Riker's 'Size Principle' and Its Application to Finnish Roll-Call Data

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1. Introduction

To base political research on the theory of games is a very recent idea. Pioneer work in this field has been carried out by William H. Riker, whose work *The Theory of Political Coalitions* from 1962 is the first coherent attempt to find a game-theoretical explanation of politically important coalition formation. The most significant result of this research is the 'size principle', which describes political decision-making as follows:

> In social situations similar to *n*-person, zero-sum games with side-payments, participants create coalitions just as large as they believe will ensure winning and no larger.[1]

This basic principle means that the size of a coalition is exclusively determined by the player's estimate of whether his course of action is winning or not. If there is a situation of complete and perfect information (i.e. the players know the potential influence of each other from the point of view of the outcome and the choices of the other players), this leads to formation of minimum winning coalitions. The implied additional assumption in this argument is naturally that the players are rational – that they choose the alternative that leads to the outcome they prefer most.

Riker presents two additional principles: the 'strategic principle' and the 'disequilibrium principle'. The strategic principle is derived from the size principle, and the disequilibrium principle from the former two principles. Thus the basis of his coalition analysis forms a deductive system we can test by testing only the size principle.

Riker's theory has been criticized on the grounds that his simplified assumptions (particularly the zero-sum condition) have made its application to the real world difficult.[2] The intention of this paper is not, however, to examine the basis of Riker's theory but to extend its basic tenet, i.e. the size principle of a winning coalition and to apply this to the decision-making mechanism of a multi-party system and also

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2. A Set-Theoretical Description of a Multi-Party System

Consider a given party $\mathrm{A_j}$ in a decision-making situation in parliament. Its MP's have three decision alternatives (only MP's actually present are being observed): i) to vote for the motion, ii) to vote against the motion, or iii) to abstain from voting.

In order to find a more sensible starting point for set-theoretical concepts, we can examine the situation in another way. For one thing we know that some of the party's MP's might be absent from the voting. Absent MP's of party $\mathrm{A_j}$ are assigned the symbol $\mathrm{A_{Pj}}$. Therefore the number of its MP's in the voting situation is naturally $\mathrm{A'_j = A_j - A_{Pj}}$. On the grounds of the MP's choice, $\mathrm{A'_j}$ can further be divided into three exclusive sets:

a. The majority of the party: A_j (more than half its MP's),
b. The minority of the party: $\bar{A}_j$ ('inner complement') (less than half its MP's),
c. The abstainers of the party: $\mathrm{A_{Bj}}$ (those who were present but did not vote).

Definitions a and b correspond to decision alternatives i and ii, and definition c is equivalent to decision alternative iii. Thus we get the following definitions:

Definition 1: A given party $\mathrm{A_j}$ can be divided in a given voting situation on the basis of the choice of its MP's into four subsets as follows:

$\mathrm{A_j} = A_j \cup \bar{A}_j \cup \mathrm{A_{Bj}} \cup \mathrm{A_{Pj}} = \mathrm{A'_j} \cup \mathrm{A_{Pj}}$ and $A_j \cap \bar{A}_j \cap \mathrm{A_{Bj}} \cap \mathrm{A_{Pj}} = \phi$.[3]

We can expand the definition to a case where there are r parties $(\mathrm{A_1}, \ldots, \mathrm{A_r})$. We get the description shown in Table I.

Table I. A Set-Theoretical Description of a Multi-Party System in a Voting Situation

		Corresponding union sets
The majority of the party	$A_1, A_2, \ldots, A_j, \ldots, A_r$	$\bigcup_{i=1}^{r} A_i$
The 'inner complement' of the party	$\bar{A}_1, \bar{A}_2, \ldots, \bar{A}_j, \ldots, \bar{A}_r$	$\bigcup_{l=1}^{r} \bar{A}_l$
The abstainers of the party	$\mathrm{A_{B1}, A_{B2}, \ldots, A_{Bj}, \ldots, A_{Br}}$	$\bigcup_{m=1}^{r} \mathrm{A_{Bm}}$
The absentees of the party	$\mathrm{A_{P1}, A_{P2}, \ldots, A_{Pj}, \ldots, A_{Pr}}$	$\bigcup_{n=1}^{r} \mathrm{A_{Pn}}$
The party as a whole	$\mathrm{A_1, A_2, \ldots, A_j, \ldots, A_r}$	$\bigcup_{p=1}^{r} \mathrm{A_p}$

In Table I the rows show how the parties divide into subsets, and columns the corresponding subsets of the parties.

