OUTLINES - CRITICAL PRACTICE STUDIES

• Vol. 26, Special Issue • 2024 • (57-77) •
www.outlines.dk

Math identity and Cultural Historical Activity Theory: storying the math trajectories of pre-service teachers in urban settings

Socorro Orozco

California State University, Los Angeles, USA

Aria Razfar

University of Illinois Chicago, USA

Abstract

The purpose of this exploratory study is to further investigate the cultural contexts under which pre-service teachers develop their mathematics identities. We reviewed pre-service teachers' mathematics autobiographies that include a description of their math abilities, their perceptions of what it means to be a good student of mathematics and the impact of parental/caregiver and teacher relationships on the development of their mathematics identity. The analysis revealed three types of mathematics identities: positive, negative, and neutral and/or ambivalent. It also identified two dimensions of mathematics identity development, the social and the internal. We present implications to teacher preparation including the need to incorporate narrative eliciting assignments in mathematics methods courses.

Keywords: mathematics identity; narrative analysis; Cultural Historical Activity Theory; mathematics autobiography; pre-service teachers.

Introduction

Although the work on identity on its own is vast, that of mathematics identity has only received attention much more recently. Even more recent is the focus on understanding mathematics identity with the purpose of shaping teacher preparation curriculum (Lutovac & Kaasila, 2013). This work generally centers negative mathematics identity as shaped by toxic school experiences and often treats it as something that must be resolved on an individual level through positive classroom experiences only (Lutovac & Kaasila, 2014; Boaler & Selling, 2017). This line of research has also focused on the work that can be done within the self to transform self-understandings with the goal of improving performance in certain mathematical tasks (e.g., test taking) (Pizzie et al., 2020).

The purpose of the current exploratory study was to further investigate the cultural contexts under which one develops a mathematics identity to inform teacher preparation and address achievement gaps. We analyzed pre-service teachers' narratives from an elementary mathematics methods class to identify specific lived experiences that lay the foundation for taking a stance on one's mathematics identity. These narratives provided a comprehensive view of identity (Peacock & Holland, 1993) because they included various cultural contexts (school, home, community) and relationships (teachers, parent/caregiver, peers) that contributed to math identity formation. The analysis revealed specific practices for learning mathematics and intricacies of the cultural contexts that served as meaning making tools (Leont'ev, 1979) that resulted in their current feelings about mathematics and potential impact in their future work as teachers. To this end, we used Cultural Historical Activity Theory (CHAT) (Engeström, 2001) to frame the study and to identify a unit of analysis, specific parts of the narratives that we refer to as episodes. The following research questions guided the study:

1. Do pre-service teachers indicate a positive, negative, neutral, and/or ambivalent mathematics identity? How?
2. Do the pre-service teachers see the learning of mathematics as primarily an individualistic or a social activity?
3. What types of socialization practices do pre-service teachers identify as significant types of mathematical learning?
4. What semiotic (meaning making) tools do the pre-service teachers apply to navigate the social experiences while learning mathematics?

Persisting inequities in mathematics achievement

The overall mathematics achievement steady increase in the United States over the last few decades was significantly disrupted by the catastrophic events due to the COVID-19 pandemic (NCES, n.d.). When taking a closer look, we can see that, although narrowing, there has been a significant gap between the performance of White and Black and White and Latinx students (NCES, 2019). There is also a strong relationship between students' social class and their mathematical achievement (Quaye & Pomeroy, 2022; Hernandez, 2014) that places working-class and poor students at great disadvantage. From an intersectional lens, poor and working-class Black and Latinx students have been put in an increasingly precarious position concerning mathematical achievement.

When it comes to teacher preparation, some research has shown that overall elementary pre-service teachers feel "adequately prepared to teach mathematics" (Rosas, 2011). However, when investigating the relationship between math anxiety and teaching self-efficacy, Greshman (2009) found a significant negative relationship between the two and concluded that pre-service teachers with higher levels of mathematics anxiety tended to have lower levels of teacher self-efficacy. The inequities related to mathematics education are also seen in other measures within teacher preparation. In California, for example, Black and Latinx pre-service teachers have the lowest passing rates of the ethnicities reported in all the mathematics standardized tests for teachers (California Commission on Teacher Credentialing, 2022). An even greater difference was reported in 2020-2021 where only 25% passing rate was reported for Black pre-service teachers and 40% for Latinx compared to 64% for all examinees and 71.4% for White students (California Commission on Teacher Credentialing, 2022).

This general deficit-based take on the mathematics achievement gap fails to provide insight into new possibilities for creating change. Instead, some suggest ways to challenge the mathematics achievement gap discourse by focusing on current empowering work taking place in many educational spaces (Miller, 2016; Carey, 2014). For example, Miller (2016) reminds us of the importance of having teachers and students engage in critical discourses by making connections between social injustices and mathematical content. Also, Carey (2014) proposes challenging the labels we use to compare students (e.g., proficient, below proficient, etc.) by engaging in sociocultural analyses of the impact such labels have on the perceptions we create for the different groups.

