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On the Properties of Voting Systems

Hannu Nurmi, University of Turku

The article focuses on the problem of choosing the 'best' voting procedure for making collective decisions. The procedures discussed are simple majority rule, Borda count, approval voting, and maximin method. The first three have been axiomatized while the maximin method has not yet been given an axiomatic characterization. The properties, in terms of which the goodness of the procedures is assessed, are dictatorship, consistency, path independence, weak axiom of revealed preference, Pareto optimality, and manipulability. It turns out that the picture emerging from the comparison of the procedures in terms of these properties is most favorable to the approval voting.

1. Introduction

Modern interest in the properties of voting systems as devices for aggregating individual wills goes back at least two hundred years. The studies of Condorcet on 'homo suffragans' are fairly well-known to any student of social choice theory at least as far as the Condorcet effect is concerned. What is perhaps less well-known is that the impetus to Condorcet's work came partly from the interesting memoir of Jean-Charles de Borda dated 16 June 1770 and called Mémoire sur les Elections au Scrutin. The memoir was eventually published in 1781 in Mémoires de l'Académie Royale des Sciences. As one perhaps could expect from a scholar of the late Enlightenment period, the aim of the memoir was to design a pure and just voting system based on majority rule (see De Grazia 1953; Granger 1956: 118–120). The starting point was the observation that under certain circumstances, the collective choice rule picking the plurality winning alternative is highly unsatisfactory. In particular, if there are three alternatives A, B and C to choose from, the alternative A may carry plurality of votes if each voter is allowed to cast one and only one vote, and yet the majority of voters may prefer B or C or both to A. Of course, the situation is more paradoxical if the majority prefers both B and C to A because this means that *any* of the non-winning alternatives would be preferred by a majority to the winning one. This case is now called the strong Borda paradox (Colman & Pountney 1978). But even the weak case in which only one of the alternatives is

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1. Introduction

Modern interest in the properties of voting systems as devices for aggregating individual wills goes back at least two hundred years. The studies of Condorcet on 'homo suffragans' are fairly well-known to any student of social choice theory at least as far as the Condorcet effect is concerned. What is perhaps less well-known is that the impetus to Condorcet's work came partly from the interesting memoir of Jean-Charles de Borda dated 16 June 1770 and called Mémoire sur les Elections au Scrutin. The memoir was eventually published in 1781 in Mémoires de l'Académie Royale des Sciences. As one perhaps could expect from a scholar of the late Enlightenment period, the aim of the memoir was to design a pure and just voting system based on majority rule (see De Grazia 1953; Granger 1956: 118–120). The starting point was the observation that under certain circumstances, the collective choice rule picking the plurality winning alternative is highly unsatisfactory. In particular, if there are three alternatives A, B and C to choose from, the alternative A may carry plurality of votes if each voter is allowed to cast one and only one vote, and yet the majority of voters may prefer B or C or both to A. Of course, the situation is more paradoxical if the majority prefers both B and C to A because this means that *any* of the non-winning alternatives would be preferred by a majority to the winning one. This case is now called the strong Borda paradox (Colman & Pountney 1978). But even the weak case in which only one of the alternatives is preferred to the winning one by a simple majority suggests that plurality voting has undesirable properties.

What Borda proposed as a way to overcome these difficulties was called by him the election by order of merit, and the Borda count by modern writers. The method consists of adding the scores given to each alternative and identifying the winner as the alternative having the largest score. The scores, in turn, are given according to the voter strong preference orderings so that the least preferred alternative gets 1 point from the voter, the penultimate one 2 points etc. The points given to each alternative are then added to get the score of the alternative.

The Borda count was first very well received by the Royal (French) Academy of Sciences. Indeed, it was even applied for a while in the election of officers to the Academy (see De Grazia 1953). However, the method is subject to the criticism that it does not necessarily choose the Condorcet winning alternative (i. e. an alternative defeating all the others by a simple majority when pairwisely confronted with them) when one exists. Moreover, the Borda count is amenable to strategic manipulation (see e.g. Niemi & Riker 1976). It is characteristic of the spirit in which the voting procedure discussions were conducted in 18th century France that Borda, when notified about the strategic manipulability of his procedure, replied that his system was intended for honest men only. It is a pity that we cannot overlook the strategic manipulability as a property of some importance in assessing the merits of voting systems. It is, however, but one of the many properties brought to light by the studies conducted in modern times in the Enlightenment-type spirit of devising rational social institutions. As is often the case, the results of these studies have revealed the relative merits of voting systems rather than absolute ones. However, the sheer amount of work published in the field should put us in a position to give partial answers to Borda's and Condorcet's problem of what is a just and effective way of aggregating individual wills.