On the basis of definition 1 above we can state that

$$\bigcup_{p=1}^{r} A_p = (\bigcup_{l=1}^{r} A_l) \cup (\bigcup_{l=1}^{r} \bar{A}_l) \cup (\bigcup_{m=1}^{r} A_{Bm}) \cup (\bigcup_{n=1}^{r} A_{Pn}),$$

where

$$(\bigcup_{l=1}^{r} A_l) \cap (\bigcup_{l=1}^{r} \bar{A}_l) \cap (\bigcup_{m=1}^{r} A_{Bm}) \cap (\bigcup_{n=1}^{r} A_{Pn}) = \phi.$$

We can observe that the number of all possible subsets of the parties is $4 \cdot r$, where r is the number of parties. For example, if there are eight parties, there are althogether 32 subsets to be considered in the analysis.

The concepts of winning and losing coalitions have a central role in Riker's theory. The set of all winning coalitions, W, can – in the case of a parliamentary roll-call analysis – be defined as a set of coalitions that determine for the whole representative body the policies in given issues. Similarly the set of losing coalitions, L, can be defined as those coalitions that cannot determine the policies of the representative body. Operationally they naturally are defined as the class of those coalitions that have won in some roll-call vote and the class of those coalitions that have lost in some roll-call vote.

In addition to W and L we find two other potential types of coalitions in analyzing the roll-call vote. These are the coalitions of those who abstain and those who are absent. We can symbolize the corresponding sets by B and P. From Table I we can read that $B_i = \bigcup_{m=1}^{r} A_{Bm}$ and $P_i = \bigcup_{n=1}^{r} A_{Bn}$. Furthermore, we can see that the union set of a given winning coalition, W_i, and a given losing coalition, L_i, consists of the elements of the union set $(\bigcup_{l=1}^{r} A_l) \cup (\bigcup_{l=1}^{r} \bar{A}_l)$. In a given roll-call situation this corresponds to the distribution of ayes and nays. We can now define the winning and losing coalition in a multi-party system.

Definition 2: The winning coalition, W_i, is a coalition that determines the policy of the representative body and consists of the majorities of parties l,..., k and the inner complements of the parties k+l,..., r, when the number of the parties is r. Formulated in symbols, that is

$$W_i = (\bigcup_{l=1}^{k} A_l) \cup (\bigcup_{l=k+1}^{r} \bar{A}_l).$$

Definition 3: The losing coalition, L_i, is a coalition that cannot determine the policies of the representative body and consists of the inner complements of the parties l,..., k and the majorities of the parties k+l,..., r. Or, in symbols:

$$L_i = (\bigcup_{l=k+1}^{r} A_l) \cup (\bigcup_{l=1}^{k} \bar{A}_l).$$

Definitions 2 and 3 are easy to understand when we observe decision-making at the party level. According to definition 1, a faction of MP's in party A_j can oppose the majority choice. If, for example, A_j's majority A_j belongs to the winning coalition this party's 'inner complement' $\bar{A}_j$ therefore belongs to the losing coalition, because A_j and $\bar{A}_j$ cannot take the same side in the voting. Taking this into consideration in the case of each party we arrive at the above definitions 2 and 3, according to which we can further conclude that $W_i \cap L_i = \phi$.

According to the definitions of winning and losing coalitions we can now arrive at a useful concept of coalition structure:

Definition 4: Coalition structure, CS_i,[4] is a union set formed from the winning and the losing coalition.[5] This can be formulated as

$$CS_i = W_i \cup L_i.$$

3. Riker's Size Principle in a Multi-Party System

If we can consider a roll-call vote an example of the type of situation where Riker's basic assumption holds, we can expect that the coalitions formed there would be minimum winning coalitions. Riker defines a minimum winning coalition as follows:

> Let S_p be a set and $S_p \in W^{min}$ (W^{min}= a set of all minimum winning coalitions), then $(S_p - 1) \notin W^{min}$.

According to this definition, subtracting even one member from a winning coalition means that it ceases to be winning. If we assign to a winning coalition the symbol W_i and to the corresponding minimum winning coalition the symbol W_i^{min}, then the size of a winning coalition is defined by the parameter $F = W_i - W_i^{min}$, which shows how near the winning coalition is to a minimum one.

If we inspect a set of aye and nay votes-union set $(\bigcup_{j=1}^{r} A_j) \cup (\bigcup_{j=1}^{r} \bar{A}_j)$ and assume that the total number of votes is n, if n is an even number, the minimum winning coalition, according to simple majority rule, is

$$W_i^{min} = \frac{n}{2} + 1. \quad (1)$$

If n in turn is an odd number the minimum winning coalition is

$$W_i^{min} = \frac{n+1}{2}. \quad (2)$$

From these definitions we then get equivalent F values

$$F = W_i - W_i^{min} = \frac{W_i - L_i}{2} - 1 \quad (1)$$

(2) $$F = W_i - W_i^{min} = \frac{W_i - L_i - 1}{2}.$$

(In calculating F we have taken the condition $W_i + L_i = n$ into consideration.)

