A Simulation Analysis of Selected Voting Procedures*

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The formal model on which our simulations are based is for obvious reasons rather crude. Viewing the real world, however, we observe that any collective choice situation consists of a set of decision-participants, a set of alternatives and a set of preference statements.

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The set of decision-participants or 'voters' (the 'who cluster) can be structured into seven different systems of voting blocs if the number of blocs is assumed to be less than six. *The set of alternatives* can be structured according to the number of dimensions that exist in a set. Finally we can restrict *the set of possible preference statements* in a number of different ways.

A *situation* is defined by a specific system of voting blocs, a certain number of dimensions and certain restrictions on the set of possible preference statements. In each situation we randomly create 10,000 preference patterns.[3] In each pattern certain properties of certain voting procedures are tested.

We must stress that this simulation approach is the only realistic way to establish such complicating features as the strategic potential of voting blocs under specific voting procedures. This can be done in the simulation by making all possible changes in a specific type of preference statement from a voting bloc.

The concepts used in this section are quite familiar to any political scientist working in the fields of probability modeling, theories of voting, and collective choice. They might not, however, be familiar to readers who have had only limited experience in these research areas. Hence, in the following section we discuss our interpretation of concepts like probability modeling, preference statements, voting blocs, strategic potentials, simulated changes, etc.

2. Concepts Used

Probability Modeling

The simulation analysis presented below is one of many possible applications of formal probability models to collective decision-making processes. The formal model within which our simulations take place was first published as one of sixteen papers[4] showing in detail the many and varied uses of probability theory in the development of formal models of collective decision-making. They also reveal the considerable potentiality and utility of probabilistic models and – like all models – illustrate the hazards and liabilities that attend this approach. The pros and cons of probability modeling are extensively discussed by Niemi and Weisberg in their introduction and conclusion.

What might be stressed as the main advantage of *this* investigation is the ability to study a vast number of different preference situations which otherwise would not be amenable to treatment. My investigation of Scandinavian roll-calls shows the impossibility of investigating the strategic aspects of the elimination procedure on the basis of empirical data.[5] The main disadvantage is the inability to justify particular probability assumptions.

Preference Statements and Voting Blocs

A preference statement is regarded here primarily as a representation of a decision participant's evaluation of a set of alternatives. Obviously a set of preference statements cannot be transformed into a collective choice unless they are comparable. The effect of this demand is to give relevant preference statements a very crude character. In parliamentary decision-making, collective choices made through roll-calls are usually based on pair-wise ordinal preferences. That is, the outcome is based on a crude 'better-worse' evaluation of the alternatives which in many cases would be deemed *not* to represent the preferences of individual participants in an adequate way.

The nature of our study, however, is such that the problem of whether the preferences of decision-participants are adequately represented by, say, a rank ordering is not a relevant problem here. We simply note the fact that collective choices *are made* by representing the preferences in a specific way. *Our problem* is to analyze the properties of different voting procedures when individual or bloc preferences are represented by a specific type of preference statement. We will, however, assume that every participant could, on demand, produce a complete rank ordering of the alternatives.

The decision-participants of this study are *voting blocs,* where a bloc consists of voters who have agreed to evaluate the alternatives under consideration in the same way. A voting bloc could, of course, be a single individual.

Decision Principles

The procedures to be analyzed in this study are based on the majority principle as defined below. We also define an equality principle, which will be a useful tool for comparing the various voting procedures.

Our definition of the *majority principle* is based on two concepts: the majority alternative and the cyclical set. We define a *majority alternative* as one that obtains a majority of the votes in pair-wise comparison with all other alternatives on the agenda. That is, if and only if a particular alternative obtains a majority when paired against all other alternatives, each in turn, then it is a majority alternative. An example illustrates this definition.

Suppose we have an electorate of three voters (I, II, III) which must decide upon a collective policy from among three alternatives (a_1, a_2, a_3). The voters' preference orderings are:

I	*II*	*III*
a_1	a_2	a_3
a_2	a_1	a_1
a_3	a_3	a_2

That is, reading down the table, voter I prefers a_1 most, prefers a_2 next, and a_3 last. Consider a_1. For the dyad (a_1, a_2), the preference orderings indicate that voters I and III choose a_1. For the dyad (a_1, a_3), voters I and II choose a_1. Thus, a_1 obtains a majority when paired against each of the remaining alternatives and, according to our definition, is the majority alternative. Note that a_1 is not most preferred by a majority of the voters. Only voter I most prefers a_1. Of course, if a majority of voters, like I, ranked a_1 highest in their preference orderings, then it would satisfy our definition of majority alternative. We simply observe that this is not necessary, as the example indicates.

Some sets of preference orderings do not, however, possess a majority alternative. Consider the following arrangement:

I	*II*	*III*
a_1	a_2	a_3
a_2	a_3	a_1
a_3	a_1	a_2

Alternative a_1 is preferred by a majority to a_2 (I, III), but not to a_3. Voters II and III prefer a_3 to a_1. Thus, a_1 is not a majority alternative. Neither is a_2 since, as we have already seen, a_1 is preferred by a majority to it. That leaves a_3, which is preferred to a_1 (II, III) but voters I and II prefer a_2 to a_3. We have then an unusual result: a_1 is preferred to a_2, a_2 to a_3, and a_3 to a_1. The alternatives form a cycle with regard to majorities. A set of rank orderings where no majority alternative exists will be denoted *a cyclical set.*

By our definition *the majority principle* states that if a majority alternative exists, then this alternative should be chosen as the outcome. When the set of rank orderings is circular, however, majorities are frustrated in their preferences for a collective choice and no alternative is the obvious selection by the majority principle.

Finally the *equality principle* states that the probability that a bloc's most preferred alternative wins should be proportional to the weight of the bloc. This equality principle will be a useful tool in an attempt to test the 'fairness' of some of the procedures which are introduced in the next section.

Voting Procedures

The voting procedures to be analyzed in this study are the elimination and the quality procedure as defined below. In almost any case a tie for first place can occur, and some tie-breaking mechanism is needed (such as a chairman who only votes in case of a tie or a rule that the status quo wins in case of a tie). Since some such mechanism always exists, we do not deal further with ties.

The elimination procedure. The initial poll is taken between the first two alternatives in a predetermined voting order. A second poll is taken between the winner of the first poll and the next alternative in the voting order. This process continues through the entire list of alternatives. In the end one of the alternatives emerges as the winner. In some 'amendment' variants of the elimination procedure the set of alternatives is not defined when the first vote is taken – that is, amendments can be put forward during the voting process. This does not, however, change the basic structure of the procedure. Nevertheless the distinction between a well-defined set of alternatives coupled with a predetermined voting order and this amendment variant should be kept in mind.

The quality procedure. The voters register a complete rank ordering of the alternatives under consideration, and the outcome is selected on the basis of these rank orderings. If a majority alternative exists, this alternative is the winner. If a cycle exists a voting order is determined at random and the elimination procedure is used to determine an outcome. The main distinction between the elimination procedure and the quality procedure is that when the latter procedure is used the voters do not know the voting order when they record their preferences.

Strategic Potentials, Moves, and Countermoves

We noted above that cyclical sets have a potentially devastating effect on majority rule. Empirical investigations show, however, that cyclical sets seem to occur less frequently than is expected from theoretical calculations.[6] This is probably due, first, to the fact that cyclical majorities can only occur when alternatives are viewed multidimensionally and, second, that these sets themselves tend to be open to compromise and logrolling. Thus when potential cycles exist, they are 'solved' by the decision processes that precede the actual voting.

What must be explored, however, is the possibility that the cycles can be used to promote the interests of individual voters or blocs. This possibility depends, in the case of the elimination procedure, on the fact that if the set is cyclical then the outcome will to a large extent be determined by the voting order. In fact, in the case of three alternatives the last alternative will always win.[7] With this in mind consider the example on p. 39 where a_1 is the majority alternative. A simple analysis shows that voter II is the only one who can change the set of rank orderings into a cyclical one. This is done by changing his rank ordering to a_2, a_3, a_1. This strategic potential can be utilized only when there is a favorable voting order, here a_1, a_3, a_2 (or a_3, a_1, a_2). With this voting order, in the first poll, between a_1 and a_3, voter II should vote for a_3 contrary to his 'true' rank ordering. In the last poll, between a_2 and a_3, voter I is forced to vote for a_2 since otherwise the outcome will be a_3, I's least preferred alternative. Note that voter I has no alternative but to vote for a_1 in the first poll and for a_2 in the second. Voter III, however, can make the only possible strategic countermove by voting in the first poll for a_1 against his own primary alternative a_3.

A specific voting order might thus give a voter or a bloc of voters a strategic possibility. The most favorable voting orders are those in which the voter's primary preference is the last alternative in the voting order (cf. below p. 46).[8] To try to take advantage of the paradox by voting contrary to one's 'true' preferences is here denoted *a strategic move.* All strategic moves are primarily directed against the majority alternative. Hence that majority who prefer the majority alternative to the 'pushed' alternative can always make *a completely effective strategic countermove* by changing the majority alternative into an alternative most preferred by a majority of the voters. This will, however, imply that one (some) bloc(s) must vote against its (their) 'true' first, and sometimes second, third, etc., preferences.

An empirical investigation of roll-calls in the Swedish and Finnish Parliaments, where the elimination procedure is used,[9] shows that the strategic move is a tool used from time to time with the desired effect. For obvious reasons, however, the frequency of strategic countermoves is impossible to establish.

In this paper the *strategic potentials* of a specific procedure will be established by analyzing if and to what extent changes in the preference statement can change the winning probability of different blocs in different situations. The simulation analysis of strategic potentials of the elimination procedure and the quality procedure must, however, be based upon a thorough knowledge of the cyclical sets. Accordingly we must classify the cyclical sets on the basis of the internal majority relations. We must, moreover, establish in each type of cyclical set three different probabilities or sets of probabilities, as follows:

1. The probability, if the quality procedure is used, that each alternative will be the winner.
2. The probability, if the elimination procedure is used, that the last alternative in the voting order will be the winner, the probability for the next to last, etc.
3. The probability, if the elimination procedure is used, that a specific alternative will be the winner when it is the last alternative in the voting order.

