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Referendum Design: An Exercise in Applied Social Choice Theory

Hannu Nurmi*

The formal status of the referendum institution in democratic systems varies from consultative and optional to mandatory and binding. Despite the formal status, it can be argued that in all systems the legislators can hardly ignore the referendum results. This article presents two arguments: (1) The results of social choice theory suggest that the number of alternatives in referenda should to be restricted to two in order to avoid severe problems of interpretation and agenda manipulation. (2) The consultative referendum system may lead to quite dramatic norm conflicts for both legislators and voters. Taken together, these two arguments imply that referenda are appropriate only in cases where there is a natural way to dichotomize the issue at hand. Moreover, whenever a referendum is called, its result should be binding.

Introduction

Practically all European democracies occasionally resort to referendum even though the institutions responsible for the day-to-day running of the political system are representative. In other words, direct and indirect democracy occasionally coexist. The formal status of referendum varies considerably. At one extreme we have consultative and optional referenda, i.e. the result of the referendum vote does not bind the legislators in any legal sense, and it is entirely up to the appropriate authorities to decide whether to resort to a referendum. At the other extreme are systems in which the authorities have very little or no leeway in calling for a referendum and in which the result of the vote is formally binding (see e.g. Suksi 1993).

Despite its ubiquity, the theoretical foundations of the referendum institution are weak. Often a referendum is called for because the representative institutions deem it necessary. The question posed in this article is whether those foundations could be strengthened by taking into account the results obtained in social choice theory. Since this theory is notorious for its negative results, the prospects of such an approach do not seem promising, *prima facie*. As I will endeavor to show in the following, this appearance is

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Despite its ubiquity, the theoretical foundations of the referendum institution are weak. Often a referendum is called for because the representative institutions deem it necessary. The question posed in this article is whether those foundations could be strengthened by taking into account the results obtained in social choice theory. Since this theory is notorious for its negative results, the prospects of such an approach do not seem promising, *prima facie*. As I will endeavor to show in the following, this appearance is

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deceptive. Many important insights into the possibilities and limitations of referenda can be gained from social choice theory.

More specifically, I will make and discuss three claims concerning referenda. Firstly, I will argue that in cases where a correct decision exists, the referendum institution is not necessarily inferior to the opinion of experts, provided that they occasionally make mistakes. Secondly, we will show that the number of alternatives in referenda plays a crucial role. In referenda with more than two alternatives, the problems of interpreting ballot results, strategic manipulation possibilities, etc. loom large. Thirdly, I will argue that referenda should not be consultative because that could lead to serious conflict between two *per se* justifiable theories of democracy. The main conclusion is that direct forms of democracy, such as referenda, have an important role in representational democracies if due attention is paid to their proper arrangements.

The Implications of Condorcet's Jury Theorem

Perhaps the earliest profoundly positive result in social choice theory is due to Marquis de Condorcet (1785). The question Condorcet focused on is the following: assuming that each individual has a given probability of being right, what is the probability that the majority of a group consisting of such individuals is right?

Assume for a moment that each individual has an identical probability p of being right. This probability could be interpreted as the relative frequency of correct "yes" or "no" answers to a long sequence of questions of which the correctness of the answers can be determined afterwards. Let us focus on such a question that calls for either a "yes" or a "no" answer and assume that the number of persons who have given the right answer is x. Under the additional assumption that the persons vote independently of each other, we get, by applying the binomial probability formula, the probability that among n individuals exactly x have given the right answer:

$$f(x) = p^x \ (1-p)^{n-x}$$

P denotes the probability that the group using the simple majority rule gives the right answer. In other words, P is the probability that more than 50 percent of the group members will vote "yes" (or "no") when "yes" ("no") is the right answer. Obviously,

$$P = \sum_{x=n'}^{n} \binom{n}{x} \ p^x \ (1-p)^{n-x}$$

Here $n' = (n + 1)/2$. The distribution of P can be approximated by the normal

Table 1. The Probability of the Majority Being Right in a Dichotomous Choice Situation (p = the arithmetic mean of individual competencies; n = size of the group)

	p				
	0.5050	0.5500	0.7500	0.9000	0.9750
n 3	0.5075	0.5748	0.8438	0.9720	0.9982
5	0.5094	0.5931	0.8965	0.9914	0.9998
7	0.5109	0.6083	0.9294	0.9973	0.9999
9	0.5123	0.6214	0.9510	0.9991	0.9999
15	0.5154	0.6514	0.9873	0.9999	0.9999
25	0.5199	0.6924	0.9981	0.9999	0.9999
75	0.5345	0.8079	0.9999	0.9999	0.9999
250	0.5628	0.9440	0.9999	0.9999	0.9999
1000	0.6241	0.9993	0.9999	0.9999	0.9999

Source: Miller (1986).

distribution with mean np and variance np(1 − p). Thus, we obtain

$$P = 1 - G((n/2 - np)/\sqrt{np(1-p)}) = G((p - 0.5)/\sqrt{p(1-p)/n}$$

Here G(y) is the area under the density curve of normal distribution from $-\infty$ to y.

