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Politics, Economics and the Measurement of Power

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To what extent is political power fundamentally different from or, alternatively, comparable to economic power? While it is true that the basic institutions of democratic political life – the electoral arena and the sovereign representative assembly – differ from such capitalist economic institutions as the market and the joint-stock company, the logic of the power game which takes place in both settings is quite similar. In both institutions power will be a function of the capacity to enter decisive coalitions with other players: individuals, political parties, stockholders or groups of stockholders. Power indices may therefore be employed in order to reveal aspects of the strategic gaming that takes place both in representative assemblies and at yearly stockholders' meetings. This article discusses and compares various quantitative measures of voting power in the two kinds of voting bodies.

In an interesting article employing power indices, Anders Westholm (1992) launches some challenging suggestions and conclusions about the relationship between political and economic institutions. Westholm argues, among other things, that the discipline of political science should broaden its scope of analysis to include those social systems in the environment of the political system that either have an appreciable impact upon politics or the modelling of which may have theoretical implications for the science of politics. The joint-stock company in an advanced capitalist democracy is a case satisfying both requirements as well as posing some important practical problems.

Westholm's claims are worth comment, because they must in some respects be considered controversial. At the same time, a discussion of Westholm's claims may bring out more clearly the potential strength of power indices for social science analysis. The purpose of this article is to show how the index method may be used to illuminate not only theoretical problems but also practically relevant issues in decision-making. Before looking at these indices, however, a few remarks are in order regarding the set of institutional arrangements relating to the joint-stock company as these are subsequently to be analysed in quantitative terms.

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In an interesting article employing power indices, Anders Westholm (1992) launches some challenging suggestions and conclusions about the relationship between political and economic institutions. Westholm argues, among other things, that the discipline of political science should broaden its scope of analysis to include those social systems in the environment of the political system that either have an appreciable impact upon politics or the modelling of which may have theoretical implications for the science of politics. The joint-stock company in an advanced capitalist democracy is a case satisfying both requirements as well as posing some important practical problems.

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What is a Joint-stock Company?

A joint-stock company, states Westholm, is a peculiar social institution, because it serves as both the subject and object of exchange. It is similar to a "democratic polity" insofar as crucial decisions are made by a majority vote at stockholder meetings of the company, but it is also like the market, because voting rights may be purchased or sold. Thus, stockholders resemble both voters in politics and actors in the market.

Yet, it is confusing to identify the capitalist institution of an economic entity such as a joint-stock company, which is characterized by limited liability, as an electoral arena (cf. Williamson 1985). It would seem more accurate to compare a joint-stock company with a representative assembly and the market with electoral choice in the national elections. At stockholder meetings, coalitions are formed, but this also applies to representative assemblies. Westholm states that distinctive for a joint-stock company is that "the rights to control can be bought as well as sold" (Westholm 1992, 195). But there is, after all, also something called "log-rolling" in politics.

Westholm furthermore states that

> . . . actors eligible to enter the stock market consist of individuals as well as corporate actors of various kinds... While this is true of most other markets as well, it nevertheless makes the stock market peculiar in that the set of goods traded thereby comes to overlap with the set of actors between which trading takes place. (1992, 196)

On this point as well we think Westholm's remarks are a bit misleading. Democratic politics is built upon the one man, one vote rule, a rule which has its parallel in a joint-stock company in the form of a one issue of stock, one vote rule. But it does not follow that the number of voting rights sold and purchased in a stock market is the same as the number of persons and organizations that trade. In this respect, in other words, there is a major difference between capitalist institutions and democratic institutions – viz. that of property rights which may be aggregated in a fashion that is not comparable to the one man, one vote rule – which has tremendous consequences (cf. Williamson 1985). Of course, in representative assemblies votes can be traded by both individuals and political parties operating as a collectivity. But once again, it would seem that the joint-stock company is most akin to the representative assembly rather than the electoral arena at large.

The Paradox of Voting

We are also inclined to disagree with Westholm regarding the paradox of voting, a characteristic of national elections which hardly occurs at stockholder meetings of joint-stock companies. Although the individual voter's contribution to the collective outcome is typically infinitesimal, being discounted by a factor $1/N$, where N is the size of the electorate, turnout at general elections nonetheless tends to be relatively high (50 percent or more). In the case of stockholders meetings, by comparison, very few stockholders generally attend. How can this difference be accounted for?