According to Riker's hypothesis, F's value should be zero if the size of a winning coalition is to be minimal. This is the case in (1) when $W_i - L_i = 2$ and in (2) when $W_i - L_i = 1$.[6]

It was implicitly assumed above that an individual MP is an independent decision-making unit. As we know from many studies concerning representative assemblies, this assumption is very unrealistic.[7] It is more reasonable to take the party as the primary decision-making unit. Then we can take into consideration the party's 'cohesion' in a decision situation, that is, its propensity to act unanimously in roll-call votes. Before defining any measures for F it is necessary first to explain the set-theoretical basis of parliamentary voting behavior.

If we take a given CS_i, we can distinguish between the theoretical and empirical values, W_i^{theor} and W_i^{emp}, and L_i^{theor} and L_i^{emp} of W_i and L_i. Similarly we can define, for a given CS_i, the values F^{theor} and F^{emp} (we take here only the case when n is even):

$$F_i^{theor} = \frac{W_i^{theor} - L_i^{theor}}{2} - 1 \text{ and } F_i^{emp} = \frac{W_i^{emp} - L_i^{emp}}{2} - 1.$$

W_i^{theor} and L_i^{theor} can be determined in a given CS_i when $\bigcup_{l=1}^{r} \bar{A}_l = \phi$, $\bigcup_{m=1}^{r} A_{Bm} = \phi$, and $\bigcup_{n=1}^{r} A_{Pn} = \phi$, i.e. when we do not consider those who either oppose the majority of the party, abstain, or are absent. The set-theoretical conditions for determining F^{theor} in a given CS_i are given in Table II.

Table II. Set-Theoretical Conditions Determining F_i^{theor} in a Given Coalition Structure CS_i

		Corresponding union sets
The majority of the party	$A_1 = A_1, \ldots, A_j = A_j, \ldots, A_r = A_r$	$\bigcup_{l=1}^{r} A_l = \bigcup_{p=1}^{r} A_p$
The inner complement of the party	$\bar{A}_1 = \phi, \ldots, \bar{A}_j = \phi, \ldots, \bar{A}_r = \phi$	$\bigcup_{l=1}^{r} \bar{A}_l = \phi$
The abstainers of the party	$A_{Bj} = \phi, \ldots, A_{Bj} = \phi, \ldots, A_{Br} = \phi$	$\bigcup_{m=1}^{r} A_{Bm} = \phi$
The absentees of the party	$A_{P1} = \phi, \ldots, A_{Pj} = \phi, \ldots, A_{Pr} = \phi$	$\bigcup_{n=1}^{r} A_{Pn} = \phi$
The party as a whole	$A_1, \ldots, A_j, \ldots, A_r$	$\bigcup_{p=1}^{r} A_p$

A party's F_i^{theor} in CS_i is thus determined according to its number of seats in parliament. We can thus calculate a priori, for any possible coalition structure, a theoretical parameter expressing the size of a winning coalition. If the total number of parties is r, the number of coalition structures formed from these parties is 2^{r-1}. If we examine the determining of F^{emp} in the same coalition structure CS_i, we cannot be certain that $\bigcup_{l=1}^{r} \bar{A}_l$, $\bigcup_{m=1}^{r} A_{Bm}$, and $\bigcup_{n=1}^{r} A_{Pn}$ are necessarily empty sets, although it is empirically possible. For example, in the Finnish Parliament there has rarely existed a roll-call vote where no MP of any party has either voted against the party majority, abstained from voting, or been absent. Thus we can give the set-theoretical conditions that determine F_i^{emp}'s value in the coalition structure CS_i:

$$\bigcup_{l=1}^{r} \bar{A}_l \geqslant \phi, \ \bigcup_{m=1}^{r} A_{Bm} \geqslant \phi, \text{ and } \bigcup_{n=1}^{r} A_{Pn} \geqslant \phi.$$

From this definition it can be seen immediately that if the sets above are empty in CS_i, then $F_i^{theor} = F_i^{emp}$.

Next we must find a relation that connects W_i^{emp} to W_i^{theor} and similarly L_i^{emp} to L_i^{theor}.

Let us assume that parties in the winning coalition are 1, . . ., k, their absent MP's union set is P_{Wi}, their abstaining MP's union set is B_{Wi}, and the union set of their inner complements is $\bar{A}_{Wi}$, so that $W_i^{emp} = W_i^{theor} - P_{Wi} - B_{Wi} - \bar{A}_{Wi} + \bar{A}_{Li}$.

