## Electoral Justice as a Criterion for Different Systems of Proportional Representation

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Balinski and Young (1978) have recently extended the traditional analysis by introducing different criteria for proportional representation systems. They started from the assumption that the only ultimate choice of

a method should be based on its *properties*. It is worth noting that Balinski and Young concentrate only on electoral rules; the other important components of electoral systems are not taken into account. They introduced the following notions: methods that are inducive to stability, methods enchouraging coalitions, and methods encouraging schisms (Balinski and Young 1978, 849). Not one of the commonest methods of P. R. fulfilled these requirements.

This article is focused on developing some empirical criteria for P.R. systems. In the first section, a criterion of electoral justice is defined. In contrast to the approach of Balanski and Young, our criterion is purely empirical; it is hard to imagine how electoral justice could be studied theoretically. In this respect the approach originally presented by Balinski and Young is extended in an empirical direction.

# 1. 'Electoral Justice' as a Criterion for Proportional Representation

We can start our analysis by describing in Figure 1 some critical points in the functioning of democratic choice process and decision-making. An electoral system can be defined as a transformation system which converts parties' votes into parliamentary seats. Studying properties of this transformation process entails analysing the criteria of proportionality. Let the vote and seat shares of parties  $A_1, A_2, \ldots, A_n$  be  $v_1, v_2, \ldots, v_n$  and  $s_1, s_2, \ldots s_n$  respectively. The exact proportionality holds if and only if, for all the parties

$$v_i = s_i \tag{1}$$

The analysis concerning proportionality concentrates only on the transformation process mentioned. There are many variables affecting the vote distribution between parties. In Figure 1 these factors are symbolized by  $x_1, \ldots x_n$ . However, these variables are excluded from the analysis.

Equation (1) represents the condition of exact proportionality. On the basis of Equation (1) the *measure of proportionality* is easily defined:

$$D = \frac{1}{2} \sum_{i=1}^{n} |v_i - s_i|$$
 (2)

Equation (2) is widely used in empirical studies of proportionality (see Loosemore and Hanby 1971). The upper limit of the D index is one (therefore coefficient ½). The minimum value is zero and this lower limit is reached when Condition (1) holds.

In comparison to the proportionality approach, the analysis of electoral

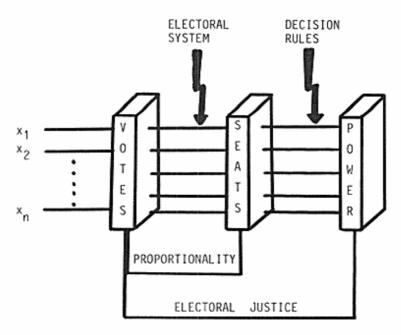


Figure 1. A scheme for defining the proportionality and electoral justice for different systems of P.R.

justice goes one step further. The basic idea is to introduce the decision-making process into the model. Thus one can consider not only the relationship between parties' seats and votes but also between their decision-making capacity and vote share. This implies that the coalition formation process is also included. The final goal of a party is to maximize not only its parliamentary seats but also its power in decision-making. The relation between these two goals is, of course, apparent; to increase the number of seats is to increase the decision-making capacity. But this relationship is not exactly linear, as we shall see later.

By introducing the decision-making system into the model, we in fact take another transformation process into account. Different decision rules form a transformation system which changes the parties' seat shares into their potential power shares in parliamentary decision-making. A vote share of 0.20 does not necessarily mean that the capacity of this party to influence decisions would also be 0.20. Therefore the seat share of a given party is a poor indicator of its power in parliamentary decision-making.

If the seat share is not a sufficient indicator of power, what is? In recent years, interest in different power indices has grown markedly. Already over twenty years ago Shapley and Shubik (1954) presented a power index, which has many empirical applications. More recently Banzhaf (1965; 1968) and Coleman (1971) have defined their variants of power indicies. At the moment a comprehensive study is being carried out

concerning the theoretical properties of these three indices, although the question of which is best has not yet been solved. However, regardless of their order of suitability, we can symbolize the power share of a party i by  $p_i$ . Electoral justice is defined by the following condition

$$v_i = p_i \tag{3}$$

for all the parties (i = 1, ..., n). Electoral justice is thus defined as the relation between a party's vote and its power share. We may describe an electoral system as just when it guarantees every party as great a possibility to influence decision-making (measured by power indices) as is, its share of votes. The index of electoral justice is easily defined analogously to Equation (2) as follows

$$J = \frac{1}{2} \sum_{i=1}^{n} \left| v_i - p_i \right|$$
 (4)

The J index also receives values between zero and one, as does the D index.