There is a significant amount of future elementary teachers whose mathematics abilities are not where they need to be. For this subset of the pre-service teacher population we should aim, at the very least, to create an environment where they can begin to unpack their relationship with mathematics during teacher preparation so that they can commit to improve their skills once they are in the classroom. Even though we know inequities in mathematics achievement will not be resolved by improving teachers' skills, we hope that changes in their mindsets will lead to more effective teaching practices. If teachers do not feel competent about their math teaching skills, they tend to avoid the subject when they teach and students tend to have lower achievement levels (Ramirez et al., 2018). By centering autobiographical reflection in mathematics methods courses we can engage in more holistic approaches to teacher preparation.

CHAT as a Conceptual Framework

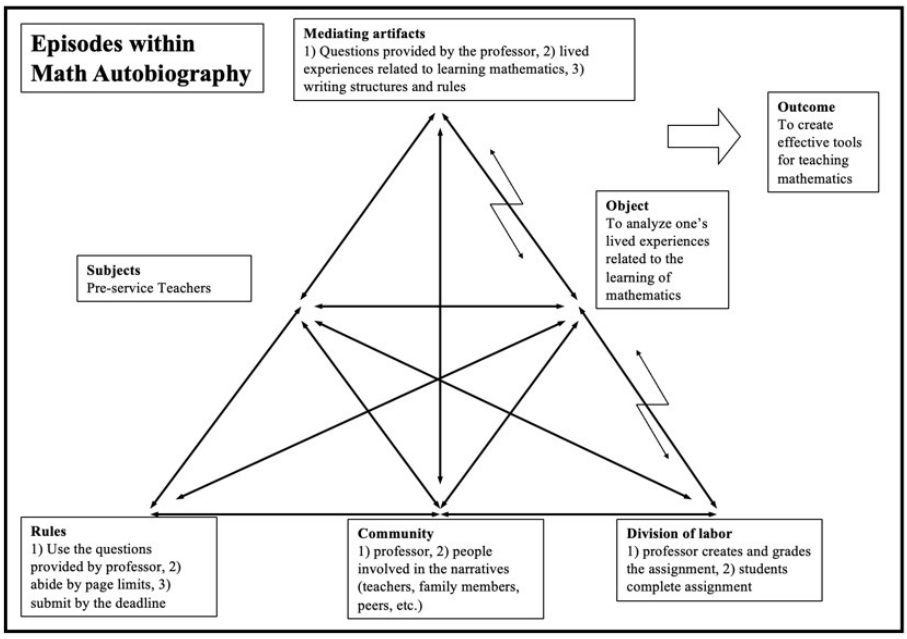
This exploratory study draws on Cultural Historical Activity Theory (CHAT) (Engeström, 2001; Engeström & Sannino, 2021; Grimalt-Álvaro & Ametller, 2021) to analyze pre-service teachers' experiences learning mathematics and their appraisal of the mediating artifacts that guided such experiences. The CHAT activity system or activity system triangular model provided the structure for organizing the unit of analysis and to deepen our understanding of the nuances of teacher-student, caregiver-student, and mathematics-student relationships and cultural practices that shape our math identities (see Figure 1). The activity then is the act of reflecting on specific experiences about learning mathematics. The unit of analysis is an *episode* (as defined in Razfar & Rumenapp, 2014), a section of the narrative (*math autobiography assignment*) with a clear beginning and conclusion where the subject (the pre-service teacher) signals a change in their story. The

object is to analyze lived experiences related to learning mathematics and the primary mediating tools is the math autobiography assignment. Furthermore, CHAT provided the interpretive lens for identifying the social interactions that ultimately yielded qualitative cognitive transformation, the transfer of knowledge from one to another. In dissecting these narratives, we zoomed into specific episodes (defined in methodology section) that let us know how pre-service teachers learned to take a stance about their mathematics identity, particularly those that had long-lasting effects on their mathematics identities. Also, we noted the spaces inside and outside schooling where these mathematical experiences take place and beliefs about mathematical abilities are transferred from one person to another. These spaces point out the cultural and historical context, people who are part of them, and the actions and operations within the activity system, which give us a unique view of the application of the mediating tools (Roth, 2012).

The extensive work of Cultural Historical research, mathematics identity, and storying paved the way for us to be able to frame the use of pre-service teachers' narratives within their unique, current sociocultural contexts and the ways in which these can guide future decision-making processes, particularly their future role as teachers (Black et al., 2010; Roth, 2012; Uffen et al., 2022). Because of our various levels of experience with these topics we were able to weave the different threads of understanding by making meaningful connections between classroom practices and research strategies. Our collective knowledge of each of these topics allowed us to reach new understandings to support teacher preparation.