This relation can be understood so that sets P_{Wi}, B_{Wi}, and A_{Wi} reduce a theoretical winning coalition. From definition 2 above we know that A_{Li} increases W_i^{emp}'s value. If the equivalent subsets of a losing coalition are assigned the symbols P_{Li}, B_{Li}, and A_{Li}, there exists analogously the relation

$$L_i^{emp} = L_i^{theor} - P_{Li} - B_{Li} - \bar{A}_{Li} + \bar{A}_{Wi}.$$

On the basis of the above formulations we can get the formula for calculating F_i^{emp}:

$$F_i^{emp} = \frac{W_i^{emp} - L_i^{emp}}{2} - 1$$

$$= \frac{(W_i^{theor} - P_{Wi} - B_{Wi} - \bar{A}_{Wi} + \bar{A}_{Li}) - (L_i^{theor} - P_{Li} - B_{Li} - \bar{A}_{Li} + \bar{A}_{Wi})}{2} - 1$$

$$= \frac{W_i^{theor} - L_i^{theor} - [(P_{Wi} - P_{Li}) + (B_{Wi} - B_{Li}) + 2(\bar{A}_{Wi} - \bar{A}_{Li})]}{2} - 1.$$

From the formula we can easily see that F_i^{emp} values can vary much within a given coalition structure CS_i (F_i^{theor} being always constant), because F_i^{emp} is the function of

$$\bigcup_{l=1}^{r} \bar{A}_l, \ \bigcup_{m=1}^{r} A_{Bm}, \text{ and } \bigcup_{n=1}^{r} A_{Pn}.$$

In order to test Riker's theory within a given coalition structure CS_i, we can define $\Delta F_i = F_i^{theor} - F_i^{emp}$. ΔF_i can thus be calculated with the formula

$$\Delta F_i = F_i^{theor} - F_i^{emp} = \frac{(P_{Wi} - P_{Li}) + (B_{Wi} - B_{Li}) + 2\,(\bar{A}_{Wi} - \bar{A}_{Li})}{2}.$$

The bigger P_{Wi}, B_{Wi}, and $\bar{A}_{Wi}$ are when compared with P_{Li}, B_{Li}, and $\bar{A}_{Li}$, the smaller F_i^{emp} is when compared with F_i^{theor}. $\Delta F_i = 0$ only when

$$P_{Wi} + B_{Wi} + 2\,\bar{A}_{Wi} = P_{Li} + B_{Li} + 2\,\bar{A}_{Li}.$$

If $F_i^{theor} > F_i^{emp}$, then $\Delta F_i > 0$, and the size of winning coalition follows Riker's theory because $\Delta F_i > 0$ expresses a tendency toward a minimum winning coalition. If $F_i^{theor} < F_i^{emp}$, then $\Delta F_i < 0$, and the empirical coalition is bigger than could be theoretically expected. The size of a winning coalition approaches the maximum winning coalition, and the result is contradictory to Riker's theory.

The elucidation of the set-theoretical analysis of parliamentary voting behavior is important in order to extend Riker's theory. The size principle can be extended within this frame of reference, as we have indicated above, to the analysis of the internal variations of different coalition structures, i.e. it is applicable to the analysis of multi-party systems. It is interesting to note that a two-party system is a special case of this analysis. There the sets P_{Wi}, B_{Wi}, $\bar{A}_{Wi}$ and P_{Li}, B_{Li}, $\bar{A}_{Li}$ describe directly the subsets of majority and minority parties in a voting situation.

4. An Empirical Application

The above extension of Riker's analysis for the study of multi-party systems was tested with Finnish roll-call vote data. For the analysis 450 roll-call votes in the Finnish one-chamber Parliament were sampled from the period of 1964–1966. Because Finland had three different Cabinets during this time, the sample was stratified so that there were 150 roll-call votes from the period of each Cabinet. This stratification allows Riker's principle to be tested under different conditions, because the Cabinets differed considerably in their composition. Thus the first (Prime Minister Lehto) was a party politically neutral, so-called 'civil servant Cabinet', the second (Prime Minister Virolainen) a bourgeois coalition Cabinet, and the last (Prime Minister Paasio) a socialist majority Cabinet.

