Models for Change in Voting Behavior

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1. Introduction

This article suggests one general and three specific models for the description of changes in the distribution of votes by political party for the geographic subareas within a given area. Data from Danish parliamentary elections from 1960 to 1968 are chosen to show how the models work.

The main topics discussed are: What is *uniform change?* and What is *differential change?* Uniform change occurs when the support for a particular political party changes by the same amount in the same direction in all subareas, while differential change refers to deviation from uniform change in support for that party.

In addition to the description and testing of models for uniform and differential change, this article suggests a macro-sociological theory of voting behavior for interpretation of the statistical models.

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2. A Model for Uniform Change

A tendency to uniform change, that is, approximately the same amount of change in all geographical subareas, has frequently been observed for parties in democratic countries. This tendency was observed in many Danish political districts 1960–1968, in four successive elections to the Danish parliament which have been analyzed by Borre and Stehouwer.[2]

The present question is: How can we measure the amount of change in the support for a political party in a geographical subarea?

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A very common method is to take the proportion received by the party, of all potential votes in that subarea, as a measure of the 'support' for that party. Using this approach to construct a probability model, we take the proportion of votes for party h in subarea i in election t as an estimate of $p_{it}^{(h)}$, the probability that one voter chosen at random from the subarea has cast his vote for party h. That is, the stochastic equality

$$p_{it}^{(h)} \approx \frac{a_{it}^{(h)}}{N_{it}} \tag{1}$$

where $a_{it}^{(h)}$ is the number of people voting for party h, and N_{it} is the number of voters in the subarea in election t (the category of non-voters, i.e. those eligible to vote who did not, is regarded as a separate 'party').

From this formula, there are several ways of formulating a model for uniform change.

Borre and Stehouwer[3] compute the difference in the party's proportion of votes in two successive elections and use this difference as a measure of the amount of change. In probabilistic terms, the *model for uniform change* (between elections 1 and 2) is then

$$p_{i2}^{(h)} - p_{i1}^{(h)} = c^{(h)};\ i = 1, 2, \ldots, k \tag{2}$$

that is, the same *difference*, $c^{(h)}$, is observed in all k subareas.

This model, however, fits badly with the actual Danish election statistics. Small parties, in particular, show systematic deviations from this model – for these parties the difference tends to be *proportional* to the proportion of votes in election 1. This suggests another model for uniform change:

$$\frac{p_{i2}^{(h)}}{p_{i1}^{(h)}} = c^{(h)};\ i = 1, 2, \ldots, k \tag{3}$$

that is, the same *ratio*, $c^{(h)}$, is observed in all subareas.

This model has been used by Tage Bild[4] to analyze the (very small) vote for the Danish Communist Party; it can only be applied to very small political parties because of one obvious weakness: formula (3) does not imply the logical condition

$$\sum_{h=1}^{m} p_{it}^{(h)} = 1. \tag{4}$$

That is, the model does not imply that the sum of the proportions received by all parties in one subarea in an election must equal one. With such a model one can get the following results: Suppose the votes for a party in one subarea increase from 10 percent to 20 percent of the total vote. Suppose that in another subarea the same party begins with 50 percent of all votes cast. Then, according to formula (3), in the second subarea this party's support must double to 100 percent of the vote in order to show uniform change, without regard to what happens to competing parties.

Therefore it seems more reasonable to use a model which describes the support for one party *relative* to the support for other parties. This support for each party will be described by a parameter $\lambda_{it}^{(h)}$ which is not known a priori for any of the parties. The model suggested is:

$$p_{it}^{(h)} = \frac{\lambda_{it}^{(h)}}{\lambda_{it}^{(1)} + \lambda_{it}^{(2)} + \ldots + \lambda_{it}^{(m)}} = \frac{\lambda_{it}^{(h)}}{\gamma_{it}} . \tag{5}$$

That is, the probability of voting for party h is the parameter $\lambda_{it}^{(h)}$ divided by the sum, γ_{it}, of the parameters for all parties receiving votes in that election.

Furthermore, let us define *uniform change* as the condition that

$$\frac{\lambda_{i2}^{(h)}}{\lambda_{i1}^{(h)}} \approx c^{(h)}; \; i = 1, 2, \ldots, k . \tag{6}$$

That is, the same *ratio*, $c^{(h)}$, is assumed to appear in all subareas for a particular party.