2. Characterization of Voting Procedures

A straightforward way to describe a voting system which gives simultaneously a considerable amount of information about its theoretical properties is to present a set of axioms for the system, i.e. the set of necessary and sufficient conditions for the voting system in question. Of course, the axiomatization is not always readily achievable and many voting systems have so far defied attempts at axiomatic characterizations. Still, a few methods have been axiomatized. Upon investigating them, we can structure the problem of Borda and Condorcet so as to be able to say in which respects the voting systems differ along various dimensions of comparison.

The first axiomatized voting procedure to be discussed is the simple majority rule for which May (1952) gave the set of necessary and sufficient conditions. More specifically, May listed a set of conditions each necessary and jointly sufficient for a ballot-counting procedure f to be a simple majority rule (see also Plott 1976: 558–560). The procedure f is function mapping the n-tuples of individual ballots of the n voters into the set $\{-1, 0, 1\}$. To characterize the simple majority rule, it is assumed that only two alternatives, viz. x and y, are considered. Denote a vote for x by 1 and a vote for y by −1 and an abstention by 0. An analogous interpretation can be given to the elements in the range of f: −1 means that y is chosen, 0 means that there is a tie, and 1 indicates that x is chosen by the collectivity. The domain of f is then the n-fold Cartesian product set $\{-1, 0, 1\}^n$. With these notations the definition of the simple majority rule is as follows:

if (i) $B_1 + \ldots + B_n > 0$ implies $f(B_1, \ldots, B_n) = 1$,
(ii) $B_1 + \ldots + B_n = 0$ implies $f(B_1, \ldots, B_n) = 0$, and
(iii) $B_1 + \ldots + B_n < 0$ implies $f(B_1, \ldots, B_n) = 1$,

where B_i ($i = 1, \ldots, n$) indicates the ballot of the individual i, then f is the simple majority rule.

May shows that the simple majority rule f so defined has the following properties and, conversely, any f having all those properties is a simple majority rule:

1. Decisiveness: i.e. f is defined for any possible n-tuple of individual ballots,
2. Symmetry: the permutation of individual ballots does not affect the social choice, that is, if $(B'_1, \ldots, B'_n)$ is a permutation of $(B_1, \ldots, B_n)$, then $f(B'_1, \ldots, B'_n) = f(B_1, \ldots, B_n)$,
3. Duality: if $B_i = -B'_i$ for all $i = 1, \ldots, n$, then $f(B_1, \ldots, B_n) = -f(B'_1, \ldots, B'_n)$. In other words, suppose that there are two voting situations I and II such that in I x (or y, respectively) is chosen under f. Suppose now that in II all those who voted for x decide to vote for y and all those who voted for y decide to vote for x. Suppose moreover that those who abstain in I also abstain in II. Under these assumptions the procedure having the duality property chooses y (or x).
4. Strong monotonicity: if $B_i \geqq B'_i$ for all $i = 1, \ldots, n$ the strict inequality holding for at least one individual and if $f(B'_1, \ldots, B'_n) = 0$ or 1, then $f(B_1, \ldots, B_n) = 1$.

When adopting the simple majority rule, one is therefore committed to the above properties 1.–4. We observe that the rule has been defined for a two-candidate situation only. Thus, the problems related to the organization of several consecutive ballots or considering more than two candidates simultaneously are not dealt with. And yet these problems are the most crucial ones in devising just and effective voting systems. We now turn to procedures that deal with the multi-candidate case.

The Borda count is obviously applicable when more than two candidates are considered. To analyze it, we adopt some additional notation. Let $C: A \times R_1 \times \ldots \times R_n \rightarrow \mathcal{A}$ be the social choice function where $\mathcal{A}$ is the set of subsets of A. In other words, C assigns to any n+1 −tuple consisting of the set of alternatives A and n individual weak preference relations over A, the set of socially best alternatives. The theoretically interesting part of a voting procedure is the choice function which represents it. The social choice function representing the Borda count has the following properties (Young 1974):