# The Problem of Measuring Power in Parliamentary Decision-Making

Defining and measuring 'power' are among the most elusive tasks of political science. It is difficult to conceive of a precise definition of power which would satisfy all political scientists, and it is almost as difficult to find even a satisfactory measure of power.

The earliest index of potential power in decision-making is that defined by Shapley and Shubik (1954). The simplest form of the Shapley power share  $\emptyset_i$  for the i-th party is

$$\emptyset_{i} = \frac{r_{i}}{n!} \tag{5}$$

where r<sub>i</sub> is the number of permutations in which the i-th party is pivotal in changing a minority into a majority, and n! is the number of all possible permutations of n parties. The index values add up to unity by definition. Although the Shapley value is primarily based on the permutations of the players, its coalition-theoretical interpretation is also evident. It can easily be shown that the Shapley index is defined also in terms of critical defections from winning coalitions (for details see Laakso 1978). The difference between the Shapley index and those presented by Banzhaf and

Coleman is that the former gives each critical defection of an actor a 'weight', while in the Banzhaf and Coleman indices each critical defection has an equal weight (Banzhaf 1965 and 1968, Coleman 1971).

All the measures presented so far have some unfavourable properties evaluated in measurement-theoretical investigations. Nurmi (1978) has shown that none of the indices mentioned above can be considered measures of power because they do not satisfy the condition of additivity. Compared to the Shapley and Banzhaf indices, the Coleman index has several disadvantageous features (see for details Nurmi 1978).

Which index should we choose? The measurement-theoretical analysis of power indices did not find any differences between the Shapley and Banzhaf indices, which both best fitted the criteria presented (Nurmi 1978). The choice must be based on other criteria. In defining electoral justice the role of the parliamentary decision rules becomes critical. What is our choice of index if this additional variable is taken into account? In an earlier study (Laakso 1978) I have shown that the Banzhaf index gives values against 'common sense' when considering different parties' situation regarding different decision rules. In contrast, the Shapley index does not behave in this paradoxical manner. Because the decision rules are in a very central position when measuring the electoral justice of different systems of P.R., the choice of the Shapley index is meaningful.

If the problem of measuring power is solved in the manner described how should we take the whole decision-making system into account? The most general decision rule is, of course, a simple majority. But this decision rule is not by far the most important. For example, in decisions concerning constitutional reform, the majority required in most countries exceeds a simple majority. It seems reasonable to include all decision rules in our model. Thus by applying the Shapley value, the index of power is defined as follows

$$p_{i} = \frac{1}{m} \sum_{i=1}^{n} \emptyset_{i}$$
 (6)

The index of decisional power is defined as a mean of the Shapley values in each decision rule in use. The numerical value of this new power index (pi) is without empirical meaning and it should only be used in testing the electoral justice of different systems of P.R. In turn, the Shapley values in a given decision rule are very informative in drawing conclusions about parties' potential capacity to influence decision-making (see Laakso 1975). Therefore, in this article only the simple majority was chosen to

represent all the other decision rules. Our index of power is thus defined

$$p_i = Q_i^{SM} \tag{7}$$

where  $\emptyset_i^{SM}$  is the power share for the i-th party on simple majority. Empirical values were also calculated using Equation (6), but there was no change in the interpretation of results.

### 3. The Hypotheses

The starting point illustrated in Figure 1 allows us to make comparisons either between electoral systems or between electoral rules of P.R. An electoral rule concerns the method of allocating seats, and is thus only one component of an electoral system.

The following empirical application is based on a comparison of different electoral rules. For this to be possible, all the other components of an electoral system must be kept constant. Returning to Figure 1, this means that the vote shares of the parties should also be constant (variables  $x_1$ , . . . ,  $x_n$  excluded from the analysis). The only changing variable is the electoral rule. We shall consider the impact of this variable on both the proportionality and the electoral justice of elections.

According to several studies, the 'proportionality order' of the most common electoral rules is (see e.g. Rae & Hanby and Loosemore 1971):

the simple quota rule (most proportional)

the Danish method

Sainte Laguë

the modified Sainte Laguë

d'Hondt

Imperial (most disproportional)

Theoretical analysis has shown that the quota rules are much more proportional than the number series methods (see e.g. Laakso 1979). In the 'family' of number series methods mentioned above, the Danish method is the most proportional. The Sainte Laguë rules applied in a modified form in the Scandinavian countries (Denmark, Norway, and Sweden) are more proportional than the d'Hondt method, which also has a wide application (Iceland, Finland, Belgium, etc.). On the basis of theoretical results obtained, we can present Hypothesis 1:

Hypothesis 1: The theoretical proportionality order of electoral rules should hold true also in real parliamentary elections.