According to Condorcet's jury theorem, there are three distinguishable cases:

(1) If $0.5 < p < 1$ and $n > 2$, then $P > p$, P increases with n and when n approaches infinity, P converges to 1.
(2) If $0 < p < 0.5$ and $n > 2$, then $P < p$, P decreases with the increase of n and P approaches 0 when n approaches infinity.
(3) If $p = 0.5$, then $P = 0.5$, for all values of n (Miller 1986).

The normal approximation can be used whenever the value of p is not near 0 or 1. Table 1 gives an idea of how fast P approaches 1 when n increases for various values of p.

The message of the theorem is clear: the majority is more reliable than the average citizen if the latter is more often right than wrong and if the probability of being right is the same for all citizens. Indeed, the majority becomes omniscient when the number of individuals increases. The assumption that the probability that each citizen is right is larger than 1/2 is essential, because if this is not the case, then P approaches 0, i.e. it becomes certain that the majority is wrong.

The applicability of Condorcet's jury theorem is, however, seriously limited by the assumption that each individual is equally competent, i.e. has

the same probability of being right. In the following we shall focus on various results based on less stringent assumptions.

The Generalized Jury Theorem

The generalization of the above theorem into what is called *embellished Condorcet jury theorem* yields the following result (Miller 1986). Suppose that each individual i is characterized by probability p_i of being right. If now $1/2 < p' < 1$ and $n > 2$, then $P > p'$, and P approaches 1 as n approaches infinity. Here $p' = \sum_i p_i/n$, i.e. p' is the arithmetic mean of the individual probabilities of being right.

In this theorem the individuals do not necessarily have identical competencies. Furthermore, they are not all required to be more often right than wrong. What is assumed instead is that the arithmetic mean of the individual competencies is larger than 1/2.

This result contradicts, at least *prima facie*, Dahl's (1970, 34) contention: ". . . whenever you believe that 1 is significantly more competent than 2 or 3 to make a decision that will seriously affect you, you will want the decision to be made by 1. You will not want it to be made by 2 or 3, *nor by any majority of 1, 2, and 3.*"

The generalized Condorcet theorem maintains that it might well be in our interest that the decision be made according to the opinion of the majority of 1, 2 and 3, assuming, of course, that in matters of great importance to us, we want to maximize the correctness of the decision outcomes.

The generalized Condorcet theorem demonstrates that it is perhaps not necessary to be overly concerned about the use of referenda in matters which may have been decided by experts, e.g. economico-political alliances. Adding a sufficient number of minimally competent decision makers improves the quality of decision outcomes no matter how good it was before. The theorem also tells us that unless the expert is really good, i.e. unless he/she has the probability 1 of being right, the majority of the group has a greater competence than the expert, provided a sufficient number of minimally competent people is added. "Minimally competent" here refers to the probability greater than 1/2 of being right.

Condorcet Theorems and Referenda

In the preceding we have discussed situations in which a "right decision" exists. In political contexts where almost by definition right decisions do not exist, the application of Condorcet theorems calls for additional assumptions. The most important assumption is that we can meaningfully use the notion of

the voters' true interest. Furthermore, we assume that any decision reached by the group is or is not in accordance with the voters' true interest. Hence, we label a voting decision "right" if and only if it is in accordance with the true interests of the voter in question. This situation differs from the preceding one in that we assume that the voters' true interests differ (Miller 1986).

We now define the competence of a voter as his/her probability of casting a right vote (i.e. "yes" if "yes" is in accordance with his/her true interests and "no" if "no" is in accordance with his/her true interests). Furthermore, we define the success of the decision process as follows: the decision process succeeds only when the collective decision coincides with the true interests of the majority of voters (ibid.).

As in the first version of Condorcet's jury theorem, we assume that each voter is equally competent, i.e. his/her probability of voting according to his/her true interests is p. Now n_A denotes the number of voters whose true interest is A. Similarly n_B denotes the number of voters whose true interest is B. Without essential loss of generality, we assume that $n_A > n_B$, i.e. the number of voters whose true interest is that A is chosen is larger than the number of voters whose true interest is that B is chosen. With x as the number of votes for A, the expected value of x is:

$$E(x) = n_A p + n_B(1 - p).$$

Since $n_A > n_B$ by assumption, $n_A = n/2 + a$ and $p = 1/2 + b$ where a and $b > 0$. Consequently

$$E(x) = (n/2 + a)(1/2 + b) + (n/2 - a)(1/2 - b) = n/2 + 2ab.$$

Thus, the expected number of votes for A is larger than 1/2. In other words, whenever the individual competencies are identical and larger than 1/2, we can expect the decision process to succeed, i.e. to result in an outcome that coincides with the true interests of the majority of voters.