As an example, consider our cyclical situation above on p. 40.

1. Each alternative has a probability of one in three of being the winner if the voting order is determined according to the quality procedure, *i.e. at random*.
2. The last alternative in the voting order will always be the winner.
3. Each alternative will win when it is the last alternative in the voting order.

The first set of probabilities is computed since it represents the random variant of the elimination procedure, here denoted the quality procedure. The second set of probabilities is computed since it enables us to show the general effect of the voting order in cyclical situations. Finally the third set is computed since it represents the important variant of the elimination procedure in which one bloc of voters has the power to make its own alternative the last one in the voting order.

In our simulations we will use the first and the last set of probabilities to establish upper limits on the winning probability of a bloc's primary preference in three types of situations. The situations are a) no strategic moves are made, b) strategic moves from one bloc are made whenever possible but there are no countermoves, and c) when a successful strategic move is made, certain countermoves are carried out.

These simulations will thus enable us to establish the extent to which the elimination procedure and the quality procedure produce outcomes in accordance with the equality principle with or without strategic moves and countermoves.[10]

These simulations will also give us an idea of the strategic potentials of each of these procedures. In addition, the frequency with which favorable changes can be made by a voting bloc in sets where a majority alternative exists will give us an idea of the extent to which the two procedures are in accord with the majority principle.

The classification of cyclical sets and the establishment of the three sets of probabilities are carried out in the next section.

3. A Classification of Cyclical Sets

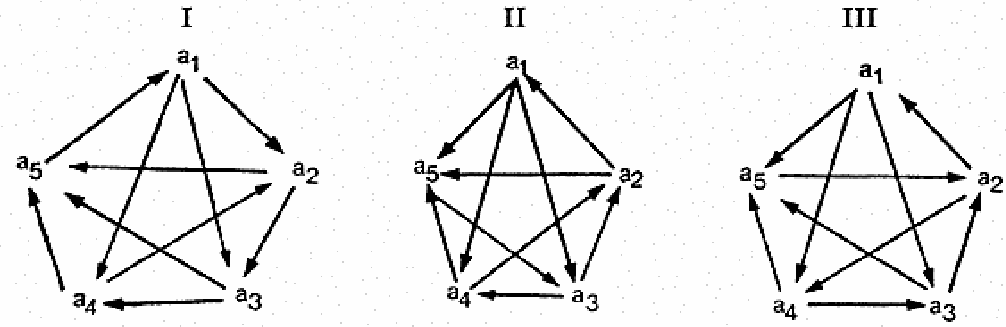
A classification of cyclical sets on the basis of the internal majority relations is a necessary first step in any attempt to analyze their effects. The classification is limited to sets of alternatives with up to five alternatives and is carried out in two steps. The first step is based on the distribution of majority relations, while the second step utilizes indirect majority relations.

Consider the first step. The distribution of majority relations in any set of preference orderings can be evaluated as follows. Mark each alternative with the total number of its 'positive' majority relations. We will then have a cyclical set if and only if each alternative is marked $\leq n - 2$, where n equals the number of alternatives.

Proof: Since no alternative can have a majority relation with itself, no alternative can ever be marked $> n - 1$. If one alternative is marked $n - 1$ it is a majority alternative since the figure $n - 1$ implies that the alternative obtains a majority when paired against every other alternative. If all alternatives are marked $\leq n - 2$, this implies a cyclical set since each alternative has at least one majority against it.

One possible distribution in a set consisting of five alternatives is: $a_1 = 3$, $a_2 = 2$, $a_3 = 2$, $a_4 = 2$, and $a_5 = 1$. A closer study of different representatives of this distribution shows that the position of alternatives with the same number of direct majority relations may be entirely different due to the indirect relations.

Ex.



Each of these three cases is a representative of the given distribution. The positions of the alternatives are, however, different in each case. Consider, for example, a_5. In case I this alternative has a majority over the strongest alternative, a_1. In case II, it has a majority over a_3, which in turn has majorities over the two other 'two-majority' alternatives, a_2 and a_4. In case III, finally, a_5 has a majority over a_2, which has majorities over one 'three-majority' alternative (a_1) and one 'two-majority' alternative (a_4).

The second step in the classification is based on these differences in the indirect majority relations. The importance of these relations is best illustrated by analyzing how the voting result would differ in the three cases under the elimination procedure. An analysis of the $5! = 120$ possible voting orders in each of the three cases results in the following three distributions of winning alternatives:

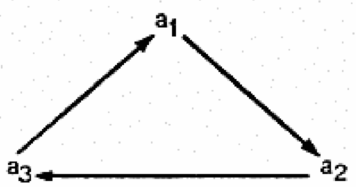
I.	$a_1 = 54$,	$a_2 = 14$,	$a_3 = 14$,	$a_4 = 14$, and $a_5 = 24$
II.	$a_1 = 46$,	$a_2 = 32$,	$a_3 = 22$,	$a_4 = 14$, and $a_5 = 6$
III.	$a_1 = 40$,	$a_2 = 36$,	$a_3 = 22$,	$a_4 = 12$, and $a_5 = 10$

In other words, if a set of preference orderings has a structure as set forth in case I, and if the elimination procedure is used, then a_1 will be the winner in 54 of the 120 possible voting orders, a_2 will be the winner in 14, etc. Such an analysis of all possible voting orders will produce the three sets of probabilities defined in section 2.5.

Hence we first make a systematic classification of cyclical sets (for three, four, and five alternatives) and then analyze each voting order in each case in order to establish the winner and its position in the voting order. The results of the second step are shown in Table I below.

When a set of *three* alternatives is cyclical there is only one case, viz. the symmetrical 'circle.'

Ex.



Direct majority relations
$a_1 = 1$
$a_2 = 1$
$a_3 = 1$

That is, each alternative is marked 1, and any circle can be transformed into the one above by a re-labeling of the alternatives.

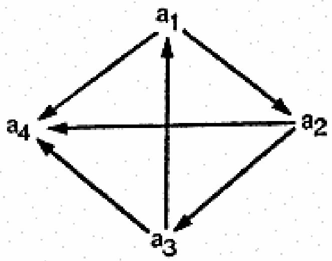
When a set of *four* alternatives is cyclical we have two distributions, since no alternative may be marked $> n - 2 = 2$. Six majority relations can only be distributed among four alternatives in the following two ways:[11]

1. $a_1 = 2$
   $a_2 = 2$
   $a_3 = 2$
   $a_4 = 0$

2. $a_1 = 1$
   $a_2 = 2$
   $a_3 = 2$
   $a_4 = 1$

The first distribution is analogous to the case of three alternatives. That is, we have a symmetrical 'circle' among three alternatives and a fourth alternative with no 'positive' majority.

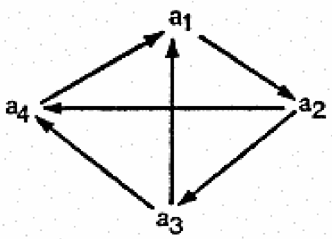
Ex.



$a_1 = 2$
$a_2 = 2$
$a_3 = 2$
$a_4 = 0$

The second distribution is slightly more complicated. The uniqueness of this case can be shown[12] by the following study of the majority relations.

Ex.



$a_1 = 1$
$a_2 = 2$
$a_3 = 2$
$a_4 = 1$

Of the two alternatives, a_1 and a_4, which have only one majority, one alternative necessarily has a majority over the other, e.g. a_4 over a_1. Then a_1 must have a majority over a_2 or a_3, e.g. a_2. This will establish a unique structure: a_2 has a majority over a_3 and a_4, and a_3 has a majority over a_4 and a_1. Hence only one case exists.

Due to lack of space we do not, here, report our analysis of sets of five alternatives.[13]

In Table I we present the results of an analysis of each[14] voting order in each case to establish the winner and its position in the voting order.

On the basis of this information the probabilities defined on p. 42 are worked out. The first probability, i.e. the random probability, equals the number of times the alternative would win divided by the total number of voting orders. The third probability requires some simple summations and divisions. Expressed in words, the probability of winning when an alternative is last in the voting order equals the number of voting orders when it is the last alternative, and it wins, divided by the total number of voting orders when it is last. Notice that the structure of the elimination procedure implies that an alternative that is first or second

Table I. Basic Winning Alternatives and Probabilities in Classes of Cyclical Sets

	Number of voting orders in which the winning alternative is:			Probability of each alternative winning:	
	the third from last alternative	the next to last alternative	the last alternative	when the voting order is selected at random	when it is the last alternative in the voting order
Three alternative cases					
Alt.					
a_1	0	0	2	1/3	1.0
a_2	0	0	2	1/3	1.0
a_3	0	0	2	1/3	1.0
	0	0	6		
Four alternative cases					
Case 1					
a_1	0	2	6	1/3	1.0
a_2	0	2	6	1/3	1.0
a_3	0	2	6	1/3	1.0
a_4	0	0	0	0	0
	0	6	18		
Case 2					
a_1	0	0	6	1/4	1.0
a_2	0	4	6	5/12	1.0
a_3	0	2	4	1/4	2/3
a_4	0	0	2	1/12	1/3
	0	6	18		

in the voting order can never be the winner in a cycle. This occurs since those alternatives by definition have at least one majority against them. Hence these two probabilities always equal zero and can be excluded from the table.

5. A Presentation of the Elimination and the Quality Procedures

The Elimination Procedure

The native country of the elimination procedure is Great Britain. Thus in addition to Sweden and Finland, the procedure is used in the English-speaking countries and in previous English possessions (def. see p. 40).