Third generation CHAT (Engeström, 2001; Wake et al., 2016) made it possible for us to study the lived experiences by analyzing the use of conceptual tools in two different, but simultaneously occurring, activity systems. One of these activity systems exists internally and points to the internal cognitive processes that occurred because of interacting with others (i.e., how pre-service teachers explain what was on their mind when recalling different experiences). The other activity system is external, and it is explained as the actions the pre-service teachers took under the pressure to follow a specific social script. Third generation CHAT thus provided the methodological structure for data analysis process. In addition, the concept of dialectics (Lave et al., 1984) enabled explaining the contradictions between the two activity systems as shared by the pre-service teacher (e.g., when they describe situations where beliefs and practice are incongruent).

Figure 1. Activity System for analyzing distinct episodes within each Math Autobiography Narrative



Authors' Positionality

We arrived at the work of math identity development because of convergent interests in both issues surrounding mathematics achievement throughout the educational pipeline, and in Cultural Historical Activity Theory (CHAT) as a framework for qualitative educational research. Our relationship was established through a mentoring program for early career faculty with an interest in cultural historical research, which led to a rich exploration of our current projects and the ways CHAT helps us shape them. The context allowed us to examine our approaches to using CHAT to guide educational research in general, and in particular classroom instruction and teacher preparation because of our current professional positions. This examination eventually resulted in us focusing on mathematics identity as the most significant and timely topic as it is conducive to deeper understanding of the persistent inequities related to mathematics achievement that have been exacerbated by the COVID-19 pandemic. Moreover, we chose to explore the mathematics identities of pre-service teachers using narrative analysis methodology and autobiographical explorations in teacher preparation because of our desire to expand qualitative research tools as reflected in our previous work (Orozco, 2022; Razfar (2013), Razfar & Rumenapp, 2014).

We draw from our extensive teaching experience in the mathematics classroom, in teacher preparation, and learning sciences. Our conclusions are aligned with and grounded on behavior and learning patterns we have observed in our classrooms through our innumerable interactions with pre-service teachers and how these are reflected in mathematics identity research. We aim to contribute to research in this area by looking deeper into how one develops either a positive or negative mathematics identity and how this may inform future classroom practice (Lutovac & Kaasila, 2014). We focus on both, the ways in which pre-service teachers explain how they internally process information from their social environments and their outward expression.

Our research interest is fueled by our deep respect for the intimate relationships we form with future teachers in our work as professors and supervisors of fieldwork teacher preparation programs. These relationships are the door to a world of perspectives on the impact of mathematics-related experiences both inside and outside of the formal schooling spaces that help us gain collective understandings of mathematics identity as situated within teacher preparation. The narratives presented here are one the mediating tools of the teacher-student relationship intentionally chosen to guide pre-service teachers' understandings of the teaching and learning of mathematics and how this shapes social reproduction (Douglas, & Attewell, 2017).

Methodology

We used a narrative analysis approach (Razfar & Rumenapp, 2014) to study pre-service teachers' math autobiographies (n=25). These were originally collected as a class assignment for an elementary mathematics methods class and IRB approval was obtained approximately one year after the class ended in August of 2021. Each narrative was 4 pages in length (approximately 100 pages were analyzed). The class assignment was titled *Math Autobiography,* and it was structured as a self-reflection that asked students to describe different aspects of their mathematics identity by responding to the following four prompts:

1. How do you describe yourself as a math learner? What is your perception of your math ability?
2. During your K-12 schooling experiences, how did you perceive/understand your role as a math student?
3. During your K-12 schooling experiences, what characteristics resembled a good math student?
4. Describe your family dynamics and that of the community in which you were raised and the impact it had on your perception of your math ability?

Design

The narratives selected for this exploratory study are a subset from a larger study that is focused on the use of autobiographical writing assignments as a form of culturally relevant instructional practices to teach mathematics methods. The autobiographical nature of the assignment and the use of specific prompts allowed for pre-service teachers to story (Phillip et al., 2018) the trajectories of their relationship with mathematics using personal examples from various times in their lives and from different contexts (e.g., formal schooling, home, community spaces, etc.) and to identify practices, experience, and contexts that had a significant impact on their current relationship with mathematics. Storying helped us preserve the authenticity of the narratives by reminding us of the human beings who created them (Peacock & Holland, 1993; Raj, 2022). By centering the stories of pre-service teachers as they reflected on the impact of early schooling and home experiences, we were able to draw conclusions about the cultural practices that created lasting memories, positive and negative, that became the lessons to be shared in their future classrooms (Cole & Engeström, 1993).