Westholm looks for an answer with the aid of a game-theoretic approach, whereas we think the answer is in fact a matter of transaction costs. The costs of mobilizing what is called the ocean of small stockholders into a power coalition acting jointly are typically very high, whereas the transaction cost to vote for each single individual is very small indeed. Such a transaction cost interpretation is supported empirically by the relatively few occurrences of the mobilization of "ocean power" even where the decisions were of great importance or controversial (cf. Milnor & Shapley 1978).

The fact that property rights are involved in capitalist institutions and not Lincolnian voting rights also explains why participation at joint-stock company meetings is so different from participation in elections or in parliamentary sessions. Westholm argues that participation in terms of percentage of votes at joint-stock stockholder meetings will follow the percentage of votes which the major owners of stock possess, a proposition he deduces from the generalization that the degree of ownership concentration determines the autonomy of the executive of the company. However, the Westholm implication is a truism while the Berle and Means' hypothesis regarding separation of ownership from control in a modern joint-stock company, is not (cf. Berle & Means 1947). That the major stockholders in a joint-stock company have a dominating strategy, i.e. participate at stockholder meetings, is not a consequence of the power distribution according to the power indices, but a premise for the entire framework. One acquires major shares of stock because they stand for economic assets.

Given these conditions, how can we employ the power indices for non-trivial purposes? Is it possible to model the logic of economic power as well as that of political power with the same instrument, the power index?

Measuring Economic Power

In relation to the joint-stock company, economic power is first and foremost a function of the ownership of shares of various types which may be translated into a crucial factor – a given number of votes at the stockholder meeting. Thus, the degree of control and the dispersion of control among stockholders is a function of the relative distribution of votes. Thus far the argument is simple. But, how is the differential power of various stockholders to be measured? Power among stockholders may be measured by a variety of methods, which we will describe and discuss below using a representative sample of Swedish firms (cf. Sundquist 1985).[1]

First, there is the simple concentration measure, C_n, defined as the proportion of the total number of votes held by the n largest stockholders. The weakness of this measure lies in the lack of a well-defined theoretical rule for the choice of n, the number of stockholders to include. This introduces ambiguity in the measure since a different choice of n may result in a different ordering of the firms with respect to the degree of concentration. In short, a more refined weighting procedure is needed which reacts to changes in the number of stockholders as well as to changes in the sizes of shareholdings.

A second alternative, the Herfindahl index of concentration, which is defined as the sum of squares of shareholding proportions, provides such a measure (cf. Adelman 1969). The measure efficiently weights the importance of the size of the different shareholdings. An intuitive understanding of this measure is given by its commonly used reciprocal, the *numbers-equivalent*, i.e. the number of equally sized shareholding that would result in the same index. The Herfindahl index may also be defined as the arithmetic mean of the proportions added to the sum of the squared deviations from this mean, that is, the variance of the proportions times the number of stockholders considered (see Appendix). However, because of the measure's dependency on the variance, there is no simple relation between the number of stockholders and the value of the Herfindahl index.

A third measure, based on a voting model, has been suggested by Cubbin & Leech (1983). In this case the degree of control embodies the concept of a probability of winning votes at major stockholder meetings as well as a probability that a stockholder votes. The model is then used to determine the size of a controlling shareholding (see Appendix). In this manner, we arrive at two different scores; one shows the proportion of the total vote that is necessary for the control of the stockholder meeting, the other the actual proportion of votes held by the largest group of stockholders.

Table 1 contains illustrations of the three measures of economic power discussed above as applied to a set of data relating to Swedish business companies. The companies in the sample are split into three subgroups. The first group, (1), consists of companies where the largest stockholder, as shown under the column denoted C_1, alone controls the company with his or her votes. No coalition of other major stockholders can threaten this position. In the second group, (2), the largest stockholder still controls the company, but a coalition of other major stockholders, $C_n - C_1$, may threaten him or her. Finally, the last group, (3), consists of companies

Table 1. Three Measures of Economic Power.