The elimination procedure yields varied ordinal information about the preferences of the voters. It is usually possible to reconstruct some complete rank orderings. The important trait is, however, that it is never possible to establish the existence of a cyclical set since less than all pairwise votes are taken.[15] An empirical analysis[16] of roll-calls from the Swedish and Finnish Parliaments I made has, however, made the existence of 'real' and 'strategic' cyclical sets very likely.

Despite their empirical likelihood, empirical data do not furnish sufficient information to establish fully the properties of this procedure. Hence we are forced to use hypothetical situations. On the basis of these situations we can conclude that the existence of cyclical sets is what causes problems under the elimination procedure. A *majority alternative* will always be chosen when the elimination procedure is used.[17] A cyclical set, however, is troublesome both when it occurs naturally and when it is used as a tool to promote the interests of individual voters.

This fact led us to classify cyclical sets on the basis of the internal majority relations and to establish three different probability distributions in each type of cyclical set (section 3). The equality aspect and the strategic potentials of the elimination procedure will be thoroughly analyzed in our simulation section.

We must stress the experimental character of our analysis. In actual parliamentary decision-making, negative properties of decision procedures will be to some extent neutralized by countermoves by parliamentary members. That is, 'the rules of the game' are one determining factor when the members decide upon their actions. The reluctance on the part of parliamentarians in Denmark and Norway to allow more than two alternatives in the final voting process is one highly plausible effect of the properties of the successive[18] procedure.

When the elimination procedure is used, one factor that tends to reduce the number of alternatives is the use of strategic countermoves as defined above on p. 41. In situations where a 'real' cycle would tend to materialize despite the inhibiting factors mentioned on p. 41, we find another type of move that will tend to diminish the number of alternatives that reaches the voting phase. Consider once more our cyclical situation on p. 40. Assuming sufficient information, all three voters will know that if a_1 is given last place in the voting order, this alternative will be the winner unless some move is made. In such a situation voter II can and should vote for a_3 in the first roll-call – that is, *against his own primary preference*. Otherwise a_1, which is considered worst by voter II, will be the winner. There is, however, a definite reluctance on the part of parliamentary members to vote against their primary preference. This fact will ensure that this type of move will usually be taken care of in the decision process preceding the voting phase. The implicit disadvantage, contrary to what was generally found above, for those alternatives in a cycle having positions late in the voting order should, however, be kept in mind.

The Quality Procedure

This procedure is probably first considered by parliamentarians accustomed to the elimination method. The quality procedure has been discussed above on p. 41 and will be thoroughly analyzed in our simulation section.

However, it should be stressed here that this procedure sharply diminishes strategic considerations but does not eliminate them. A simple example will help clarify this point.

Ex. Suppose there are three voters, I, II, and III, and three alternatives, a_1, a_2 and a_3.

I	II	III
a_1	a_2	a_3
a_2	a_1	a_1
a_3	a_3	a_2

This is the same situation as on p. 39. We concluded on p. 41 that voter II could change the set of rank orderings into a cyclical one (by changing his rank ordering to a_2, a_3, a_1). We also concluded that this strategic potential could best be utilized with a favorable voting order.

If the voting order is not known in advance, II can still make the strategic move that changes the set into a cyclical one. Then, however, he will run a considerable risk, since after such a move a_1, a_2, and a_3 will each have the same chance of winning (assuming no countermoves by III).

Seemingly II would run the risk of a_3 winning only if he favored a_2 very much and/or if he did not dislike a_3 that much. Under either of these conditions, however, a strategic move might be made.

Two conclusions can be drawn from this example:

a. The quality procedure is not completely satisfactory if we want a procedure in which there are no strategic possibilities.
b. The intensity of preferences plays a significant role in collective decision-making, and the problem of constructing acceptable procedures based on more differentiated preference statements would seem to be highly relevant. An introductory discussion and analysis of this very complicated problem has been carried out.[19]

6. The Theoretical Model

A Classification of Systems of Voting Blocs

In this section we present an exhaustive and mutually exclusive classification of systems of voting blocs, where each bloc consists of voters who have agreed to rank the alternatives in the same way. The classification will be carried out for assemblies with up to five voting blocs. Throughout the analysis blocs will be considered cohesive. Since our interest lies primarily in situations governed by majority rule, the classification is based on the ability of the blocs to form majority coalitions. A motivating assumption is that, *ceteris paribus,* it is easier to form a two-bloc coalition than a three-bloc coalition, a three-bloc coalition than a four-bloc one, and so on.

A simple example helps clarify our approach. Listed below are four different systems, each consisting of four or five blocs.

Ex.					
I.	A = 0.4	B = 0.3	C = 0.25	D = 0.05	
II.	A = 0.3	B = 0.3	C = 0.3	D = 0.1	
III.	A = 0.4	B = 0.2	C = 0.2	D = 0.2	
IV.	A = 0.4	B = 0.2	C = 0.2	D = 0.15	E = 0.05

The weight attached to each bloc is simply the proportion of assembly members who belong to that bloc. In each system, of course, the weights add up to one.

Note that these systems differ in the ability of various blocs to form majority coalitions. In the first system, blocs A, B, and C can form two-bloc majority coalitions freely among themselves. Bloc D, on the other hand, is unable to transform any coalition into a winning one. That is, no coalition rises from a minority to a majority position when D enters. The second system is exactly the same as the first with regard to the ability of the blocs to form majority coalitions. In the third system bloc A has a uniquely advantageous position in that it can form two-bloc majority coalitions with any of the other blocs in the system. The other blocs can form two-bloc majority coalitions only with A. The fourth system is the same as the third except that the fifth bloc (E) is unable to create any majority coalition.[20]

The following classification is thus based on the ability of the blocs to form majorities alone or in coalition with one or more of the other blocs in the system. The proof of the exhaustive and mutually exclusive nature of the classification is found in Bjurulf.[21]

1. *Two blocs* in the voting process

Except when both blocs have equal weights, one bloc will hold a clear majority with a weight > 0.5. Cases in which both blocs (or coalition of blocs) have a weight of 0.5 will be regarded as a special case here and below. Since tie-breaking mechanisms are usually available, these cases will not be treated further.

2. *Three blocs* in the voting process
   a. One bloc has a clear majority. (Ex. A = 0.6, B = 0.3, and C = 0.1.)
   b. All two-bloc coalitions have a clear majority. (We will call this the three-bloc case.) (Ex. A = 0.4, B = 0.4, and C = 0.2.)

3. *Four blocs* in the voting process
   a. One bloc has a clear majority. (Ex. A = 0.6, B = 0.2, C = 0.1, and D = 0.1.)
   b. The three-bloc case: All two-bloc coalitions among the three largest blocs have a clear majority. The fourth bloc cannot contribute to a majority coalition (i.e. by changing a minority coalition to a majority one). (Ex. A = 0.4, B = 0.28, C = 0.28, and D = 0.04.)
   c. The largest bloc is the only one that can create a two-bloc majority. (Ex. A = 0.4, B = 0.2, C = 0.2, and D = 0.2.)

4. *Five blocs* in the voting process
   a. One bloc has a clear majority. (Ex. A = 0.60, B = 0.15, C = 0.10, D = 0.10, and E = 0.05.)
   b. The three-bloc case: All two-bloc coalitions among the three largest blocs have a clear majority. Blocs D and E cannot contribute to a majority coalition. That is, no three-bloc majority can be created in which two of the three blocs do not belong to the three largest, and thus already have a clear majority position. (Ex. A = 0.30, B = 0.30, C = 0.30, D = 0.05, and E = 0.05.)
   c. All two-bloc coalitions that include A have a clear majority. No three-bloc coalition without A has a clear majority position because a two-bloc coalition containing A can always form in opposition to this. (Ex. A = 0.45, B = 0.15, C = 0.15, D = 0.15, and E = 0.10.)
   d. The smallest bloc cannot contribute to a majority coalition. Otherwise, all two-bloc coalitions that include A have a clear majority. (Ex. A = 0.40, B = 0.20, C = 0.20, D = 0.15, and E = 0.05.)
   e. A is necessary for the formation of a two-bloc majority. This can, however, only be done with B or C. Either A or (B + C) is thus necessary and sufficient for the existence of a three-bloc majority. (Ex. A = 0.40, B = 0.24, C = 0.24, D = 0.07, and E = 0.05.)
   f. A and B are necessary for the formation of a two-bloc majority. All three-bloc coalitions containing A or B form a majority. (Ex. A = 0.40, B = 0.40, C = 0.08, D = 0.07, and E = 0.05.)
   g. No two-bloc majority can be created. All three-bloc coalitions form a majority. (Ex. A = 0.21, B = 0.20, C = 0.20, D = 0.20, and E = 0.19.) A special case in this category is when all blocs have equal weights. This is equivalent to the case of five individuals as considered by previous investigators.

The Dimensionality of the Preference Orderings

A group of preference orderings can always be characterized by a set of dimensions on which individuals and alternatives are placed. Occasionally all of the preference orderings will 'fit' a single dimension. This case is important, of course, because of Black's (1958, Chap. IV) well-known finding[22] that the voting paradox cannot occur when preference orderings are single-peaked (which is equivalent to unidimensionality).[23] Most often, however, two or more dimensions are needed to fully characterize the preference orderings. While it is notoriously difficult to work with more than one dimension, some aspects of the multidimensional situation have recently succumbed to the analysis of Davis, Hinich, and Ordeshook.[24] In probabilistic analyses, however, little attention has so far been paid to the dimensionality of the preference orderings. One exception to this is Niemi,[25] who worked with the proportion of individuals having single-peaked preference functions. Here we utilize a different approach, which we hope will be useful both in this and other applications.