Douglas W. Rae has shown in his famous study 'The political consequences of electoral laws' (1967) that all electoral systems tend to reduce the fractionalization of party systems. Therefore, according to Rae

$$F_v > F_s$$
 (8)

where  $F_v$  = party system fractionalization at the vote level and  $F_s$  = party system fractionalization at the seat level. The Fv and  $F_s$  indices used by Rae are calculated as follows

$$F_{v} = 1 - \sum_{i=1}^{n} v_{i}^{2}$$
 (9)

and

$$F_{S} = 1 - \sum_{i=1}^{n} S_{i}^{2}$$
 (10)

where vi and si are the vote and seat share of i-th party respectively.

Rae's indices have been applied widely. However, his measures are very difficult to interpret. Therefore, new measures have been presented which on the one hand are based on fractionalization indices, but which on the other hand are also reasonably simple to interpret. These new indices are called the effective number of parties and are symbolized by Nv (the vote level) and Ns (the seat level). The formulas for these indices are (Laakso and Taagepera 1979):

$$N_{v} = \begin{bmatrix} \sum_{i=1}^{n} v_{i}^{2} \end{bmatrix}^{-1} \tag{11}$$

and

$$N_{S} = \left[\sum_{i=1}^{n} S_{i}^{2}\right]^{-1}$$
 (12)

It is easily shown that N = 1/(1-F).

If we reformulate Rae's hypothesis we may state:

Hypothesis 2: The effective number of parties at the vote level should exceed the effective number of parties at the seat level.

#### Formally Nv>Ns

This hypothesis makes it possible to draw several conclusions also about the proportionality of elections. If elections have exact proportionality,  $N_v = N_s$ .

Many empirical applications based on power indices have shown that potential power tends to increase with an actor's size. The more representatives a party can gain, the more it can increase its power as compared to its seat share. Thus, it is obvious from earlier studies that:

Hypothesis 3: The effective number of parties at the seat level should exceed the effective number of parties at the potential power level.

Formally N<sub>s</sub>>N<sub>p</sub>

From Hypotheses 2 and 3 we can conclude that

$$N_v > N_s > N_p$$

The decrease of the effective number of parties should be dependent on the proportionality of different electoral methods.

On the basis of Hypotheses 2 and 3 we can also conclude:

Hypothesis 4: The electoral justice values (J) should exceed the index of proportionality (D).

The justice of electoral rules should depend on their proportionality.

The first part of Hypothesis 4 is self-evident, because potential power cumulates to large parties. The second part of Hypothesis 4 is in fact obvious on the basis of the hypotheses presented earlier. However, the nature of this dependency is in theory difficult to ascertain. Moreover, many studies have shown that the dependency of potential power is not a linear function of actor size (see e.g. Laakso 1975).

Hypotheses 1-4 allow us to make some conclusions about the proportionality and justice of electoral systems. Perfect proportionality requires that D = O. This result implies  $N_v = N_s$ . On the other hand, if  $N_v = N_s$ , the D index is not necessarily = O (compare Equations 11 and 12).

The requirement of perfect justice means that J = O. Naturally, this implies  $N_v = N_p$ . What is the relation between perfect proportionality and perfect justice? Are perfectly proportional elections also perfectly just? It is easily shown that if J = O, this implies that  $N_v \neq N_s$ ,  $N_s \neq N_p$ , and D > O. Let us assume perfect proportionality (D = O). Rhis implies that  $N_v = N_s$ . Because the power indices are based on coalitions of parties, the power share of parties is not equal to their seat share. Thus  $N_s \neq N_p$ . (The only exception is the situation in which all parties have the same seat share; this can hardly be expected in real situations.)  $N_v = N_s$  and  $N_s \neq N_p$  imply that J > O. We can therefore present the following conclusion: Perfect proportional elections are never perfectly just. The reverse also holds true.

The results presented above show that the criteria of proportionality and justice are different. Because potential power cumulates to large parties, perfect justice presupposes that  $N_v \le N_s$ . This implies that electoral methods give a bonus to small and middle-sized parties. The prerequisite for electoral justice is thus electoral disproportionality (D>O).