Group	Company	Herfindahl index	Controlling shareholding	C_1 Largest shareholding	C_n Concentration ratio
(1)	Atlas Copco	0.1417	21.5	35.3	
	Esab	0.2813	33.4	49.0	
	Gota (GotaGruppen)	0.1981	18.9	43.0	
	Lundbergs	0.8124	4.1	90.1	
	Nobel Industrier	0.4256	12.4	64.8	
	Pharmacia	0.2239	20.2	45.7	
	Stora	0.1278	19.1	33.8	
	Trelleborg	0.3532	16.3	58.6	
(2)	Aga	0.1228	26.9	31.0	C_8-C_1 = 32.6
	Euroc	0.1700	27.8	37.6	C_9-C_1 = 38.0
	Siab	0.2913	36.9	49.1	C_{14}-C_1 = 49.1
(3)	Alfa-Laval	0.1739	45.1	31.4	C_2 = 56.8
	Asea	0.0757	23.5	23.5	C_2 = 30.4
	Astra	0.0529	29.2	14.6	C_3 = 33.4
	Electrolux	0.4470	51.0	48.6	C_2 = 94.5
	Ericsson	0.3433	51.0	42.5	C_2 = 82.6
	MoDo	0.2518	49.2	40.3	C_2 = 62.6
	Saab-Scania	0.0684	24.0	21.7	C_2 = 28.2
	Sandvik	0.1219	31.0	29.4	C_2 = 40.5
	SCA	0.1666	36.2	34.4	C_2 = 54.2
	S-E-Banken	0.0342	26.9	8.7	C_4 = 29.5
	SHB	0.0361	25.4	11.1	C_4 = 30.1
	Skandia	0.0628	33.8	14.3	C_3 = 34.5
	Skanska	0.1138	31.4	27.8	C_2 = 42.7
	SKF	0.2186	51.0	34.8	C_2 = 64.1
	Volvo	0.0703	31.2	18.5	C_3 = 40.1

Group 1: The share of votes of the largest stockholder, C_1, is greater than the score needed for a controlling share of the votes.

Group 2: As for group 1, however, a coalition of the next $n - 1$ stockholders may threaten the largest stockholder with their share of the votes, C_n-C_1.

Group 3: A controlling block, C_n, is only obtained by a coalition of the n largest stockholders.

where no single stockholder controls the company, but a coalition of the largest stockholders, C_n, will in many cases be able to form a controlling group.

We find, for example, that for Atlas Copco the fairly low Herfindahl index indicates that power is quite diffuse, corresponding to a situation where seven equally powerful owners each have roughly 14 percent of the votes. As for Lundbergs, by comparison, the index score indicates a sharp concentration as one owner has about 80 percent of the vote. The average score for all companies listed in Table 1 is roughly 0.14, meaning that economic power is not very concentrated. As noted, such a result is produced by a case in which seven owners split the votes among themselves in each company. Thus, the Herfindahl index does not indicate a concentration of ownership among most major Swedish companies.

Yet when we use the probabilistic model, suggested by Cubbin and Leech, the findings are quite different. Here we see that in group (1) there is one dominating sphere, whereas in group (2) we again have one dominating owner but control may be challenged. In group (3), there is better correspondence between the two measures as one sphere alone cannot exercise control. Thus, we find that the Herfindahl index and the probabilistic voting model contain different information. With this, let us proceed further to game theoretical modelling with its power indices.

Economic power can also be tapped by means of a measure of different actors' degree of control over the outcomes of a *voting game*,[2] that is, by various power indices. The two most common measures for such situations are the Shapley–Shubik and the Banzhaf indices respectively. Both measure the a priori ability of an actor to affect the outcome of a game. The influence, or power, that actors may have on other actors is not considered. Thus, in our sample of stockholders this would include such relations as shares divided among different members of a family, cross-ownership, pyramiding, friendship and/or partnership, all of whom are expected to cast their votes in unison. Where such relations are identified, players are grouped together into spheres of common interests.

An early discussion of voting and power was provided by Shapley & Shubik (1954). Based on the theory of *simple games*,[3] i.e., *weighted majority games*[4] they defined a power index which is actually the probability that a voter will cast the decisive vote and create a majority, provided the voting is sequential.

In the mid-1960s Banzhaf (1965, 1968) subsequently proposed another measure of power based on coalitions of voters. In this case the power of a voter is the probability that a voter can alter the decision of a coalition by changing his vote. Banzhaf assumed a dichotomous voting situation with equal probability of voting "yes" or "no".