We have in Tables III and IV the coalition structures that appeared in roll-call votes during these Cabinets.[8] As we mentioned above r political parties can be divided in 2^{r-1} ways into two opposing coalitions (the winning and the losing). As there are eight parties in the period studied, the number of potential coalition structures is $2^{8-1} = 128$. From the period of Lehto's Cabinet we find 29 coalition structures (22.8 percent of the potential maximum), from Virolainen's period 23 (18.0 percent of the maximum), and from Paasio's period 31 (24.3 percent of the maximum). This indicates that the coalition structures (which coalitions 'can be

Table III. Coalition Structures and Their Frequencies during Lehto's and Virolainen's Cabinets

	CS_1	CS_2	CS_3	CS_4	CS_5	CS_6	CS_7	CS_8	CS_9	CS_{10}	CS_{11}	CS_{12}
AP	W	W	W	W	W	W	W	L	W	W	L	L
NCP	W	W	W	W	L	W	W	W	L	W	L	L
LI	W	W	L	L	L	W	W	L	L	W	W	L
SPP	W	W	L	W	W	W	L	W	L	W	W	L
FPP	W	W	W	W	L	W	L	W	L	W	L	W
DLPF	L	L	L	L	W	W	W	W	W	W	W	W
SDP	L	W	L	L	W	W	L	W	W	W	W	W
SUWS	L	L	L	L	W	W	W	W	W	L	W	W
Lehto's Cabinet	49	33	1	11	5	3	1	1	4	1	1	4
Virolainen's Cabinet	71	21	–	20	–	1	–	1	2	–	1	2

	CS_{13}	CS_{14}	CS_{15}	CS_{16}	CS_{17}	CS_{18}	CS_{19}	CS_{20}	CS_{21}	CS_{22}	CS_{23}	CS_{24}
AP	L	W	W	W	W	L	L	W	W	W	W	W
NCP	L	L	W	L	W	W	W	W	W	L	W	W
LI	L	L	W	W	W	W	W	W	L	W	L	L
SPP	W	W	W	W	L	L	W	W	W	L	W	W
FPP	L	W	W	W	W	W	W	L	W	W	L	W
DLPF	W	W	L	W	L	L	W	L	W	W	L	L
SDP	W	W	W	W	W	W	W	W	L	W	L	W
SUWS	W	W	W	W	L	L	W	L	W	W	L	L
Lehto's Cabinet	1	2	11	1	1	3	4	1	1	1	3	1
Virolainen's Cabinet	–	–	9	2	1	–	3	–	–	–	2	–

	CS_{25}	CS_{26}	CS_{27}	CS_{28}	CS_{29}	CS_{30}	CS_{31}	CS_{32}	CS_{33}	CS_{34}
AP	L	L	W	W	W	W	W	W	L	L
NCP	L	L	L	W	W	W	L	W	W	W
LI	W	L	L	L	W	L	W	W	W	–
SPP	W	L	L	L	W	W	L	L	L	W
FPP	W	L	W	W	L	W	L	W	W	W
DLPF	W	W	W	W	L	W	W	W	L	W
SDP	W	W	W	W	L	W	L	W	W	W
SUWS	W	L	W	W	W	W	W	W	W	L
Lehto's Cabinet	1	1	2	1	1	–	–	–	–	–
Virolainens' Cabinet	–	1	1	1	–	6	2	1	1	1

W=party belonged to winning coalition; L=party belonged to losing coalition.

Abbreviations for Tables III and IV:
AP=Agrarian Party
CP=Center Party (former AP)
DLPF=Democratic League of the People of Finland
FPP=Finnish People's Party
FRP=Finnish Rural Party
LI=Liberal Independent
LPP=Liberal People's Party
NCP=National Coalition Party
SDP=Social Democratic Party
SPP=Swedish People's Party
SUWS=Social Democratic Union of Workers and Small Farmers

made') is normatively controlled. The same is reflected in the fact that only few coalition structures appear more frequently. The impact of the Cabinet coalition on the frequency of the roll-call coalitions is naturally significant. During Virolainen's Cabinet the correspondence was 13.3 percent (CS_4) and during Paasio's Cabinet it was 40.0 percent (CS'_3). We can see that the Cabinet coalition does not alone determine the roll-call coalitions.[9]

In order to test Riker's size principle, all $\Delta F_i = F_i^{theor} - F_i^{emp}$ for the 450 roll-call votes were calculated. These are given in Table V where we can see that F_i^{theor} is greater than F_i^{emp} because ΔF_i is most often positive. This is significant because it holds for the period of all three Cabinets. Negative ΔF_i values (that contradict Riker's theory) can be found on only a few occasions in different coalition structures: eight during Lehto's Cabinet, seven during Virolainen's Cabinet, and five during Paasio's Cabinet. Generally these negative values appear in coalition structures which are uncommon. Thus in Lehto's period the range of the presence of these coalition structures was 1–4, in Virolainen's period 1–2, and during Paasio's period 1–11. Because the frequencies of these coalition structures are generally close to one, one cannot consider the appearance of negative ΔF_i in them as strong evidence against Riker's theory. Only the frequency of CS'_1 (11) during Paasio's period and the negative ΔF_i can be considered a significant deviation from Riker's size principle. Because ΔF_i^{theor} in this coalition structure is only three, we can explain this exception by