### 4. Empirical Application

The empirical application of this paper is focused on the study of different methods of P.R. regarding proportionality and justice of elections and the effective number of parties at the seat and the power share level. The empirical data cover Finnish parliamentary elections from 1945 to 1972. Finland applies the d'Hondt rule with electoral alliances in quite large constituencies. The Parliament has 200 representatives.

In addition to the d'Hondt methods, the Sainte Laguë and modified Sainte Laguë methods as well as Droop's quota rule are applied to the same election data. Thus the only variable which changes is the electoral rule. This makes it possible to draw conclusions about the proportionality and justice of different electoral methods. All the other components of the electoral system are thus kept constant (e.g. electoral district size, parliamentary size). The effective number of parties at the vote level (Nv) is constant.

The results of nine elections are presented in Table 1. According to the mean of the D index, the proportionality order of electoral rules is:

Sainte Laguë (most proportional)
mod. Sainte Laguë
Droop's quota
d'Hondt
d'Hondt without
electoral alliances (least proportional)

There is a slight discrepancy between our empirical and our theoretical results. Theoretical calculations have shown that the quota methods are more proportional than the number series methods (e.g. Laakso 1979). According to the electoral results in Table 1, Droop's quota method is more disproportional than the Sainte Laguë methods. The difference is, however, very small. The proportionality order presented above only holds true in four elections. There are slight differences in order between the Sainte Laguë, the modified Sainte Laguë, and Droop's quota rules in five elections (1951, 1958, 1962, 1966, 1972).

Table 1. The proportionality (D) and justice (J) of Finnish parliamentary elections from 1945 to 1972 applying different electoral methods to the same election data.

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	d'Hondt	d'Hondt without elect.all.	Sainte Laguë	modified Sainte Laguë	Droop's quota
1945	.042	.061	.034	.034	.034
1948	.045	.045	.024	.024	.034
1951	.023	.047	.019	.016	.017
1954	.032	.056	.028	.028	.031
1958	.038	.043	.018	.027	.025
1962	.068	.081	.031	.045	.040
1966	.045	.069	.024	.019	.019
1970	.056	.072	.026	.026	.032
1972	.043	.067	.022	.018	.027
mean	.044	.060	.025	.026	.029

the J index

	d'Hondt	d'Hondt without elect.all.	Sainte Laguë	modified Sainte Laguë	Droop's quota
1945	.158	.401	.190	.190	.160
1948	.303	.303	.270	.270	.303
1951	.226	.274	.172	.172	.226
1954	.194	.361	.194	.194	.194
1958	.181	.200	.158	.166	.166
1962	.248	.318	.158	.159	.154
1966	.246	.407	.148	.154	.154
1970	.174	.220	.124	.124	.124
1972	.122	.216	.106	.106	.106
mean	.206	.300	.169	.171	.176

According to the results, the elections analysed are generally quite proportional. The discrepancy between parties' vote and seat shares is between 2.5 and 6.0 per cent, depending on the electoral rule applied. The comparison of the Sainte Laguë methods and d'Hondt shows that the modification of the Sainte Laguë rule changes the proportionality of elections only slightly. This result contradicts our theoretical calculations which show that the modified St. Laguë lies clearly between the Sainte Laguë and d'Hondt rules. The increase of disproportionality is marked when applying d'Hondt, as electoral alliances are forbidden.

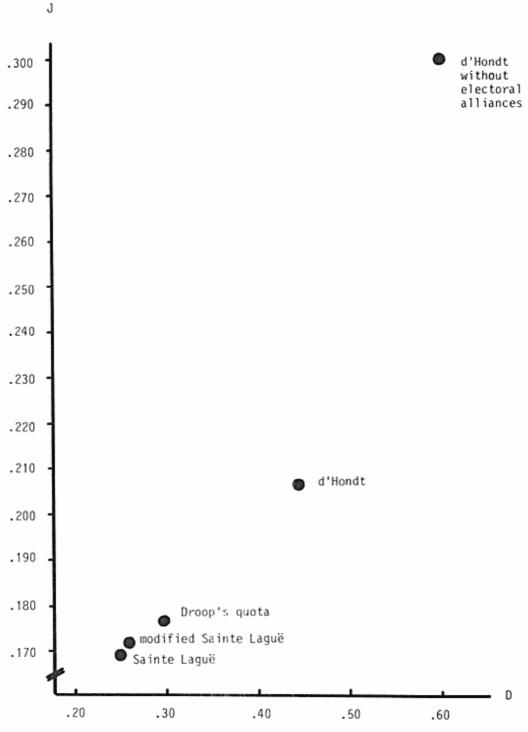


Figure 2. The J index as a function of the D index in Finnish parliamentary elections from 1945 to 1972 applying different electoral methods to the same election data (the mean values).