Formula (6) can also be expressed by

$$\lambda_{it}^{(h)} = \xi_i^{(h)} \, \varepsilon_t^{(h)} \tag{7}$$

where $\lambda_{it}^{(h)}$ is separated into two parameters, one independent of elections, the other independent of subareas.

That this formula implies (6) can be seen by the fact that

$$\frac{\lambda_{i2}^{(h)}}{\lambda_{i1}^{(h)}} = \frac{\xi_i^{(h)}\varepsilon_2^{(h)}}{\xi_i^{(h)}\varepsilon_1^{(h)}} = \frac{\varepsilon_2^{(h)}}{\varepsilon_1^{(h)}} \; ; \; i = 1, 2, \ldots, k . \tag{8}$$

That is, the ratio between two support parameters is the same for all subareas. The two assumptions (5) and (7) can now be gathered into one formula:

$$p_{it}^{(h)} = \frac{\xi_i^{(h)}\varepsilon_t^{(h)}}{\xi_i^{(1)}\varepsilon_t^{(1)} + \xi_i^{(2)}\varepsilon_t^{(2)} + \ldots + \xi_i^{(m)}\varepsilon_t^{(m)}} = \frac{\xi_i^{(h)}\varepsilon_t^{(h)}}{\gamma_{it}} . \tag{9}$$

This model is a version of Rasch's general model of measurement.[5] The model can be tested, and the size of the parameters ξ and ε can be estimated from the data.

If desired, the model can be applied to only n out of all m parties. The formula then becomes

$$p_{it}^{(h)} = \frac{\xi_i^{(h)}\varepsilon_t^{(h)}}{\xi_i^{(1)}\varepsilon_t^{(1)} + \ldots + \xi_i^{(n)}\varepsilon_t^{(n)}} = \frac{\xi_i^{(h)}\varepsilon_t^{(h)}}{\gamma_{it}} \tag{10}$$

where $p_{it}^{(h)}$ can be estimated by the proportion voting for party h in relation to the proportion voting for one of the parties 1, . . . , n.

For generalization below of the model, a purely formal logarithmic transformation of the parameters can be made:

$$\theta_i^{(h)} = \log \xi_i^{(h)}; \ \sigma_t^{(h)} = \log \varepsilon_t^{(h)} . \tag{11}$$

Thus (10) can be written

$$p_{it}^{(h)} = \frac{1}{\gamma_{it}} e^{\theta_i^{(h)} + \sigma_t^{(h)}} . \tag{12}$$

Statistical methods for testing this model and estimating its parameters are described by the present author.[6]

The only test shown here will be a preliminary graphic test of model (12) on data from the elections of 1960, 1964, 1966, and 1968 for 41 subareas within the municipality of Copenhagen. The analysis will be limited to those eight parties which participated in all four elections: A Social Democrats, B Radicals, C Conservatives, D Liberals, E Justice Party, F Socialist People's Party, K Communists, U Independent Party.