1. Neutrality. Let a_1, a_2, a_3 and a_4 be any four distinct elements in the set A of alternatives and let $\{R_i\}$ and $\{R'_i\}$ $(i = 1, \ldots, n)$ be two n-tuples of individual weak preference relations such that $\forall i$: $a_1 R_i a_2$ if and only if $a_3 R'_i a_4$, and $a_2 R_i a_1$ if and only if $a_4 R'_i a_3$. Then for a procedure C to be neutral it is both necessary and sufficient that $a_1 \varepsilon C(A, R_1, \ldots, R_n)$ if and only if $a_3 \varepsilon C(A, R'_1, \ldots, R'_n)$, and $a_2 \varepsilon C(A, R_1, \ldots, R_n)$ if and only if $a_4 \varepsilon C(A, R'_1, \ldots, R'_n)$. The intuitive interpretation of this property is that the relabelling of alternatives does not affect the social choice. In other words, the procedure does not discriminate for or against any alternative in A.
2. Consistency. Let $\{R_i\}$ and $\{R'_j\}$ $(i=1,\ldots,n; j=1, \ldots, m)$ be two sets of preference relations of two disjoint sets of individuals N and M, respectively. Suppose that $C(A, R_1, \ldots, R_n) \cap C(A, R'_1, \ldots, R'_m) \neq \emptyset$. If now $C(A, R_1, \ldots, R_n) \cap C(A, R'_1, \ldots, R'_m) = C(A, R_1, \ldots, R_n, R'_1, \ldots, R'_m)$, then C is consistent. In other words, if two groups of people acting separately choose from the set A some common elements when applying the procedure C, then when acting together they should choose exactly the common elements in order for C to be consistent.
3. Faithfulness. Consider a voting body consisting of one individual i whose most preferred alternative is a_j. Then C is said to be faithful if and only if $C(A, R_i) = a_j$.
4. Cancellation property. Let $R = \{R_1, \ldots, R_n\}$ be any n-tuple of individual preference relations and let $N(a_i, a_j)$ be the number of those voters who strictly prefer a_i to a_j. If now $N(a_i, a_j) = N(a_j, a_i)$ for all a_i, a_j in A and C chooses the entire set A, then C has the cancellation property. In other words, under f a number of voters having a given preference over a given pair of alternatives can be cancelled out by the same number of voters having the opposite preference.

Young's (1974) result states that there is one and only one social choice function having the above properties 1.–4. and that function is the Borda count. Young's proof of the result has later been simplified by Hansson & Sahlquist (1976) (see also Smith 1973 and Young 1975).

A voting procedure of more recent origin than the Borda count is the approval voting. It is also applicable in multicandidate contests (Brams & Fishburn 1978). The customary way of describing the approval voting is to resort to the ballot counting choice function f as was done above in connection with the simple majority rule. In approval voting each voter votes for as many candidates as he likes without ranking the choices. Intuitively, by giving a vote to a candidate, the voter approves him. The winner is the candidate set receiving the largest number of votes.

More precisely the approval voting function f can be characterized as follows (see Fishburn 1978; Fishburn 1979). Let $\mathcal{A}$ be the set of all subsets of the alternative set A and let g: $\mathcal{A} \rightarrow \mathbb{N}$ be the ballot response profile, where $\mathbb{N}$ is the set of natural numbers. For any subset of A, g indicates the number of voters voting for that subset. The set of all ballot response profiles is denoted by G. Let now $g(x) = \Sigma\, g(B)$ for $x \in B$. g(x) gives the number of voters whose ballots contain x. For a fixed g, let f(g) denote the social choice set. The approval voting function is now defined as follows: $f(g) = \{x \in A \mid g(x) \geqq g(y)$ for all y in A$\}$ for all g in G.

Fishburn (1978) shows that the approval voting function can be axiomatized by means of the following conditions:

1. Neutrality, i.e. the permutation of ballots does not affect the social choice (cf. above).
2. Consistency: for any g and g' in G: $f(g) \cap f(g') \neq \emptyset$ implies that $f(g + g') = f(g) \cap f(g')$ (cf. above).
3. Disjoint equality: if there are only two voters with ballot sets C and C', respectively, in G such that $C \cap C' = \emptyset$, then $f(g) = C \cup C'$. That is, if there are only two voters who have distinct approved candidate sets, the social choice set consists of the union of the sets.