This is not to say that everybody who voted for A got their true interest served. Among those who voted for A may be voters who voted against their true interest. Similarly, among those who did not vote for A may have been voters whose true interest would have called for a vote for A. What the result says is that the majority of all voters can be expected to get their interests served. We introduce the following notation: $p^* = E(x)/n$, i.e. p^* is the expected proportion of votes for A. What is the probability that the majority decision coincides with the true interests of the majority? Of course, this is one of the fundamental problems of democracy. x(A) denotes the number of voters who vote for A and whose true interest is served by A. x(B), on the other hand, is the number of those voters whose true interest is B but who

erroneously vote for A. Hence, $x = x(A) + x(B)$. Applying the binomial formula we obtain:

$$f(x(A)) = \binom{(n_A)}{x(A)} p^{x(A)} \quad (1-p)^{n_A - x(A)}$$

$$f(x(B)) = \binom{(n_B)}{x(B)} (1-p)^{x(B)} \quad p^{n_B - x(B)}$$

We can now compute the probability that the choice process succeeds, i.e. that the majority decision coincides with that of the true interests of the majority. This equals the probability P* that x is larger than 50 percent of the voters. Thus,

$$P^* = \sum_{x=n'}^{n} \quad \sum_{x(B)=k}^{n_B} \quad \binom{n_A}{x(A)} \quad \binom{n_B}{x(B)} \quad p^{n_B + x(A) - x(B)} \quad (1-p)^{n_A - x(A) + x(B)}$$

Here $n' = (n+1)/2$ and $k = x - n_A$, if $x - n_A > 0$, and $k = 0$ otherwise. P* may be approximated with normal distribution with mean $n_A p + n_B(1-p) = E(x) = np^*$ and variance $n_A p(1-p) + n_B p(1-p) = np(1-p)$ (Miller 1986, 180). Thus, we get the following generalization of Condorcet's jury theorem:

If $1/2 < p < 1$ and $n > 2$, then for any proportion n_A/n (1) $P^* > p^*$, (2) P* increases with n and (3) P* converges to 1 when n increases without limit.

In other words, Condorcet's original theorem is valid in this case as well. The increase in the number of decision makers increases the probability that the decisions reached by the majority coincide with the true interests of the majority. Moreover, the probability that the majority decision is right is larger than the arithmetic mean of the individual probabilities.

In the preceding calculations we have relaxed the electorate homogeneity assumption. It turns out that this assumption is not necessary for Condorcet's result which states that the majority of a group is more competent than its average member and approaches omniscience when the size of the group increases. We need the assumption that the arithmetic mean of individual probabilities of being right exceeds 1/2 and the distribution of competencies is symmetric. (In case all individuals are minimally competent, we do not need the assumption concerning the distribution).

However, the above holds only if the individuals are independent of each other. If this is not the case, we cannot apply the binomial formula and its normal distribution approximation. Since the independence assumption seems somewhat unrealistic in some circumstances, we focus on whether Condorcet's jury theorem holds in situations where the voters are to some

extent dependent on each other due to, say, common ideology or group discipline.

Dependent Voters

Our discussion on dependence is based on system reliability theory. This theory aims at finding the probabilities that systems operate properly if certain portions of their components fail. The construction of the majority systems model is based on the assumption that the system is composed of several components so that it works if and only if the majority of its components work. Once each component has been given a probability that it works properly, we can analyze the reliability of majority systems under various assumptions concerning the interdependence of components (Boland 1989; Boland, Proschan and Tong 1989). For our discussion the components can be interpreted as voters or jurors and the proper functioning of a component as the event that the juror is right.

We assume that the system components are Y, X_1, . . ., X_{2m}. Each component is simultaneously interpreted as a dichotomous variable so that e.g. $X_i = 1$ means that X_i-component works properly and $X_i = 0$ means that X_i fails. We assume that $p(Y = 1) = p(X_i = 1) = p$, for all $i = 1, \ldots, 2m$. Thus, each component has the same probability of failing. Moreover, $q = 1 - p$. The conditional probabilities are:

$$p(X_i = 1|Y = 1) = p + rq \text{ and}$$

$$p(X_i = 1|Y = 0) = p - rp, \text{ where } i = 1, \ldots, 2m.$$

Here the parameter r measures the interdependence or correlation between X_i and Y. Obviously, when $r = 1$, the probability that when X_i gets the value 1 also Y gets the value 1, is 1. On the other hand, when $r = 0$, the conditional probabilities of X_i equal their absolute probabilities, i.e. they are independent of Y. It is noteworthy that it is possible to describe positive dependence and independence in this model, whereas negative dependence cannot be described.