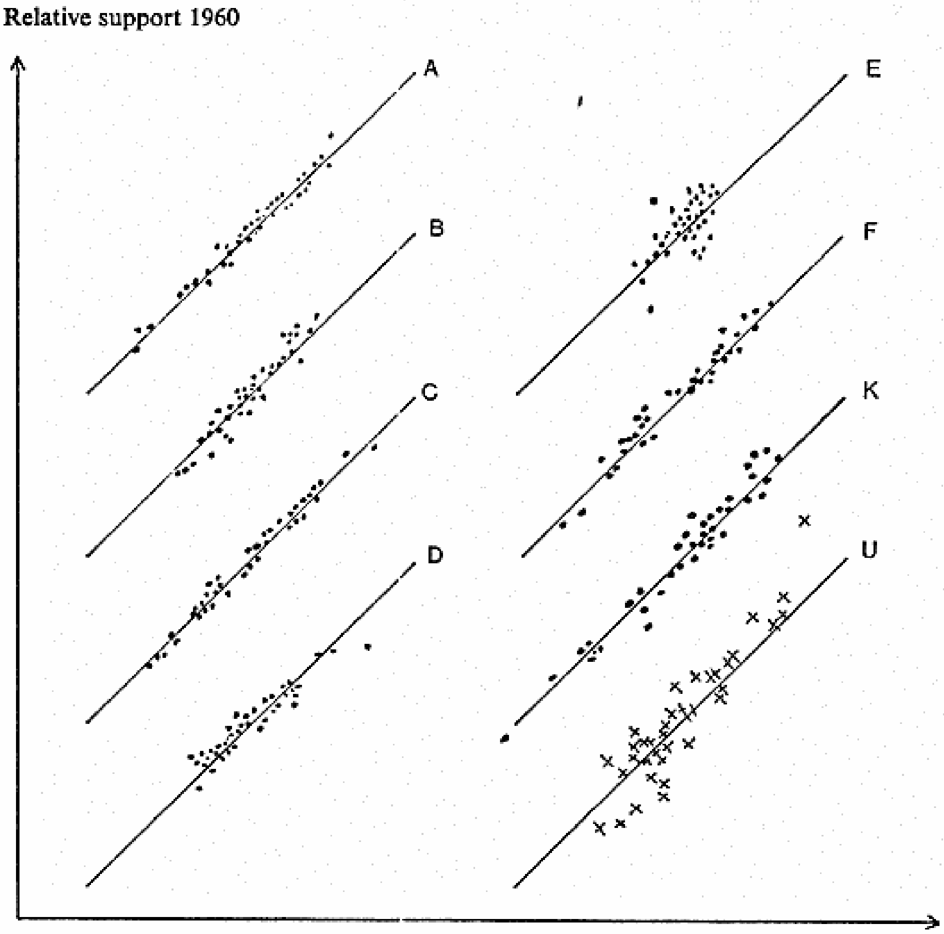

Figure 1. Mean Relative Support 1960–1968.

Following the procedure described in Appendix 1, Figure 1 plots, for each of the 41 subareas, the relative support each party received in 1960 (measured by $l_{it}^{(h.)} - l_{.t}^{(h.)}$; see Appendix 1) against the mean of the party's relative support in all 4 elections (measured by $l_{i.}^{(h.)} - l_{..}^{(h.)}$). If model (12) for uniform change fits the data, then the points should deviate only at random from a straight line with slope 1 for each party. (For convenience the graphs in Figure 1 are referred to different origins.)

The data in Figure 1 show a remarkably close fit with the model, with two minor exceptions: parties E and U show greater deviations from their respective lines than the other parties. But one would expect this statistically because parties E and U receive very few votes. More interesting are the systematic deviations for parties B and U, where the points indicate better fit with straight lines having slopes greater than 1. (These deviations suggest a rule for generalizing this model to a model for differential change — see section 3 below).

$\theta_i^{(h)}$ measures the relative support for party h in subarea i independent of time (that is, independent of elections), while $\sigma_t^{(h)}$ measures the relative support for party h at election t independent of space (that is, independent of the subareas).

Values of $\sigma_t^{(h)}$ are indicated in Figure 2 for each election. Note for example that the relative support for party A (Social Democrats) appears fairly constant, support for B (Radicals) increased throughout the period, and support for U (Independent Party) decreased throughout the period.

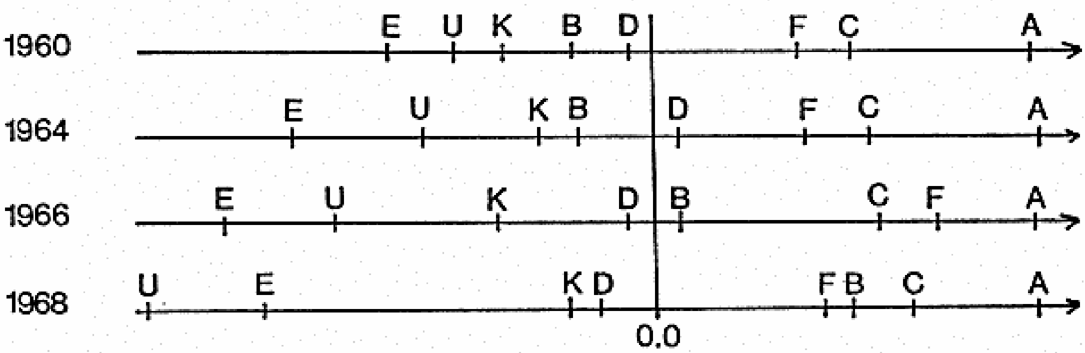

Figure 2. Relative Support for Each Party, $\sigma_t^{(h)}$.