Participants

The participants in this study were 25 pre-service teachers in an elementary-level teacher preparation program. They each submitted a *Math Autobiography* as a class assignment in their mathematics methods course. Although the *Math Autobiography* assignment did not require students to list demographic data, the great majority of them provided some of it in their responses. Most of the students come from two-parent homes (19/25) and more than half of them (14/25) mentioned having at least one sibling. Those who mentioned having siblings shared that this had some connection to their learning and teaching of mathematics (e.g., received help from them or practiced math by helping them). Only six of them provided a racial/ethnic self-identification (five Latinx/Hispanic/Mexican and one Asian), but the university is a Hispanic Serving Institution with over 70% Latinx students). Five of them have at least one college-educated parent and all of them explained that this was a source of inspiration and that they were able to receive help with math homework. Six of them identified as either immigrants or children of immigrants and all shared similar experiences regarding the lack of knowledge of the American schooling system.

Data Analysis

The first part of the analysis consisted of line by line reading of each narrative to identify a specific stance towards mathematics. Then, we used the research questions to create categories based on the pre-service teachers' experiences with the topic of mathematics identity to help us disaggregate the data further. For example, question one enabled the responses to be divided into three categories: positive, negative, and neutral and/or ambivalent. During the initial analysis we were able to identify codes that denoted either a positive or negative mathematics identity such as "I have always been" or "I have never been." There was a third category for the narratives that took a moral stance but that was neither positive nor negative. We named this category neutral and/or ambivalent. The stories in this category had strong feelings towards, whether positive or negative, but did not necessarily act accordingly. For example, in one of the stories placed in this category the pre-service teacher shared negative feelings towards mathematics but explained that because of the pressure to succeed academically they forced themselves to achieve.

In the second part of the analysis, we zoomed into the narratives by using Razfar et al. (2023) narrative continuum to analyze specific *episodes* (see Appendix I). The narrative continuum uses seven narrative dimensions to explain who is telling the story, how the story is being told, and the intentions behind telling the story (see Appendix II). An *episode* is a part of the narrative where there is a significant shift (Razfar & Rumenapp, 2014); a point in the narrative that signals a change in the orientation within the story. The following is an example of an episode:

> When I was a senior in high school, I had a math teacher who created a bond and relationships with all of the students...Building this relationship with my teacher made me accomplish more than I ever thought I could in a math class. (High School Episode).

Noting these specific parts of the narratives helped us identify significant elements within each story and, which led to collective understandings by identifying common experiences, these are reported as themes in the next section.

Finally, the coding process allowed us to see between 2 and 4 episodes within each story and most were described using temporal signals such as moving from one grade level to another or by proving a range (e.g., from kinder to the 3rd grade)—the structure of the prompts provided for the assignment contributed to the organization. One salient episode present in more than half of the pre-service teachers' stories was their current experiences with education either as students in a teacher preparation program or as paraprofessionals in schools. This was significant because we could see motivating factors to heal or transform their mathematics identity, especially for those labeled with negative or neutral and/or ambivalent. See for example the following excerpt:

> Now that I am on my journey [to] becoming a teacher, I feel like this really shaped my mathematical ability…As a future educator, I am more aware of how students need us to be patient towards them. They need us to try different teaching methods to meet their educational needs...I strive to support my students to feel confident in asking a question, without the fear of getting scolded. (Credential Program Episode).

Here we can observe a desire to provide better learning experiences than their own, and they include specific practices they see as beneficial (e.g., asking questions). This indicates a level of awareness about future behavior will be different than what they experienced.

Findings

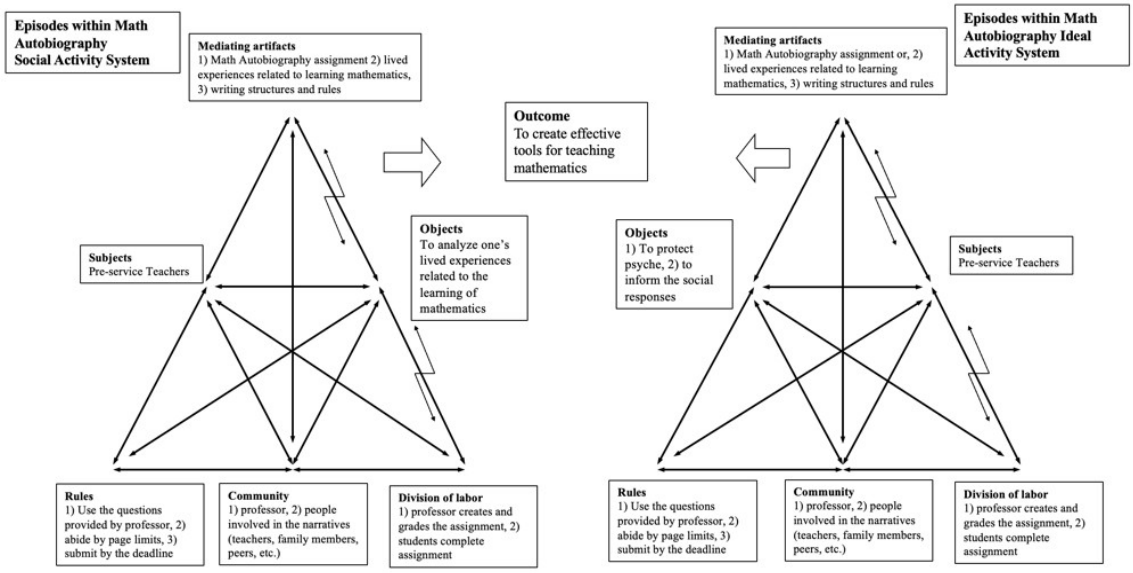
We were able to identify a great range of unique, home and schooling cultural practices geared towards the learning of mathematics (Nicol et al., 2020). In turn, descriptions of participation in these cultural practices provide insight into the living structures through which individuals develop attitudes and dispositions towards mathematics (Hernandez-Martinez & Vos, 2018; Alderton, 2020). Although many best practices in mathematics instruction have been widely studied in recent decades, these do not generally focus on teacher preparation (Boaler, 2002). Even those that have a focus on the professional development of pre-service teachers focus on what teachers can do with their own students (Barba, 2022). The current study provides insight into specific ways we can support pre-service teachers. The following two themes emerged: 1) *Negotiating the social world and the internal cognitive processes,* and 2) *Deeper knowledge of the characteristics of positive, negative, and neutral and/or ambivalent math identity.*