Table IV. Coalition Structures and Their Frequencies during Paasio's Cabinet

	CS'_1	CS'_2	CS'_3	CS'_4	CS'_5	CS'_6	CS'_7	CS'_8	CS'_9	CS'_{10}	CS'_{11}	CS'_{12}	CS'_{13}	CS'_{14}	CS'_{15}	CS'_{16}
CP	L	L	W	W	W	W	W	L	L	B*	L	W	W	W	L	W
NCP	L	L	L	W	W	L	L	L	L	B*	W	L	L	L	W	W
FRP	L	W	L	L	L	L	W	W	W	B*	W	W	–	W	L	W
SPP	L	L	L	W	W	L	L	W	L	B*	L	W	L	W	L	W
LPP	L	W	L	W	W	W	L	L	L	B*	W	W	L	L	W	W
DLPF	W	W	W	L	W	W	W	W	W	B*	W	W	W	W	W	W
SDP	W	W	W	W	W	W	W	W	W	B*	W	W	W	W	W	W
SUWS	W	W	W	L	L	W	W	W	W	B*	W	W	W	W	W	W
Freq.	11	1	60	1	1	1	13	1	2	1	2	2	5	9	2	6

	CS'_{17}	CS'_{18}	CS'_{19}	CS'_{20}	CS'_{21}	CS'_{22}	CS'_{23}	CS'_{24}	CS'_{25}	CS'_{26}	CS'_{27}	CS'_{28}	CS'_{29}	CS'_{30}	CS'_{31}
CP	W	L	L	W	W	W	W	W	W	L	L	L	W	W	L
NCP	L	L	W	W	W	W	L	W	W	W	W	W	L	W	W
FRP	L	L	W	W	W	L	W	W	–	L	L	W	L	L	L
SPP	W	W	W	W	W	W	L	L	L	L	W	W	W	L	W
LPP	L	L	L	L	W	W	L	W	W	L	W	W	W	W	L
DLPF	W	W	W	W	L	W	W	W	W	W	W	W	W	W	W
SDP	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
SUWS	W	W	W	W	W	W	W	W	W	W	W	W	L	L	W
Freq.	5	1	1	1	3	7	2	4	1	1	1	1	1	2	1

B*=party belonged to blocking coalition (i.e. there was a tie vote); W=party belonged to winning coalition; L=party belonged to losing coalition.

resorting to Riker's 'information principle'. The parties that belong to the winning coalition have minimized the risk of losing the vote by taking care that the F_i^{emp} stays persistently higher than its expected value. As we remember Riker assumes that W_i^{emp} tends toward minimum because the actors try to maximize their own benefits, but this rationality has greater predictive power in situations where information is more perfect than in situations of uncertainty. We can use the same 'information principle' to explain the negative ΔF_i values in the cases CS_{11}, CS_{12}, and CS_{23} during Lehto's and Virolainen's Cabinets and CS'_9 during Paasio's Cabinet.

When ΔF_i is positive this means that $P_{Wi}+B_{Wi}+2\bar{A}_{Wi}>P_{Li}+B_{Li}+2\bar{A}_{Li}$. The decreasing of F_i^{emp} toward the minimum winning coalition increases the benefits of the MP's that belong to W_i^{emp}, according to Riker's theory. Furthermore MP's,

Table V. ΔF_i Values in Different Coalition Structures puring Lehto's, Virolainen's, and Paasio's Cabinets