These, then, are the necessary and sufficient conditions for f to be the approval voting function. In other words, when adopting the approval voting procedure, we eo ipso commit ourselves to the above conditions 1–3. While the simple majority rule is defined for two candidate contests only, both the Borda count and the approval voting can be used to solve the thorny problem of how to choose the socially best alternatives. Before assessing their performance with respect to various criteria, let us briefly outline a non-axiomatized voting procedure, the maximin method, which can also be used in multi-candidate contests.

The method was originally introduced by Simpson (1969) as the minimax method. Subsequently, Kramer (1977) elaborated it as well as proving some interesting theorems about its properties. We shall discuss the method in a slightly modified form in which it can be called the maximin method for quite obvious reasons (see Nurmi 1980).

Let the number of candidates be n and let $[r_{ij}]$ be an n by n matrix where $r_{ij} = N(a_i, a_j)$, for all $i, j = 1, 2, \ldots, n$ and $i \neq j$. Here $N(a_i, a_j)$ denotes the number of voters preferring a_i to a_j in the pairwise comparison of the two candidates. Now define $\bar{r}_i = \min_j N(a_i, a_j)$ and $\bar{r} = \max_i \bar{r}_i$. We say that a_k (ε A) belongs to the maximin set A_M iff $\bar{r} = \bar{r}_k$. In other words, for each alternative we look for its toughest competitor; either one which defeats it by the largest number of votes or which is defeated by it by the smallest margin, and count the number of votes given to it in that particular comparison. The alternatives which get the maximum number of votes in their toughest contest belong to the maximin set. The maximin method has not been axiomatized although many of its necessary conditions are known. We shall discuss some of them in the following as we turn to the systematic comparison of the above voting procedures (for a similar approach, see Straffin 1979).

3. What Properties Are Desirable?

3.1. Non-dictatorship

Quite obviously if one is interested in *voting* procedures one does not deem dictatorial procedures desirable. And yet to determine whether a procedure is dictatorial or not may not be quite straightforward. A combination of quite innocent-looking properties may imply a dictatorship. A case in point is the combination of the independence of infeasible alternatives and what is called the weak axiom of revealed preference (WARP) (see Plott 1976, 550). The former property requires that only the change of the voter preferences concerning the feasible alternatives can change the social choice. The latter, in turn, is a kind of consistency property to be discussed shortly. Any procedure having these two properties is dictatorial, i.e. the procedure chooses as if there were an individual whose preference always determines the social choice. Thus, if the independence of infeasible alternatives and WARP are viewed as consistency properties, non-dictatorship is not compatible with both of these forms of consistency.

Perhaps a somewhat more general result on the dictatorship of social choice function was independently proven by Gibbard (1973) and Satterthwaite (1975) (see also Gärdenfors 1977). It states that any resolute social choice function with at least three possible values (choices) is either manipulable or dictatorial. To appreciate the coverage of this theorem, we need to specify what is meant by resoluteness: a social choice function is resolute if its range consists of singleton sets of alternatives only. In other words, the social choice set must consist of no more than one alternative. Obviously the theorem does not apply to the simple majority rule, the Borda count, approval voting or the maximin method as any of these rules can result in a tie. In general, it can be seen that none of these procedures is dictatorial because under each of them the social choice remains invariant under the permutation of voters, i.e. the procedures are anonymous, thereby excluding the possibility of a dictator.

3.2. Consistency

One of the 'obvious' properties required of a social decision process is consistency, at least in so far as inconsistency is usually deemed an unacceptable feature of both individual and collective decisions. However, it is only after having been confronted with various specifications of consistency that one is likely to appreciate the complexity of the issue. Let us take as our point of departure the ordinary language specification of consistency in which it is stated that no one can consistently argue that p and non-p is the case where p stands for a proposition. Translated into the context of collective decisions, this requirement would obviously prohibit choosing a_1 and non-a_1 at the same time. But what does it mean? Surely, any social choice function satisfies trivially (by definition) the requirement of being non-contradictory when applied in a given decision situation. But if we interpret consistency to mean stability in the sense that the individual preferences and the set of alternatives being fixed, the procedure always chooses the same set of alternatives, then we can observe that the simple majority rule with a pairwise comparison of alternatives is not consistent. In other words, the well-known parliamentary voting procedure is not consistent unless restrictions are set upon the agenda building. This observation is of course just another way of stating the Condorcet paradox, but more generally when the core of the voting game is empty and the voter preferences satisfy some fairly mild conditions and, moreover, the set of alternatives can be represented as a multidimensional real space, then any alternative can be rendered a majority winner, as McKelvey (1976; 1979) has shown.