3. Models for Differential Change

Examination of $\theta_i^{(h)}$-values (relative support in a particular subarea) reveals two groups of positively correlated parties: A, F, K (worker-supported parties) and B, C, D, U (non-worker-supported parties), each group correlating negatively with the other. This suggests the assumption:

$$\theta_i^{(h)} = \theta_i \varphi^{(h)} \cdot \tag{13}$$

That is, the $\theta_i^{(h)}$-values of party h are proportional to a parameter, θ_i, for the subarea θ_i, which is independent of the party, with a coefficient, $\varphi^{(h)}$, specific to the party. Formula (13) inserted into (12) gives

$$p_{it}^{(h)} = \frac{1}{\gamma_{it}} e^{\theta_i \varphi^{(h)} + \sigma_t^{(h)}} . \tag{14}$$

This model (the distribution analysis model of Rasch) can be tested on each election separately by maximum-likelihood estimation procedure.[7] A method for preliminary graphic testing of the model is given by Christiansen and Stene[8] and is applied to the Copenhagen data for 1960 in Figure 3. For each party, the residue from equal support in each area, measured by $r_{it}^{(h)}$ (see Appendix 2), is plotted against the sign-weighted mean residue from equal support (measured by $r_{it}^{(*)}$).

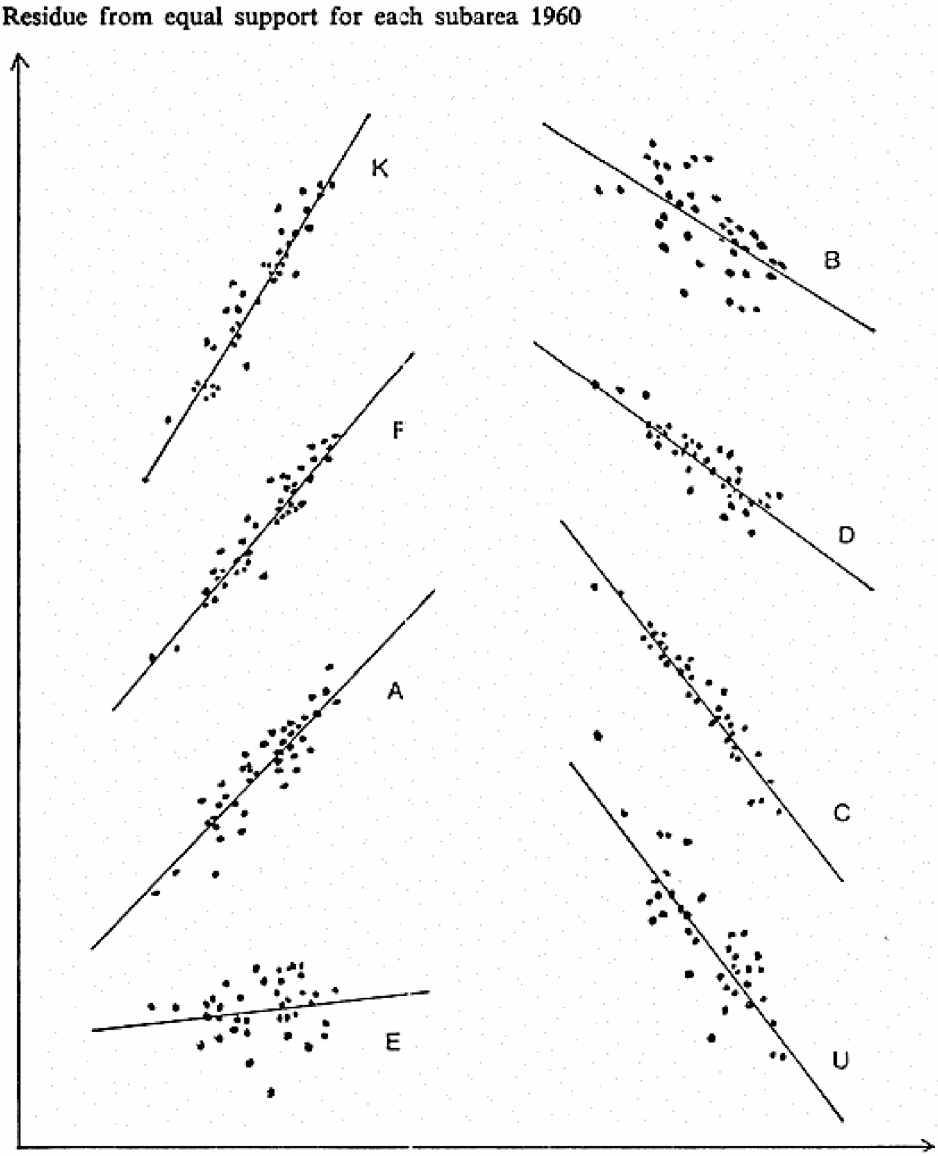

Figure 3. Sign-Weighted Mean Residue 1960

If the model fits the data, the points for each party should only deviate at random from a straight line with slope proportional to $\varphi^{(h)}$. (For convenience the graphs in Figure 3 are referred to different origins.)