	Lehto's Cabinet			Virolainen's Cabinet			Paasio's Cabinet			
CS_i	F_i^{theor}	F_i^{emp}	ΔF_i	F_i^{theor}	F_i^{emp}	ΔF_i	CS'_i	F_i^{theor}	F_i^{emp}	ΔF_i
CS_1	12	10.7	1.3	12	11.0	1.0	CS'_1	3	8.3	— 5.5
CS_2	50	35.7	14.3	50	37.5	12.5	CS'_2	12	11.0	1.0
CS_3	2	4.0	— 2.0	2	-	-	CS'_3	52	40.2	11.8
CS_4	11	7.8	3.2	11	10.5	0.5	CS'_4	50	24.0	26.0
CS_5	53	41.0	12.0	53	-	-	CS'_5	91	59.0	32.0
CS_6	99	81.3	17.7	99	85.0	14.0	CS'_6	60	55.0	5.0
CS_7	34	27.0	7.0	34	-	-	CS'_7	53	46.2	6.8
CS_8	46	32.0	14.0	46	7.0	39.0	CS'_8	16	14.0	2.0
CS_9	39	35.3	3.7	39	17.5	21.5	CS'_9	4	14.0	—10.0
CS_{10}	97	59.0	38.0	97	-	-	CS'_{10}	3	1.0	2.0
CS_{11}	2	20.0	—18.0	2	7.0	— 5.0	CS'_{11}	38	25.5	12.5
CS_{12}	0	6.8	— 6.8	0	9.5	— 9.5	CS'_{12}	73	56.5	16.5
CS_{13}	1	7.0	— 6.0	1	-	-	CS'_{13}	52	47.0	5.0
CS_{14}	66	35.0	31.0	66	-	-	CS'_{14}	65	53.7	11.3
CS_{15}	52	38.0	14.0	52	37.0	15.0	CS'_{15}	37	18.5	18.5
CS_{16}	67	18.0	49.0	67	47.5	19.5	CS'_{16}	99	67.3	31.7
CS_{17}	36	33.0	3.0	36	19.0	17.0	CS'_{17}	64	43.4	20.6
CS_{18}	15	26.3	—11.3	15	-	-	CS'_{18}	15	27.0	—12.0
CS_{19}	47	37.0	10.0	47	45.7	1.3	CS'_{19}	42	42.0	0.0
CS_{20}	37	23.0	14.0	37	-	-	CS'_{20}	91	68.0	23.0
CS_{21}	60	59.0	1.0	60	-	-	CS'_{21}	58	61.4	— 3.4
CS_{22}	53	28.0	25.0	53	-	-	CS'_{22}	98	72.0	26.0
CS_{23}	1	3.3	— 2.3	1	10.0	— 9.0	CS'_{23}	53	54.5	— 1.5
CS_{24}	49	42.0	7.0	49	-	-	CS'_{24}	87	69.5	17.5
CS_{25}	15	19.0	— 4.0	15	-	-	CS'_{25}	87	68.0	19.0
CS_{26}	14	18.0	— 4.0	14	15.0	— 1.0	CS'_{26}	29	26.0	3.0
CS_{27}	52	42.5	9.5	52	55.0	— 3.0	CS'_{27}	49	40.0	9.0
CS_{28}	84	36.0	48.0	84	73.0	11.0	CS'_{28}	50	40.0	10.0
CS_{20}	48	29.0	19.0	48	-	-	CS'_{29}	65	65.0	0.0
CS_{30}	-	-	-	98	61.7	36.3	CS'_{30}	79	60.0	19.0
CS_{31}	-	-	-	2	21.5	—19.5	CS'_{31}	41	27.0	14.0
CS_{32}	-	-	-	85	62.0	23.0				
CS_{33}	-	-	-	13	15.0	— 2.0				
CS_{34}	-	-	-	45	22.0	23.0				

by belonging to P_{Wi} and B_{Wi} can avoid the responsibility of the consequences of the winning decision. The analogous interpretation can be made about the subsets of P_{Li} and B_{Li}. However, when the ΔF_i is positive this trend is greater in the winning coalition. The subsets of $\bar{A}_{Wi}$ and $\bar{A}_{Li}$ reflect the willingness of certain MP's to maximize their own benefits by opposing the majority of their parties. This may under certain conditions also be useful for their parties: the flexibility in the discipline in this respect may pay back in the following elections in terms of more votes.[10] Because $\Delta F_i > 0$, the maximization of the individual benefits of MP's takes place more often when they belong to the winning coalition.

To sum up, we can say that the results of our analysis within given coalition structures are very closely congruent with Riker's coalition theory, although the tendency to form minimum winning coalitions can be due to other factors than Riker assumes.

5. Conclusions

The set-theoretical description above aims at providing a logical foundation for the analysis of parliamentary voting behavior. We have also formulated it to give a conceptual frame within which one can extend Riker's theory of coalition behavior and test it reliably with roll-call data. The results support Riker's theory (its size principle) but also lead us to look for an underlying causal mechanism rather than to assume individual rationality, which Riker bases his theory upon.

The parameter ΔF_i used to test Riker's theory indicates the deviation of the observed empirical F-value from the theoretical F-value. We can still improve this measure by changing it from absolute to relative. This naturally will change the content of Riker's theory, which after this change could be stated as follows: In voting situations similar to n-person zero-sum games with side-payments in a given coalition structure CS_i, the relative size of W_i^{emp} to W_i^{theor} is smaller than the relative size of the losing coalition L_i^{emp} to L_i^{theor}.