So, under the above conditions, anyone controlling the agenda completely also determines the social choice. It must be observed, though, that the control required is, indeed, complete in the sense of allowing the agenda-controller to add any number of alternatives to the agenda in addition to dictating the voting sequence.

The Borda count, approval voting, and the maximin method are all consistent in this sense. That is, if in any given decision situation where the alternative set is fixed the social choice set differs from what it has been in another situation with the same alternative set, the difference must be due to a change in the individual preference relations.

But consistency can also be understood as referring to situations in which either the alternative set or the voter set vary. In the context of the axiomatization of the Borda count and the approval voting, the consistency axiom has the latter interpretation: i.e. it requires that two groups when choosing from a fixed set of alternatives should fulfill a certain criterion in order to be consistent. The Borda count and approval voting thus satisfy the consistency condition in this sense. It can be noticed immediately that the simple majority rule with a fixed alternative set is also consistent if the agenda remains fixed. The maximin method, on the other hand, is not consistent (see Nurmi 1980)

Another type of consistency property is path independence. It is a requirement that regardless of the way in which we partition our alternative set into two subsets, the social choice set should be the same when we make a choice from a set consisting of the social choice set of one of the subsets and the other subset, and when the choice is made from the original alternative set. In symbols:

$C(A, R_1, \ldots, R_n) = C(C(A_1, R_1, \ldots, R_n) \cup A_2, R_1, \ldots, R_n)$ for all $A_1, A_2 \subset A$ such that $A = A_1 \cup A_2$ with $A_1 \cap A_2 = \emptyset$.

Of course, controlling the agenda is tantamount to controlling the 'path' in the relevant sense. Therefore, whenever the agenda control amounts to the control of the social choice, we are dealing with a path dependent procedure. Clearly, the simple majority rule is not a path independent procedure. It is well-known that the Borda count violates the condition of the independence of irrelevant alternatives. In particular, the number of alternatives considered affects the social choice under the Borda count. Consequently, by a suitable partitioning of the set A, one can affect the social choice from a subset of A and, thereafter, the 'overall' Borda winning set. Hence, the Borda count obviously is not a path independent procedure.

That the approval voting is path independent is obvious. The individual ballots remaining the same, any partitioning of the alternative sets leaves the set of voters voting for each subset of alternatives unaffected. When the set of winners of the subset is considered together with the complement of the subset, the number of votes ('approvals') given to each of the considered alternatives is no more or less than are given when the entire alternative set is considered simultaneously – provided, of course, that the preferences of the voters do not change and that the voting is sincere. Another way of looking at this feature of approval voting is to observe that the procedure satisfies the independence of irrelevant alternatives condition. Consequently, the removal of some of the irrelevant – i.e. socially not approved – alternatives does not affect the social choice between the rest of the alternatives.

The maximin method, on the other hand, does not have this consistency property. This can be seen upon noticing that it is not independent of irrelevant alternatives (Nurmi 1980). The winners in each subset are determined on inspecting their toughest competitors. Hence, the winners of some of the subset contests do not necessarily include all the winners of the overall contest.

Path independence thus differentiates the voting procedures discussed here. It is, however, worth pointing out that although a seemingly 'nice' property, path independence is incompatible with some other intuitively plausible properties. For instance, path independence, sincere voting, Pareto optimality and non-oligarchy have been shown to be incompatible (Plott 1976, 575). So at least one notion of group rationality seems somewhat difficult to combine with one particular interpretation of consistency.

There is one further concept that could be thought of as referring to an aspect of the intuitive notion of consistency, viz. WARP. Stated informally, WARP says the following. Suppose that when a collective choice is made, it turns out that the alternative set A included some unrealistic or otherwise unrealizable alternatives and that a new choice is made among the realizable ones A'. A procedure satisfying WARP now has the property that if the preferences remain the same and an alternative belonging to A' is in the choice set when the set A is considered, then the choice set of A includes all the alternatives chosen from A' (and perhaps some others not in A') (Plott 1976: 549).