The data fit the model quite well, but not as closely as in the previous analysis. The best fit is observed for parties C, F, K and the worst fit for parties B, E.

For the Copenhagen data it was found that θ_i as estimated separately for each election is approximately constant throughout the period 1960–1968.[9] Also it is related approximately linearly to a measure for over-representation of manual workers in each subarea. Therefore (see equation (13)), $\varphi^{(h)}$ is interpreted as 'the degree of manual-worker orientation of party h'. Figure 4 shows values of $\varphi^{(h)}$ as estimated separately for each election.

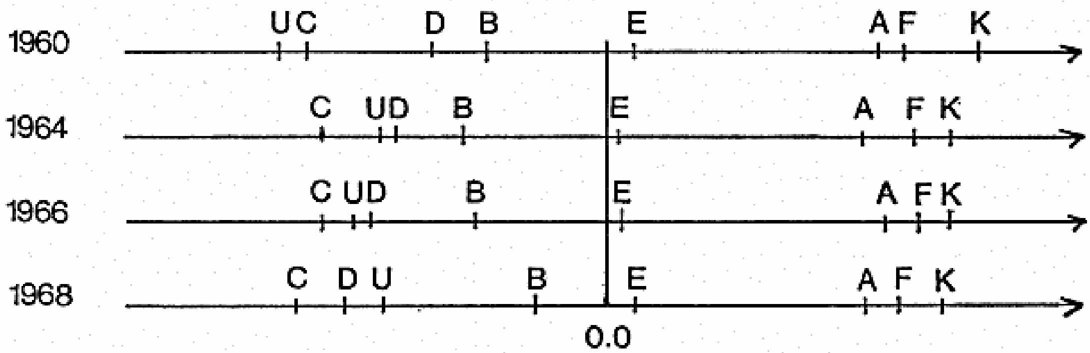

Figure 4. Manual-Worker Orientation, $\varphi^{(h)}$.

Minor changes in $\varphi^{(h)}$ are observed over time, and this indicates that $\varphi^{(h)}$ is time-dependent. As expected, the parties traditionally in favor of more equal income (A, F, K) have positive values of $\varphi^{(h)}$, while the more conservative parties (U, C, D) have negative values of $\varphi^{(h)}$. Party E (Justice Party) is in be ween, and it appears that B (Radicals) is approaching zero at the end of the period investigated.

When, as in this example, $\varphi^{(h)}$ is dependent on time, formula (14) must be generalized into a model for *differential change:*

(15)
$$p_{it}^{(h)} = \frac{1}{\gamma_{it}} e^{\theta_i \varphi_t^{(h)} + \sigma_t^{(h)}} .$$

This differential change is described by $\varphi_t^{(h)}$, the orientation of party h at election t on the dimension measured by θ_i. In this example, the dimension θ_i has been identified with manual-worker orientation.

The model can be further generalized into a model for differential change with *more* than one dimension:

(16)
$$p_{it}^{(h)} = \frac{1}{\gamma_{it}} e^{\Theta_i \cdot \Phi_t^{(h)} + \sigma_t^{(h)}} .$$

In this formula Θ_i: $(\theta_{i1}, \theta_{i2}, \ldots, \theta_{ir})$ and $\Phi_t^{(h)}$: $(\varphi_{t1}^{(h)}, \varphi_{t2}^{(h)}, \ldots, \varphi_{tr}^{(h)})$ are vectors of dimension r, and

$$(17) \qquad \Theta_i \cdot \Phi_t^{(h)} = \theta_{i1}\,\varphi_{t1}^{(h)} + \theta_{i2}\,\varphi_{t2}^{(h)} + \ldots + \theta_{ir}\,\varphi_{tr}^{(h)} \ .$$

As in factor analysis, tests of this model are not very conclusive when the number of dimensions is not known.