If we inspect the winning coalition W_i, the deviation of W_i^{emp} from the W_i^{theor} is caused by the subsets of P_{Wi}, B_{Wi}, $\bar{A}_{Wi}$, and $\bar{A}_{Li}$. In the same way the deviation of L_i^{emp} from L_i^{theor} is determined by P_{Li}, B_{Li}, $\bar{A}_{Li}$, and $\bar{A}_{Wi}$. The greater the subsets P_{Wi}, B_{Wi}, $\bar{A}_{Wi}$ are and the smaller the subset $\bar{A}_{Li}$ is, the closer W_i^{emp} is to W_i^{min}.[11] In the relative measure one should not take $\bar{A}_{Li}$ into consideration, because it boosts the size of the winning coalition irrespective of the will of its members.[12] If we divide the union set of $(P_{Wi}+B_{Wi}+\bar{A}_{Wi})$ with W_i^{theor} and multiply the result by 100 we get the amount that these subsets decrease the winning coalition as a percentage. The same can be done in case of the losing coalition. Thus we will get the parameter $\Delta F_i^{\oplus}$ that measures the relative size of the winning coalition.

$$\Delta F_i^{\oplus} = 100 \left(\frac{P_{Wi} + B_{Wi} + \bar{A}_{Wi}}{W_i^{theor}} - \frac{P_{Li} + B_{Li} + \bar{A}_{Li}}{L_i^{theor}} \right).$$

If we take into consideration the equations $W_i^{emp} = W_i^{theor} - P_{Wi} - B_{Wi} - \bar{A}_{Wi} + \bar{A}_{Li}$ and $L_i^{emp} = L_i^{theor} - P_{Li} - B_{Li} - \bar{A}_{Li} + \bar{A}_{Wi}$ we will get

$$\Delta F^{\oplus} = 100 \left(\frac{L_i^{emp} - \bar{A}_{Wi}}{L_i^{theor}} - \frac{W_i^{emp} - \bar{A}_{Li}}{W_i^{theor}} \right).$$

$\Delta F_i^{\oplus}$ has been derived so that when $\Delta F_i^{\oplus} > 0$, the results support the above 'improved' Riker's hypothesis. We have not calculated the parameter values according to this measure but it seems to be necessary to develop Riker's analysis in this direction in further studies that apply it to parliamentary decision-making.

NOTES

1. W. H. Riker, *The Theory of Political Coalitions*, New Haven and London: Yale University Press, 1962, pp. 32–33.
2. Cf. C. Adrian and C. Press, 'Decision Costs in Coalition Formation', *American Political Science Review 62* (1968), pp. 556–563; J. L. Bernd (ed.), *Mathematical Applications in Political Science, II*, Dallas, Texas: Southern Methodist University Press, 1966, p. 165.
3. The symbols ∪, ∩, ϕ are used in their customary meaning. See e.g. S. Miettinen, *Logiikan perusteet I* (Elements of Logic I), Helsinki: Yioppilastuki ry, 1971.
4. The set of all coalition structures can be symbolized by CS.
5. This formulation corresponds to the customary concept of 'group combination', of Tables III and IV.
6. If there are minority or majority rules in case of certain roll-call votes the calculation of F-values becomes more complicated; see M. Laakso, *Poliittinen koalitioteoria politiikan tutkimuksessa* (Political Theory of Coalition in Political Science Research), Unpublished Master's Thesis, University of Helsinki, Institute of Political Science, Helsinki, 1971, p. 47.
7. See e.g. P. Nyholm, *Suomen eduskuntaryhmien koheesio vuosien 1948–51 vaalikaudella ja vuoden 1954 valtiopäivillä* (The Cohesion of Finnish Parliamentary Groups during the Sessions 1948–51 and the Session 1954), Bidrag til kännedom av Finlands natur och folk utgivna av Finska Vetenskaps-Societeten, H. 106, Helsinki, 1961.
8. The coalition structures of Paasio's period are symbolized by CS' because they were composed of different parties than during Lehto's and Virolainen's periods.
9. For a more detailed analysis see Laakso, *op.cit.*, pp. 72–74.
10. It is worthwhile to note that the parties that had relatively large 'inner complements' in our data for a Cabinet period turned out the greatest winners in the elections that followed this period. Thus the Social Democrats won in 1966 and the Conservative Party in 1970.
11. A weakness of the ΔF_i parameter used here (and also the weakness of Riker's theory) is that if nearly all parties belong to the winning coalition then $\Delta F_i > 0$ because $(P_{Wi} + B_{Wi} + 2\bar{A}_{Wi})$ is natural greater than $(P_{Li} + B_{Li} + 2\bar{A}_{Li})$.
12. Similarly $\bar{A}_{Wi}$ cannot be taken into consideration when calculating the losing coalition.