Negotiating the social world and the internal cognitive processes

Third generation CHAT made it possible to draw conclusions about the lasting effects of specific life events on math identity (Black et al., 2010; Boaler & Selling, 2017; Engeström & Sannino, 2021) and how pre-service teachers contend with making sense of practices and their beliefs about their mathematics abilities. By studying life events more closely we were able to identify mathematics identity development as two separate activity

systems occurring simultaneously: one that takes place in the social world and the other that focuses on internal cognitive process (*ideal*) (see Figure 2). Examining this social-internal relationship made it possible to name inequitable learning processes that contribute to persisting mathematical achievement gaps.

Figure 2. Social and ideal activity systems of episodes within Math Autobiography Narratives



We observed that the storying process for almost every narrative had similar parts including how they would act in the social world and the internal cognitive processes shared as the retelling of their private speech. This provided details about the formation of one's mathematics identity as it unfolded. We also noted a distinction between the two activity systems regarding the object. In the social activity system, it was clear that the object was focused on completing the task (to provide an analysis of their experiences), while the ideal activity system was much more focused on providing protection for the psyche and to guide the social response. The social response was guided by an understanding of the social agreements (e.g., you must provide an answer when the teacher asks a question) and in some instances were contradictory to the private speech. For example, one of the pre-service teachers shared:

> Even when my friends would try to tutor me, they would show their frustrations and made me feel incompetent. Because of this I would lie to them and tell them that I finally understood the material but really, I had no clue. I was so worried with social acceptance that I ignored my academic needs. (K-12 Experiences Episode).

The clear separation between what you do in the social world versus the internal cognitive processes provides the reasons why mathematics identity remains relatively constant. In this case, the pre-service teacher shares their desire to comply with the social scripts of classroom behavior and how long it should take to understand a math task, which was contradictory to their actual needs (Grimalt-Álvaro & Ametller, 2021). Another pre-service teacher explained how they contended with what they imagined were the social expectations around learning and doing mathematics:

> My role [as a math student] was to absorb what was being taught and then be able to replicate it using the same steps and procedures as taught by the teacher…I feared not knowing the correct answers when put on the spot because that somehow meant that I was dumb. I often hid my hands under the desk as I counted with my fingers. (Growing up Episode).

In both examples we can see how the pre-service teachers struggled to align their beliefs about how they should act when doing mathematics (based on classroom expectations) and how they were experiencing the activity internally. Although the social and internal activity systems are almost identical, we can see that the actions and the private speech are not necessarily aligned (Roth, 2012). The conscious actions of staying quiet when the teacher asks if there are any questions or saying "I understand" when struggling to make sense of mathematical tasks are informed by well-embedded subconscious operations linked to intelligence and social acceptance.

Deeper knowledge of the characteristics of positive, negative, and neutral and/or ambivalent math identity

After coding the stories using the narrative continuum it became clear that all pre-service teachers took one moral stance when defining their relationship with mathematics: positive, negative, neutral and/or ambivalent. There were very specific distinctions in both the behavior and the ways of thinking for each of the three groups. The table below summarizes the characteristics for each group and separates them between those occurring in the social and in the internal activity systems (see Table 1). We concluded that for all three groups mathematics identity is established in early life (elementary years) and reinforced through schooling and home activity.

