Scandinavian Political Studies, Vol. 10 – No. 2, 1987
ISSN 0080-6757

The Contradictions of Rational Abstention: Counterfinality, Voting, and Games without a Solution*

Jon Hovi, University of Oslo

This paper considers the theory of rational abstention from the angle of Elster's categories of social contradictions. It is demonstrated that Downsian expected utility maximizing voters will be trapped in a state of counterfinality, which is one of these categories. Realizing this, citizens may alter their behavioral assumptions, and come to base their decisions on strategic thinking. However, this leads into another contradiction in Elster's sense, since in general, the Voting Game has no solution. The latter conclusion is shown to hold true regardless of the number of candidates involved, and irrespective of the number of rounds in the election. The lesson to draw is that neither Bayesian decision theory nor the theory of games can provide satisfactory advice for the rational citizen in voting situations, and, by implication, decision criteria other than those offered by these theories may be of some relevance. However, several possible criteria exist, and unless satisfactory predictions can be made as to how the various criteria are distributed in the electorate, the theory must remain indeterminate as far as predictions of voter turnout are concerned.

In his book *Logic and Society*, Jon Elster (1978) distinguishes between three forms of 'real' social contradictions. The first is related to production of unintended consequences resulting when individual agents act on the basis of assumptions that cannot be generalized. This category, which implies a contradiction between intended and achieved results, is called *counterfinality*. A famous example is the so-called 'cobweb' case, to be discussed at some length in the third section.

The second form is referred to as *suboptimality*, and has to do with situations where the agents intendedly produce results which everyone involved *knows* are inferior to some other (achievable) state of the world. Thus, suboptimality can be considered a contradiction between the potential and the real. A standard example, of course, is the Prisoner's Dilemma.

The final category of contradictions is that of *games without a solution*, which – according to Elster – has features in common with each of the

* I am indebted to Raino Malnes, Knut Midgaard, Tone Ognedal and Bjørn Erik Rasch for useful suggestions.

Scandinavian Political Studies, Vol. 10 – No. 2, 1987
ISSN 0080-6757

The Contradictions of Rational Abstention: Counterfinality, Voting, and Games without a Solution*

Jon Hovi, University of Oslo

This paper considers the theory of rational abstention from the angle of Elster's categories of social contradictions. It is demonstrated that Downsian expected utility maximizing voters will be trapped in a state of counterfinality, which is one of these categories. Realizing this, citizens may alter their behavioral assumptions, and come to base their decisions on strategic thinking. However, this leads into another contradiction in Elster's sense, since in general, the Voting Game has no solution. The latter conclusion is shown to hold true regardless of the number of candidates involved, and irrespective of the number of rounds in the election. The lesson to draw is that neither Bayesian decision theory nor the theory of games can provide satisfactory advice for the rational citizen in voting situations, and, by implication, decision criteria other than those offered by these theories may be of some relevance. However, several possible criteria exist, and unless satisfactory predictions can be made as to how the various criteria are distributed in the electorate, the theory must remain indeterminate as far as predictions of voter turnout are concerned.

In his book *Logic and Society*, Jon Elster (1978) distinguishes between three forms of 'real' social contradictions. The first is related to production of unintended consequences resulting when individual agents act on the basis of assumptions that cannot be generalized. This category, which implies a contradiction between intended and achieved results, is called *counterfinality*. A famous example is the so-called 'cobweb' case, to be discussed at some length in the third section.

The second form is referred to as *suboptimality*, and has to do with situations where the agents intendedly produce results which everyone involved *knows* are inferior to some other (achievable) state of the world. Thus, suboptimality can be considered a contradiction between the potential and the real. A standard example, of course, is the Prisoner's Dilemma.

The final category of contradictions is that of *games without a solution*, which – according to Elster – has features in common with each of the

* I am indebted to Raino Malnes, Knut Midgaard, Tone Ognedal and Bjørn Erik Rasch for useful suggestions.

other two forms mentioned above. An example is 'The Dollar Auction Game' (Shubik 1971), in which a number of individuals bid for an object of given value (say, a 100-dollar bill). The bids differ, however, from ordinary auctions in that all participants have to pay their highest bid, regardless of whether they win or not. If there are no constraints on the amount a participant is allowed to bid, rational individuals will keep on outbidding each other indefinitely – the value of the object being no upper limit for the amount the participants will be willing to bid.[1]

In this paper, I will demonstrate the possibility that if a number of individuals become aware that they are in a situation where one of these contradictions applies, they may change their behavioral assumptions in a manner that leads them into another contradiction. The case to be considered is the paradox of voting that has troubled political scientists even since the publication of Anthony Downs' *An Economic Theory of Democracy* in 1957. In the next section, I sketch the main features of the Downsian (or expected utility) theory on the rationality of abstention. In the third, it is argued that if the behavioral assumptions of this theory are correct, the agents of the model (i.e. the electorate) will be in a state of counterfinality. Given that the agents realize this, they may come to base their decisions on strategic reasoning, rather than on Downsian assumptions. However, this leads to another contradiction in Elster's sense, because the resulting game is – except under very special and highly unlikely circumstances – a game without a solution. In the fourth section, it is demonstrated that this conclusion holds both in the Downsian two-candidate case, and also in the more general case with m candidates. The latter has so far received surprisingly little attention, in spite of the overwhelming magnitude of the literature in this field.

An important implication of the discussion, then, is that *neither* expected utility maximizing *nor* game theory can provide satisfactory advice for the rational citizen in voting situations. Hence, there is a need for alternative decision criteria in such cases. A few examples of such criteria are briefly discussed toward the end of the paper.

The Downsian Theory of Rational Abstention

The paradox of large-scale participation in national elections was first formulated by Anthony Downs (1957, 260–276), and all subsequent papers on the subject draw heavily on this seminal work. The model can be summarized as follows. Two candidates, *a* and *b*, are running for office. Each member of the electorate has to make a decision on whether to vote for *a*, to vote for *b*, or to abstain. Since it is sometimes better, and never worse, to vote for your most preferred candidate rather than for his opponent,[2] the choice can – for all practical purposes – be considered a binary one: Either a given citizen votes for the candidate he favors most, or he abstains. According to Downs, a rational citizen will decide to vote if (and only if) the expected utility from voting is greater than the expected utility from abstention. Using the notation of Gordon Tullock (1967, 109), this means that the rational citizen will vote (for his most preferred candidate) only provided that the following inequality holds true:

$$BD - C_v > 0$$

B = benefit expected to be derived from success of your party or candidate (i.e. the 'party differential').
D = likelihood that your vote will make a difference.
C_v = the cost of voting.

The argument now runs as follows. In national elections, D is likely to be extremely small. According to Tullock, in an American presidential election D is less than one in ten million. Hence, even if the party differential is fairly large, it is unlikely that the product BD will be greater than the cost of voting (C_v). For example, if B is \$10 000 (a number that is probably well above the average citizen's party differential), and C_v is \$1 (a number that is almost certainly below most people's cost of participation), the costs outweigh the (expected) gains by a thousand to one. The seemingly obvious conclusion is therefore that a rational citizen will decide to abstain, meaning that if everyone acts under the assumptions stated in the theory, participation in national elections will be equal (or at least close) to zero.[3] The paradox, of course, is that in most real-world elections, more than 50 per cent of the electorate go to the polls.

This apparent contradiction between rationalistic theory on the one hand, and empirical evidence on the other, has triggered several attempts to revise the theory in order to provide predictions that are compatible with available data. Two main strategies can be distinguished within this literature. First, suggestions have been made that the payoff-functions of real-world citizens include factors that are not considered in Downs' original model. For example, it is commonly held that there is some positive utility associated with the act of voting *per se*, stemming from a sense of duty, moral obligations, or some other form of social benefits. Variants of this view can be found within the works of Goodin & Roberts (1975), Riker & Ordeshook (1968), and even Downs (1957) himself, to mention only a few. Formally, this assumption can be incorporated into the calculus by adding some positive factor *W* into the payoff-function, or – as Tullock (1967) indicates – by assuming that the cost of voting is *negative* (i.e. it costs something to *abstain* from voting). Another related possibility is that the calculus includes considerations that are in some sense altruistic. This view typically argues that it is not merely selfish concerns that motivate people for political actions. Rather, the average citizen is held to consider the consequences for society at large, or possibly for a somewhat smaller reference group, when estimating the party differential between the various candidates. Hence, the B-term should be replaced by $B_p + B_c$, '. . . with B_p representing the direct benefits that the voter expects to receive, and B_c representing the benefits he will receive because he gains some satisfaction from other people being benefited' (Tullock 1967, 111). The argument is that since B_c may outnumber B_p many times, the expected utility from voting may be positive, even if D is quite small. An important variant of this view is that of Margolis (1982), who envisions individual choice as based on a dual calculus, stemming from the assumption that Mr. Smith – the average citizen – is made up of a G(roup)–Smith and a S(elf)–Smith. The former considers costs and benefits for society at large, while the latter takes account of the consequences for Smith himself only. Because S-interests are only marginally affected by the act of voting, decisions are likely to be determined by G-interests in such cases, meaning that when G-interests and S-interests conflict, an individual may well be expected to vote *contrary* to his self-interest.[4]

The other theoretical revision of Downs' work proceeds along a different line of reasoning. While typically rejecting the idea that voting is motivated by considerations such as altruism or morality, it argues that the decision to vote is *not* based in expected utility maximizing, but rather in the application of some other decision criterion (Ferejohn & Fiorina 1974, 1975; see also the comments in *American Political Science Review* 1975, 908–919, 926–928). Particular attention has been given to the minimax regret principle, according to which an individual should choose the alternative that minimizes the difference between (1) the best result achievable had he known the actions of other agents beforehand, and (2) the actual state of the world. The maximum regret from abstention occurs if your most preferred candidate loses the election by a margin of exactly one vote, while the maximum regret from voting occurs whenever either candidate obtains a majority of more than one vote. In the former case, the 'regret' equals $B - C_v$, while in the latter case it is C_v. If $B > 2C_v$, then, the minimax regret criterion urges the citizen to vote.[5] Hence, the introduction of this decision rule apparently resolves the paradox raised by Downs' theory.