Boland demonstrates that the probability that the majority of components work properly decreases with the increase of correlation. Applying this result to voting contexts we can argue that the probability that the majority is right decreases when voters' dependence on one "leader" (variable Y) increases. However, as long as the correlation between the voters and the leader is less than 1, the probability that the majority is right exceeds that of a single voter. Hence, in Boland's model the interdependence between voters does not affect the essence of Condorcet's theorem.

Table 2. The Majority Competencies (mc) for Individual Competence p = 0.6 for Varying Group Sizes and for Three Different Values of h

n				
5	h	−0.08	0	0.08
	mc	0.7221	0.6826	0.6587
9	h	−0.04	0	0.04
	mc	0.7784	0.7334	0.7084
41	h	−0.01	0	0.01
	mc	0.955	0.905	0.867

Source: Berg (1993).

A more general approach to modeling interdependence is developed by Berg (1993) who replaces the binomial distribution with Pólya-Eggenberger or beta-binomial distribution which is a generalization of the binomial one. According to this distribution, the probability of exactly x jurors being right is:

$$b_n(x; p, h) = \binom{n}{x} \quad p^{[x,h]}(1-p)^{[n-x,h]}/1^{[n,h]}$$

Here $h > \max(-p/(n-1), -(1-p)/(n-1))$. The expressions in brackets indicate increasing factorials: $p^{[x,h]} = p(p+h)(p+2h). \ . \ .(p+(x-1)h)$. Obviously, when $h = 0$, this distribution is reduced to a binomial one. The correlation between any two voters is $h/(h+1)$. Thus, h can be interpreted as a dependence parameter. Because of the dichotomous nature of the variables, the value of correlation is smaller than 1 and larger than $(p-1)/(n+p-2)$.

Table 2 borrowed from Berg (1993) indicates the variation of the majority competence for small values of h. Clearly, when the absolute value of the interdependence is small, the majority competence increases if the interdependence is negative, whereas it decreases if the dependence is positive. Berg (1993, 92–93) shows that this is the case whenever $p > 1/2$. Thus, we can conclude that positive interdependence between voters decreases the majority competence from its value under the independence assumption.

Despite this observation, the main content of Condorcet's jury theorem remains intact in beta-binomial distributions. Thus, with $p > 1/2$ and for fixed value of h, the probability that the majority decision is right increases with the increase of the number of voters. Moreover, whenever $1/2 < p < 1$, the majority competence always exceeds that of individual p.

The preceding discussion on the variations of Condorcet's jury theorem reveals that even in contexts where it is meaningful to speak about correct and incorrect decisions, the group choice using majority rule is not

necessarily inferior to the expert choice, unless the expert is perfect and the group consists of individuals who are not even minimally competent. By its very nature, the theorem deals with a dichotomous choice situation. One might therefore ask what will happen to all these nice results when the set of alternatives from which a choice has to be made consists of several alternatives. As we shall see shortly, the increase in the number of alternatives subjected to a referendum opens a Pandora's box of problems.

More than Two Alternatives

Modern social choice theory normally takes its point of departure in the assumption that individuals can place the alternatives in an order of preference (allowing for possible ties). Although this assumption is by no means innocuous, we shall not go into its justifiability here (but see Nurmi 1991). Once we have accepted this assumption, however grudgingly, we can see how the increase from two to three or more alternatives indeed makes it harder to arrange a referendum in a satisfactory way (for the properties of the majority rule in dichotomous situations, see May 1952 and Rae 1969). There are two difficulties involved: (1) interpreting the voting result and (2) the strategic behavior of the voters and agenda setters.

Problems of Interpretation

The so-called Condorcet paradox or the phenomenon of cyclic majority preferences is perhaps the earliest demonstration of possible difficulties in choice settings involving more than two alternatives (see e.g. Gehrlein 1983). In a nutshell, the paradox consists of the possibility that given three alternatives and three equally large groups, any single choice frustrates the majority of voters. Thus, unless all alternatives can be chosen, it may be downright impossible to tell which alternative is collectively best (see also Nurmi 1987).

Cyclic majorities are, however, just one of many difficulties involved in multi-alternative settings. An almost equally well-known difficulty is that even in the absence of cyclic majorities and in situations where a Condorcet winner exists (i.e. an alternative defeating all the others in pairwise contests) there may be other considerations that call for the choice of some other alternative. These considerations were first proposed by Borda in the late 18th century (DeGrazia 1953). The main source of difficulty is that there are at least two *prima facie* plausible intuitions regarding which choice should be deemed best, and these intuitions sometimes conflict. Borda's intuition says that the choice of any alternative from a set should depend on its ranking in individual preferences. Condorcet's intuition, on the other hand,

Example 1. Difficulties in Determining the Collectively Best Alternative

Group 1	Group 2	Group 3
A	B	C
C	C	B
B	A	A

maintains that pairwise comparisons of alternatives using the majority rule should determine the choice. I shall call Borda's intuition positional and Condorcet's binary.