Those with a clear positive mathematics identity used phrases such as "I've always liked math" or "math has always been my strongest subject" and they seemed to have very nurturing experiences overall. Their expectations of the social environment seemed to be met and they felt connected to the cultural processes and the teachers. They also demonstrated overall trust for the teachers' capacity to teach math well and participated fully in the different classroom activities. The following excerpt from one of the narratives exemplifies this experience:

> Since I believed I was building my knowledge, I always believed the teacher had valuable information that I needed to continue my growth…First thing we would do is go over our completed homework...The remainder of class was spent on practicing what we just learned while the teacher was available for guidance if you needed or wanted it. This structure to the math class was perfect and made sense to me. (K-12 Experiences Episode).

The experiences shared by those with negative mathematics identities were almost the opposite than those with a positive mathematics identity. In general, they were less willing to take risks and did not trust that the teachers or the institutions to best take care of their needs. They used phrases like "I've always struggled with math" or "I have never been confident" to describe themselves as math learners. These statements were directly connected to specific memories about encountering challenges when being exposed to mathematics. In the following episode you can see the pre-service teacher making connections between the concepts of grades and developing believes about one's mathematics abilities:

> Growing up, mathematics was difficult for me. I struggled in elementary school to fully comprehend topics such as decimals, fractions, and long division. I specifically remember I had received my first C in the 4th grade for math…After my teacher and parents discussed the need for improvement in my math grade, I realized that I was going to need more support. This made me feel discouraged and I thought very low of my abilities as a math learner. (Elementary School Episode).

These findings are aligned with previous work on using narrative analysis to analyze aspects of mathematics identity and the use of math autobiographies with pre-service teachers (Kaasila, 2007, McCulloch et al., 2013). However, two details not previously brought up are: 1) that both groups (positive and negative mathematics identity) shared the same level of commitment towards becoming effective teachers, and 2) that both groups shared similar practices engaging in mathematical tasks in the social world, but differed in how these were processed internally. Both groups spoke about working hard at different and expending great effort to do well, but those with positive mathematics identities connected this to feelings of satisfaction whereas those with negative mathematics identities to feelings of frustration.

The neutral and/or ambivalent group presented unique characteristics that differentiated them from the two other types. The stories did not seem to have strong feelings as the positive and negative groups did. They used words like "average" and "sufficient" to describe their mathematics abilities. In most of the cases these did not refer to a particular level of achievement but rather to their attitude towards mathematics—that seemed to be indifferent. Some shared, for example, getting good grades in math but not enjoying the practice. Others talked about struggling with mathematics but having enough skills to remain persistent and work through their classes. They also attributed their enjoyment when doing mathematics to certain conditions such as the teacher making it fun or being exposed to the right number of examples:

> It is definitely not my strongest quality, but I would say that I am persistent in my efforts. Math was a subject I struggled to comprehend all throughout my K-12 schooling experience. I always tried extremely hard to understand and do my best when it came to math and my family was always there to support me when I needed it. I wanted to excel in the subject but always felt that it wasn't my strength because of the amount of time it would take me to solve a problem. (Growing up Episode).

Another unique characteristic among this group was that they excelled at mathematics but were aware that they lacked depth of knowledge of the content. This signaled a unique stance that revealed a distinct connection to mathematics than those with a positive mathematics identity. The following episode exemplifies this stance:

> In addition to my slow pace, what held me back from excelling in math was not understanding the overall concept, the "WHY?" I knew that I needed to follow certain rules and procedures, but I didn't understand the reasoning behind it. (Growing up Episode).

This differentiation is significant because it provides insight into how one relates to mathematics. Those with a positive mathematics identity have a deeper understanding of mathematics, a more conceptual understanding. Pre-service teachers in this group point to

general procedural understanding of mathematics which helped get through classes without creating negative feelings about their intellectual capacity.

Table 1. Summary of social and internal characteristics of the three types of mathematics identities

	Social	Internal
Type of mathematics identity		
Positive	- More willing to take risks - No fear of making mistakes - Seeks a community of learners - Approaches tests and other assessments as part of the process	- Believes math is useful - Sees real-life applications - Trust teachers are teaching the best way possible - Higher metacognition
Negative	- Less willing to take risks - Resorts to memorization - Avoids engaging in mathematical tasks - Less willing to ask for help - Grades have more negative effects (not motivating)	- Negative self-talk - Doubtful of their skills - Does not believe math is useful in "real life" - Does not trust teachers can help - Attributes classroom dynamics to teachers' capacity to connect with students - Doubts teachers' capacity to teach math well - Attributes lack of learning math to schools not doing enough to meet their needs - Does not "fit the criteria" for being a good math student
Neutral or Ambivalent	- Success depends on whether the teacher is engaging or on having	- Forced to learn, but cannot not engage critically

	time to practice - Can apply mathematics to daily life, but does not consider that to be a talent - Trusting of classroom practices - Does not seek help, but accepts it when it is presented as an option	- Trusts their effort - Relies on memorization - Love/hate relationship with mathematics - Follows steps presented by the teacher - Considers their own learning style (I can learn math if the approach matches my learning style)