In our variant, bloc preferences are locked into certain positions on the dimensions by assuming that each bloc has one specific primary preference. For example, suppose the single dimensions are a_1, a_2, a_3, a_4, a_5 and a_1, a_3, a_5, a_2, a_4. Each bloc is assigned one alternative as the primary preference of that bloc, e.g. $A = a_1$, $B = a_2$, etc. Then when each bloc is assigned a preference ordering, the set from which the selection is made must satisfy two requirements: 1) single-peakedness, and 2) one specific alternative as the primary preference, Thus, in the example, the preference ordering of bloc A must be a_1, a_2, a_3, a_4, a_5 or a_1, a_3, a_5 , a_2, a_4. The preference ordering of bloc B will be a_2, a_1, a_3, a_4, a_5 or a_2, a_3, a_1, a_4, a_5 or a_2, a_3, a_4, a_1, a_5 or a_2, a_3, a_4, a_5, a_1 or a_2, a_4, a_5, a_3, a_1 or a_2, a_5, a_4, a_3, a_1 or a_2, a_5, a_3, a_4, a_1 or a_2, a_5, a_3, a_1, a_4.

In this way we can analyze the effects of having the blocs situated more or less uniformly along a left to right scale. The 'dimensional' analysis will start with all blocs positioned along one scale. This situation is analyzed by studying the degenerate case in which the two dimensions are identical. Having analyzed this case we will turn to some of the other cases, viewing them as greater or lesser deviations from the left to right scale.

Assumptions about Conflict and Tied Ranks

In addition to the systems of voting blocs and the dimensional character of the preference orderings, two other factors will be varied in the simulation. First, we will make several assumptions about the possibility that two or more blocs have the same first choice. Theories of decision-making are most concerned with situations of conflict. Hence cases in which two large blocs (especially the two largest) have the same first preference are of somewhat less interest. In parts of the simulation, therefore, we will require that some or all of the voting blocs have different first choices.

Secondly, for a part of the simulation we wish to introduce the possibility of ties in the preference orderings. We will do this in a limited way here by assuming either that preferences below the first choice may be tied or that preferences other than the first and last choices may be tied. These assumptions strike us as a fairly realistic initial way of bringing in tied ranks, since we presume that best and worst alternatives are among the easiest to distinguish. The probability of different types of ties under these assumptions is given in footnote.[26]

7. A Computer Simulation of Strategic and Equality Aspects of the Elimination Procedures

The probability distributions we are seeking can be closely approximated by a simulation in which sets of individual (or, in this case, bloc) preference orderings are chosen at random and then analyzed. An excellent description of the detailed procedure is given by Klahr,[27] although in his case the analytical objective was cyclical social orderings. As in other probabilistic analyses, we assume throughout that all individual (bloc) preference orderings are transitive and that in voting on a pair of alternatives each bloc votes for the alternative that is higher in its preference ordering. We also assume that all preference orderings are equally likely – subject to the constraints discussed above.

A Computer Analysis of the Elimination Procedure

In our analysis of *the strategic and equality aspects of two variants of the elimination procedure* we have in each case computed four distributions of the winning probability of each bloc. The *first* distribution is the winning probability of the different blocs in 10,000[28] randomly generated sets of preference orderings when the voting order is decided at random *after* the registration of the preference statements and assuming no strategic moves. In the first situation to be analyzed, where all blocs have different first choices (Table II), this distribution should add up to 1.00 (except for rounding error). The *second* distribution is the same as the first except that the entry for each bloc represents the probability of winning when *that* bloc in non-winning positions has tried *all*[29] possible strategic moves and no other strategic moves or countermoves have been made. Obviously this distribution usually adds to more than 1.00. The same is true of the *third* distribution where the winning probability for each bloc is calculated as if *that* bloc had the power to make its own first alternative the last one in the voting order. The *last* distribution is the same as the third except that each probability presumes that the bloc under consideration has tried all possible strategic moves (when it is in a non-winning position) and no other strategic moves or countermoves have been made. We should emphasize that we have used the probabilities given in Table I when computing the winning probabilities shown in Table II. That is, we have calculated the strategic *potentials* given a bloc with emphasis on the success of the first choice. Whether a bloc facing a ⅓ probability of succeeding by the strategic move, a ⅓ probability of no change, and a ⅓ probability of a worse alternative will in fact make the strategic move is a problem dependent on intensity of preferences and on psychological factors. Here we simply add ⅓ to the bloc's winning probability in an attempt to calculate the magnitude of the *potential* strategic moves.

Our first results are derived from the situation in which no dimensional assumptions are made, where the five voting blocs have different first choices, and where no tied ranks are permitted. The results for each system of voting blocs are given in Table III.

The first[30] case in Table II is the three-bloc case (cf. p. 50). Recall that since no ties are permitted, any distribution within the case will yield the same result.[31] Accordingly, in the first case the three largest blocs have the same winning probability (within the sampling error of ± 0.01).[32]

Table II. Winning Probabilities for Each Bloc, with or without Strategic Moves, within systems of Five Voting Blocs, for Five Alternatives, with All Blocs Having Different First Choices, No Ties

Party	A	B	C	D	E
Weight (Case b)	0.40	0.31	0.25	0.03	0.01
Winning probability *without* strategic moves and the voting order decided *at random*	0.29	0.30	0.29	0.06	0.06
Winning probability *with* strategic moves and the voting order decided *at random*	0.38	0.39	0.38	0.06	0.06
Winning probability *without* strategic moves and *each party in control of the voting order*	0,47	0,48	0,47	0,13	0,14
Winning probability *with* strategic moves and *each party in control of the voting order*	0.78	0.79	0.79	0.13	0.14
Weight (Case c)	0.47	0.21	0.15	0.10	0.07
Winning probability *without* strategic moves and the voting order decided *at random*	0.72	0.07	0.07	0.07	0.07
Winning probability *with* strategic moves and the voting order decided *at random*	0.76	0.10	0.10	0.10	0.10
Winning probability *without* strategic moves and *each party in control of the voting order*	0.88	0.16	0.15	0.16	0.16
Winning probability *with* strategic moves and *each party in control of the voting order*	0.996	0.26	0.25	0.27	0.26
Weight (Case d)	0.46	0.30	0.15	0.07	0.02
Winning probability *without* strategic moves and the voting order decided *at random*	0.54	0.14	0.13	0.14	0.05
Winning probability *with* strategic moves and the voting order decided *at random*	0.61	0.19	0.19	0.19	0.05
Winning probability *without* strategic moves and *each party in control of the voting order*	0.74	0.27	0.26	0.28	0.13
Winning probability *with* strategic moves and *each party in control of the voting order*	0.97	0.45	0.45	0.46	0.13
Weight (Case e)	0.46	0.30	0.19	0.03	0.02
Winning probability *without* strategic moves and the voting order decided *at random*	0.41	0.20	0.20	0.10	0.09
Winning probability *with* strategic moves and the voting order decided *at random*	0.49	0.27	0.28	0.12	0.11
Winning probability *without* strategic moves and *each party in control of the voting order*	0.62	0.37	0.37	0.22	0.20
Winning probability *with* strategic moves and *each party in control of the voting order*	0.91	0.63	0.63	0.29	0.28
Weight (Case f)	0.40	0.36	0.09	0.08	0.07
Winning probability *without* strategic moves and the voting order decided *at random*	0.29	0.28	0.14	0.15	0.14
Winning probability *with* strategic moves and the voting order decided *at random*	0.37	0.36	0.18	0.19	0.18
Winning probability *without* strategic moves and *each party in control of the voting order*	0.49	0.47	0.28	0.28	0.28
Winning probability *with* strategic moves and *each party in control of the voting order*	0.78	0.78	0.43	0.44	0.43

Party	A	B	C	D	E
Weight (Case g)	0.20	0.20	0.20	0.20	0.20
Winning probability *without* strategic moves and the voting order decided *at random*	0.20	0.21	0.20	0.20	0.20
Winning probability *with* strategic moves and the voting order decided *at random*	0.26	0.27	0.26	0.26	0.26
Winning probability *without* strategic moves and *each party in control of the voting order*	0.35	0.36	0.35	0.35	0.35
Winning probability *with* strategic moves and *each party in control of the voting order*	0.57	0.58	0.57	0.57	0.58

Comparing the four distributions, note that the two 'dummy blocs' D and E have no strategic potentials. Their winning probabilities relate solely to the positions of their primary preferences in the preference statements of blocs A–C. Turning to the result for the three main blocs we note *that when the voting order is decided at random, even with strategic moves, these blocs cannot reach the winning probability obtained when they control the voting order.*[33] The strategic gain is about 10 percent. The gain when controlling the voting order, however, is about 20 percent. Furthermore, when a bloc is in control of the voting order and employs strategic voting, the gain is especially large.

For cases c–g we find a moderate increasing strategic gain when the voting order is decided at random. The gain is, however, always below the 10 percent gain found in the three-bloc case b.

For cases c and d (closely resembling the Scandinavian situation) we should note that when the largest bloc votes strategically and has control of the voting order it will almost always win.

Case f resembles the case of three balancing blocs, case b. Here there are two large balancing blocs. These two blocs and the three blocs in the b case have almost identical winning probabilities in all four distributions. The results of the remaining three blocs in the f case, however, do not resemble the results for the dummy blocs in the b case. In case f the smallest blocs have considerable strategic potentials.

For cases b, e, f, and g we should note that the first strategic distribution is fairly close to the actual distribution of weights. Such is not true for cases c and d (even though the last two distributions differ even more).

This situation with five alternatives in the final voting process is important in an attempt to uncover some of the problems facing the blocs if each one wants to bring its own solution to the voting process. The fact that in the Swedish and Finnish Parliaments five alternatives frequently remain in the phase just preceding the voting emphasizes this.

Obviously, however, the situation with three alternatives is also of vital theoretical and empirical interest. In Table III sets of three alternatives are first analyzed in a type of situation closely resembling the preceding one.