Example 1 – admittedly an extreme example – shows that with three alternatives we may get into serious trouble in interpreting the voting result.

Thus, Group 1 considers A best, C second best and B worst. The rest of the preference profile is interpreted similarly. Let us assume that Group 1 consists of 40 percent, Group 2 of 35 percent, and Group 3 of 25 percent of the voters. Then one positional criterion, viz. plurality, would call for A to be chosen since it is ranked first by more voters than any other alternative. Condorcet's binary intuition, on the other hand, would dictate the choice of C since it would beat all the other alternatives in pairwise contests if the majority criterion determined the winner. To make matters worse, in a plurality runoff, B would win. Thus any of these three alternatives could win, depending on the intuition used to define the notion of winning.

The point of the example is to show that in some contexts the decision rule is as important a determinant of the social choices as the preferences of the electorate. This observation applies to alternative sets consisting of more than two elements. If only two alternatives are considered, then all three intuitions mentioned in the example converge to the same alternative and consequently there will be no interpretation problems.

Misrepresentation of Preferences

Subjecting multiple decision alternatives to referendum poses yet another problem, namely a possible undermining of the very rationale of referendum by giving the voters incentives not to reveal their true opinions about the alternatives at hand. This problem always arises when opinion polls are publicized. Of course, this is hardly a novelty. Some two thousand years ago, Pliny the Younger is said to have pondered the problem of how to guarantee the best possible voting outcome in the Roman Senate after realizing that his favorite candidate was not within reach and also knowing quite a bit about the distribution of opinions among other senators (Farquharson 1969).

Consider the above example again. If Group 3 voters knew that the plurality method was being used to determine the winning alternative and should they become aware of the distribution of opinions in the electorate, they would have a strong incentive not to vote for their favorite, alternative C, (even though C is the Condorcet winner), but they would vote for B to prevent the least favorite alternative (A) from being elected.

Social choice theory tells us that strategic misrepresentation of preferences is hardly avoidable. More specifically, according to the Gibbard-Satterthwaite theorem, every non-trivial, universal and single-valued social choice function is either manipulable (i.e. vulnerable to strategic misrepresentation of preferences) or dictatorial (Gibbard 1973; Satterthwaite 1975). In other words, no matter which single-valued choice function (i.e. procedure which always results in a single winning alternative) we resort to, we may encounter situations in which fully informed voters would benefit from not acting in accordance with their true preferences. Although the procedures most often used in voting are not single-valued but allow for ties, it can be shown that all the most common voting schemes are vulnerable to misrepresentation of preferences (Nurmi 1984).

It is typically very difficult to find out whether preference misrepresentation has actually taken place in practical decision making situations. A relatively uncontroversial piece of evidence is the behavior of some groups – most notably the Communists – in the 1956 Finnish electoral college (see Lagerspetz 1993 for a most perceptive and detailed account. See also Tsebelis 1990). In parliaments – e.g. the Finnish one – that use the amendment system, legislative majorities almost routinely resort to the preference misrepresentation in order to maneuver the weakest possible contestant to confront their favorite candidate in the final pairwise vote. Rasch (1987) argues that there is practically no evidence of preference misrepresentation in the Norwegian parliament, presumably due to its different voting system.

What bearing does the number of alternatives have on this problem? With two alternatives, the incentives to misrepresent one's preferences disappear. Since there are two alternatives, there are two voting strategies for each voter: vote for your favorite or vote for the alternative. It is easy to see that it is never beneficial for a voter to choose the latter strategy, because if his/her vote makes any difference at all, this strategy will be to his/her disadvantage.

Agenda Manipulation

Suppose a referendum is arranged to decide whether a country should sign a treaty with a politico-economic alliance of countries, and suppose, moreover, that in addition to the basic alternatives "yes" (A) and "no" (B) there is a

Example 2. A hypothetical Referendum with Three Alternatives

45% of voters	30% of voters	25% of voters
A	B	C
C	C	B
B	A	A

third alternative: "no, unless . . ." (C). Consider a hypothetical distribution of opinions in which 45 percent of the voters support A, 30 percent support B, and 25 percent support C.

The question now arises as to whether it is possible to infer something more about voter preferences. Under the present interpretation of the alternatives, we can make fairly straightforward inferences. To wit, for those who support the treaty as well as those who favor rejection, the second best alternative would seem to be C, qualified rejection. Somewhat less obvious is how those in favor of C would rank A vis-à-vis B. If we assume that they would rank B second since it would seem closer to C, we get the profile shown in Example 2.