The trouble with all these revisions of Downs' theory is that they involve assumptions that are made more or less ad hoc. Consider first the possibility that people vote because they derive some satisfaction from the act of voting per se. As Brian Barry (1970, 16) notes, this assumption begs the question of why some people have this motivation, while others do not. To provide a satisfactory explanation, one would have to develop a general theory of when people act instrumentally, and when the act itself is the motivating force.

Similarly, the proposition that voting can be explained on the basis of altruism is unsatisfactory unless it can be derived from a general theory of when people act on altruistic grounds, and when they behave out of selfish motives. (It remains somewhat unclear whether or not Margolis' work qualifies as such a theory. See McLean (1986) for an evaluation.) However, there are also more fundamental reasons for believing that altruism cannot resolve the paradox raised by Downs' theory. In fact, it is shown in the fourth section that in the game-theoretic version of the problem, self-interested and altruistic citizens vote (and abstain) under essentially identical conditions.

Finally, the assumption that citizens' choices are made on the basis of the minimax regret principle must be discussed as ad hoc, if it cannot be shown to be consistent with the axioms of rational choice theory. Within this theory, there is an a priori presumption that decisions are made on the basis of one of the following two decision rules:

Expected utility maximizing. This criterion is used whenever the problem is one of decision-making under risk, or – as a special case – under certainty.
Identification of equilibrium strategies. This principle applies if the problem is of a strategic nature.

The reasoning behind this presumption is that these two decision rules are those most immediately following from the axioms of the theory. Moreover, in situations where neither of these criteria are applicable, there are several available principles to which a decision-maker might turn. A satisfactory explanation of why the decision to vote should be based in the minimax regret principle must therefore demonstrate that the following two propositions hold true:

(a) Traditional formulas for rational choice break down when applied to the decision to vote.
(b) Among the many possible decision rules that individuals may resort to when standard procedures break down, the minimax regret criterion is the one most likely to be applied by rational agents in voting situations.

In the following two sections, I provide some evidence that the first proposition is in fact true – given that the citizens consider voting a strictly instrumental act. However, it remains to demonstrate the validity of the second proposition. Some of the difficulties in providing such a demonstration will be briefly commented upon in the final section.

Voting and Counterfinality

As already mentioned, counterfinality stems from the simultaneous use by many individuals of behavioral assumptions that cannot be generalized, i.e.

assumptions that are self-defeating when everyone involved relies upon them to be true. In other words, counterfinality has to do with incompatible belief systems (Elster 1978, 106). Consider the cobweb example briefly mentioned in the introduction to this paper. This refers to a situation where a number of peasants individually must decide the level of production for the following season. The demand curve for the relevant product is assumed to have the usual decreasing shape. Each producer will of course settle for a large level of supply if the price is expected to be high, and for a more limited level if the price is supposed to be low. In the original model, all producers are assumed to act under the presumption that the price that can be obtained in the following season will be identical to the one observed in the market in the present season. This means that a high price at the time when decisions are made implies a large level of production for the following season. However, if everyone acts in this manner, the price will be low in the following season, if it is high when decisions are made. Hence, we have a case of counterfinality, since the assumption (with regard to price) on which the individual producer relies, becomes untrue when everyone believes it to be correct. But since the peasants' expectations will not be fulfilled the original model may be said to rest on behavioral assumptions that are rather unsatisfactory – at least if they are supposed to be valid over an extended period of time. It is then reasonable to believe that each decision-maker will learn from his mistakes, and also draw the inference that everybody else will do so as well. Possibly, they will then come to base their decisions on strategic reasoning, i.e., arrive at the conclusion that the price in the following season will be determined by the aggregate output of *all* the producers in the market. Consequently, an optimal decision must be derived from a calculus of what others can be expected to do. If this argument is accepted, then game theory obviously is the proper framework for analyzing the producers' decisions.

An analogous line of reasoning can be established for voting decisions. If every member of the electorate acts under the assumption that his vote is highly unlikely to be pivotal for the outcome, the assumption is faulty, and the citizens' expectations will not be fulfilled. A necessary condition for the assumption to be correct, is that the turnout will be quite large. However, if everyone believes that this will be the case, Downsian citizens will come to the conclusion that it doesn't pay to go to the polls, and by implication, nobody will. Of course, with zero turnout, any single vote would be decisive for the outcome of the election, contrary to everyone's expectations. Again, the lesson to draw is that a rational decision must be based on an analysis of what other agents can be expected to do, meaning that strategic reasoning is once again relevant.

The rather brief comments made in this section should suffice to indicate that simple Downsian expected utility-calculations are seriously flawed as a basis for a rational decision of whether or not to participate in an election. In the next section, I shall consider the question of whether strategic reasoning can 'solve' the contradiction of counterfinality produced in a Downsian world.

The Voting Game

In this section, I shall first consider a game-theoretic version of elections that involve only two candidates or parties. Next, the consequences of allowing for m candidates will be investigated. In both cases the election is assumed to be decided in a single round. However, toward the end of the section, some brief inferences about multi-round elections will be made.

Two Candidates

Restated in game-theoretic terms, the Downsian model of a two-candidate election can be spelled out as follows.[6] Two candidates, *a* and *b*, are running for office. The electorate consists of two groups, A and B, where all n_A members of A prefer *a* to *b*, while the opposite holds true for the n_B members of B. All citizens, then, have two available strategies: Either to vote for one's most preferred candidate, or to abstain. All players are supposed to consider voting a strictly instrumental act. It is irrelevant, however, whether a given citizen's preference for a particular candidate stems from self-interest or from some kind of altruism. As noted in the second section, altruistic sentiments typically tend to influence the size of the party differential, i.e., the intensity with which a given candidate is preferred to the other. The following argument, however, rests entirely on ordinal preferences only, meaning that the behavioral implications of the model will not be influenced in any way by change in preference intensities.[7]

Before proceeding, three points should be made. First, the game is best analyzed as a *one-shot-game*. This is because elections are normally held at intervals of several years. Hence, the number of players may vary from election to election. Furthermore, even if the number of players is constant, the players' identities may not be the same, because of deaths, migration, first-time voters, etc. Finally, many citizens' preferences might change from one election to another, so that n_A and n_B cannot be assumed constant over time.

The second point to be made is that an election is basically a *non-cooperative game*, since enforceable contracts cannot be made among the citizens as to who shall vote and who shall not, let alone agreements on how votes are to be cast. And even if we disregard the problem of enforcement, it must be admitted that any such agreements would be highly unlikely in a nation-wide election, because the vast number of people, together with the geographical distances involved, would tend to raise transaction costs to a level where it might well outweigh any gains such agreements could bring about. Note also that such collaboration involves great free-rider incentives, since any benefits it might eventually yield accrue equally to all supporters of the relevant candidate, regardless of whether or not they bear any of the costs of collaboration.

Finally, since the distribution of votes is typically not published until after the polling stations are closed, no individual can expect to know the decisions of other players at the time he has to make his own choice. Consequently, I shall assume that the game is played with *simultaneous*, rather than with sequential moves.

Our problem is now whether or not the game under consideration has a solution in the strict sense. To answer this question, it is necessary to specify what happens in the case of tie. I shall first consider the case where one of the candidates (candidate *a*) wins the election with certainty if a tie occurs. This means that the rules of the election are such that *a* comes into office if and only if the number of individuals in A that decide to vote is equal to or greater than the corresponding number in B. Otherwise, *b* is the winner of the election.