A test can be made, however, in the special case where *only one* of the elements of $\Phi_t^{(h)}$ is dependent on time, that is, differential change occurs in only one dimension. By convenient renaming of parameters, we can obtain:

$$(18) \qquad \Theta_i \cdot \Phi_t^{(h)} = \theta_{i1}\,\varphi_{t1}^{(h)} + (\theta_{i2}\,\varphi_2^{(h)} + \ldots + \theta_{ir}\,\varphi_r^{(h)}) = \alpha_i\,\beta_t^{(h)} + \theta_i^{(h)}$$

that is, only the first term is dependent on time. When this expression is inserted into equation (16), the model becomes

$$(19) \qquad p_{it}^{(h)} = \frac{1}{\gamma_{it}}\, e^{\theta_i^{(h)} + \sigma_t^{(h)} + \alpha_i\,\beta_t^{(h)}} \ .$$

If this model for differential change is compared with the model for uniform change, formula (12), it turns out that (19) differs from (12) only by adding the one-dimensional term $\alpha_i\beta_t^{(h)}$ to the exponent.

This model fits the Copenhagen data quite well. (A simple preliminary test follows the procedure of Appendix 2, in computing the residues from the model in equation (12)). The model also fits the regional distribution of data from the same elections, previously published by Stehouwer and Borre in *Scandinavian Political Studies.*[10]

For use with these data, the analysis includes eight political parties, plus the 'party' of non-voters. The close fit of the model with these data can be illustrated by the small differences between the actual and the 'computed' proportions (computed on the basis of estimated parameters) voting for each party in each subarea (region) in each election. The differences are in no instance greater than 2 percent, and are greater than 1 percent in only 17 out of 396 instances.[11]

Table I. Values of α_i, the Differential Change Dimension

Subarea i	α_i
1. Copenhagen, working-class constituencies	.455
2. Copenhagen, middle-class constituencies	.369
3. Copenhagen, suburbs and surroundings	.269
4. Five largest provincial towns	.298
5. Other urban constituencies, the Islands	–.165
6. Other urban constituencies, East Jutland	.069
7. Other urban constituencies, West and North Jutland	–.234
8. Rural constituencies, the Islands	–.261
9. Rural constituencies, East Jutland	–.308
10. Rural constituencies, West and North Jutland	–.270
11. South Jutland	–.254

Table I shows that the differential change dimension α_i is strongly positively correlated with degree of urbanization. Therefore (see formula (19)), we will call $\beta_t^{(h)}$ 'the degree of *urban orientation* of party h at election t'. Figure 5 shows values of $\beta_t^{(h)}$ for each election.

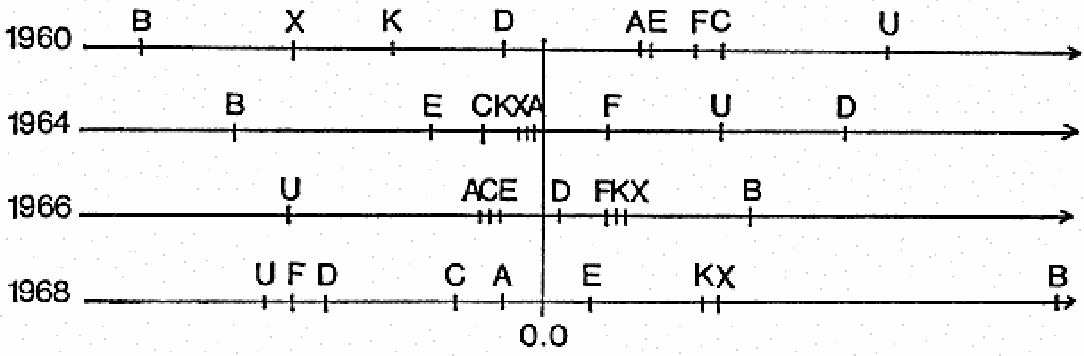

Figure 5. Urban Orientation $\beta_t^{(h)}$ *(x: non-voters).*

Party B (Radicals) shows a marked increase in urban orientation, and there is a general tendency to either increase or decrease throughout the period for most parties. Party D (Liberals) is a clear exception to this rule.

4. A Macro-Sociological Theory for Voting Behavior

It has surprised me that most of the electoral data I have worked with so far can be described with such simple models. It is characteristic of the models that they describe the distribution of votes in *geographic populations*, with parameters either *independent of time* (independent of elections) or *independent of space* (independent of subareas within the total area).