With plurality voting A wins, but one would be hard pressed to call A a clear winner. As one can readily observe, A would be defeated by both B and C in pairwise contests with a majority of votes. However, even more problematic is the fact that the above profile may be a result of agenda manipulation. It is possible that the agenda setter is interested in guaranteeing the victory of A and, upon learning from e.g. opinion polls that A would be defeated (45 percent–55 percent) in a confrontation with B, the other basic alternative, the agenda setter tailors a new "compromise" alternative (C) which is as "close" to B as possible. Thus, the support for B can be split and the result looks like a pretty clear victory for A.

In Example 2 the support for the rejection alternative is split. Of course, the same stratagem could be used if the objective was to weaken the support of the pro-treaty alternative. Nothing in this example hinges on the interpretation of the alternatives except the assumption that the "distance" between C and B is smaller than the distance between C and A.

Prima facie, the example would seem to rest on rather specific assumptions concerning the vote distribution. Most of these are, however, needed just for illustrative purposes. Any setting in which the voters supporting A and B, A and C, as well as B and C comprise more than 50 percent of the electorate is open for agenda manipulation similar to the above example.

Accordingly, only settings in which A, B or C is supported by more than 50 percent of the voters are immune to the problems of interpretation and agenda manipulation.

With two alternatives, these types of agenda manipulation possibilities are absent. This is not to say that all kinds of agenda manipulations would *eo ipso* be excluded. The wording of alternatives is always important. There is, however, very little one can say in general about this problematique.

The Nordic referenda concerning membership of the European Union suggest another important strategic feature, i.e. the timing of a referendum vis-à-vis other similar referenda. Of course, any conclusion based on counterfactual assumptions – for example that the order of Nordic referenda had been Norway, Sweden, Finland instead of Finland, Sweden, Norway – is bound to be conjectural, but Hovi and Hellevik's analyses (1994 and 1996) give pretty strong indications that the outcomes would have been different had the sequence of referenda been different. That the political leaders also saw this possibility is clear from the public debate preceding the Nordic referenda.

Direct and Indirect Democracy

One of the advantages of representative or indirect democracy over direct democracy is that in the former the elected representatives may devote their full attention to and gain expertise in public affairs, while the rest of the population may concentrate on other matters most of the time. These advantages may, however, turn into disadvantages if the politicians form coalitions against voters and taxpayers and aim at maximizing their own revenue at the cost of the latter (Brennan and Buchanan 1980). One of the principal means by which the politicians may exploit their position vis-à-vis the voters is to control the agenda (Frey 1992).

Frey argues that direct citizen participation may in some cases break up the politicians' coalitions against the voters. However, as Frey also points out, non-binding referenda and plebiscites may have the opposite effect, viz. to consolidate the power of the politicians. Our preceding discussion leads to the conclusion that even binding referenda may have this consolidating effect unless specific precautions are taken. In particular, the possibility of the agenda setters to split "natural" alternatives into "compromise" alternatives is a serious threat to the interpretation of the referendum result. Any attempt to strengthen democracy by introducing direct forms of citizen participation has to take into account the agenda control issues. This is, however, not to deny the validity of Frey's argument against non-binding referenda. On the contrary, additional reasons for sharing Frey's view are presented in the next section.

Another recent writer, Budge (1993), comments on Riker's (1982) and McLean's (1990) interpretations of social choice results. As we know, Riker builds his argument against a "populist" view of democracy on the paucity

of voting equilibria and the ubiquity of strategic manipulation possibilities. Briefly, his conclusion is that the notion of the will of the people is useless in guiding our search for good democratic decision procedures for the simple reason that there is no such thing as the will of the people. In typical circumstances, the electoral outcomes are not equilibria and, hence, there are always other outcomes that would better reflect "the will of the people." The omnipresence of strategic manipulation possibilities poses a more serious problem, namely finding individual preferences in a reliable way. Thus, we cannot even determine whether there is an equilibrium outcome in the voting game since the voter preferences revealed in voting may be "distorted" by strategic considerations.

Now, Budge (1993, 153) points out that "paradoxes and voting cycles seem just as likely to emerge under representative democracy and in legislatures as in large populations." In a way this seems correct, but only on the condition that the behavioral assumptions concerning the voters in legislatures and electorates at large are the same. Budge's conclusion becomes unconvincing if the legislators are assumed to be better versed in the mechanisms of voting than the voters in general. More specifically, the legislators are much less likely to resort to "sincere" voting than the voters. Rather, they are likely to be strategic or "sophisticated" in their behavior. Moreover, they are more likely to spot attempts at agenda manipulation than the voters in general.