To grasp the logic of the situation, it may be of some use to start with a simple three-person model. Suppose the electorate consists of only three persons, of which player 1 supports candidate *a*, while the other two prefer *b*. The game can be depicted as shown in Figure 1. It is not difficult to see that the game in Figure 1 doesn't contain any equilibrium, and, consequently, that it lacks a solution in the strict sense. In particular, it is clear that the outcome where nobody goes to the polls cannot be an equilibrium. This outcome implies victory for *a*, while either of the players 2 and 3 could have secured the office for *b* – given that player 1 abstains. Thus, if the outcome of zero turnout should emerge, both supporters of *b* would have a reason to regret their decisions. However, neither of the outcomes $(\bar{V}, V, \bar{V})$ or $(\bar{V}, \bar{V}, V)$ can be an equilibrium, since if one of these occurs, player 1 would have a reason to regret that he didn't vote. Furthermore, if both player 1 and either of the players 2 or 3 vote (i.e. there is only one abstainer), we have another tie, meaning that *a* is the winner, and that both members of B would regret their choices. Similarly, with maximum turnout, player 1's vote would be totally in vain, meaning that if both members of B decide to vote, it is best for player 1 to stay at home. If he did, $(\bar{V}, V, V)$ would be the outcome. But then either member of B would be better off by abstaining (*b* is the winner in any case), so this outcome cannot be an equilibrium either. And if only player 2 (or 3) decides to vote, player 1 would regret that he didn't, while if he did vote, both members of B would be frustrated, etc. In short, no matter what outcome emerges, at least one player will have a reason to regret his choice of strategy.

Through this kind of reasoning, then, it is easy to see that the game in Figure 1 cannot have any equilibrium. However, once the number of players increases, this type of proof quickly becomes extremely complex, and virtually impossible to work with. Fortunately, there is a much simpler way of seeing that the game cannot have any equilibria, which applies

Player 3		V		$\bar{V}$	
Player 2		V	$\bar{V}$	V	$\bar{V}$
Player 1	V	3 3 1	1 2 3	2 1 3	2 2 3
	$\bar{V}$	3 3 2	3 4 2	4 3 2	2 2 4

Key: $\bar{V}$ = Abstain
V = Vote
4 = Preferred candidate wins election; cost of voting saved (best).
3 = Preferred candidate wins election; cost of voting spent (second best).
2 = Preferred candidate loses election; cost of voting saved (second worst).
1 = Preferred candidate loses election; cost of voting spent (worst).

Fig. 1. The Voting Game. Three Players.

irrespective of the number of players involved. Under the rules stated above, all possible outcomes can be classified in two categories. As shown in Figure 2, any outcome will either mean victory for *a*, or it implies victory for *b*), a similar pattern of regret would occur. All members of A who went their votes have a reason to regret their decisions, since they have spent the cost of voting in vain. Furthermore, unless the victory is secured by a tie, every voting member of A has (individually) a reason for regret, since his vote is then not pivotal for *a*'s victory. And if the result *is* a tie, all abstaining members of B would regret that they didn't vote, since a single additional vote for *b* would then be pivotal for bringing this candidate into office.

If an outcome of the latter category emerges (i.e. one that implies victory for *b*), a similar pattern of regret would occur. All members of A who went to the ballots will feel sorry that they did. If the margin of victory exceeds one vote, the same goes for those who voted for *b*. And finally, provided the margin is exactly one vote, abstaining members of A have a reason to regret their decisions.

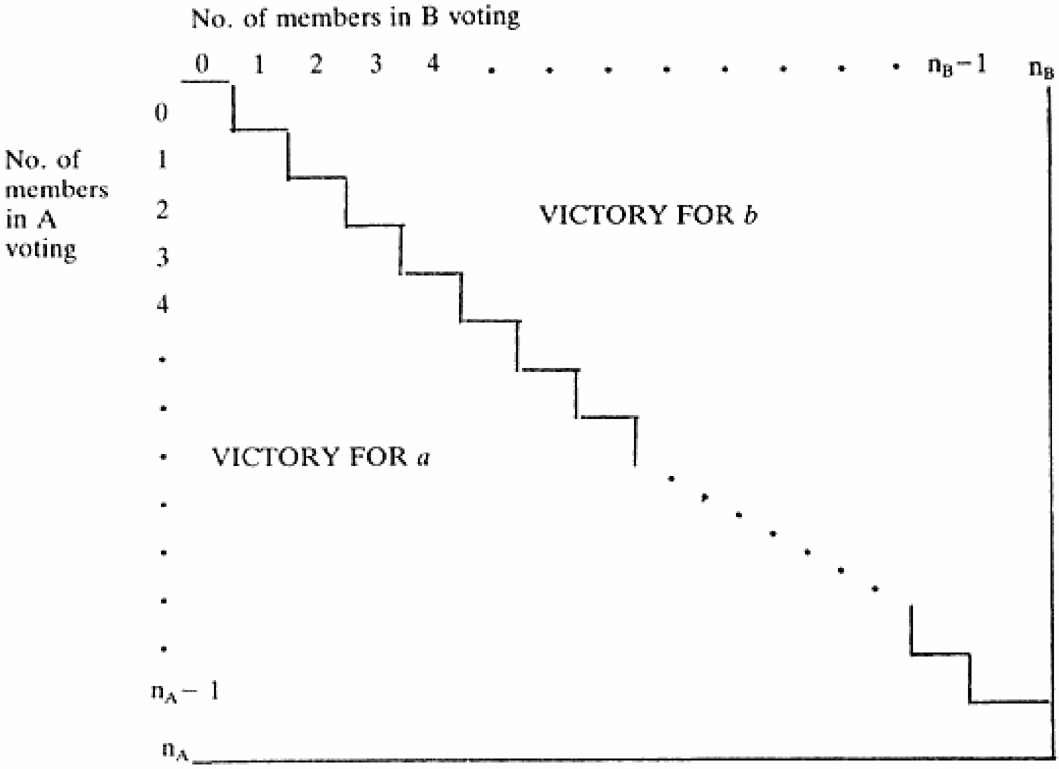

Fig. 2. The Voting Game. Outcome Matrix. N Players.

Hence, irrespective of the number of players involved, it holds true that for any possible outcome, at least one – and sometimes all – player(s) could have done better by choosing differently, given that other citizens' decisions are held constant. This is simply another way of saying that no outcome can possibly be an equilibrium. This result is quite interesting, because it contradicts the conventional wisdom that if the citizens are self-interested, and consider voting instrumentally, turnout will be close or equal to zero. The game-theoretic approach shows that it is at least not unconditionally irrational to vote – even if the electorate is very large, and irrespective of whether the individuals are motivated by self-interest or by altruism – and that zero turnout is not an equilibrium (in the sense of Nash). However, it must be added that since no other outcome is an equilibrium either, it is impossible to identify *any* strategy as the rational course of action in The Voting Game.

So far, the problem of a tie has been treated somewhat unsatisfactorily, since in many works on this subject (see, for instance, Ferejohn & Fiorina 1974), ties are supposed to be decided by the flipping of a fair coin, or by some other random device. I shall now consider the consequences of introducing this assumption into the game-theoretic framework.

In this case, there are three different categories of outcomes; victory for *a*; victory for *b*; and ties. Outcomes that imply certain victory for one of the parties still cannot be equilibria, for reasons similar to those mentioned previously. The only possibility we need to consider, then, is that one or several of the tied outcomes are equilibria. To explore this possibility further, one additional distinction will be made. When ties are decided by a random device, it is of some importance whether or not the two groups of supporters are identical in size, i.e., whether or not it is possible for one group (e.g. A) to raise a majority irrespective of the turnout in the other group. Hence, we must draw a distinction between cases where $n_A = n_B$ on the one hand, and cases where $n_A > n_B$ on the other.

Consider first the latter possibility. In this case, the game still lacks a solution in the strict sense, since no tie can possibly be an equilibrium (and, we have already noted, nor can any other outcome). If total turnout is zero, everyone would have preferred to vote, since a single vote would then be decisive. Furthermore, any tie with positive turnout would leave every abstainer frustrated, because his vote would have been pivotal for bringing 'his' candidate into office with certainty. This goes even for the case where all members of B (the smaller group) go to the polls, in which case the remaining abstainer(s) in A would have a reason for regret.

Then, turn to cases where the electoral groups have exactly equally many members. In this case, there exists a unique Nash equilibrium, namely that all players vote. No other tie can be an equilibrium, because if any such outcome emerges, all abstainers would regret that they didn't vote, thereby securing a certain victory for whatever candidate they prefer. A maximum turnout of $n_A + n_B$, however, *is* an equilibrium, because every single vote is then in a sense pivotal for bringing the tie about, while no further abstainers exist, who might regret their not voting. This conclusion may be said to lend some support to the conventional wisdom that the 'closer' the election is expected to be (i.e. the greater the probability the citizens attach to the possibility that $n_A = n_B$), the greater the likelihood that any given individual will decide to vote, and more generally, the greater the aggregate turnout. It may be noted that with strictly instrumentally oriented citizens, maximum turnout is strongly Pareto-dominated by the outcome with zero turnout. This is due to the fact that from an instrumental point of view, the two outcomes are equivalent (the result of the election is the same), while only the latter outcome allows the citizens to save their costs of voting.[8]

To sum up, then, we may say that the two-candidate Voting Game has no equilibrium if ties generate a winner with certainty. If ties are decided by a random device, there is still no equilibrium in the game, except for the case where the number of supporters of the two candidates is identical. In the latter case, the game has a unique equilibrium, and consequently a solution in the strict sense. It may be of some interest that in this particular instance the game-theoretic approach yields predictions that are very different from those obtained by Downs: The equilibrium is *not* zero turnout, but rather that every single member of the electorate goes to the polls. However, it is obvious that cases of this category are hard to find in real-world elections. It should be remembered that a difference of one single supporter is enough to guarantee that no equilibrium exists. It therefore seems reasonable to conclude that in most elections the introduction of strategic thinking – stemming from the contradiction of counterfinality produced by Downsian assumptions – will lead to another contradiction in Elster's sense, since the Voting Game – except under highly unlikely circumstances – has no solution.