Table III. Winning Probabilities for Each Bloc, with or without Strategic Moves within Systems[34] of Five Voting Blocs, for Three Alternatives, with the Three Largest Blocs Having Different First Choices. No Ties

Party	A	B	C	D	E
Weight (Case b_2)	0.40	0.31	0.25	0.03	0.01
Winning probability *without* strategic moves and the voting order decided *at random*	0.34	0.33	0.34	0.33	0.33
Winning probability *with* strategic moves and the voting order decided *at random*	0.42	0.41	0.42	0.33	0.33
Winning probability *without* strategic moves and *each party in control of the voting order*	0.50	0.49	0.50	0.50	0.49
Winning probability *with* strategic moves and *each party in control of the voting order*	0.75	0.74	0.75	0.50	0.49
Weight (Case c_2)	0.47	0.21	0.15	0.10	0.07
Winning probability *without* strategic moves and the voting order decided *at random*	0.81	0.10	0.10	0.46	0.45
Winning probability *with* strategic moves and the voting order decided *at random*	0.84	0.12	0.11	0.47	0.46
Winning probability *without* strategic moves and *each party in control of the voting order*	0.88	0.17	0.16	0.52	0.52
Winning probability *with* strategic moves and *each party in control of the voting order*	0.97	0.23	0.22	0.56	0.56
Weight (Case d_2)	0.46	0.30	0.15	0.07	0.02
Winning probability *without* strategic moves and the voting order decided *at random*	0.64	0.18	0.18	0.55	0.34
Winning probability *with* strategic moves and the voting order decided *at random*	0.70	0.22	0.22	0.58	0.34
Winning probability *without* strategic moves and *each party in control of the voting order*	0.76	0.29	0.29	0.67	0.45
Winning probability *with* strategic moves and *each party in control of the voting order*	0.92	0.41	0.42	0.74	0.45
Weight (Case e_2)	0.46	0.30	0.19	0.03	0.02
Winning probability *without* strategic moves and the voting order decided *at random*	0.48	0.26	0.27	0.43	0.43
Winning probability *with* strategic moves and the voting order decided *at random*	0.55	0.32	0.33	0.44	0.45
Winning probability *without* strategic moves and *each party in control of the voting order*	0.63	0.41	0.42	0.58	0.59
Winning probability *with* strategic moves and *each party in control of the voting order*	0.85	0.60	0.61	0.62	0.62
Weight (Case f_2)	0.40	0.36	0.09	0.08	0.07
Winning probability *without* strategic moves and the voting order decided *at random*	0.40	0.39	0.22	0.53	0.54
Winning probability *with* strategic moves and the voting order decided *at random*	0.45	0.45	0.25	0.55	0.56
Winning probability *without* strategic moves and *each party in control of the voting order*	0.52	0.51	0.33	0.65	0.66
Winning probability *with* strategic moves and *each party in control of the voting order*	0.68	0.68	0.45	0.72	0.72

Party	A	B	C	D	E
Weight (Case g_2)	0.20	0.20	0.20	0.20	0.20
Winning probability *without* strategic moves and the voting order decided *at random*	0.33	0.32	0.34	0.62	0.62
Winning probability *with* strategic moves and the voting order decided *at random*	0.37	0.36	0.38	0.65	0.65
Winning probability *without* strategic moves and *each party in control of the voting order*	0.42	0.41	0.43	0.71	0.70
Winning probability *with* strategic moves and *each party in control of the voting order*	0.52	0.51	0.52	0.79	0.79

In Table III we should remember (e.g. in case b_2) that only the three largest blocs have conflicting first choices and that each of the dummy blocs has a first choice identical to the first choice of one of the big blocs. Accordingly a winning probability of 0.33 for D and E is to be expected. Obviously the first three probabilities in the first distribution should add up to 1.00.

In general we find that the first probability distribution is somewhat higher and that the last is lower but that all the tendencies found in the five-alternative situation are also found in the three-alternative situation.

The next change in the parameters of our voting system is to permit indifference between alternatives – i.e. ties are allowed in the bloc's preference statements. This adds two complicating factors. First, we cannot eliminate tied ranks in the social ordering simply by seeing that no pairs of blocs have weights that add up to exactly 0.50. This is most easily seen in the case of equal weights, in which each bloc in the five-bloc system has a weight of 0.20. With no indifference permitted, a unique majority alternative always exists. With ties, however, we may have a weight of 0.40 for one alternative, 0.40 for another, and 0.20 indifferent. We will see, however, that this problem usually does not change the results very much. A second complication is that our classification of systems of voting blocs would have to be considerably expanded to take care of all the 'subcases' that can arise when ties are permitted. For example, the following two examples are both e systems:

e_1: A = 0.31, B = O.24, C = 0.23, D = 0.12, and E = 0.10.
e_2: A = 0.46, B = 0.30, C = 0.19, D = 0.03, and E = 0.02.

However, if bloc A is indifferent between alternatives 1 and 2, D and E can contribute to a majority coalition in e_1 but not in e_2. Rather than creating an elaborate classification we have used two subcases for each of our previously defined cases.

The results of the first simulation with ties permitted[35] and three alternatives are given in Tables IV and V.

Table IV. Winning Probabilities for Each Bloc, with or without Strategic Moves in Subcases of Systems of Five Voting Blocs, for Three Alternatives, with the Three largest Blocs Having Different First Choices, Ties Permitted

Party	A	B	C	D	E
Weight (Case b_2)	0.40	0.31	0.25	0.03	0.01
Winning probability *without* strategic moves and the voting order decided *at random*	0.47	0.32	0.21	0.33	0.36
Winning probability *with* strategic moves and the voting order decided *at random*	0.55	0.42	0.29	0.33	0.34
Winning probability *without* strategic moves and *each party in control of the voting order*	0.62	0.47	0.36	0.48	0.49
Winning probability *with* strategic moves and *each party in control of the voting order*	0.86	0.77	0.60	0.48	0.49
Weight (Case c_2)	0.47	0.21	0.15	0.10	0.07
Winning probability *without* strategic moves and the voting order decided *at random*	0.87	0.07	0.06	0.42	0.42
Winning probability *with* strategic moves and the voting order decided *at random*	0.90	0.09	0.08	0.43	0.43
Winning probability *without* strategic moves and *each party in control of the voting order*	0.91	0.11	0.10	0.47	0.46
Winning probability *with* strategic moves and *each party in control of the voting order*	0.99	0.17	0.15	0.50	0.49
Weight (Case d_2)	0.46	0.30	0.15	0.07	0.02
Winning probability *without* strategic moves and the voting order decided *at random*	0.72	0.16	0.12	0.50	0.35
Winning probability *with* strategic moves and the voting order decided *at random*	0.77	0.22	0.16	0.52	0.35
Winning probability *without* strategic moves and *each party in control of the voting order*	0.81	0.25	0.21	0.59	0.44
Winning probability *with* strategic moves and *each party in control of the voting order*	0.95	0.42	0.33	0.65	0.44
Weight (Case e_2)	0.46	0.30	0.19	0.03	0.02
Winning probability *without* strategic moves and the voting order decided *at random*	0.58	0.25	0.17	0.41	0.41
Winning probability *with* strategic moves and the voting order decided *at random*	0.64	0.33	0.23	0.42	0.42
Winning probability *without* strategic moves and *each party in control of the voting order*	0.70	0.38	0.30	0.54	0.54
Winning probability *with* strategic moves and *each party in control of the voting order*	0.90	0.62	0.47	0.57	0.56
Weight (Case f_2)	0.46	0.36	0.09	0.08	0.07
Winning probability *without* strategic moves and the voting order decided *at random*	0.49	0.37	0.14	0.52	0.51
Winning probability *with* strategic moves and the voting order decided *at random*	0.55	0.43	0.17	0.54	0.53
Winning probability *without* strategic moves and *each party in control of the voting order*	0.60	0.48	0.25	0.63	0.62
Winning probability *with* strategic moves and *each party in control of the voting order*	0.77	0.67	0.35	0.69	0.68

Party	A	B	C	D	E
Weight (Case g_2)	0.20	0.20	0.20	0.20	0.20
Winning probability *without* strategic moves and the voting order decided *at random*	0.32	0.32	0.34	0.62	0.62
Winning probability *with* strategic moves and the voting order decided *at random*	0.35	0.35	0.36	0.64	0.65
Winning probability *without* strategic moves and *each party in control of the voting order*	0.39	0.38	0.40	0.68	0.69
Winning probability *with* strategic moves and *each party in control of the voting order*	0.46	0.46	0.47	0.77	0.77

V

Party	A	B	C	D	E
Weight (Case b_3)	0.29	0.27	0.25	0.10	0.09
Winning probability *without* strategic moves and the voting order decided *at random*	0.39	0.34	0.27	0.41	0.40
Winning probability *with* strategic moves and the voting order decided *at random*	0.47	0.42	0.36	0.42	0.42
Winning probability *without* strategic moves and *each party in control of the voting order*	0.55	0.49	0.43	0.56	0.56
Winning probability *with* strategic moves and *each party in control of the voting order*	0.79	0.75	0.69	0.60	0.60
Weight (Case c_3)	0.37	0.18	0.16	0.15	0.14
Winning probability *without* strategic moves and the voting order decided *at random*	0.76	0.13	0.11	0.49	0.49
Winning probability *with* strategic moves and the voting order decided *at random*	0.81	0.15	0.13	0.50	0.51
Winning probability *without* strategic moves and *each party in control of the voting order*	0.83	0.20	0.18	0.56	0.56
Winning probability *with* strategic moves and *each party in control of the voting order*	0.96	0.26	0.24	0.60	0.61
Weight (Case d_3)	0.31	0.23	0.21	0.20	0.05
Winning probability *without* strategic moves and the voting order decided *at random*	0.55	0.22	0.22	0.65	0.33
Winning probability *with* strategic moves and the voting order decided *at random*	0.62	0.27	0.26	0.68	0.33
Winning probability *without* strategic moves and *each party in control of the voting order*	0.66	0.32	0.32	0.75	0.43
Winning probability *with* strategic moves and *each party in control of the voting order*	0.86	0.45	0.45	0.84	0.43
Weight (Case e_3)	0.31	0.24	0.23	0.12	0.10
Winning probability *without* strategic moves and the voting order decided *at random*	0.49	0.26	0.25	0.50	0.45
Winning probability *with* strategic moves and the voting order decided *at random*	0.56	0.33	0.32	0.52	0.47
Winning probability *without* strategic moves and *each party in control of the voting order*	0.62	0.39	0.38	0.62	0.58
Winning probability *with* strategic moves and *each party in control of the voting order*	0.84	0.59	0.59	0.69	0.64