Does this undermine Budge's argument? I think it does. Sophisticated voting essentially restricts the manipulation possibilities of the agenda setters, especially in legislative contexts where binary procedures are used. McKelvey's (1979) famous chaos result on the majority voting outcomes in spatial voting games rests on the assumption of sincere voting. When this assumption is replaced with the sophisticated voting assumption, results are much more positive in the sense that the outcomes can be predicted to converge to plausible subsets of the alternative set. Thus, for example Miller (1980) shows that the uncovered set – which is always a subset of Pareto optimal outcomes – contains all the alternatives that can result from sophisticated voting. Banks (1985) demonstrates that we can accurately characterize the outcomes that may result from sophisticated voting. Sophisticated voting games simply do not result in the same chaos as sincere voting games. Consequently, it can be argued that legislators, by virtue of knowing their working instrument (voting), are better able to avoid the paradoxes than voters in general. This conclusion would – albeit indirectly – support Sartori's (1987) view of representative democracy.

The preceding discussion shows that relatively few problems arise in referenda with two alternatives. In the next section we argue that combining referenda with representational democracy poses some additional problems

Example 3. Ostrogorski's Paradox

Voters	Issue 1	Issue 2	Issue 3
A (20%)	X	X	Y
B (20%)	X	Y	X
C (20%)	Y	X	X
D (40%)	Y	Y	Y

which can, however, be overcome through the requirement that the referenda be binding, i.e. not consultative.

Combining Referenda with Representative Democracy

From the voter's point of view, the referenda are both an additional avenue for expressing one's opinion about specific issues and a complication in the voter calculus. They complicate matters insofar as during the election of the representatives the voter often does not know which issues will be subjected to a referendum during the parliamentary term of office. The more important an issue is, the harder it is for the voter to decide who to vote for if the voter's opinion on this issue diverges from that of his/her party or candidate. If the voter knows that a referendum will be called on this important issue, the voter has the luxury of being able to vote both for his/her favorite party or candidate and for his/her stand on the issue. But most often this kind of knowledge is not available at the time of the election.

In countries that resort to consultative non-binding referenda, a particular problem of great importance may be encountered, namely deciding which is more authoritative: the referendum outcome or the parliamentary voting outcome. To wit, it may happen that the majority of voters favor an opinion and the majority of the representatives its negation. *Prima facie*, this problem seems to be related to two well-known paradoxes: the Ostrogorski paradox and Simpson's paradox (Rae and Daudt 1976; Daudt and Rae 1978; Lagerspetz 1996; Cohen 1986; Good and Mittal 1987; Saari 1990). Example 3, by Daudt and Rae (1978, 338), illustrates Ostrogorski's paradox.

In Example 3 there are two parties, X and Y, and voters are divided into four groups, A, B, C and D. The first three groups comprise 20 percent of the electorate each, while group D comprises 40 percent of the electorate. The entries in the table indicate which party is favored by the voters. For example, voters in group A favor X over Y on issue 1.

Now, if a voter's choice of party is determined on the basis of which party is regarded as better on most issues (assuming that the issues are of equal importance), then the voters in A, B and C vote for X, whereas the voters in

Example 4. Simpson's Paradox

	X-treatment	Standard Treatment
Evanston	33% (100 out of 300)	30% (30 out of 100)
Chicago	50% (50 out of 100)	46% (140 out of 300)

Example 5. The Paradox of Representation

	Party A			Party B		
	MP1	. . .	MP6	MP7	MP8	MP9
yes	5	. . .	5	11	11	11
no	6	. . .	6	0	0	0

D vote for Y. Thus, 60 percent of the voters choose X. However, considering the issues one at a time, one immediately observes that on each issue, party Y is supported by 60 percent of the voters. Clearly, it makes a huge difference whether the issues are voted on one at a time or simultaneously as a party platform.

Example 4 shows Saari's (1990) illustration of Simpson's paradox.[1] In an effort to determine whether treatment X can cure the common cold, groups of people in Evanston and Chicago were subjected to different treatments. The example demonstrates the recovery rate of various groups. Clearly, X-treatment seems more efficient than the standard treatment, since the recovery rates are higher in the left than in the right column. However, an aggregation of the data supports the opposite conclusion, i.e. that 170 out of 400 regained health under standard treatment, compared to only 150 out of 400 under X-treatment.

In referendum contexts we encounter the following paradox of representation. Let us assume that the parliament consists of 9 members and that there are 99 voters so that each group of 11 voters elects one MP. Party A has 6 out of 9 or 2/3 of the parliament seats (MP1, . . ., MP6), while party B has 3 out of 9 or 1/3 of the seats (MP7, MP8, MP9). We assume that the system is proportional, i.e. 2/3 of the electorate supports party A and 1/3 party B. Now a referendum is arranged in which the voters are asked to answer either "yes" or "no" to a question. The distribution of votes in both parliamentary elections and the referendum is indicated in Example 5. Clearly "yes" wins the referendum with 63 votes out of 99.