M Candidates

Up to this point, I have only considered cases where the number of candidates running for office equals two. However, since in many real-world elections there are more than two parties, it may be of some interest to investigate the consequences of relaxing this assumption.

The model can now be summarized as follows. A finite number m candidates are running for office. The electorate, of size n, is divided into z camps, each of which has a different preference ordering over the various candidates. Each member of the electorate can choose to vote for any candidate he wishes, or to abstain. Although it is still true that no citizen will vote for the candidate he prefers least, we can now no longer assume that each player faces a binary choice. Fortunately, however, this does not complicate things too much.

As in the two-candidate case, it is now of some importance what happens in the case of a tie. Again, I shall first consider the case where any possible tie generates a winner with certainty. Of course, with more than two candidates involved, there are many possible ties that may emerge. For example, with three candidates, *a*, *b*, and *c*, there are four possible categories of ties that are of any interest:

$$V_a = V_b > V_c$$
$$V_a = V_c > V_b$$
$$V_b = V_c > V_a$$
$$V_a = V_b = V_c$$

Here V_i denotes the number of votes cast in favor of candidate i ($i = a, b, c$).

We assume, then, that a set of rules has been specified, which guarantee that any outcome generates a winner with certainty. In particular, it is common knowledge which candidate is the winner for every tie that can possibly occur. It is then easy to verify that no equilibrium can exist in the game. Suppose candidate *a* wins the election. Then all votes cast in favor of any other candidate are totally in vain, meaning that all citizens voting for other candidates would have been better off staying at home. Furthermore, given that the victory is *not* secured by a tie, everyone voting for *a* would individually have been better off if they had not gone to the polls. And provided the victory *is* secured by a tie, all abstainers who prefer another candidate involved in the tie to *a* would regret not having cast their votes. Similarly, those having cast their votes in favor of one of the candidates receiving fewer votes than did those involved in the tie, would regret their choices of strategy. Now, since the same reasoning can be applied no matter which candidate comes out the winner, it follows that no outcome whatsoever can be an equilibrium when ties generate a winner with certainty. So far, then, our conclusions are equivalent to those obtained for the case with two candidates only.

What happens in the m-candidate case when ties are decided by a random device? It turns out that this depends in a rather complex manner on the exact number of candidates, as well as on the precise constellation of preferences in the electorate. I shall therefore confine the discussion to a few points regarding the three-candidate case. It is assumed that ties are decided by a random device yielding an equal probability of coming into office for all candidates being involved in the tie. This means, of course, that the higher the number of candidates involved in a given tie, the less likely it is that any particular candidate will come out the winner.

Before we proceed, it is necessary to introduce some additional notation. The three candidates are referred to as *a*, *b*, and *c*, respectively. To simplify matters somewhat, I shall make the (unimportant) assumption that the candidates' programs differ only along one single political dimension, so that only four rankings of the candidates are possible within the electorate. (To be more specific, the citizens' preferences are assumed to be *single-peaked.*) If *a* and *c* can be placed at the two extremes of the relevant dimension (e.g. *a* is a spokesman for a leftist position and *c* advocates a rightist position), while *b*'s program is somewhere in between, these rankings are:

a	*b*	*c*	(held by group A, of size n_A)
c	*b*	*a*	(held by group C, of size n_C)
b	*c*	*a*	(held by group BC, of size n_{BC})
b	*a*	*c*	(held by group BA, of size n_{BA})

Note that the first group of citizens (i.e., those ranking the candidates in the order *a b c*) is labeled A, while the second, third, and fourth groups are referred to as C, BC, and BA, respectively. The various groups are of size n_A, n_C, n_{BC}, and n_{BA}, where $n_A + n_C + n_{BC} + n_{BA} = n$. Note also that the subscript BC indicates that members of the relevant group have *b* as a first preference, and rank *c* second. The subscript BA can be interpreted accordingly. As for the first two groups, a two-letter subscript is unnecessary, since the assumption of single-peaked preferences guarantees that a first preference for *a* or *c* implies that *b* is ranked second.

Recall that in the two-candidate game, a distinction was drawn between cases where the two groups of supporters are identical in size on the one hand, and cases where the groups differ in size on the other. A corresponding distinction must now be made for the three-candidate game. To do this, it is convenient to introduce the concept of a 'coalition'. A coalition is said to arise whenever a number of individuals vote for the same candidate. However, not all conceivable coalitions are of any interest for our purposes. It will be shown below that in equilibrium, all members of a given group (e.g. A) must vote for the same candidate. Hence, the only coalitions that are of any interest here are those that either include all members of a given group, or none at all. I shall call such coalitions 'essential coalitions'. A distinction should then be drawn between cases where two or more essential coalitions may arise that are identical in size on the one hand, and cases where this is not possible on the other. It may be noted that in the two-candidate game, essential coalitions of equal size may only arise if $n_A = n_B$.

I shall now first assume that the configuration of preferences is such that essential coalitions of identical size cannot possibly arise. Figure 3 offers a complete specification of the logically possible constellations of preferences that are *excluded* by this assumption. In cases of this category, there is still no equilibrium in the game: Since essential coalitions of equal size are not possible, a tie can only result if (a) somebody abstains, or (b) someone casts his vote in vain. In the former case, the abstainer(s) would have a reason for regret, since a single vote would then have been pivotal for bringing the relevant citizen's first or second preference into office with certainty (note that at least one of these candidates must be involved in any possible tie). In the latter case, which means that at least one citizen votes for a candidate who receives fewer votes than do those involved in the tie, those having cast their votes in this manner will regret that they didn't vote otherwise.

The only remaining possibility we have to consider, then, is that two or more essential coalitions may arise, that are identical in size. To investigate this possibility further, recall that in the two-candidate case, only ties involving universal turnout can be equilibria, since for any other tie there must be abstainers whose vote would have been pivotal for bringing the preferred candidate into office with certainty. With three candidates included in the model, we have to add that only ties where all citizens' votes are pivotal for bringing the tie about can be equilibria. This is because whenever somebody casts his vote in favor of a candidate not involved in the tie, the vote will be in vain. Provided that we exclude the possibility of indifference, then, all possible equilibria must have one of the following sets of properties:

$V_a = V_b = \frac{1}{2}n;\ V_c = 0$
or $V_a = V_c = \frac{1}{2}n;\ V_b = 0$
or $V_b = V_c = \frac{1}{2}n;\ V_a = 0$
or $V_a = V_b = V_c = \frac{1}{3}n$

V_i = The number of votes cast in favor of candicate i ($i = a, b, c$).

At this point, we know that any possible equilibrium must satisfy the following conditions:

(i) The relevant outcome is a tie.
(ii) The relevant outcome involves universal turnout.
(iii) No citizen votes for his least preferred candidate.
(iv) If the relevant outcome involves a two-candidate tie, the third candidate receives no votes whatsoever.

In addition, we may now state one further condition:

(v) If a given citizen's first preference is included in the relevant tie, the citizen votes for this candidate.

That condition (v) must hold for any possible equilibrium, can be seen in a very simple way. Consider any tie that involves a given citizen's first preference. If this outcome is an equilibrium, then (by condition (iii)) the relevant citizen's strategy choice cannot be to vote for his third preference. But by condition (ii), he must vote. Hence, his vote has to be cast in favor of either his first or his second preference. Since by condition (i), the outcome must be a tie, voting second preference cannot be optimal, if one's first preference is involved in the resulting tie. The obvious reason is that by voting otherwise (or staying at home) the citizen could have secured a certain victory for his most preferred candidate. Hence, no outcome can possibly be an equilibrium unless condition (v) holds.

From conditions (i) through (v) it follows that in equilibrium, all members of a given group (e.g. A) must vote for the same candidate. For example, consider group A. By condition (i), any equilibrium has to be a tie. Candidate *a* (the first preference of the members of A) may or may not be included in the tie. If he is, then (by condition (v)) all members of A must vote for *a*, if the relevant outcome is to be an equilibrium. And if he isn't, all members of A must (by conditions (ii), (iii), and (iv)) vote for *b*.

This means that to identify possible equilibria, we need only consider the logical possibilities of preference configurations in which two or three (combinations of) groups are of equal size. In other words, this is the only way that essential coalitions with equally many members can possibly arise.

Altogether, there are 13 such possibilities in our three-candidate game. However, most of these can be dismissed immediately, since they can only generate ties involving universal turnout (i.e. satisfy conditions (i) and (ii)) if one or several of the conditions (iii) through (v) are violated. Hence, these cases cannot possibly produce a game involving an equilibrium. Figure 3 lists the various logical possibilities, and states the condition(s) that must be violated if the specified case is to yield a tie involving universal turnout. If no such violations are needed, it is indicated by 'none'.

