaining four relations (DI, S–NI, NI, S–NI–DO) to vertices of the octahedron can be defined as follows. If the current cycle is not the final one (i.e., further processing is required), the current augmentation of the interpretant (S–DI) may become a sign-object (cf. qualisign) in the next cycle, in order for a generation of the final interpretant (S–NI). Otherwise, the sign-object must involve information about a final approximation of the expression of the dynamic interpretant. This information can be represented as an abstract 'state', by the rheme sign aspect (DI), and as a habitual 'event', by the legisign aspect (NI). The process may terminate, by generating a representation of the relation between the sign, its object, and interpretant, following (a final) information augmentation (S–NI–DO).

³⁰ In the S–DI, the DO is involved.

According to the above analysis, the relations, DI, S–NI, NI, S–NI–DO, can be mapped to a single sign aspect of Peirce’s classification, as well as to a single node of the cutting plane of the octahedron. The above mapping can be extended to a categorical relation. To this end we observe that, by virtue of the involvement of the above four relations in the information augmentation process, those relations may represent properties of the interpreting system itself. We also notice that the octahedron may represent relations involved in action–reaction phenomena in three ways: as a firstness (cf. ‘action’), a secondness (cf. cutting plane), and a thirdness (cf. ‘reaction’). From this, we may conclude that a categorical relation of the above four notions can be defined by completing their mapping by a first and a third element, represented by the nodes ‘action’, and ‘reaction’, respectively. See Fig. 18.

This completes our analysis of a relation between a Newtonian model of action–reaction phenomena and Peirce’s theory of signs. Our model is restricted to relational interpretation (cf. secondness). A representation of sign interpretation as a firstness, and a thirdness are beyond our scope.³¹

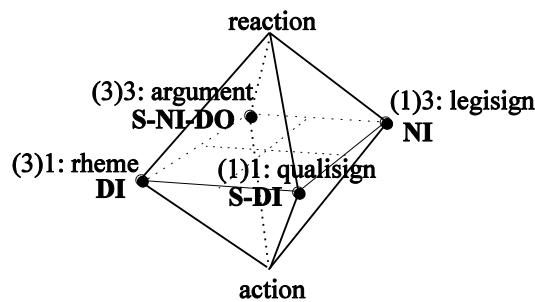


Figure 18: A mapping of DI, S–DI, NI, S–NI–DO, to relations of the octahedron, e.g., S–DI is mapped to the relation ‘action’–qualisign–‘reaction’

³¹Through representing the relation involved in firstness and thirdness, by a single node, the octahedron may be conceived to be a representation of the lattice multiplication $q_3 * q_2 * q_1$.

7 Summary

We suggest that a model of meaningful information processing can be given by introducing a model of action–reaction phenomena and embedding it in a Peircean theory of signs. Following Peirce, we assume that phenomena, as well as their representations by relations can be distinguished in three categories. Peirce maintained that from his categories everything else, including his signs and sign aspects, can be derived. In this paper we show that our model has the potential of representing three types of relation, consisting of 10, 28, and 66 elements, that are analogous to Peirce’s three classifications of signs. This implies the possibility of a common representation for Peirce’s different classifications. Peirce’s sign theory is considered by many to be a theory of the knowable (the types of distinction that can be signified by signs). By virtue of the above relation with Peircean semiotics, and because of the fundamental nature of signs, our approach has the potential for a uniform modeling of information processing in any domain, theoretically. Past research in natural language processing, ‘naive’ logic and ‘naive’ reasoning has shown that the above hypothesis may hold, and that the developed models may be practical as well.

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References

- Adriaans, P. 2009. Between order and chaos: The quest for meaningful information. *Theory of Computing Systems.*, 650–674.
- Adriaans, P., Allo, P., C, B., Primiero, G., Rovin, B., Sarbo, J., et al. 2010. Philosophical aspects of the foundations of models of information. In J. Seligman (Ed.), *Workshop on the philosophy of the information and computing sciences*. Leiden (The Netherlands): Lorentz Center.
- Barr, M., & Wells, C. 1990. *Category theory for computing science*. Prentice-Hall.

- Bense, M. 1976. Das System der Theoretischen Semiotik. *Semiosis*(1), 24–28.
- Birkhoff, G., & Bartee, T. 1970. *Modern applied algebra*. New York: McGraw-Hill.
- Burch, R. 2011. Peirces 10, 28, and 66 sign-types: The simplest mathematics. *Semiotica*, 184 (1/4), 93–98.
- Davey, B., & Priestley, H. 1990. *Introduction to lattices and order*. Cambridge University Press.
- Farias, P., & Queiroz, J. 2003. On diagrams for peirces 10, 28, and 66 classes of signs. *Semiotica*, 147 (1/4), 165–184.
- Harnad, S. 1987. *Categorical Perception: The groundwork of cognition*. Cambridge: Cambridge University Press.
- McCarthy, J., & Hayes, P. 1969. Some philosophical problems from the standpoint of artificial intelligence. In B. Meltzer & D. Michie (Eds.), *Machine intelligence 4* (pp. 463–502). Edinburgh University Press.
- Newton, I. 1999/1687. *The Principia*. Berkeley: University of California Press.
- Peirce, C. 1865–1909. *Logic Notebook*. Houghton Library, Harvard university: Unpublished manuscript on microfilm.
- Peirce, C. 1932. *Collected papers of Charles Sanders Peirce*. Cambridge: Harvard University Press.
- Peirce, C. 1992, 1998. *The Essential Peirce: Selected Philosophical Writings, 2 volumes*. Bloomington: Peirce Edition Project, Indiana University Press.
- Sanders, G. 1970. Peirce's sixty-six signs? *Transactions of the Charles S. Peirce Society*, 6 (1), 3–16.
- Sarbo, J. 1996. Lattice embedding. In P. Eklund, G. Ellis, & G. Mann (Eds.), *Conceptual structures: Knowledge representation as interlingua (ICCS'96)* (Vol. 1115, pp. 293–307).
- Sarbo, J., Farkas, J., & van Breemen, A. 2011. *Knowledge in Formation: A Computational Theory of Interpretation*. Berlin: Springer (eBook: <http://dx.doi.org/10.1007/978-3-642-17089-8>).
- van Breemen, A. 2012. *Knowledge in Formation: The Peircean, semiotic background*. Radboud University Nijmegen: PhD Thesis (forthcoming).

- Vitanyi, P. 2006. Meaningful information. *IEEE Transactions on Information Theory*, 52 (10), 4617–4626.
- Walther, E. 1979. *Allgemeine Zeichenlehre: Einführung in die Grundlagen der Semiotik*. Stuttgart: Deutche Verlags-Anstalt.
- Weiss, P., & Burks, A. 1945. Peirce's sixty-six signs. *The Journal of Philosophy*, 42 (14), 383–388.