Now if we interpret the simple majority rule as the parliamentary voting procedure in a multi-candidate contest, we can observe that WARP is not satisfied even if agenda manipulation is excluded. Suppose that A_1 is the choice set from A' and A_2 is the choice set from A. Suppose moreover that $A_1 \cap A_2 \neq \emptyset$. The question now is whether $A_1 \subseteq A_2$ or not. Obviously some of the candidates in A_1 may be defeated in pairwise comparisons with some of the candidates in A_2 while some may survive. Hence, it is not necessary that $A_1 \subseteq A_2$. Sen's (1970, 17) well-known example of a game championship is a case in point: WARP requires that if some Pakistani is a world champion, then all champions of Pakistan must be champions of the world. Obviously, if championships are determined on the basis of the pairwise comparisons of alternatives, WARP is not necessarily satisfied.

That the Borda count does not satisfy WARP is perhaps most easily seen by noticing that in order to have the WARP property, the procedure must be such that an overall winning alternative is also the winner in every subset to which it belongs. The Borda count does not have this property (see, e.g. Riker & Ordeshook 1973: 88 for a counterexample).

Assuming that the individual preferences remain the same when the choice is made from A and when it is made from A', it is clear that the approval voting satisfies WARP. Indeed, this is a logical consequence of 'approving' of candidates by giving them votes and 'disapproving' of them by not giving them votes. As for the maximin method, it also satisfies WARP as has been shown elsewhere (Nurmi 1980).

Having now discussed several consistency properties of the voting procedures, let us turn to another related property, viz. rationality. While consistency is usually deemed to be a necessary condition of individual rationality, we shall now deal with a system level notion, group rationality.

3.3. Pareto optimality

One of the most extensively discussed theoretical properties in welfare economics and social choice is Pareto optimality. Even though it is doubtful whether considerations pertaining to the justice of social institutions or arrangements are reflected in the criterion of Pareto optimality, it is difficult to deny that a failure to satisfy it – when other perhaps more 'important' criteria are fulfilled – would be collectively irrational.

Stated in the context of voting procedures Pareto optimality criterion says that if everyone in a decision-making body prefers x to y, then y is not to be chosen (Plott 1976, 528). Prima facie, this principle does not appear too stringent, but it is worth noticing that we are now trying to determine whether our procedures necessarily exclude Pareto suboptimal choices.

It has been shown by McKelvey (1976; 1979) that there is nothing in the nature of the simple majority rule to exclude the possibility of Pareto suboptimal choices. Indeed, provided that the core is empty and that the voters have preferences representable by continuous utility functions, any outcome can be rendered the majority winner if the policy space is the Re^n- space ($n = 2, 3, \ldots$).

The Borda count, on the other hand, clearly fulfills the above stated condition of Pareto optimality. This is due to the fact that if x is preferred to y by everyone, then it obviously gets a higher total score than y. Consequently, y cannot be chosen.

The assessment of the approval voting is less straightforward. It is obvious that if everyone approves x (i.e. gives x a vote) and disapproves y (i.e. gives no vote to y), then y cannot be chosen. More generally, if the preferences are dichotomous, the Pareto optimality of the outcome is guaranteed. But the dichotomous nature is not necessary for this result to hold. It also holds in cases where each voter has more than two equivalence classes of alternatives provided that at least one voter puts x in a class of approved alternatives and y in a class of disapproved ones. So, we conclude that if the preference between x and y is interpreted in the relevant sense, the approval voting is Pareto optimal.

That also the maximin method satisfies the Pareto principle can be seen from the following. Suppose that x is preferred to y by everyone. Then obviously the minimum number of votes for y in all pairwise comparisons is 0. An alternative with this property cannot be chosen by the maximin method unless all the other alternatives have the same property. This, in turn, means that there is a unanimity cycle in the collective preference through every alternative. Should this be the case, the Pareto optimal set would clearly be empty. Thus, the maximin method satisfies Pareto optimality.

The above properties deal with procedures from the rationality point of view with the implicit assumption that the inputs of the procedures – i.e. the individual votes or preferences – are given in a fashion that is unproblematic for the assessment of the procedures. But there are reasons to argue that the voting procedures cannot satisfactorily be evaluated without an explicit consideration of the strategic aspects of the voting situations.

3.4. Manipulation and implementation

It is well-known from the collective goods problematique that the sincere revelation of preferences may not always be beneficial for the individuals participating in collective decision making. From the view-point of the voting procedures this possibility has to be taken into account because the possibility of strategic manipulation – when present – is what makes the procedures unstable. Let us make a distinction between the following properties:

(i) the dominant strategy property, and
(ii) sincerity of the procedure.