The Shapley–Shubik index is a theoretical measure founded on three well-defined but quite simple mathematical axioms. These axioms give an intuitive support to the interpretation of the Shapley–Shubik index as a real value as well as constraining the index to be an additive measure, i.e., the Shapley–Shubik indices always sum to unity. Thus, the sum of individual power indices will be a plausible measure of coalition power. The Banzhaf index, on the other hand, originates from empirical studies of weighted voting and multi-member electoral districts, and fairness of representation in voting situations. While lacking the additive property, it has an axiomatic founding (Dubey & Shapley 1979) and gains its appeal from being associated with a more straightforward probability model (see Appendix).

The strength of these power indices is that they include the modelling of strategic opportunities which owners face in a situation of power interaction as found at stock-holder meetings. Thus, it is possible to take into account the fact that a large number of stockholders are so small that they will only rarely attend the yearly meetings, forming what is called the ocean in *oceanic games*.[5] When there is no account taken of the votes of the ocean then the power index models a *truncated game*, i.e., a weighted majority game consisting of only the major stockholders. However, one may also wish to include ocean power in the game, assuming that the ocean of minor stockholders is present. For purposes of illustration, the Banzhaf index and two different types of Shapley–Shubik indices, truncated and non-truncated, were calculated for selected Swedish companies and are shown in Table 2.

To begin with, it may be observed that there are significant differences between the models of ocean games and those of truncated games for these six major companies. Including the ocean in the game does reduce the power of the major owners. If, in addition, the ocean could act collectively with one unanimous vote, then they would clearly be the strongest player. Yet, making the empirical assumption that the ocean is not present is correct most of the time, which implies that we shall look at the truncated games.

Furthermore, there is a quite different but very revealing picture of economic power displayed in the data (Tables 1–2). For three of the companies, one single group manages to control almost every possible decision with only just about 25 percent of the votes. Neither the Herfindahl index nor the probabilistic model captures the contribution of strategic action to the wielding of economic power. When there are a few players with roughly the same share of votes, as in one case, then economic power will once again not be proportional to the share of the votes. The larger the share, the higher the amount of economic power, disproportionately.

Summing Up: Economic Power

The application of the various measures to data on the distribution of votes among stockholders in Swedish companies results in an interpretation problem. First, the power scores for one and the same company vary considerably. This is most interesting, as it is often believed that the alternative power indices tend to coincide. Second, the two overall measures – the Herfindahl index and the probabilistic voting score – do not give the same results. Third, the two versions of the Shapley–Shubik index – the ocean game version and the truncated game version – are not in agreement. Finally, the Banzhaf non-normalized index and the Shapley–Shubik truncated game version also differ.

Table 2. Power Indices as Applied to Selected Swedish Companies, 1985.

Company	i	w	φ_1	φ_2	β	Company	i	w	φ_1	φ_2	β
Atlas Copco	1	35.3	0.515	1.000	1.000	Lundbergs	1	90.1	1.000	1.000	1.000
	2	12.1	0.077				2	1.3			
	3	3.2	0.025				3	1.1			
	4	2.4	0.019				4	1.0			
	5	1.5	0.012				ocean	6.5			
	ocean	45.5	0.352								
Asea	1	23.5	0.298	0.602	0.940	Volvo	1	18.5	0.214	0.341	0.641
	2	6.9	0.066	0.079	0.061		2	12.1	0.126	0.179	0.328
	3	6.8	0.065	0.079	0.061		3	9.5	0.096	0.160	0.289
	4	4.9	0.046	0.057	0.057		4	6.3	0.062	0.088	0.164
	5	4.0	0.037	0.052	0.053		5	5.0	0.048	0.074	0.133
	6	3.2	0.030	0.035	0.041		6	4.8	0.046	0.074	0.133
	7	2.3	0.021	0.028	0.029		7	4.3	0.041	0.060	0.102
	8	1.7	0.016	0.020	0.022		8	1.5	0.014	0.026	0.039
	9	1.5	0.014	0.020	0.022		ocean	38.0	0.352		
	10	1.4	0.013	0.020	0.022						
	11	1.1	0.010	0.010	0.010						
	ocean	42.7	0.385								
S-E-Banken	1	8.7	0.093	0.210	0.441	Skandia	1	14.3	0.158	0.247	0.531
	2	8.4	0.089	0.191	0.402		2	10.3	0.108	0.160	0.336
	3	7.6	0.080	0.179	0.379		3	9.9	0.103	0.153	0.324
	4	4.8	0.049	0.105	0.223		4	7.4	0.075	0.110	0.238
	5	4.7	0.048	0.097	0.207		5	7.1	0.072	0.103	0.227
	6	4.0	0.040	0.081	0.176		6	6.1	0.061	0.084	0.188
	7	2.2	0.022	0.044	0.090		7	3.3	0.032	0.041	0.090
	8	1.5	0.015	0.035	0.070		8	2.8	0.027	0.041	0.090
	9	1.5	0.015	0.035	0.070		9	1.5	0.014	0.021	0.047
	10	1.2	0.012	0.025	0.051		10	1.5	0.014	0.021	0.047
	ocean	55.4	0.538				11	1.5	0.014	0.021	0.047
							ocean	34.3	0.322		