Party	A	B	C	D	E
Weight (Case f_5)	0.31	0.20	0.18	0.16	0.15
Winning probability *without* strategic moves and the voting order decided *at random*	0.46	0.35	0.19	0.56	0.56
Winning probability *with* strategic moves and the voting order decided *at random*	0.52	0.40	0.23	0.58	0.58
Winning probability *without* strategic moves and *each party in control of the voting order*	0.57	0.45	0.29	0.66	0.66
Winning probability *with* strategic moves and *each party in control of the voting order*	0.74	0.61	0.40	0.74	0.74
Weight (Case g_1)	0.24	0.23	0.21	0.18	0.14
Winning probability *without* strategic moves and the voting order decided *at random*	0.33	0.33	0.34	0.68	0.54
Winning probability *with* strategic moves and the voting order decided *at random*	0.37	0.37	0.38	0.72	0.56
Winning probability *without* strategic moves and *each party in control of the voting order*	0.42	0.42	0.43	0.77	0.63
Winning probability *with* strategic moves and *each party in control of the voting order*	0.54	0.54	0.54	0.87	0.69

The main difference between Table III and Table IV is found in the first probability distribution. We find that the tied preferences tend to increase the winning probability of the largest bloc. As in Table III, the sum of the probabilities for the three largest blocs should be 1.00.

Turning to the strategic gains, however, and comparing them with the gains found in Table III, we find that the increases in distributions 1 through 4 are very similar. The differences can mostly be attributed to the generally high values of the probabilities in Table IV.

In Table V other subcases represent our previously defined cases. The effect of this is that the sharply reduced weight for the largest party diminishes the first winning probability. The increases from the same basic winning probability were found to be almost identical. In general the basic tendencies found in Table II are still with us. That is, neither the decrease in number of alternatives nor the introduction of indifference changes the tendencies.

Our last simulation in the analysis of the elimination variants was meant to incorporate the dimensional character of the preference orderings as outlined above. A thorough investigation would, however, consume a tremendous amount of computer time. Hence we will have to postpone a complete analysis until the next computer generation is installed.

We have, however, made a thorough investigation of the unidimensional situation. Since it is highly probable that the two-dimensional situation would yield results 'in between' the unidimensional case and the multidimensional case analyzed above, we feel that this constraint is acceptable.

Table VI. Winning Probability with or without Strategic Moves within Systems of Five Voting Blocs, for Five Alternatives, with All Blocs Having Different First Choices and All Blocs Having Single-Peaked Preferences

Parties ordered according to their positions on a left to right scale	A	B	C	D	E
Weight (Case g_2)	0.20	0.20	0.20	0.20	0.20
Winning probability *without* strategic moves and the voting order decided *at random*	0.00	0.00	1.00	0.00	0.00
Winning probability *with* strategic moves and the voting order decided *at random*	0.03	0.18	1.00	0.18	0.03
Winning probability *without* strategic moves and *each party in control of the voting order*	0.00	0.00	1.00	0.00	0.00
Winning probability *with* strategic moves and *each party in control of the voting order*	0.11	0.52	1.00	0.52	0.11

	A	B	C	D	E
Weight (Case g_1)	0.24	0.23	0.14	0.18	0.21
Winning probability *without* strategic moves and the voting order decided *at random*	0.00	0.00	1.00	0.00	0.00
Winning probability *with* strategic moves and the voting order decided *at random*	0.03	0.19	1.00	0.18	0.03
Winning probability *without* strategic moves and *each party in control of the voting order*	0.00	0.00	1.00	0.00	0.00
Winning probability *with* strategic moves and *each party in control of the voting order*	0.11	0.53	1.00	0.51	0.11

In Table VI we try to clarify how the analysis of the unidimensional situation is presented. Since no ties are permitted, any subcase within our seven main cases will yeild the same result. In Table VI we have picked out two subcases in the g case. In the first subcase (g_2) the blocs have the positions ABCDE on a left to right scale. Since all blocs have single-peaked preference statements[36] the A bloc can only have the preference statement a_1, a_2, a_3, a_4, a_5 and the E bloc only a_5, a_4, a_3, a_2, a_1. For the other blocs we make a random selection among the possible ones. That is, for bloc B one of a_2, a_1, a_3, a_4, a_5 or a_2, a_3, a_1, a_4, a_5 or a_2, a_3, a_4, a_1, a_5 or a_2, a_3, a_4, a_5, a_1 is randomly selected. When computing the strategic potentials, however, all 24 possible preference statements were investigated to see which one produces the best result assuming that the blocs primarily concentrate on getting their first preference as the outcome. The same four distributions were computed as above.

The result for case g_2 is a verification of Black's theorem that 'if the member's curves are single-peaked 0_{med} will be able to get a simple majority over any other motions a_1, a_2, . . . a_m put forward.'[37] Accordingly the first preference of the C bloc will always emerge as the majority alternative since bloc C holds the median position on the scale. It automatically follows that no strategic moves can improve the position of bloc C.

The strategic potentials are concentrated around the median power center. We find again that the strategic potential when the voting order is selected at random after the voters have registered their preferences is limited when compared with the strategic potential for a bloc in control of the voting order.

As predicted, the results for the g_1 case are identical (within the margin of error) to those for g_2. Notice, however, that in this case the smallest bloc (E) holds the median position. That is, the blocs are ordered ABEDC. Since all of the g subcases are identical from the point of view of creating majorities, the positions and not the distributions of weight determine the winning probabilities.

Table VII

Party	A	B	C	D	E
Weight (Case b_2)	0.41	0.31	0.25	0.03	0.01
Winning probability *without* strategic moves and the voting order decided *at random*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and the voting order decided *at random*	0.11	1.00	0.25	0.00	0.00
Winning probability *without* strategic moves and *each party in control of the voting order*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and *each party in control of the voting order*	0.32	1.00	0.74	0.00	0.00
Weight (Case c_2)	0.47	0.21	0.15	0.10	0.07
Winning probability *without* strategic moves and the voting order decided *at random*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and the voting order decided *at random*	0.28	1.00	0.25	0.11	0.02
Winning probability *without* strategic moves and *each party in control of the voting order*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and *each party in control of the voting order*	0.91	1.00	0.75	0.42	0.09
Weight (Case d_2)	0.40	0.30	0.15	0.07	0.02
Winning probability *without* strategic moves and the voting order decided *at random*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and the voting order decided *at random*	0.21	1.00	0.25	0.11	0.00
Winning probability *without* strategic moves and *each party in control of the voting order*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and *each party in control of the voting order*	0.63	1.00	0.75	0.42	0.00
Weight (Case e_2)	0.46	0.30	0.19	0.03	0.02
Winning probability *without* strategic moves and the voting order decided *at random*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and the voting order decided *at random*	0.21	1.00	0.25	0.00	0.00
Winning probability *without* strategic moves and *each party in control of the voting order*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and *each party in control of the voting order*	0.64	1.00	0.74	0.00	0.00

Party	A	B	C	D	E
Weight (Case f_2)	0.40	0.36	0.09	0.08	0.07
Winning probability *without* strategic moves and the voting order decided *at random*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and the voting order decided *at random*	0.18	1.00	0.00	0.00	0.00
Winning probability *without* strategic moves and *each party in control of the voting order*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and *each party in control of the voting order*	0.55	1.00	0.00	0.00	0.00

Table VIII

Party	E	A	B	C	D
Weight	0.07	0.41	0.21	0.165	0.145
Winning probability *without* strategic moves and the voting order decided *at random*	0.00	0.00	1.00	0.00	0.00
Winning probability *with* strategic moves and the voting order decided *at random*	0.00	0.31	1.00	0.18	0.03
Winning probability *without* strategic moves and *each party in control of the voting order*	0.00	0.00	1.00	0.00	0.00
Winning probability *with* strategic moves and *each party in control of the voting order*	0.00	0.92	1.00	0.51	0.11

Party	E	A	C	B	D
Weight	0.05	0.47	0.16	0.20	0.12
Winning probability *without* strategic moves and the voting order decided *at random*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and the voting order decided *at random*	0.11	1.00	0.00	0.00	0.00
Winning probability *without* strategic moves and *each party in control of the voting order*	0.00	1.00	0.00	0.00	0.00
Winning probability *with* strategic moves and *each party in control of the voting order*	0.31	1.00	0.00	0.00	0.00

In Table VII we analyze one and the same ordering in our remaining five cases. The ordering analyzed is ABCDE. In all cases Black's theorem is verified. In addition, note that the strategic potential is concentrated around the median position. In all cases we found a sharp increase in strategic potential when a bloc is in control of the voting order.

Turning to the specific probabilities we should note the identical values for the C bloc for cases b_2 through e_2. This is due to the fact that in all these cases the C bloc has the opportunity to defeat the majority alternative (a_2) by voting with the A bloc (i.e. using a_1). In case f_2, however, the C bloc does not hold a majority in coalition with A. The large strategic potentials of the A bloc, especially when in control of the voting order, are clearly shown in the Table. Remember here that all these strategic potentials imply that the *majority alternative* (a_2) *would not be chosen.*

The two cases in Table VIII are especially interesting from a Swedish point of view since they closely resemble the political structure in contemporary Swedish Parliament.