Procedures having the former property allow each voter one dominant strategy. When the procedure satisfies the property (ii), in turn, it assigns to each player one dominant strategy which consists of behaving according to his (her) true preferences. Clearly, (ii) is a special case of (i). The dominant strategy property has to do with the stability of the procedure or mechanism. The problem of implementation of the social choice mechanism is customarily phrased in the following way: does a given mechanism support – i.e. choose – outcomes that are equilibria of the underlying game forms (Dasgupta et al. 1979; Ferejohn et al. 1980). The types of equilibria usually discussed are dominant strategy equilibria or Nash equilibria (see, however, Ferejohn et al. 1980). We shall focus on the former type of implementability here. In other words, we restrict ourselves to discussing whether the above voting procedures have the property that for all individual preference configurations there exists for each voter a dominant strategy such that the choice of their dominant strategies by all players yields an outcome that belongs to the choice set of the voting procedure.

Clearly, the simple majority rule is non-manipulable in the case where two alternatives only are considered. The parliamentary voting procedure, on the other hand, is amenable to the strategic misrepresentation of preferences. That it has in fact been manipulated by real-world political actors is well-known (see e.g. Bjurulf & Niemi 1978). We can easily observe that not only does the procedure lack the sincerity property but it does not even possess the dominant strategy property. In other words, it is an unstable procedure.

Also the Borda count is obviously manipulable because given that the preferences of other voters are fixed, an individual may in some cases affect the social choice by, say, giving the largest number of points to his (her) second best alternative so as to guarantee its choice when his (her) first preference does not have any chance of being chosen. Consequently, the Borda count is not sincere. Nor is it stable as can be readily observed.

The results of Brams & Fishburn (1978) show that the approval voting procedure is less manipulable than other similar voting methods (see also Fishburn 1978a). Specifically, if the number of equivalence (or indifference) classes of alternatives for each individual is 2, then the approval voting is non-manipulable, but when the number of equivalence classes is at least 3, no single ballot non-ranked voting system is non-manipulable.

Also the maximin method is subject to strategic manipulation. This can be seen from the following example (Nurmi 1980). Consider the following table in which the results of sincere voting are tabulated when the number of alternatives is 4 and the number of voters is 4:

	x_1	x_2	x_3	x_4	row minima
x_1	-	3	2	3	2
x_2	1	-	4	4	1
x_3	2	0	-	0	0
x_4	1	0	1	-	0

Here the entry on the i'th row and the j'th column $(i, j = 1, 2, 3, 4)$ gives the number votes that x_i gets in a pairwise comparison with x_j. Suppose now that one of the voters has a strict preference order over the alternatives according to which x_4 is the best, x_2 next best, x_1 the third, and x_3 the worst of the alternatives. This voter can by misrepresenting his preferences between x_1 and x_3 bring about the following matrix:

	x_1	x_2	x_3	x_4	row minima
x_1	-	3	1	3	1
x_2	1	-	4	4	1
x_3	3	0	-	3	0
x_4	1	0	1	-	0

In the sincere voting situation the choice set consists of x_1 only while in the later case it consists of x_1 and x_2. Thus, the voter has by misrepresenting his preferences brought about an outcome that is better from his view-point than the previous one.

4. Conclusion

In the preceding we have discussed several voting procedures in the light of various criteria of goodness. The following table (Table 1) summarizes the main observations. Obviously the check-list of the properties is partial and subject to criticism on various grounds. The picture that emerges from it is favorable to the approval voting. It becomes even more so when due attention is paid to the fact that when there is an 'obvious' winner, viz. the Condorcet winning alternative, the approval voting procedure always elects it if the voters vote according to their true preferences (Fishburn & Brams 1981). This feature also characterizes the maximin method (Nurmi 1980) and quite obviously the parliamentary voting procedure. In contrast, the Borda count does not necessarily choose the Condorcet winner.

One last caveat is in order: the voting methods discussed above are but a small subset of all possible methods. Similarly the criteria discussed could be augmented with many others. For an earlier account along similar lines the reader is referred to Straffin's study (1979).

Table 1. Comparison of the voting procedures with respect to various criteria of goodness.

Rule Property	Simple majority	Borda count	Approval voting	Maximin method
Dictatorship	No	No	No	No
Consistency	No	Yes	Yes	No
Path-independence	No	No	Yes	No
WARP	No	No	Yes	Yes
Pareto optimality	No	Yes	Yes	Yes
Manipulability	Yes	Yes	Yes	Yes

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