i = shareholder, ordered by size; w = percent of votes, stockholder i; φ_1 = Shapley–Shubik indices, ocean games; φ_2 = Shapley–Shubik indices, truncated games; β = Banzhaf indices.

The Herfindahl index may be interpreted as the average control of the largest stockholders in terms of a certain share of the votes. The meaning of the probability scores is the size of the controlling vote and its percentage of the total votes. It suggests that there may be a controlling stockholder even though the overall distribution of power, as measured by the Herfindahl index, is not even close to 1.

The game theoretical measures of power may be interpreted as the capacity of various groups to be decisive when votes are cast at a stockholders' meeting. The capacity to be decisive is the same as the probability to be decisive, which depends on how the various groups participating in a stockholders' meeting are identified. And there is no way to specify this a priori. Sometimes part of the ocean may turn up and act as a united group, sometimes the ocean is totally absent. A posteriori, the latter assumption tends to be the correct one.

At the same time, the power index scores, in particular the truncated game scores, model economic power in a different way from the other measures. Since it takes into account the tactics and strategies of the major owners in relation to their capacity to form winning coalitions with other owners, large or small, its picture of economic power is to be preferred. What comes out nicely is the finding that quantitative voting games do not fulfil the requirement that power should reflect the proportional differences between the control of votes. What then, about political power in voting contexts?

Measuring Political Power

The concept of political power is complex and has been the object of different interpretations. Power has, for example, been treated as a causal relation, or influence, or persuasion. Here, we look at political power in the voting context. A group or a committee make decisions by voting for or against a proposition, the outcome depending on how many votes are required for a proposition to be accepted or rejected. We take the European Community as an example, because the decision rules of its bodies have attracted considerable attention recently. Is it possible to throw light on the complicated question of how quantitative voting rules in the Community relate to the distribution of power among member states?

As of 1992 the European Community, EC, has twelve members – Belgium, France, Italy, Luxemburg, Netherlands and Germany, (BRD), which founded the community in 1958, in addition to Denmark, Ireland and the United Kingdom who became members in 1972, Greece in 1981, and, lastly, Portugal and Spain in 1986. The EC is run by the Council of Ministers, the Commission, the Parliament and the Court. The Council of

Ministers remains the supreme decision body in the EC. Each member is represented by the minister responsible for the question under consideration, e.g. when environmental issues are discussed each member is represented by the minister responsible for environmental issues, and so forth. Decisions taken by the Council are compulsory for the member states.

The decision rules which prevail are prescribed in the Treaty of Rome, paragraph 148. Decisions in the council are made either *unanimously*, with one vote for each member country on important issues (e.g. when new members are to be be elected), or, increasingly by *simple majority*, where each member has one vote or by *qualified majority*, where members have different numbers of votes and a decision requires 54 out of the total number of 76 votes.

Since the qualified majority rule is also used with regard to important issues, we must understand what quantitative voting means in political contexts. There are two problems here: first, how to introduce differential numbers of votes for member states of different population sizes, and, second, what is the effect of quantitative voting schemes on the voting power of member states? Some principle of fairness should presumably be employed when the number of votes is distributed among the member states. At the EC one could distribute the number of votes to each member strictly in proportion to the size of the population. Such a procedure would imply that the democratic rule of one man/one vote has been extended upwards to cover also a confederation, such as the EC. But will such a proportional rule result in equal power?