Configuration of Preferences	Condition(s) violated
3-candidate ties:	
I. $n_A = n_{BA} + n_{BC} = n_C$	None
$n_A + n_{BA} = n_{BC} = n_C$	(v)
$n_A + n_{BC} = n_{BA} = n_C$	(iii)
$n_A + n_C = n_{BA} = n_{BC}$	(iii)
$n_A = n_{BA} = n_C + n_{BC}$	(iii)
$n_A = n_{BC} = n_C + n_{BA}$	(iii)
2-candidate ties:	
II. $n_A = n_C + n_{BA} + n_{BC}$	None
$n_A + n_C = n_{BC} + n_{BA}$	(iii)
$n_A + n_C + n_{BC} = n_{BA}$	(iii)/(iv)
$n_A + n_C + n_{BA} = n_{BC}$	(iii)/(iv)
III. $n_A + n_{BA} + n_{BC} = n_C$	None
IV. $n_A + n_{BA} = n_{BC} + n_C$	None
$n_A + n_{BC} = n_{BA} + n_C$	(iii)/(iv)

Fig. 3. Possible Configurations of Preferences that May Yield Ties with Universal Turnout in the Three-Candidate Case.

It can be seen from Figure 3 that only four possible configurations of preferences need be considered further. In addition, there is the possibility that two or more of these constellations are present simultaneously. It turns out that the game has a unique equilibrium (given that some additional conditions are satisfied, cf. the appendix) whenever one (and only one) of the (sets of) equations I through IV holds true. And if two or more equations hold simultaneously, the game has multiple equilibria, and not always a solution in the strict sense. Readers interested in the specific nature of the various equilibria are referred to the appendix of this paper. Here, it suffices to say that the constellations of preferences that can yield games involving equilibria must be considered very special – although logically possible. It is probably safe to predict that such cases are highly unlikely to be found in real-world elections. Hence, the general conclusion seems to be that even when the game involves more than two candidates, it is unlikely to have a solution.

Multi-Round Elections

So far, I have only discussed elections which are decided in a single round. However, many real-world elections involve two or more rounds. Hence, it is of some interest to consider the possible relevance of my results for such cases also.

To analyze a multi-round election, it is convenient to use so-called 'backward analysis'. This is because of the fact that to determine what is the rational course of action in an early round, we must consider the possible consequences that may follow in the final round. Of course, the final round is no different from a single-round election. Thus, we already know that this round will in general not have a solution in the strict sense – no matter what candidates eventually are left at that point. This simple fact implies that it is impossible to evaluate the consequences of the various courses of action being available in earlier rounds. Consequently, we can draw the conclusion that if the final round of a multi-round election doesn't have a solution, nor will the full game consisting of several rounds. In other words, the results reached for single-round games seems to be equally relevant for elections involving more than one round.

Non-Technical Summary

Before concluding this section, one further point should be made. The frequent use of the term 'regret' throughout this paper should *not* lead the reader to believe that the argument in any sense rests on *ex-post*-reasoning. It is simply part of a convenient way of stating the proof that the relevant game in general has no solution. To convince oneself of this, it may be useful to reconsider the argument in the following (rather informal) manner. Imagine a rational, instrumentally motivated citizen who is trying to work out whether or not he should go to the polls. He quickly realizes that to make a well-founded decision, he has to provide a satisfactory prediction of other people's choices, since casting his vote will pay only if it is pivotal. His first thought may be (*à la* Downs) that a single vote is unlikely to be decisive. As far as he can remember, no election in the past has been anywhere near a tie. Furthermore, this is what any papers he might have read on the subject tell him to believe. Consequently, he decides to stay at home. However, returning to his chair, planning to watch the election on television, he begins to wonder whether his decision was so wonderful after all. If all his fellow citizens think the way he did, nobody will vote, meaning that his single vote will be decisive if he changes his mind. But then again, everybody else can of course be expected to draw the same conclusion, so that he should not therefore necessarily go to the polls. At this point, our citizen may sensibly ask himself the following question: Is it possible to identify any combination of decisions implying that *all* my fellow citizens *and* myself do the best we can, given everybody else's decisions? Given that he ignores a few, highly unlikely possibilities (e.g. in a two-candidate election, that the candidates have exactly equally many supporters), he will – as we have already seen – come to the conclusion that there isn't. The (*ex ante*) lesson to draw, is that there is no satisfactory way of predicting whether or not a single vote will be decisive, given that the electorate consists of entirely rational and instrumentally oriented citizens.

However, this does not mean that our citizen cannot find any way of arriving at a decision. In fact, decision theory provides him with a number of possible criteria, on which he might conceivably rely. As we shall see in the following section, the difficulty for political science is to predict which one a given citizen will eventually turn to in voting situations.

Some Notes on Alternative Decision Rules in the Voting Game

A general lesson to draw from preceding sections is that Bayesian decision theory, as well as the theory of games, is of little help for the instrumentally oriented citizen who is trying to make a well-founded decision of whether or not to go to the polls. This conclusion, however, is subject to one important qualification. In the previous section, I have only considered the possibility of equilibria in *pure strategies*. It deserves to be mentioned that general theorems within the theory of games guarantee the existence of at least one equilibrium in *mixed strategies*. Any equilibrium of this kind will imply that all citizens make their decisions by the use of some random device, implying a positive probability p_i $(0 < p_i < 1)$ of citizen no. i going to the polls. Of course, with large numbers of players, this would generate a turnout somewhere between 0 and 100 per cent.[10] Although this is consistent with available data, mixed strategy equilibria are unlikely to be of much relevance for a positive theory of voting decisions. This is due to the fact that the necessary calculations are highly complex, and must be based on information that cannot be assumed common knowledge. To see why, note that an optimal mixed strategy in general depends on the cardinal utilities associated with the various outcomes for *all* players. This means that if a given citizen is to be able to make the necessary calculations, he would have to know the party differentials and voting costs of all his fellow citizens. The fact that these variables are private information, combined with the huge number of players involved, indicates that for all practical purposes, mixed strategy equilibria may safely be dismissed as irrelevant for the study of voting decisions.

We have already argued that in situations where standard rational decision criteria break down, there may be a reason to believe that decision makers will turn to other principles for making their choices. As noted in the second section of this paper, one such criterion – the minimax regret principle – has received particular attention in the case of voting. If all agents adhere to this principle, everyone will go to the polls – exception taken for citizens whose voting costs are exceptionally high, or who are physically hindered from casting their votes.

The trouble with the assumption that rational citizens base their voting decisions on the minimax regret principle, is that we have no sound theoretical basis for the belief that this criterion has any prominence whatsoever over other possible decision rules. Consider for example the maximin principle. According to this principle, a decision maker should choose the alternative whose worst possible payoff is best. The worst result that can be obtained for a citizen if he goes to the polls is $-C_v$, which occurs whenever his preferred candidate loses the election, in spite of him casting his vote. Abstention, on the other hand, yields a minimum payoff of 0, also occurring when the preferred candidate loses the election. Consequently, the maximin principle recommends abstention as the best course of action.

If we cannot give good reasons why either of these criteria must be considered irrelevant, then, we are at best left with a theory that is *indeterminate* – unless we are able to predict which individuals are likely to be maximum regret minimizers, and which are minimum payoff maximizers. Ferejohn & Fiorina (1974) argue that these kinds of predictions are in fact possible, on the ground that the maxim in principle may be considered a more pessimistic criterion than minimax regret:

> Maximin decision makers never vote. Aren't the poor and culturally deprived more likely to be maximin decision makers than the rich and educated? (Ferejohn & Fiorina 1974, 535)

Suppose it is true that wealthy and prosperous citizens rely on more optimistic decision rules than do the poor and culturally deprived. Since the minimax regret principle must be considered rather pessimistic as well, this would indicate that even more optimistic decision rules may be of some relevance – at least for people belonging to the upper classes. One such possibility is the maximax criterion, according to which a decision maker should choose the alternative whose best possible payoff is best. Now, the best payoff that can be obtained by voting is $B - C_v$, while the best possible payoff from abstention is B. Hence, a maximax decision maker always abstains, implying that Ferejohn & Fiorina's presumption that the tendency to vote is monotonically related to some pessimism-optimism-dimension, may be questioned.[11]

At this point, the discussion could have been extended to include further decision principles, in addition to those considered above. However, I have chosen not to do so, since it would add little to our main conclusion. The point is simply that in the case of voting, a number of decision criteria may be relevant. Unless we can somehow single out one of them, or – alternatively – provide satisfactory predictions of the distribution of decision rules within the electorate, the theory must remain open-ended, in the sense that exact predictions of voter turnout cannot be reached. Of course, I would not deem such extentions of the theory impossible, but I do think it fair to say that any such attempts would have to go beyond the limits of political science and economics.

Conclusion

This paper has argued that in the case of voting, Downsian expected utility maximizing leads the electorate into an instance of counterfinality. Given that the decision makers realize this, they may change their behavioral assumptions, and possibly come to base their decisions on strategic reasoning. However, since the Voting Game is – except under highly unlikely circumstances – a game without a solution, this change of assumptions leads from one contradiction (in Elster's sense) to another. Hence, neither Bayesian decision theory nor the theory of games can provide satisfactory advice in voting situations, and by implication, other decision rules may be of some relevance. However, several possible criteria exist, meaning that unless we can somehow single out one of them, or provide predictions of how the various principles are distributed within the electorate, rationalistic theory must remain open-ended, as far as predictions of voter turnout are concerned. It remains an open question whether or not this indeterminateness (which is not unique within the realm of rationalistic theory) should be considered a weakness of the theory.