The second case is a good representation of the present (1972) Swedish political context in cases where one dimension predominates, although many people would argue that the B and the C blocs should change positions. In any case, when acting separately the B through D blocs are dummy blocs in this unidimensional structure. Empirical investigations of Swedish roll-calls seem to indicate that this is a situation frequently occurring, although multidimensional situations also occur quite often.[38]

If there is a change in the present Swedish situation, the first case in Table VIII is the most probable new organization. This case will be further discussed below. Here it is sufficient to emphasize the high strategic potential of the largest bloc when it is in control of the voting order.

8. Summary

A significant potential for strategic moves inherent in the elimination procedure would be sharply reduced (but not eliminated) when using the variant known as the quality procedure. This is the main conclusion of analyzing the results of our simulation study.

Turning to these results we will first summarize the *general* effects. Then we will report the specific effects of voting blocs and other features on the strategic and equality aspects.

Two basic variations (in my opinion the most important variants)[39] of the elimination procedure were analyzed. In the *first* type one bloc has the power to make its own first alternative the last one in the voting order. In the *second* type the voting order is randomly selected either before or after all voters have registered their preference statements.

In the first variant there was considerable potential for strategic manipulation of the results in all situations in which no bloc controlled more than 50 percent of the first choices. The opportunities for strategic voting were found to be in rough proportion to the numerical strength of the blocs. This variation of the elimination procedure bears a strong and unexpected resemblance to the plurality procedure, in which only the first preferences determine the outcome. The plurality procedure is, however, cruder since it might, for example, have a result that is considered the worst one by more than half of the voters.[40] This is never possible when using the elimination procedure since the last vote will ensure that the worst alternative is always eliminated.

The resemblance to the plurality procedure is most pronounced in cases in which we have one dominating bloc (the c, d, and e cases). In these cases we find that the strategic winning probability for the largest bloc is generally higher than 0.90.

For the c case the winning probability might even reach 0.99. However, in the cases with two (f), three (b), or five (g) balancing blocs the less crude character of the first variant of the elimination procedure will, in spite of the frequent potential for strategic voting, insure a better adherence to the equality principle.

The strategic potentialities, especially when the largest bloc is in control[41] of the voting order, will tend to limit sharply the number of alternatives brought into the voting process. Hence issues are mostly decided in processes taking place prior to the final voting, and it is likely that strategic potentials will be most significant in this phase of the decision process.

Parliamentarians, since they often determine how the voting order should be selected, might view strategic voting as a natural and indispensable part of political life – even if they recognize the negative implications of strict majoritarian or egalitarian viewpoints. Such parliamentarians might reasonably prefer the variant of the elimination procedure in which the voting order is decided at random *but known* to the voters when they register their preferences. This variant would probably drastically increase strategic maneuvers in a multiparty system. This conclusion is based on the fact that under a random selection of the voting order, all blocs holding a certain 'majority-coalition potential' would have an opportunity to promote their primary preferences through strategic voting. Of course in each specific situation the voting order that happened to be randomly selected would play a crucial role. *In the long run*, however, this variant would produce outcomes more in accordance with the equality principle than does the first variant.

A vast change in the properties of the elimination procedure occurs when the voting order is selected at random but *not* known to the voters when they register their preferences (the quality procedure). We found that such a procedure results in very limited strategic possibilities, even if we assume that blocs take advantage of strategic voting whenever possible. That is, we regard it as very doubtful that a bloc confronted with a ⅓ probability of a better result, a ⅓ probability of no change, and a ⅓ probability of a worse result would make a strategic move. Nonetheless, in our calculations we have assumed that a strategic move would be made, and we have counted the positive probability (e.g. ⅓) in calculating probabilities of strategic voting. We have not, however, made any reductions for probabilities of a worse result. Thus we have calculated the upper limit of strategic probabilities. Despite this fact, we find that strategic voting in this case leads to only moderate improvements in the winning probability.

These results suggest that all blocs (when the quality procedure is used) have a fair chance (in relation to their weight and majority position) if they bring their first choices into the voting process. This will tend to increase the number of alternatives in the voting process and will make the blocs more 'equal' in the decisions preceding the voting itself. It is obvious, however, that these properties will also make the voting more uncertain. Parliamentarians, in general, tend to regard this uncertainty as a very negative property and will usually prefer the first variant of the elimination procedure despite its negative properties from strategic and equality aspects.

Turning now to the more detailed results, we find that neither a change in number of alternatives nor the entry of tied preferences changes the general tendencies reported above. When relaxing our basic assumption of conflicting first choices among blocs, we found as expected a heavy increase in the winning probabilities when no strategies were employed. This is due to patterns in which one alternative received more than 50 percent of the primary preferences. These patterns are trivial and serve no analytical purpose and only complicate the analysis. In regard to the effects of different systems of voting blocs, we reported above the general difference between the 'balancing' cases and the 'dominating' cases. Here it is sufficient to note the different character of the smaller blocs. The dummy character of blocs D and E in case b and the balancing positions of blocs C through E in case f are obvious in our results. We should also mention that case g is slightly distinct from the other two balancing cases.

From our analysis of the unidimensional situation we should note our corroboration of Black's theorem that 'if the members' curves are single-peaked 0_{med} will be able to get a simple majority over any other motions $a_1, a_2, \ldots a_m$ put forward.'[42] More importantly, we also found that the strategic potentials are concentrated around the median power center. The limited strategic potential when the voting order is selected at random after the voters have registered their preferences as compared to the strategic potential for a bloc in control of the voting order was found here as well. Finally we illustrated the present Swedish situation and the most probable new organization (if there is a change) in unidimensional contexts. These cases illustrate the strongly dominant position of the largest bloc in the present situation and the hazardous position of a one-bloc (e.g. B) minority government in the new organization. The strategic potential of the largest bloc if in control of the voting order is very large in the new situation (0.92). Moreover the strategic potentials of blocs C and D if in control of the voting order are considerable (0.51 and 0.11, respectively). The 'random' strategic potentials are on a much lower level, and we should also remember that the strategic moves in that situation mostly imply a considerable risk.

In this article I have analyzed certain properties of two voting procedures in *certain specified situations*. Our analysis has been concentrated on properties and effects of the final voting process. The next step in the research process is to analyze other aspects of this as well as other legislative stages in an attempt to build a dynamic model of the parliamentary voting process. The predictive value of the debate preceding the voting and the predictive value of the committee report are examples of specific objects in such an attempt.

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NOTES

1. R. G. Niemi and H. F. Weisberg (eds.), *Probability Models of Collective Decision-Making*, Columbus, Ohio: 1972, p. 393.
2. A set of preference orderings is single-peaked if there is an ordering of the alternatives on the abscissa such that when utility or degree of preference is indicated by the ordinate, each preference ordering can be represented by a curve which changes its direction at most once, from up to down (i.e. has at most one peak). A dimension is said to exist in a set of alternatives if a set of voters has a complete agreement on a standard of judgment in such a way that their preference orderings will be single-peaked.
3. A sample size of 10,000 implies a 0.95 confidence level that our estimated probability is within 0.01 of the true probability (W. J. Dixon and F. J. Massey, Jr., *Introduction to Statistical Analysis*, 1957, pp. 84–85). In all simulations that follow we have sampled 10,000 sets of preference orderings.
4. B. H. Bjurulf, 'A Probabilistic Analysis of Voting Blocs and the Occurrence of the Paradox of Voting,' in Niemi and Weisberg, *op.cit.*
5. B. H. Bjurulf, *Parlamentariska voteringmetoder*, SOU 1972:15 Bilaga 4 (3), pp. 364–365.
6. *Ibid.*, and R. G. Niemi, 'Majority Decision-Making with Partial Unidimensionality,' *American Political Science Review 63* (1969), p. 494, and Niemi and Weisberg, *op.cit.*, Chapters 9–10.
7. R. D. Luce and H. Raiffa, *Games and Decisions*, New York: 1957, p. 369.
8. D. Black, *The Theory of Committees and Elections*, Cambridge: 1958.
9. See note 5.
10. Test-runs involving countermoves indicated that a vast amount of computer time would be needed. This fact and our conclusion above that a completely effective strategic countermove always exists were the main reasons behind a decision not to analyze the effects of strategic countermoves when using the elimination procedure. The amount of computer time involved is the main argument for not analyzing appropriate countermoves when the vote-count procedure is used.
11. The labels of the alternatives are assumed to be arbitrary. That is, the distributions ($a_1 = 2$, $a_2 = 2$, $a_3 = 2$, and $a_4 = 0$), ($a_1 = 2$, $a_2 = 2$, $a_3 = 0$, and $a_4 = 2$), ($a_1 = 2$, $a_2 = 0$, $a_3 = 2$, and $a_4 = 2$) or ($a_1 = 0$, $a_2 = 2$, $a_3 = 2$, and $a_4 = 2$) can by a re-labeling of the alternatives be transformed into one another. It is generally assumed here and below that a re-labeling of the alternatives is always permitted.
12. This and the other proofs in this context have not been carried to their logical extensions.

However, since they have been tested by a simulation method, I consider them adequate for our purposes.