Let us first look at the question of whether the distribution of votes reflects the size of the country in terms of population. As seen in Table 3, the number of votes is not distributed according to a proportionality principle. The actual distribution reduces the number of votes of the large countries – France, United Kingdom, Germany and Italy – and increases the number of votes of the smaller countries, especially Luxemburg. The last column in Table 3 distributes the votes according to the square root of the population, and it is striking how close it is to the actual distribution. Why proportional to the square root of the population? To tell the truth, there is no documented discussion to be found about the principles used when the distribution of votes was settled. It might be found in protocols that are not public, but this is unknown to us. Whatever the case, would, then, the square root principle satisfy the proportionality principle that power should be allocated equally to equals and unequally to unequals? Table 4 has the answer.

The interpretation of the power index values in Table 4 is straightforward: The Banzhaf value 0.140 for France means that if a coalition is randomly selected the probability is 0.140 that France is decisive in the sense that if France changes its vote, then the decision will be altered. The Shapley–

Table 3. Distribution of Votes in the Council of Ministers.

Member State	Population (millions)	Votes	Votes Proportional to population	Votes proportional to $\sqrt{\text{population}}$
France	55	10	13	10
Italy	57	10	13	10
United Kingdom	57	10	13	10
Germany	61	10	14	11
Spain	39	8	9	9
Belgium	10	5	2	4
Greece	10	5	2	4
Netherlands	15	5	4	5
Portugal	10	5	2	4
Denmark	5	3	1	3
Ireland	4	3	1	3
Luxemburg	0.37	2	0	1

Table 4. Power Indices, EC Council of Ministers[1].

Member State	Votes	Shapley–Shubik power indices	Banzhaf power indices	Rel power[2] vs. rel. no. of votes
France	10	0.134	0.140	0.98
Italy	10	0.134	0.140	0.98
United Kingdom	10	0.134	0.140	0.98
Germany	10	0.134	0.140	0.98
Spain	8	0.111	0.118	1.03
Belgium	5	0.064	0.072	1.01
Greece	5	0.064	0.072	1.01
Netherlands	5	0.064	0.072	1.01
Portugal	5	0.064	0.072	1.01
Denmark	3	0.043	0.050	1.16
Ireland	3	0.043	0.050	1.16
Luxemburg	2	0.012	0.020	0.68

[1]Qualified majority : 54 votes of 76.
[2]Banzhaf power indices.

Shubik power index may be interpreted in a similar way. It is apparent that large states have more voting power than the smaller. But is there a proportionality between the votes and the power measure? The last column in Table 4 indicates the relative voter power versus the relative number of votes.

Even though the larger members have much more voting power than the smaller ones, the major finding here is actually that they have less relative

Table 5. The Effect of Different Voting Rules as Reflected by Banzhaf Normalized Power Indices.

	Voting rules					
	One vote each		Different amount of votes			
The EC Council	Unanimity	Majority	Majority	Qualified majority	Qualified majority 8 votes min.	Votes
France	0.083	0.083	0.134	0.129	0.121	10
Germany	0.083	0.083	0.134	0.129	0.121	10
Italy	0.083	0.083	0.134	0.129	0.121	10
United Kingdom	0.083	0.083	0.134	0.129	0.121	10
Spain	0.083	0.083	0.107	0.109	0.104	8
Netherlands	0.083	0.083	0.064	0.067	0.070	5
Belgium	0.083	0.083	0.064	0.067	0.070	5
Greece	0.083	0.083	0.064	0.067	0.070	5
Portugal	0.083	0.083	0.064	0.067	0.070	5
Denmark	0.083	0.083	0.040	0.046	0.053	3
Ireland	0.083	0.083	0.040	0.046	0.053	3
Luxemburg	0.083	0.083	0.024	0.018	0.025	2

voting power than the smaller members. Hence, the square root principle implies a distribution of votes that is more favourable to the small members including tiny Luxemburg, but less favourable to the larger ones. These calculations have been based on the Banzhaf index but similar results are obtained by the Shapley–Shubik power index. The other major finding is that alternative decision rules in the Council of Ministers would lead to greatly different power scores (cf. Table 5). The following decision rules appear as potentially relevant: unanimity; majority vote where each member has one vote; majority vote with the existing distribution of votes; qualified majority without or with the additional rule that eight members must vote for a decision.