Suppose, however, that the same issue is subjected to a parliamentary vote. Then, assuming that the MPs are cognizant of the distribution of opinions of their own supporters, it is plausible to predict that they will vote

in accordance with what they think is the opinion of the majority of their supporters. Thus, MPs from party A would vote "no" and MPs from party B would vote "yes." Obviously, "no" wins by a 6–3 margin. What is the correct outcome?

Surely the MPs who vote "no" are perfectly right in arguing that they represent the views of the majority of their supporters. If the referendum resulting in "yes" is consultative, then it is in the end the ideology of representation that is of crucial importance. Is each representative supposed to represent the whole people or just his/her own supporters? If the former alternative is the case, then the legislators have clear moral reasons to honor the referendum outcome. If, however, the MPs represent their own supporters, then the referendum outcome is of no consequence to their actions.

It is worth noticing that in the above example the victory margins are considerable. In particular, the number of MPs voting "no" comprises 2/3 of the parliament. Thus, even where a qualified 2/3 majority is needed for the motion to pass, the parliament's decision may contradict more than 60 percent of the voters.

One may well ask whether there are any limits to this type of contradiction, i.e. whether a contradiction may occur no matter how large the margin is. With two alternatives ("yes" and "no") and an almost perfectly proportional legislature – i.e. each representative has roughly equal support – a situation where "yes" is supported by 2/3 of the electorate and "no" by 2/3 of the MPs is not possible. The above example approaches a situation with 2/3 majorities in both cases. However, when we focus on the distribution of "yes" and "no" votes among the supporters of A and B, it becomes evident that a 2/3 majority for "yes" in the electorate and a 2/3 majority for "no" in the parliament is not possible. Party B comprises 1/3 of the parliament. Its supporters all prefer "yes" to "no." Hence, in order for "yes" to defeat "no" by more than a 63–36 majority, the number of A or B supporters who prefer "yes" to "no" would have to be increased. But this is impossible since all voters in group B already prefer "yes" to "no," and changing any voter's preference in group A in this direction would decrease the 2/3 MP majority. Thus, a 2/3 majority for "yes" in the electorate and a 2/3 majority for "no" in the parliament cannot occur simultaneously.[2]

It should be emphasized, however, that this limit holds for only reasonably proportional systems. If in the above example each candidate of party B is supported by considerably more than 11 voters, then it cannot be excluded that 2/3 majorities exist in opposite directions. In fact, even larger contradicting majorities could be encountered.

Let us finally consider the hypothetical situation in Example 6 in which 5/6 of the MPs may quite legitimately vote "no" despite the fact that a majority of voters vote "yes."[3]

Example 6. Another Demonstration of Paradox Representation

	MPs					
	MP1	...	MP167	MP168	...	MP200
yes	7,000	...	7,000	15,000	...	15,000
no	8,000	...	8,000	0	...	0

The system is perfectly proportional and the number of voters is 3,000,000. The number of "yes" votes is $167 \times 7000 + 33 \times 15{,}000 = 1{,}664{,}000$. The number of "no" votes is 1,336,000. Thus, the result is a clear 55 percent majority for "yes." And yet, 5/6 of the MPs may quite legitimately vote "no" arguing correctly that a majority of their supporters is in favor of this alternative.

Thus, in consultative referendum systems, large majorities of MPs may face a nearly intolerable norm conflict exemplified by the paradox of representation. Similarly, large majorities of voters may legitimately feel frustrated.

Concluding Remarks

In the preceding we have discussed the referendum institution in the light of some results of social choice theory. Although the theory is notorious for its impossibility results, it can shed light on many issues that pertain to direct and indirect democracy. The main conclusion of the preceding discussion is that the number and nature of alternatives is a crucial determinant of the possibility of various paradoxes and anomalies in voting. Referenda are important instruments of expressing popular opinion, but they can relatively easily be turned into means of control unless due attention is paid to the process of agenda formation. The straightforward interpretation of referendum results calls for preferably two alternative agendas. These agendas are also invulnerable to strategic misrepresentation of preferences. If the conditions of Condorcet's jury theorem hold, we can also be assured that the probability of a correct decision – if this notion can be employed – approaches unity as the electorate grows, assuming that the voters are on the average more than minimally competent. By and large, the majority rule can be expected to work well in dichotomous choice situations.

The paradox of representations pertains precisely to those situations in which referenda are combined with a representational system. It is a serious problem, especially in systems where referenda are consultative and non-binding. Such systems may experience sizable majorities for flatly contra-

dictory alternatives in the electorate on the one hand, and in the parliament on the other. Stipulating that referenda be binding is a way of handling the paradox of representation.

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2. I have discussed the limits of various voting paradoxes at greater length in another paper (Nurmi 1995).
3. Please note that in this as well as in the preceding example, the "electorate" consists of only those voters whose MP candidate gets elected.

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