The characteristics of the three groups provide important information about the different needs of pre-service teachers as they learn to teach mathematics. In general, the three groups shared their relationship with mathematics as being mediated by grades and other achievement measures (e.g., Standardized Tests). All three groups also talked about how they experienced needing help but differed on their feelings about it. For those with a positive mathematics identity, getting help is a necessary part of learning—learning mathematics is seen as a social activity. Differently, those with a negative mathematics identity struggling and needing help from others means that you are not good at math. This group tended to speak of success in mathematics as an individual endeavor that depends more on intelligence than effort. When zooming in deeper we are able to see connections to the cultural contexts (e.g., one blamed herself for doing poorly but realized teacher turnover had a lot to do with it). This points to math identity not just being about performance, but about making sense of how the structural learning conditions impact development.

Discussion

The most significant contributions of this exploratory study are the rich descriptions of urban pre-service teachers' accounts of the development of their relationship with mathematics. More specifically, this study provided insight into the cultural practices that contribute to having a positive, negative or neutral and/or ambivalent mathematics identity. Of particular importance is the last category, which has not been previously identified in research. The pre-service teachers in the neutral and/or ambivalent category engage in mathematical tasks in a much healthier manner compared to those in the negative identity group even when they struggle. They approach mathematics with little to no judgment and they focus more on their capacities to tackle difficult tasks as a source of motivation. They see mathematics as just another subject that sometimes requires more effort, but they can count on teachers and accept help from others without experiencing the feelings of inadequacy as those with a negative

mathematics identity. This is important to note because it can inform how we provide support for pre-service teachers who struggle with mathematics. That is, we can help pre-service teachers with negative math identities see mathematics as a cultural activity rather than an individual activity that involves thinking with others (including accepting help from more knowledgeable others). We can focus on providing more opportunities for pre-service teachers to learn to see mathematics as a practical cultural tool that can be best understood if it serves to meet a specific individual need.

The use of mathematics autobiographies to elicit narratives is a very effective self-reflection where pre-service teachers can name the affordances in their environments, family histories, value systems, home-school connections, and other factors that shape their mathematics identity. In this study, they helped to identify the social (material) and the internal activity systems that occur simultaneously and where mathematics identity is unfolded. This has direct implications on teacher preparation practices as these stories can inform the development of much more genuine models for learning to teach mathematics that are rooted in ontological analyses of lived experiences. In this way, pre-service teachers can actively participate in the construction of strategies for engaging students in mathematical tasks and not just passively collect a list of effective activities. This is particularly important for teachers with negative mathematics identities because they would have the opportunity to create instructional strategies that challenge the ones that caused them trauma.

Our analysis led us to see that some classroom structures compel students to behave in ways that are supposed to demonstrate good mathematics learning practices; however, they often have the exact opposite to what they need. The narratives in this study highlight practices currently being debated in mathematics education (Barshay, 2023) because their competitive nature prevents many developing the necessary confidence to tackle mathematical tasks. Even though there is a divide in whether timed tests cause anxiety, the general conclusion is that it does for some, and its cumulative effect can have long lasting consequences. Those who receive negative messages about their math abilities from different contexts in which they participate (e.g., school, home, friends) are most affected since their fears and insecurities are confirmed in multiple spaces. As educational policy moves towards more equitable mathematics instruction, it is imperative to analyze educational experiences and processes that provide a clearer view of culturally relevant and meaningful experiences for learning and teaching mathematics beyond identifying best practices.

Appendix I

Individual Episode Coding Table

PT Name	EPISODE NAME	Transcript	Tellership	Tellability	Embeddedness	Moral Stance	Linearity	Hearability CONTENT	Solidarity CONTENT	Elicitation PROCESS	Platform/Context/Historical info - HOW WAS THE SPACE CREATED
POS PT 1	Elementary		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 1	HS and early college		1	3	1	2	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 1	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 3	Elementary		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 3	MS and HS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 7	Current (after school program tutor)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 7	K-12 experiences		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 8	K-12 experiences		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 8	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 12	K-3rd		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 12	4th-HS		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 12	Current (credential program)		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 14	K-12 experiences		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 19	K-12 experiences		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 19	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 24	K-3rd		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 24	4th-MS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 24	HS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
POS PT 24	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 2	Before moving to the US		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 2	After moving to the US		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 2	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 4	Elementary		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 4	HS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 4	Current (credential program)		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 9	HS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 9	Elementary		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 10	Elementary		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 10	Adulthood		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 10	MS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 16	Elementary		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 16	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 16	MS/HS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 18	Community College - Current		1	3	3	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 18	Elementary		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 18	HS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 20	Current		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 20	5th-7th		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 20	8th-12th		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 21	K-12		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 21	TA work		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 21	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 23	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 23	TA work		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 23	K-3rd		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 23	4th-6th		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 25	K-5th		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEG PT 25	After 5th		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 5	Elementary		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 5	MS and HS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 6	K-3rd		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 6	4th-7th		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 6	8th-10th		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 6	11th and on		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 11	before 10th grade		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 11	10th grade		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 13	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 13	3rd		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 13	6th		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 13	MS and HS		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 15	Growing up		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 15	Current (Inst. Assist.)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 17	K-12		1	3		1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 22	Current (credential program)		1	3	4	1	1	4	4	4	elementary math methods - teaching credential course - pandemic
NEU PT 22	K-12		1	3	1	1	1	4	4	4	elementary math methods - teaching credential course - pandemic