Summing Up: Political Power

The calculations above show that the power of the larger countries is reduced by the majority rule chosen. A qualified majority reduces the relative power of voters with many votes compared to voters with a small number of votes. Thus, a rule with simple majority gives more power, in the sense used here, to the larger member states. The distribution of votes in the EC Council of Ministers is close to a distribution of votes in accordance with the square root law. Then the power of a single voter in a member state with one vote is proportional to the square root of the size of the member state, which further implies that the number of representatives for that member in a multinational decision body should be proportional to the square root of its size. However, the consequence is that within the Council of Ministers voting power is not proportional to the number of votes, the larger members having surprisingly less power than the smaller ones. Thus the problem remains: is the basis for the present distribution of votes fair? We have considered some aspects of this problem by looking at alternative decision rules for the Council of Ministers. The overall finding is that political power in voting contexts behaves very much like economic power in joint-stock companies.

Conclusion

One often makes a sharp distinction between politics and economics, referring to fundamental institutional differences between markets and elections or joint-stock companies and representative assemblies. However, modelling both economic and political institutions within the same quantitative framework – by means of power indices – is warranted, because both types of power involve quantitative voting. When groups in the polity control different amounts of votes, then the principle of one man – one vote does not guarantee equality of power. This result holds for joint-stock companies as well as for representative assemblies such as the Council of Ministers in Brussels.

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Methodological Appendix

Concentration Measures

The Concentration Ratio

If S is the total amount of shares/votes, s_i is stockholder i's amount of shares/votes, and there are N stockholders ranked from largest to smallest, we compute the concentration ratio by

$$C_n = \sum_{i=1}^{n} \left(\frac{s_i}{S}\right). \tag{1}$$

The parameter C_n, $1 \leqslant C_n \leqslant n/N$, thus measures the proportion of shares/votes held by the n largest stockholders, when n is usually much smaller than N.

The Herfindahl Index

Using the same notation, we define the Herfindahl index by

$$H = \sum_{i=1}^{n} \left(\frac{s_i}{S}\right). \tag{2}$$

i.e., the sum of the squared proportions of *all* stockholders, (cf. Adelman 1969). The bounds are then given by $1 \leqslant H \leqslant 1/N_i$. For practical purposes, since actual data on the smallest stockholders are usually not available, a lower bound of the Herfindahl index is normally used where the insignificant stockholders are excluded. Concentration will increase with the value of both indices, (l) and (2).

If we denote the proportion s_i/S by p_i, then the Herfindahl index is given by:

$$H = \sum_{i=1}^{n} p_i^2$$

We note that the mean of the p_i's is equal to $1/N$. Hence, if we denote the variance of the proportions by σ_p^2, then the sum of squared deviations from the mean is given by:

$$N\sigma_p^2 = \sum_{i=1}^{N.} \left(p_i - \frac{1}{N}\right)^2 = \sum_{i=1}^{N} p_i^2 - \frac{1}{N}.$$

Thus we obtain the alternative definition of the Herfindahl index

$$H = \frac{1}{N} + N\sigma_p^2. \tag{3}$$

If we assume that all N shareholders control the same amount of shares/votes, or, equivalently, that the sum of squared deviations in (3) equals zero, then we obtain what is called the *numbers-equivalent*, $N^* = 1/H$.

The Probabilistic Voting Model

Consider N shareholdings expressed in terms of proportions,

$$p = \frac{s_i}{S}, \qquad i = 1 \ldots N.$$

Let p_1 then denote the largest bloc of shares. Furthermore, denote by p_i the probability that a voter i votes for the largest stockholder, and by φ the probability that stockholder i exercises his vote, both assumed constant. Let X_i be the number of votes in support of the largest stockholder cast by stockholder i. Votes in opposition are counted as negative. Furthermore, we make the assumption that stockholders cast their votes independently of each other. The probability distribution of the random variable X_i is then given by:

$$\begin{aligned} Pr(X_i = p_i) &= \pi\varphi, \\ Pr(X_i = 0) &= 1 - \varphi, \\ Pr(X_i = -p_i) &= (1 - \pi)\varphi. \end{aligned} \tag{4}$$

To make it simple, we assume that i votes for or against the largest stockholder with equal probability, i.e., $\pi = 1/2$. Clearly, with zero mean, the variance of X_i is φp_i^2.