This simplicity of description indicates that voting populations in these subareas are useful macro-sociological entities for description of behavioral distributions. Furthermore, it indicates that these distributions can be explained by subarea characteristics which do not change over time and by characteristics of the election which are the same for all subareas in the total area.

At this point we introduce three assumptions consistent with the models developed above.

Assumption 1. The more *the image* of a political party satisfies *the goals* of the population in one subarea, the greater *the support* for that party in that subarea.

The weakness of assumption 1 is that there is no external definition of what is to be understood by 'image' and 'goal'. This will be a subject for further research.

Assumption 2. The image of one political party at a particular election is *the same* in all subareas.

I expect this assumption to hold only in very cohesive political regions where each subarea receives approximately the same political stimuli. It would not hold, for example, in situations where local political events are important (e.g. local municipal elections).

Assumption 3. The goals of the population in one subarea *do not change* from one election to the next.

That is, the population in the subarea will be considered a macro-sociological entity with a certain distribution of individual goals. I expect this distribution to be dependent on basic socioeconomic factors that only change slowly, so that it can be regarded as constant over short periods of time. This assumption will be unrealistic during situations of violent political crisis or great socioeconomic change.

Note that assumption 3 does not apply to the individual at the micro-level. It is only *the distribution* of individual goals that is assumed constant, not the goals of the individual person.

5. Interpretations of Parameters

All models mentioned in this article are special cases of the general model, formula (16):

$$p_{it}^{(h)} = \frac{1}{\gamma_{it}} e^{\Theta_i \Phi_t^{(h)} + \sigma_t^{(h)}} .$$

The parameters of this model will now be interpreted in light of the theory in the above section.

Statistically, $\Theta_i\Phi_t^{(h)}$ describes the differentiation of the support for each party in time and space, while $\sigma_t^{(h)}$ describes the general support (independent of subareas) for each party at each election. According to the theory, $\Theta_i\Phi_t^{(h)}$ is interpreted as the degree to which the image of party h at election t satisfies goals *specific* to the population of subarea i, while $\sigma_t^{(h)}$ is interpreted as the degree to which the image of party h at election t satisfies goals *common* to all subarea populations. Θ_i is a vector describing the position of the *specific goals* of the subarea on different dimensions of orientation (e.g. manual-worker orientation, urban orientation), while $\Phi_t^{(h)}$ is a vector describing the position of each *party image* at each election on these same dimensions. To sum up, these symbols have been given the following interpretations:

Θ_i: Orientation vector of the specific goals of subarea i.

$\Phi_t^{(h)}$: Orientation vector of the image of party h at election t.

$\sigma_t^{(h)}$: Satisfaction of common goals by the image of party h at election t.

All models described can now be interpreted as special cases of the general model.

If the model for uniform change, formula (12), is compared with the general model, formula (16), it is evident that in this special case

$$\Theta_i \, \Phi_t^{(h)} = \theta_i^{(h)} \tag{20}$$

that is, the orientation vector specific to the particular party does not change, whereas the satisfaction of common goals $\sigma_t^{(h)}$ for each party might change. This model provides a close fit with most of the parties in the Copenhagen data (see section 2 above).

Similarly, when the one-dimensional model for differential change, formula (14), is compared to the general model, we obtain:

$$\Theta_i \, \Phi_t^{(h)} = \theta_i \, \varphi_t^{(h)} \tag{21}$$

that is, both orientation vectors can be described in one dimension. For the Copenhagen data, θ_i was interpreted as manual-worker orientation on the basis of ecological correlation. Only minor changes were observed in $\varphi_t^{(h)}$, the manual-worker orientation of the parties (see section 3 above).

For the special case of the multi-dimensional model for differential change, formula (19), we have

$$\Theta_i \, \Phi_t^{(h)} = \theta_i^{(h)} + \alpha_i \, \beta_t^{(h)} \tag{22}$$

that is, the parties can change in only one orientation dimension. This dimension was identified as the rural-urban dimension in the national data previously published in this yearbook.[12]

6. Conclusion

The limited Danish electoral data analyzed so far indicate that simple models of measurement, with parameters independent of either time or space, describe a surprisingly high proportion of the change in voting behavior.

I expect that models of this kind will find broad applicability in future electoral research. An important subject for further empirical and theoretical investigations will be the analysis and interpretation of the parameters estimated from the electoral data.