13. B. H. Bjurulf, 'A Simulation Analysis of Selected Voting Procedures,' accepted for publication in the *Swedish Journal of Political Science*.
14. n alternatives = n! voting orders,
i.e., 3 alternatives = 3! = 3 x 2 x 1 = 6 voting orders,
4 » = 4! = 4 x 3 x 2 x 1 = 24 voting orders, and
5 » = 5! = 5 x 4 x 3 x 2 x 1 = 120 voting orders.
15. Niemi and Weisberg, *op.cit.*, Chapters 10 and 11.
16. Bjurulf, *Parlamentariska voteringsmetoder.*
17. Black, *op.cit.*, p. 43.
18. Hence, when analyzing procedures that are embodiments of the majority principle, we use the terms 'strategic move,' 'strategic countermove,' and 'anticipating action.' The anticipating actions are, of course, countermoves. They are not, however, strategic countermoves since they are not reactions against strategic moves.
19. See note 13.
20. Duverger pointed out that 'three kinds of party can be distinguished on the basis of strength; parties with a majority bent, major parties and minor parties' (*Political Parties*, London: 1965, p. 283). I have tried to indicate with these examples that Duverger's classification can be expanded and stated more precisely.
21. Bjurulf in Niemi and Weisberg, *op.cit.*
22. Black, *op.cit.*, Chapter 4.
23. See note 2.
24. O. A. Davis, M. J. Hinich, and P. C. Ordeshook, 'An Expository Development of a Mathematical Model of the Electoral Process,' *American Political Science Review 64* (1970).
25. Niemi, *op.cit.*
26. A preference ordering with ties can be created as follows. First, if there are to be no ties for the first choice, choose a number 1–5 (for the five alternative case) randomly. Suppose it is 2. Then alternative 2 is said to be the first preference. Then randomly select four numbers with replacement from the numbers 2–5, inclusive. Suppose the numbers are 4, 3, 3, 2. We then assign alternative 1 to the fourth preference, alternatives 3 and 4 to the third preference and alternative 5 to the second preference.
The resulting preference order is thus
Alternative 2
» 5
» 3, 4
» 1
A similar method is used when ties are not allowed for the first or last choices.
With this procedure for creating ties, we have the following probabilities:

No ties for first choice		No ties for first or last choice	
P (no ties)	$= 1\ \binom{4}{1111}/4^4 = 24/256$	P (no ties)	$= 1\ \binom{3}{111}/3^3 = 6/27$
P (two tied)	$= 12\ \binom{4}{2110}/4^4 = 144/256$	P (two tied	$= 6\ \binom{3}{210}/3^3 = 18/27$
P (two tied) pairs)	$= 6\ \binom{4}{2200}/4^4 = 36/256$	P (three tied)	$= 3\ \binom{3}{300}/3^3 = 3/27.$
P (three tied)	$= 12\ \binom{4}{3100}/4^4 = 48/256$		
P (four tied)	$= 4\ \binom{4}{4000}/4^4 = 4/256$		

In these fractions, the multinominal coefficient represents a way of distributing scores to three or four alternatives. The preceding coefficient is the number of ways of obtaining this distribution. For example, in P (four tied), $\binom{4}{4000}$ means that the same random number was drawn each time and the preceding coefficient denotes the fact that this number could be any one of the numbers 2, 3, 4, or 5.

27. D. Klahr, 'A Computer Simulation of the Paradox of Voting,' *American Political Science Review 60* (1966).
28. See note 3.
29. That is, the complete set of strong rank orderings was used.
30. The case with a weight > 0.5 is the first case in our classification. However, this case was not analyzed since the result is trivial.
31. Any weighting system within each category of the classification will give the same result (except for sampling errors). The specific systems we used are listed in note 32.
32. The specific systems used are as follows:

Case	Voting bloc				
b_1	A = 0.47	B = 0.46	C = 0.045	D = 0.014	E = 0.011
c_1	0.41	0.20	0.15	0.14	0.10
d_1	0.40	0.22	0.17	0.14	0.07
e_1	0.41	0.24	0.22	0.07	0.06
f_1	0.40	0.39	0.09	0.07	0.05
g_1	0.24	0.23	0.21	0.18	0.14
b_2	0.40	0.31	0.25	0.03	0.01
c_2	0.47	0.21	0.15	0.10	0.07
d_2	0.46	0.30	0.15	0.07	0.02
e_2	0.46	0.30	0.19	0.03	0.02
f_2	0.40	0.36	0.09	0.08	0.07
g_2	0.2	0.2	0.2	0.2	0.2
b_3	0.29	0.27	0.25	0.10	0.09
c_3	0.37	0.18	0.16	0.15	0.14
d_3	0.31	0.23	0.21	0.20	0.05
e_3	0.31	0.24	0.23	0.12	0.10
f_3	0.31	0.20	0.18	0.16	0.15

In systems e_2, f_3, and g_2, ties in the social ordering can occur. In case g_2 this possibility is greatest, and the random chance of a tie in the social ordering is 0.30.
Note that in Tables II–VII the terms bloc and party have been used synonymously.

33. Control is here defined as the situation in which the bloc has the power to make its own first alternative the last one in the voting order.
34. See note 32.
35. All results, when ties are permitted, assume that preferences other than the first choice may be tied. Results based on the assumption that preferences other than the first and last choices may be tied show only expected distinctions.
36. See note 2.
37. Black, *op.cit.*, p. 18.
38. N. Stjernquist and B. Bjurulf, 'Partisammanhållning och partisamarbete. En studie av voteringarna i riksdagens andra kammare 1964 och 1966,' *Statsvetenskaplig Tidskrift 4* (1968).
39. Our two variants of the elimination procedure are in my opinion the basic types. In real-world parliaments these types are divided into many subvariants as indicated by my discussion above and in *Parlamentariska voteringsmetoder*, pp. 370 ff.
40. See note 13.
41. See note 33.
42. Black, *op.cit.*, p. 18.

of the elimination procedure despite its negative properties from strategic and equality aspects.

Turning now to the more detailed results, we find that neither a change in number of alternatives nor the entry of tied preferences changes the general tendencies reported above. When relaxing our basic assumption of conflicting first choices among blocs, we found as expected a heavy increase in the winning probabilities when no strategies were employed. This is due to patterns in which one alternative received more than 50 percent of the primary preferences. These patterns are trivial and serve no analytical purpose and only complicate the analysis. In regard to the effects of different systems of voting blocs, we reported above the general difference between the 'balancing' cases and the 'dominating' cases. Here it is sufficient to note the different character of the smaller blocs. The dummy character of blocs D and E in case b and the balancing positions of blocs C through E in case f are obvious in our results. We should also mention that case g is slightly distinct from the other two balancing cases.

From our analysis of the unidimensional situation we should note our corroboration of Black's theorem that 'if the members' curves are single-peaked 0_{med} will be able to get a simple majority over any other motions $a_1, a_2, \ldots a_m$ put forward.'[42] More importantly, we also found that the strategic potentials are concentrated around the median power center. The limited strategic potential when the voting order is selected at random after the voters have registered their preferences as compared to the strategic potential for a bloc in control of the voting order was found here as well. Finally we illustrated the present Swedish situation and the most probable new organization (if there is a change) in unidimensional contexts. These cases illustrate the strongly dominant position of the largest bloc in the present situation and the hazardous position of a one-bloc (e.g. B) minority government in the new organization. The strategic potential of the largest bloc if in control of the voting order is very large in the new situation (0.92). Moreover the strategic potentials of blocs C and D if in control of the voting order are considerable (0.51 and 0.11, respectively). The 'random' strategic potentials are on a much lower level, and we should also remember that the strategic moves in that situation mostly imply a considerable risk.

In this article I have analyzed certain properties of two voting procedures in *certain specified situations*. Our analysis has been concentrated on properties and effects of the final voting process. The next step in the research process is to analyze other aspects of this as well as other legislative stages in an attempt to build a dynamic model of the parliamentary voting process. The predictive value of the debate preceding the voting and the predictive value of the committee report are examples of specific objects in such an attempt.

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NOTES

1. R. G. Niemi and H. F. Weisberg (eds.), *Probability Models of Collective Decision-Making*, Columbus, Ohio: 1972, p. 393.
2. A set of preference orderings is single-peaked if there is an ordering of the alternatives on the abscissa such that when utility or degree of preference is indicated by the ordinate, each preference ordering can be represented by a curve which changes its direction at most once, from up to down (i.e. has at most one peak). A dimension is said to exist in a set of alternatives if a set of voters has a complete agreement on a standard of judgment in such a way that their preference orderings will be single-peaked.
3. A sample size of 10,000 implies a 0.95 confidence level that our estimated probability is within 0.01 of the true probability (W. J. Dixon and F. J. Massey, Jr., *Introduction to Statistical Analysis*, 1957, pp. 84–85). In all simulations that follow we have sampled 10,000 sets of preference orderings.
4. B. H. Bjurulf, 'A Probabilistic Analysis of Voting Blocs and the Occurrence of the Paradox of Voting,' in Niemi and Weisberg, *op.cit.*
5. B. H. Bjurulf, *Parlamentariska voteringmetoder*, SOU 1972:15 Bilaga 4 (3), pp. 364–365.
6. *Ibid.*, and R. G. Niemi, 'Majority Decision-Making with Partial Unidimensionality,' *American Political Science Review 63* (1969), p. 494, and Niemi and Weisberg, *op.cit.*, Chapters 9–10.
7. R. D. Luce and H. Raiffa, *Games and Decisions*, New York: 1957, p. 369.
8. D. Black, *The Theory of Committees and Elections*, Cambridge: 1958.
9. See note 5.
10. Test-runs involving countermoves indicated that a vast amount of computer time would be needed. This fact and our conclusion above that a completely effective strategic countermove always exists were the main reasons behind a decision not to analyze the effects of strategic countermoves when using the elimination procedure. The amount of computer time involved is the main argument for not analyzing appropriate countermoves when the vote-count procedure is used.
11. The labels of the alternatives are assumed to be arbitrary. That is, the distributions ($a_1 = 2$, $a_2 = 2$, $a_3 = 2$, and $a_4 = 0$), ($a_1 = 2$, $a_2 = 2$, $a_3 = 0$, and $a_4 = 2$), ($a_1 = 2$, $a_2 = 0$, $a_3 = 2$, and $a_4 = 2$) or ($a_1 = 0$, $a_2 = 2$, $a_3 = 2$, and $a_4 = 2$) can by a re-labeling of the alternatives be transformed into one another. It is generally assumed here and below that a re-labeling of the alternatives is always permitted.
12. This and the other proofs in this context have not been carried to their logical extensions.