Finally, it should be stressed that the conclusions stated above apply only as far as strictly instrumentally oriented agents are concerned. It is, however, irrelevant whether they are motivated by self-interest or by some kind of altruistic sentiments.

Appendix

The purpose of this appendix is to explore the specific nature of possible equilibria in the 3-candidate game. As noted in the text, only four possible

in favor. Furthermore, nothing extra can be gained if a majority greater than the minimal one attends. Finally, to each member of B there is a certain cost of attending.

In many respects, Taylor & Ward's model is identical to the one considered in the present paper. However, there is one important difference, since in my model, all members of *both* groups are considered players (I also investigate the possibility of more than two groups). This means that I have omitted the rather unsatisfactory assumption that all members of A are known to attend and cast their votes. The change of assumptions at this point has some very important consequences, since in Taylor & Ward's model, there are always multiple equilibria, while the model considered in the present paper in general contains none at all.

During my work on this paper, I became aware of an article by Palfrey & Rosenthal, in which the authors present a model of two-candidate elections that is essentially equivalent to mine. There are, however, also important differences between their work and the present paper. Most notably, Palfrey & Rosenthal are exclusively occupied with single-round two-candidate elections, while I also consider elections involving more than two candidates, and (to some extent) more than one round. Moreover, while I have chosen not to deal with mixed strategy equilibria, this is the most prominent focus of Palfrey & Rosenthal.

7. This holds true without qualifications for the cases where ties generate a winner with certainty. However, some of the conclusions regarding the cases where ties are decided by a random device are only meaningful provided that utilities can be measured on a cardinal scale. Even when this is the case, however, it is not entirely clear that altruistic citizens are more inclined to vote, than are those motivated by self-interest.
8. This conclusion may seem strongly counterintuitive to the reader, and it is easy to see that is highly sensitive to minor changes in the assumptions made regarding the players' motivations. Consider for example the possibility that the citizens have lexicographic preferences, as follows: Each player's first concern is the outcome of the election. But *given* that a particular result has come about, he prefers a greater turnout to a smaller one, 'for the sake of democracy'. Provided that the extra utility from one single additional vote doesn't exceed the cost of voting, this change of assumptions would not alter the behavioral implications of the model. However, it may easily reverse the conclusion that zero turnout is a Pareto-superior outcome.
9. The discussion in this section is predominantly relevant for two-candidate elections being decided in a single round.
10. For an in-depth analysis of mixed strategy equilibria in the two candidate game, see Palfrey & Rosenthal (1983).
11. At least, the relationship may be expected to be more complex than Ferejohn & Fiorina seem to believe, since extreme pessimists and extreme optimists alike always abstain. If it holds true that there is a positive, monotonic relationship between wealth and education on the one hand, and optimism on the other, this would indicate that the typical abstainer is either a complete underdog *or* a perfect topdog. Voters, on the other hand, might be expected to be found somewhere in between these two extremes.

REFERENCES

Barry, B. 1970. *Sociologists, Economists, and Democracy*. London: Collier-Macmillan Ltd.

Beck, N. 1975. 'The Paradox of Minimax Regret.' *American Political Science Review* 69, 918.

Downs, A. 1957. *An Economic Theory of Democracy*. New York: Harper & Row Publishers.

Elster, J. 1978. *Logic and Society. Contradictions and Possible Worlds*. Chichester: John Wiley & Sons.

Ferejohn, J. A. & Fiorina, M. P. 1974. 'The Paradox of Not Voting: A Decision Theoretic Analysis.' *American Political Science Review* 68, 525–536.

Ferejohn, J. A. & Fiorina, M. P. 1975. 'Closeness Counts Only in Horseshoes and Dancing.' *American Political Science Review* 69, 920–925.

Goodin, R. E. & Roberts, K. W. S. 1975. 'The Ethical Voter.' *American Political Science Review* 69, 916–917.
Hovi, J. 1986. 'Binary Games as Models of Public Goods Provision.' *Scandinavian Political Studies* 9, 337–360.
Margolis, H. 1982. *Selfishness, Altruism, and Rationality.* Cambridge: Cambridge University Press.
McLean, I. 1986. 'Some Recent Work in Public Choice.' *British Journal of Political Science* 16, 377–394.
Palfrey, T. R. & Rosenthal, H. 1983. 'A Strategic Calculus of Voting.' *Public Choice* 41, 7–55.
Riker, W. H. & Ordeshook, P. C. 1968. 'A Theory of the Calculus of Voting.' *American Political Science Review* 62, 25–42.
Shubik, M. 1971. 'The Dollar Auction Game: A Paradox in Non-cooperative Behavior and Escalation.' *Journal of Conflict Resolution* 15, 109–111.
Stephens, S. V. 1975. 'The Paradox of Not Voting: Comment.' *American Political Science Review* 69, 914–917.
Strom, G. S. 1975. 'On the Apparent Paradox of Participation: A New Proposal.' *American Political Science Review* 69, 908–913.
Taylor, M. & Ward, H. 1982. 'Chickens, Whales, and Lumpy Goods: Alternative Models of Public Goods Provision.' *Political Studies* 30, 351–370.
Tullock, G. 1967. *Toward a Mathematics of Politics.* Ann Arbor: The University of Michigan Press.
Tullock, G. 1975. 'The Paradox of Not Voting for Oneself.' *American Political Science Review* 69, 919.

payoff from abstention is B. Hence, a maximax decision maker always abstains, implying that Ferejohn & Fiorina's presumption that the tendency to vote is monotonically related to some pessimism-optimism-dimension, may be questioned.[11]

At this point, the discussion could have been extended to include further decision principles, in addition to those considered above. However, I have chosen not to do so, since it would add little to our main conclusion. The point is simply that in the case of voting, a number of decision criteria may be relevant. Unless we can somehow single out one of them, or – alternatively – provide satisfactory predictions of the distribution of decision rules within the electorate, the theory must remain open-ended, in the sense that exact predictions of voter turnout cannot be reached. Of course, I would not deem such extentions of the theory impossible, but I do think it fair to say that any such attempts would have to go beyond the limits of political science and economics.

Conclusion

This paper has argued that in the case of voting, Downsian expected utility maximizing leads the electorate into an instance of counterfinality. Given that the decision makers realize this, they may change their behavioral assumptions, and possibly come to base their decisions on strategic reasoning. However, since the Voting Game is – except under highly unlikely circumstances – a game without a solution, this change of assumptions leads from one contradiction (in Elster's sense) to another. Hence, neither Bayesian decision theory nor the theory of games can provide satisfactory advice in voting situations, and by implication, other decision rules may be of some relevance. However, several possible criteria exist, meaning that unless we can somehow single out one of them, or provide predictions of how the various principles are distributed within the electorate, rationalistic theory must remain open-ended, as far as predictions of voter turnout are concerned. It remains an open question whether or not this indeterminateness (which is not unique within the realm of rationalistic theory) should be considered a weakness of the theory.

Finally, it should be stressed that the conclusions stated above apply only as far as strictly instrumentally oriented agents are concerned. It is, however, irrelevant whether they are motivated by self-interest or by some kind of altruistic sentiments.

Appendix

The purpose of this appendix is to explore the specific nature of possible equilibria in the 3-candidate game. As noted in the text, only four possible configurations of preferences need be considered, namely those in which one or several of the sets (of) equations indexed I through IV in Figure 3 hold. In addition, there is the possibility that more than one equation (set) hold simultaneously. I shall first consider cases where only one equation (set) holds, and then investigate the possibility of two or more (sets of) equations holding simultaneously. As we shall see, the latter cases open for multiple equilibria in the game.

The *first* possibility is that $n_A = n_{BA} + n_{BC} = n_C$, i.e., that all three candidates are held as a first preference by equally many citizens. It is then possible that there is a unique equilibrium in the game, namely that all citizens vote sincerely (i.e. for their first preference). This outcome is an equilibrium provided that the following conditions hold true for all citizens:

(1) $\frac{1}{3}U_i(X_1) - \frac{2}{3}U_i(X_2) + \frac{1}{3}U_i(X_3) > 0$
(2) $\frac{1}{3}U_i(X_1) - \frac{1}{6}U_i(X_2) - \frac{1}{6}U_i(X_3) > C_i$

Here, $U_i(X_j)$ denotes the utility obtained for citizen no. i if the candidate he ranks j'th comes into office ($i = 1, 2, \ldots, n$; $j = 1, 2, 3$). (1) is the condition that voting first preference is better than voting second preference, when everybody else votes sincerely. And (2) similarly states the condition that voting sincerely is better than to abstain – again assuming that everybody else votes for whatever candidate they rank first, and that the configuration of preferences are as indicated. What (1) actually says is that under these assumptions, it pays to vote for your first preference, rather than for your second, provided that the additional chance of having your favorite in office is considered sufficiently valuable to outweigh the fact that voting second preference would have secured a certain victory for this candidate. It should be noted that switching from your second to your first preference increases the latter's chances of victory from zero to 1/3. However, the drawback is that it also increases the chances of the candidate you prefer least accordingly, and reduces the chances of your second preference from certain victory to a 1/3 probability of coming out the winner. The second condition can similarly be interpreted as follows: Voting sincerely is better than abstention (given the indicated configuration of preferences and that everybody else votes sincerely), if you value the resulting increase in your favorite candidate's probability of success more than it costs to go to the polls.