A margin, M, is defined as p_1 plus the difference between the votes cast for and those against the largest stockholder, Y:

$$M = p_1 + Y, \tag{5}$$

where

$$Y = \sum_{i=2}^{N} X_i$$

is a random variable with zero mean. Because of independence, the variance of Y is equal to the sum of the individual variances of the X_i's, given above, i.e.,

$$\begin{aligned} \sigma_Y^2 &= \varphi \sum_{i=2}^{N} p_i^2 \\ &= \varphi[H - p_1^2], \end{aligned} \tag{6}$$

where H is the Herfindahl index (3). Provided individually small holdings, Y is approximately normally distributed,

$$Y \sim N\left(0, \varphi \sum_{i=2}^{N} p_i^2\right).$$

Accordingly, the random variable, M, is normally distributed around the mean p_1, i.e.,

$$M \sim N\left(p_1, \varphi \sum_{i=2}^{N} p_i^2\right).$$

Hence, if p_c is the proportion of shares/votes needed for a control of degree α, the value of p_c is found by the condition $\Pr(M > 0) = \alpha$, that is, $\Pr(Y + p_c > 0) = \Pr(Y > -p_c) = \alpha$. Standardizing the normal distribution of Y, we obtain the size of a controlling, shareholding as:

$$\sigma_Y^2 = z_\alpha \sqrt{\varphi \sum_{i=2}^{N} p_i^2} \qquad (7)$$
$$= z_\alpha \sqrt{\alpha[H - p_1^2]},$$

Thus if $\alpha = 0.95$, $p_c = 1.645\sqrt{[H - p_1^2]}$. Formula (7) gives an interesting relationship between the three quantities: controlling stockholding – Herfindahl's index – size of largest stockholding.

Power Indices

The Shapley–Shubik Power Index

The Shapley–Shubik Power index relates to a probability model where the set $N = 1, 2, \ldots, n$ of players is permuted. Let S denote the set of players that precedes the player, i, in the ordering/permutation, and let i be awarded the amount $\nu(S \cup \{i\}) - \nu(S)$, namely the gain that he brings to the coalition consisting of his predecessors. It can then be shown that i's expected gain is precisely the Shapley–Shubik index.

In a *simple game*, where ν assumes only the values 0 and 1, there is exactly one player in each ordering who receives the unity gain. This player is said to be *pivotal* in the ordering. Thus, the pivotal player's value is equal to the probability that in a random ordering of all players he and his predecessors together will have enough votes to win, whereas his predecessors alone do not.

With equally likely orderings/permutations, the value of the Shapley–Shubik power index of player/stockholder $i \in N$ is indicated by:

$$\varphi_i = \sum_{N-i \supset S} \frac{s!(n-s-1)!}{n!} [\nu(S-i) - \nu(S)] \qquad (8)$$

where $n = |N|$ and $s = |S|$.

The Banzhaf Power Index

The Banzhaf power index of a voter is defined as the number of swings for a particular player of the set N. A swing for player i is a pair of sets of the form $(S, S - \{i\})$ such that S is winning while $S - i$ is not. The probability of a swing is based on a model where each player votes "aye" or "nay" with equal probability, 1/2.

Two types of swings are present, on the one hand there is the "winning" set S which turns to "losing" if player i leaves the coalition; i.e. the pair $(S, S - \{i\})$ is a swing for i. On the other hand, there is a "losing" set S that will turn to "winning" if player i joins the coalition, i.e. the pair

$(S, S \cup \{i\})$ is also a swing for i. Because of symmetry we will only consider winning sets, for convenience denoted by $S \cup \{i\}$, and, accordingly, the losing set by S. The swing $(S \cup \{i\}, S)$ is then a winning coalition turned to losing by the departure of player i.

The Banzhaf power index of player/stockholder $i \in N$ is indicated by:

$$\beta_i = \frac{1}{2^{n-1}} \sum_{N-i \supset S} [\nu(S-i) - \nu(S)] \tag{9}$$

where $n = |N|$. Thus we see that the increment in the award to the coalition is weighted differently in the two formulas (8) and (9).

resentatives for that member in a multinational decision body should be proportional to the square root of its size. However, the consequence is that within the Council of Ministers voting power is not proportional to the number of votes, the larger members having surprisingly less power than the smaller ones. Thus the problem remains: is the basis for the present distribution of votes fair? We have considered some aspects of this problem by looking at alternative decision rules for the Council of Ministers. The overall finding is that political power in voting contexts behaves very much like economic power in joint-stock companies.

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