NOTES

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2. Ole Borre and Jan Stehouwer, *Fire folketingsvalg, 1960—68*, Århus: Akademisk Boghandel, 1970.
3. Borre and Stehouwer, *op. cit.*
4. Tage N. Bild, *Om anvendelse af poissonmålingsmodellen*, København: Københavns Universitets Sociologiske Institut, 1966 (mimeo).
5. Rasch, *op. cit.*
6. Søren Risbjerg Thomsen, *Målingsmodeller for korttids-forandringer i vælgeradfærd*, Århus: Institut for Statskundskab, Aarhus Universitet, 1971 (mimeo).
7. Thomsen, *op. cit.*
8. Ulf Christiansen and Jon Stene, *2. bind af G. Rasch's lærebog i teoretisk statistik*, København: Teknisk Forlag, 1969, Vol. 2, pp. 160—277.
9. Thomsen, *op. cit.*
10. Jan Stehouwer and Ole Borre, 'Four Elections in Denmark, 1960—68', *Scandinavian Political Studies*, Vol. 4, Oslo: Universitetsforlaget, 1969.
11. Thomsen, *op.cit.*, section 7.7. (α_i approximately constant through the whole period.)
12. Stehouwer and Borre, *op. cit.*
13. Christiansen and Stene, *loc. cit.*

Appendix 1

This appendix describes the computational procedure for the graphic test of the model for uniform change. According to formulas (1) and (12), the following stochastic relation holds:

$$l_{it}^{(h)} = \log \frac{a_{it}^{(h)}}{N_{it}} \approx \theta_i^{(h)} + \sigma_t^{(h)} - \log \gamma_{it} \cdot$$

For each subarea at each election, the mean value of $l_{it}^{(h)}$ is computed:

$$l_{it}^{(\cdot)} = \frac{1}{n} \sum_{h=1}^{n} l_{it}^{(h)} \approx \theta_i^{(\cdot)} + \sigma_t^{(\cdot)} - \log \gamma_{it} = - \log \gamma_{it}$$

by arbitrarily setting the mean values $\theta_i^{(\cdot)}$, $\sigma_t^{(\cdot)}$ equal to 0. We then compute the difference

$$l_{it}^{(h\cdot)} = l_{it}^{(h)} - l_{it}^{(\cdot)} \approx \theta_i^{(h)} + \sigma_t^{(h)} \cdot$$

From this relation follows the relation

$$l_{it}^{(h\cdot)} - l_{\cdot t}^{(h\cdot)} \approx l_{i\cdot}^{(h\cdot)} - l_{\cdot\cdot}^{(h\cdot)}$$

where a dot instead of an index indicates that the mean value is computed for all values of this index. When the left-hand side of this equation is plotted against the right-hand side for each subarea, we get the graphs in Figure 1 with slope 1 for each party.

Appendix 2

The first three steps of the computational procedure for the one-dimensional model for differential change (formula (14)) follow the procedure of Appendix 1. For each election we get

$$l_{it}^{(h\cdot)} \approx \theta_i \varphi^{(h)} + \sigma_t^{(h)}$$

$$r_{it}^{(h)} = l_{it}^{(h\cdot)} - l_{\cdot t}^{(h\cdot)} \approx \theta_i \varphi^{(h)}.$$

Christiansen and Stene[13] show a method of estimating the sign of θ_i and $\varphi^{(h)}$. With those estimates we can compute the sign-weighted means (indicated by '*' instead of the index):

$$r_{*t}^{(h)} \approx \theta_* \varphi^{(h)}, \text{ where } \theta_* = \frac{1}{k} \sum_{i=1}^{k} |\theta_i|$$

$$r_{it}^{(*)} \approx \theta_i \varphi^{(*)}, \text{ where } \varphi^{(*)} = \frac{1}{n} \sum_{h=1}^{n} |\varphi^{(h)}|$$

$$r_{*t}^{(*)} \approx \theta_* \varphi^{(*)}.$$

From these we can get the stochastic relation:

$$r_{it}^{(h)} \approx \frac{r_{it}^{(*)} r_{*t}^{(h)}}{r_{*t}^{(*)}}.$$

When $r_{it}^{(h)}$ is plotted against $r_{it}^{(*)}$ for each subarea, we get the graphs in Figure 3, with slope $r_{*t}^{(h)}/r_{*t}^{(*)}$ for each party.