Provided that (1) and (2) hold for all citizens, then, the game has an equilibrium. Moreover, assuming that equation IV does not simultaneously hold (note that equations II and III are logically inconsistent with I), the equilibrium is unique. No other three-candidate tie can be an equilibrium, since any such tie must violate condition (ii) or (v) or both. And no two-candidate tie can be an equilibrium, since (unless equation IV also holds) all such ties must violate condition (ii), (iii), or (iv). Hence, in this case, the game can be said to have a solution in the strict sense, provided that (1) and (2) hold for all citizens.

The *second* possibility is that the following equation holds (equation II in figure 3): $n_A = n_{BA} + n_{BC} + n_C$. Again, it is assumed that the other equations in Figure 3 do not also hold. As in the previous case, there may now be a unique equilibrium in the game, namely that all citizens vote – votes being cast as follows: All members of A vote for *a*, while everybody else votes for *b*. The conditions for this outcome to be an equilibrium are:

(3) $\frac{1}{2}U_i(X_1) - \frac{1}{2}U_i(X_2) > C_i$ (for members of A and BA)
(4) $\frac{1}{2}U_i(X_2) - \frac{1}{2}U_i(X_3) > C_i$ (for members of C)
(5) $\frac{1}{2}U_i(X_1) - \frac{1}{2}U_i(X_3) > C_i$ (for members of BC)

The conditions specify the circumstances under which it is better to vote for whoever of the candidates *a* and *b* a citizen prefers, rather than to abstain – given that everybody else chooses the strategies associated with the specified outcome. Note that in this case, it will never pay to vote for *c* – given that everybody else votes in the manner indicated. Furthermore, any member of A will either abstain or vote for *a*, since casting his vote in favor of *b* will not influence the outcome (*b* wins regardless of whether a given member of A votes for *b* or abstains). Similarly, it never pays for a member of C, BA, or BC to vote for *a*. To see that the specified equilibrium must be unique, recall that in equilibrium, all members of a given group must vote for the same candidate, and that turnout must be universal. To obtain a tie, it is then necessary that two or more (combinations of) groups are of identical size. But when II holds, there is no way to rearrange the groups into coalitions if intercoalitional equality is to be preserved – given that there is at least one member of each group.

This, however, is not sufficient to establish that the specified equilibrium must be unique. We also have to consider the possibility that the votes may be cast otherwise, while the coalitions remain constant. Note that the members of C, BC, and BA must in equilibrium vote for *b*, if they are to be members of the same coalition. This is because whenever the votes are cast in favor of another candidate, some of the members of the coalition vote for their least preferred candidate (condition (iii) is violated). By conditions (ii) and (iii), all members of A must vote for either *a* or *b*. But by condition (i), they cannot vote for *b*, if the outcome is to be an equilibrium (this would mean that everybody cast their votes in favor of *b*). Hence, they have to vote for *a*, meaning that the specified equilibrium must be unique.

The *third* possibility we have to consider, is that the constellation of preferences is such that $n_C = n_A + n_{BA} + n_{BC}$. In this case, there may be a unique equilibrium where all members of C vote for *c*, while everybody else votes for *b*. The proof and relevant conditions are completely equivalent to those given for the case immediately above, and is therefore left out.

The *fourth* case is the one where $n_A + n_{BA} = n_C + n_{BC}$. Once again, we may have a unique equilibrium, namely the outcome where all members of A and BA vote for *a*, while everybody else votes for *c*. Even in this case I choose to leave out relevant proofs and conditions, since they are more or less equivalent to those given for previous cases.

Up to this point, it has been assumed that whenever one of the (sets of) equations I through IV holds, the others don't. I shall now add a few brief remarks on the possibility that two or more equations hold simultaneously. Note first that equations I and II can only hold simultaneously if $n = 0$. The same is true for the pair I and III. Hence, these cases are without any interest. Equations I and IV may both be true, provided that $n_{BA} = n_{BC}$. Given that the relevant conditions hold, we then have a game with two equilibria. First, there is three-candidate tie which is an equilibrium, where all three candidates receive $\frac{1}{3}$n votes, and all citizens vote sincerely. And there is another equilibrium corresponding to the one found in the fourth case above, where *a* and *c* receive $\frac{1}{2}$n votes each. All members of A and C will of course prefer the latter outcome to the former. However, this is not necessarily true for the members of BA and BC, so that in this case, the game may not have a solution in the strict sense.

Let us now turn to the possibility than $n_{BA} = n_{BC} = 0$, i.e., no citizens have *b* as a first preference. It is then easy to verify that equations II, III, and IV all reduce to $n_A = n_C$, so that whenever one of these equations hold, so do the others. Not surprisingly, the game will then – given that the relevant conditions hold – have three equilibria. First, and most obvious, the outcome where all members of A vote for *a*, and all members of C vote for *c*, is an equilibrium. Second, there is an equilibrium where all members of A vote for *b*, while all members of C vote sincerely. And finally, we also have an equilibrium in which all members of A vote for their first preference, and everybody else for *b*. The first of these equilibria should at this point need no further comments. As for the second and third, it may seem somewhat surprising that such outcomes can be equilibria, since they imply that a candidate favored by no-one gets a 50 per cent chance of coming into office, while at the same time a candidate who is favored most by half the electorate receives not votes at all! The explanation is, however, very simple. Consider a member of A. Suppose that for some reason, he expects all his fellow members of A to vote for *b*, and all members of C to vote sincerely. Since $n_A = n_C$, it is clear that unless our citizen votes for *b*, *c* will win the election with certainty. There is therefore no need to be surprised that voting for *b* may be considered optimal (a best response) by our citizen, given the (expected) behavior of others. It is, however, hard to deny that the equilibrium where all citizens vote sincerely must be considered the natural solution of the game, since it is better for all members of A if they all vote for *a* rather than for *b*, regardless of what the members of the other group might do. The situation is of course equivalent on behalf of the members of C.

A further possibility arises if only $n_{BC} = 0$. Then equations III and IV may hold simultaneously, meaning that the game may have two equilibria. It is left for the reader to verify that the relevant outcomes are those specified during the discussion of the third and fourth cases above.

Finally, it is possible that only $n_{BA} = 0$. Then equations II and IV may both be true, and again there may be two equilibria – corresponding to those identified under the discussion of the second and fourth cases above.

To sum up, we might say that when ties are decided by a random device, the m-candidate Voting Game has no equilibrium unless two or more essential coalitions can be identified, which have equally many members. When this is possible, the game may have zero, one, or several equilibria – depending on the more specific constellation of preferences within the electorate. It must be added that for the latter cases, which have been the focus of this appendix, only three-candidate elections have been considered.

NOTES

1. In fact, this holds true only provided that a first bid of $100 is not allowed. If we don't insist on this limitation, the game has a unique equilibrium – namely that the participant having the first bid offers exactly $100, while nobody else bids anything at all. This would yield a net payoff of zero to all players. However, it may be argued that as a solution of the game, this equilibrium is quite weak and rather unsatisfactory. See Shubik (1971) for further details.
2. It is *better* to vote for your most preferred candidate provided that your vote is pivotal for the outcome of the election. If it isn't, it doesn't matter who you vote for.
   Note that if the number of candidates exceeds two, it may sometimes be best to vote for a candidate that is *not* your first preference, since this may enable you to avoid an even worse candidate coming into office.
3. Tullock (1967) claims that the equilibrium is not *equal*, but rather somewhere *close* to zero turnout. Neither allegation is in fact true (unless we allow for mixed strategies), as shown in the section on the Voting Game.
4. This formulation has an intended, dual interpretation. According to Margolis, a citizen may well be expected to vote even if his self-interest tells him to abstain. And given that he decides to vote, it is perfectly possible that he votes for the candidate least favorable to his self-interest.
5. This condition applies only provided that ties generate a winner with certainty. If ties are decided by the flipping of a fair coin, the corresponding condition is that $B > 4C_v$. See Ferejohn & Fiorina (1974) for further details.
6. The game analyzed here may be considered a generalization of a related game, invented (at least to my knowledge) by Taylor & Ward (1982, 361), and formalized in Hovi (1986, 348–349). This game can briefly be described as follows. A committee is about to decide on some matter, and is split into two factions, A and B. Group A consists of s members, each of whom is *known* to attend, and vote *against* the proposal. Group B has n members ($n > s + 1$), and wants a majority in favor of the proposal. It is assumed that if a tie occurs, the proposal is rejected, making A 'the winner'. Since $n > s + 1$, it suffices that less than all members of B attend to achieve a majority

in favor. Furthermore, nothing extra can be gained if a majority greater than the minimal one attends. Finally, to each member of B there is a certain cost of attending.

In many respects, Taylor & Ward's model is identical to the one considered in the present paper. However, there is one important difference, since in my model, all members of *both* groups are considered players (I also investigate the possibility of more than two groups). This means that I have omitted the rather unsatisfactory assumption that all members of A are known to attend and cast their votes. The change of assumptions at this point has some very important consequences, since in Taylor & Ward's model, there are always multiple equilibria, while the model considered in the present paper in general contains none at all.