The justice of electoral rules shows the same order as that of proportionality. However, the J index is very large as compared to the proportionality index. The discrepancy between the potential power and the vote share of parties varies from 16.9 per cent (Sainte Laguë) to 30.0 per cent (d'Hondt without electoral alliances). The J index values are nearly the same for the Sainte Laguë, the modified Sainte Laguë, and Droop's quota rules.

In Figure 2 the J index is presented as a function of the D index. The relation is not linear as was expected, but curvilinear. This can be explained by the properties of electoral rules. The d'Hondt method strongly favours large parties. In the same way, the Shapley index (as well the other power indices) gives a large bonus to large parties. When these two transformation processes are combined, the results in Figure 2 are easy to understand.

Table 2 shows the effective number of parties at the vote, seat and power share level using different electoral rules. Hypotheses 2 and 3 led us to the following conclusion:  $N_v > N_s > N_p$ . The results obtained from our empirical election data confirm these hypotheses when the mean values of the effective number of parties are used. In single elections, however, there are exceptions from this general result. In the 1951 elections  $N_p$  even exceeds  $N_v$  with the Sainte Laguë rules.

In Figure 3 the mean values for the effective number of parties are presented. From the results we can observe that only d'Hondt rule without electoral alliances shows a linear decrease from  $N_V$  to  $N_p$ . However, the more proportional the electoral rule, the slighter is the decrease from  $N_S$  to  $N_p$  as compared to the change from  $N_V$  to  $N_S$ . This clearly demonstrates that the electoral rules more effectively reduce the effective number of parties than does the decision-making system in parliament.

The correlation between proportionality and justice on the one hand and the effective number of parties on the other hand seems to be negative. We can conclude that: The more proportional and just the election method, the greater the party system fragmentation. This result partly confirms the famous hypothesis originally presented by Mauruce Duverger (1967, 239) that:

It has been seen that simple-majority single ballot encourages the two-party system; on the contrary both the simple-majority system with second ballot and proportional representation favour multipartism.

This conclusion presented by Duverger can be stated in a slightly different

Table 2. The effective number of parties in Finnish parliamentary elections from 1945 to 1972 at the vote (N<sub>V</sub>), seat (N<sub>S</sub>) and power (N<sub>p</sub>) share level applying different electoral methods to the same election data.

N.			
	ı	v	

	N <sub>V</sub>	d'Hondt	d'Hondt without elect.all.	Sainte Laguë	modified Sainte Laguë	Droop's quota
1945	5.11	4.78	4.57	4.86	4.86	4.87
1948	4.90	4.54	4.54	4.68	4.68	4.64
1951	4.97	4.78	4.58	4.97	4.89	4.86
1954	5.01	4.71	4.53	4.85	4.81	4.72
1958	5.19	4.87	4.82	5.10	5.00	5.01
1962	5.79	5.09	4.89	5.55	5.45	5.49
1966	5.23	4.96	4.53	5.20	5.21	5.21
1970	6.12	5.56	5.42	5.91	5.91	5.89
1972	5.93	5.51	5.19	5.90	5.77	5.70
mean	5.36	4.98	4.79	5.22	5.18	5.15

 $N_p$  (simple majority)

	d'Hondt	d'Hondt without elect.all.	Sainte Laguë	modified Sainte Laguë	Droop's quota
1945	5.08	3.57	4.68	4.68	5.12
1948	4.29	4.29	4.34	4.34	4.28
1951	4.29	3.94	5.17	5.17	4.29
1954	4.29	3.57	4.29	4.29	4.29
1958	4.91	4.83	5.17	5.12	5.12
1962	4.88	4.42	5.71	5.73	5.64
1966	4.28	3.57	5.10	5.17	5.17
1970	5.36	5.18	5.68	5.68	5.68
1972	5.45	5.06	5.69	5.69	5.93
mean	4.76	4.27	5.09	5.10	5.03

form as follows: The more proportional an electoral system is, the more it increases the number of parties (especially at the seat level).

It is easy to understand that this conclusion is equal to the result obtained in this paper. However, Duverger presents no propositions about the justice of electoral systems.