During my work on this paper, I became aware of an article by Palfrey & Rosenthal, in which the authors present a model of two-candidate elections that is essentially equivalent to mine. There are, however, also important differences between their work and the present paper. Most notably, Palfrey & Rosenthal are exclusively occupied with single-round two-candidate elections, while I also consider elections involving more than two candidates, and (to some extent) more than one round. Moreover, while I have chosen not to deal with mixed strategy equilibria, this is the most prominent focus of Palfrey & Rosenthal.

7. This holds true without qualifications for the cases where ties generate a winner with certainty. However, some of the conclusions regarding the cases where ties are decided by a random device are only meaningful provided that utilities can be measured on a cardinal scale. Even when this is the case, however, it is not entirely clear that altruistic citizens are more inclined to vote, than are those motivated by self-interest.

8. This conclusion may seem strongly counterintuitive to the reader, and it is easy to see that is highly sensitive to minor changes in the assumptions made regarding the players' motivations. Consider for example the possibility that the citizens have lexicographic preferences, as follows: Each player's first concern is the outcome of the election. But *given* that a particular result has come about, he prefers a greater turnout to a smaller one, 'for the sake of democracy'. Provided that the extra utility from one single additional vote doesn't exceed the cost of voting, this change of assumptions would not alter the behavioral implications of the model. However, it may easily reverse the conclusion that zero turnout is a Pareto-superior outcome.

9. The discussion in this section is predominantly relevant for two-candidate elections being decided in a single round.

10. For an in-depth analysis of mixed strategy equilibria in the two candidate game, see Palfrey & Rosenthal (1983).

11. At least, the relationship may be expected to be more complex than Ferejohn & Fiorina seem to believe, since extreme pessimists and extreme optimists alike always abstain. If it holds true that there is a positive, monotonic relationship between wealth and education on the one hand, and optimism on the other, this would indicate that the typical abstainer is either a complete underdog *or* a perfect topdog. Voters, on the other hand, might be expected to be found somewhere in between these two extremes.

REFERENCES

Barry, B. 1970. *Sociologists, Economists, and Democracy*. London: Collier-Macmillan Ltd.

Beck, N. 1975. 'The Paradox of Minimax Regret.' *American Political Science Review* 69, 918.

Downs, A. 1957. *An Economic Theory of Democracy*. New York: Harper & Row Publishers.

Elster, J. 1978. *Logic and Society. Contradictions and Possible Worlds*. Chichester: John Wiley & Sons.

Ferejohn, J. A. & Fiorina, M. P. 1974. 'The Paradox of Not Voting: A Decision Theoretic Analysis.' *American Political Science Review* 68, 525–536.

Ferejohn, J. A. & Fiorina, M. P. 1975. 'Closeness Counts Only in Horseshoes and Dancing.' *American Political Science Review* 69, 920–925.

the equilibrium where all citizens vote sincerely must be considered the natural solution of the game, since it is better for all members of A if they all vote for *a* rather than for *b*, regardless of what the members of the other group might do. The situation is of course equivalent on behalf of the members of C.

A further possibility arises if only $n_{BC} = 0$. Then equations III and IV may hold simultaneously, meaning that the game may have two equilibria. It is left for the reader to verify that the relevant outcomes are those specified during the discussion of the third and fourth cases above.

Finally, it is possible that only $n_{BA} = 0$. Then equations II and IV may both be true, and again there may be two equilibria – corresponding to those identified under the discussion of the second and fourth cases above.

To sum up, we might say that when ties are decided by a random device, the m-candidate Voting Game has no equilibrium unless two or more essential coalitions can be identified, which have equally many members. When this is possible, the game may have zero, one, or several equilibria – depending on the more specific constellation of preferences within the electorate. It must be added that for the latter cases, which have been the focus of this appendix, only three-candidate elections have been considered.

NOTES

1. In fact, this holds true only provided that a first bid of $100 is not allowed. If we don't insist on this limitation, the game has a unique equilibrium – namely that the participant having the first bid offers exactly $100, while nobody else bids anything at all. This would yield a net payoff of zero to all players. However, it may be argued that as a solution of the game, this equilibrium is quite weak and rather unsatisfactory. See Shubik (1971) for further details.
2. It is *better* to vote for your most preferred candidate provided that your vote is pivotal for the outcome of the election. If it isn't, it doesn't matter who you vote for.
   Note that if the number of candidates exceeds two, it may sometimes be best to vote for a candidate that is *not* your first preference, since this may enable you to avoid an even worse candidate coming into office.
3. Tullock (1967) claims that the equilibrium is not *equal*, but rather somewhere *close* to zero turnout. Neither allegation is in fact true (unless we allow for mixed strategies), as shown in the section on the Voting Game.
4. This formulation has an intended, dual interpretation. According to Margolis, a citizen may well be expected to vote even if his self-interest tells him to abstain. And given that he decides to vote, it is perfectly possible that he votes for the candidate least favorable to his self-interest.
5. This condition applies only provided that ties generate a winner with certainty. If ties are decided by the flipping of a fair coin, the corresponding condition is that $B > 4C_v$. See Ferejohn & Fiorina (1974) for further details.
6. The game analyzed here may be considered a generalization of a related game, invented (at least to my knowledge) by Taylor & Ward (1982, 361), and formalized in Hovi (1986, 348–349). This game can briefly be described as follows. A committee is about to decide on some matter, and is split into two factions, A and B. Group A consists of s members, each of whom is *known* to attend, and vote *against* the proposal. Group B has n members ($n > s + 1$), and wants a majority in favor of the proposal. It is assumed that if a tie occurs, the proposal is rejected, making A 'the winner'. Since $n > s + 1$, it suffices that less than all members of B attend to achieve a majority

in favor. Furthermore, nothing extra can be gained if a majority greater than the minimal one attends. Finally, to each member of B there is a certain cost of attending.

In many respects, Taylor & Ward's model is identical to the one considered in the present paper. However, there is one important difference, since in my model, all members of *both* groups are considered players (I also investigate the possibility of more than two groups). This means that I have omitted the rather unsatisfactory assumption that all members of A are known to attend and cast their votes. The change of assumptions at this point has some very important consequences, since in Taylor & Ward's model, there are always multiple equilibria, while the model considered in the present paper in general contains none at all.

During my work on this paper, I became aware of an article by Palfrey & Rosenthal, in which the authors present a model of two-candidate elections that is essentially equivalent to mine. There are, however, also important differences between their work and the present paper. Most notably, Palfrey & Rosenthal are exclusively occupied with single-round two-candidate elections, while I also consider elections involving more than two candidates, and (to some extent) more than one round. Moreover, while I have chosen not to deal with mixed strategy equilibria, this is the most prominent focus of Palfrey & Rosenthal.

7. This holds true without qualifications for the cases where ties generate a winner with certainty. However, some of the conclusions regarding the cases where ties are decided by a random device are only meaningful provided that utilities can be measured on a cardinal scale. Even when this is the case, however, it is not entirely clear that altruistic citizens are more inclined to vote, than are those motivated by self-interest.
8. This conclusion may seem strongly counterintuitive to the reader, and it is easy to see that is highly sensitive to minor changes in the assumptions made regarding the players' motivations. Consider for example the possibility that the citizens have lexicographic preferences, as follows: Each player's first concern is the outcome of the election. But *given* that a particular result has come about, he prefers a greater turnout to a smaller one, 'for the sake of democracy'. Provided that the extra utility from one single additional vote doesn't exceed the cost of voting, this change of assumptions would not alter the behavioral implications of the model. However, it may easily reverse the conclusion that zero turnout is a Pareto-superior outcome.
9. The discussion in this section is predominantly relevant for two-candidate elections being decided in a single round.
10. For an in-depth analysis of mixed strategy equilibria in the two candidate game, see Palfrey & Rosenthal (1983).
11. At least, the relationship may be expected to be more complex than Ferejohn & Fiorina seem to believe, since extreme pessimists and extreme optimists alike always abstain. If it holds true that there is a positive, monotonic relationship between wealth and education on the one hand, and optimism on the other, this would indicate that the typical abstainer is either a complete underdog *or* a perfect topdog. Voters, on the other hand, might be expected to be found somewhere in between these two extremes.

REFERENCES

Barry, B. 1970. *Sociologists, Economists, and Democracy*. London: Collier-Macmillan Ltd.

Beck, N. 1975. 'The Paradox of Minimax Regret.' *American Political Science Review* 69, 918.

Downs, A. 1957. *An Economic Theory of Democracy*. New York: Harper & Row Publishers.

Elster, J. 1978. *Logic and Society. Contradictions and Possible Worlds*. Chichester: John Wiley & Sons.

Ferejohn, J. A. & Fiorina, M. P. 1974. 'The Paradox of Not Voting: A Decision Theoretic Analysis.' *American Political Science Review* 68, 525–536.

Ferejohn, J. A. & Fiorina, M. P. 1975. 'Closeness Counts Only in Horseshoes and Dancing.' *American Political Science Review* 69, 920–925.