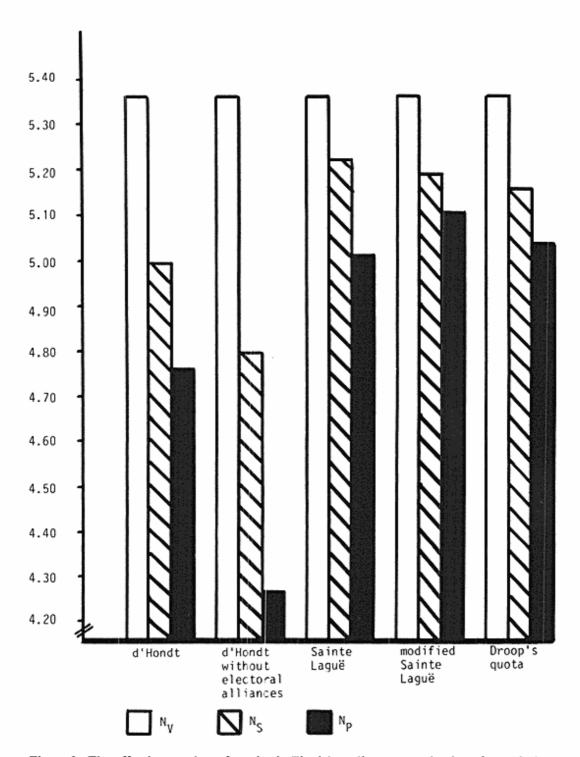


Figure 3. The effective number of parties in Finnish parliamentary elections from 1945 to 1972 at the vote (Nv), seat (Ns) and the power (Np) share level applying different electoral methods to the same election data (the mean values)

#### Conclusion

There is nowadays an increasing interest in the study of the properties of different electoral systems. Traditionally, the analysis of proportionality has received most attention. This article introduces the notion of electoral justice which is defined as the relationship between parties' votes and their power shares. Perfect justice presupposes that every party has a decisional power (measured by the Shapley index) equal to its vote share. On the basis of previous studies, four hypotheses are presented. The empirical data consist of Finnish parliamentary elections from 1945 to 1972. In addition to the d'Hondt rule which Finland has applied since 1906, the Sainte Laguë method, the modified Sanite Laguë method, and Droop's quota rule are applied to the same election data. The resultant constancy of the vote share of parties allows us to study proportionality, justice, and the effective number of parties at the seat and power share level when different electoral rules are used.

The empirical results concerning the order of proportionality differ slightly from those expected on the basis of theoretical calculations (Hypothesis 1). Droop's quota which in theory is more proportional than the Sainte Laguë rules appears to be slightly more disproportional empirically. The difference between these electoral methods, however, is very small. The order established with regard to proportionality (Sainte Laguë, modified Sainte Laguë, Droop's quota, d'Hondt) also holds true for the justice of electoral rules. However, the J index values (electoral justice) greatly exceed those measured by the D index (proportionality). In fact, this was to be expected, as decisional power cumulates very strongly to large parties (Hypothesis 4).

The effective number of parties at the vote, seat, and power share level is calculated. According to Rae's hypothesis it was assumed that electoral rules decrease the effective number of parties (Hypothesis 2). The results fit this hypothesis perfectly; the effective number of parties at the seat level ( $N_s$ ) is always smaller than at the vote share level, irrespective of the electoral rule applied. Because electoral systems generally tend to favour large parties, and because the decision-making system also increases this tendency, it was hypothesized that the effective number of parties at the seat level ( $N_s$ ) should exceed the effective number of parties at the power share level ( $N_p$ ) (hypothesis 3). The hypothesis holds true for mean values of  $N_s$  and  $N_p$ , but in single elections there are numerous exceptions to this general result. The correlation between the effective number of parties and the proportionality and justice of elections is negative. This result can

be interpreted in terms of Duverger's famous hypothesis that the ideal forms of representative democracy lead to greater party fragmentation.

While the analysis of different electoral systems has recently aroused wide interest among political scientists, since the publication of Rae's classical book there have been few empirical studies. In light of the considerable theoretical progress in this field in recent years, the time is perhaps ripe for directing attention to its empirical applications.

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Shapley, L. S. and Shubik, M. 1954. 'A Method of Evaluating the Distribution of Power in a Committee System, American Political Science Review 48, 787-792. be interpreted in terms of Duverger's famous hypothesis that the ideal forms of representative democracy lead to greater party fragmentation.

While the analysis of different electoral systems has recently aroused wide interest among political scientists, since the publication of Rae's classical book there have been few empirical studies. In light of the considerable theoretical progress in this field in recent years, the time is perhaps ripe for directing attention to its empirical applications.

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