Appendix II

Narrative Continuum Rubric for Math Autobiography Episodes				
Dimension	1	2	3	4
Tellership *Storytelling perspective*	One teller	Two tellers; OR One teller with one interviewer who interjects an opinion or story; OR One teller who relays their story and the story of someone else; OR One interviewer with two tellers	Three tellers (see scenarios applied to level 2 to calculate)	Four or more tellers (see scenarios applied to level 2 to calculate)
Tellability *Range of audience and platform*	Very public (e.g., broadcasted or published on CNN) with intention to be read widely	Published on social media (or lesser-known website) and available to anyone	Private interview with permission to share for specific purpose; OR Private writing with the intention to be published for future generations, OR Support group	Very private (e.g., personal diary)
Embeddedness *Time parameter*	Happened in the distant past and came to resolution (perspectives have changed, able to laugh at this), clear lessons learned; key words: *"long ago, different times, different era"*	Happened in the past relevant and mostly resolved, some lessons learned, some lessons emerging, key words/phrases: *"Now I know that…"*	Happening in the present, key words: *"I am still dealing with…";*	Happening now, e.g., a teller describes an interjection that takes place DURING the telling
Moral Stance *Ethical position*	Singular moral stance, posture of certainty 100%, including certainty around advocacy; key words: "*sure, certain, definitely*"	Singular moral stance, posture of certainty less definitive but still high over 50%, key words: "mostly, almost sure"	Two moral stances, e.g. "*I don't feel comfortable talking about Long Covid, but I think I should be able to*."	Three or more moral stances including conflicting stances; Posture of certainty is very low, key words: "*I don't know, I'm so confused*"
Linearity *Sequence*	Sequential step-by-step progression, key words: "*first, second, third*, etc." (100% linear)	Elements that are linear with some disruptions including flashbacks and/or flash forwards including explicit time markers (50% or more linear)	Time disrupted, movement between the present, past, and future, some time stamps may be present (50% or more circular)	No time markers, circular description (100% circular)
Hearability *Content*	Negative impact; Key words: interrupting, accusations, demeaning	Negative impact slightly more moderated; No indication of hearing in a conversation e.g. neutral comments on the surface that may cause harm, dismissive, gaslighting, tokenistic epistemic reality of patient is denied; Includes inaction and words unspoken	Neutral to positive impact, e.g. "*I hear you*."	Positive impact "*I hear you and I want you to say more*." "*You could be right, let's look into this*."

Solidarity *Content*	Parasitic, negative action, misdiagnosis, harmful treatment	Symbolic - actions that are not helpful and may be harmful	Action goes beyond listening with a recommendation for positive action	Action goes beyond listening taking action alongside someone

(Razfar et al., 2024)

References

Alderton, J. (2020). Kelly's story: transformative identity work in primary mathematics teacher education. *Gender and Education*, *32*(2), 145–160. https://doi.org/10.1080/09540253.2017.1336204

Barba. K. (2022). Mathematical Identity and the Role of the Educator. *Journal of Mathematics Education at Teachers College*, *13*(1), 7–13. https://doi.org/10.52214/jmetc.v13i1.9187

Barshay, J. (2023, May 30). Proof Points: Do math drills help children learn? The Hechinger Report. https://hechingerreport.org/proof-points-do-math-drills-help-childrenlearn/#:~:text=Children%20in%20the%20speed%20group,Fuchs%2C%20who%20led%20the%20study

Black, L., Williams, J., Hernandez-Martinez, P., Davis, P., Pampaka, M., & Wake, G. (2010). Developing a "Leading Identity": The Relationship between Students' Mathematical Identities and Their Career and Higher Education Aspirations. *Educational Studies in Mathematics*, *73*(1), 55–72. https://doi.org/10.1007/s10649-009-9217-x

Boaler, J. (2002). The Development of Disciplinary Relationships: Knowledge, Practice and Identity in Mathematics Classrooms. *For the Learning of Mathematics*, *22*(1), 42–47.

Boaler. J. (2014). Research Suggests that Timed Tests Cause Math Anxiety. *Teaching Children Mathematics*, *20*(8), 469–474. https://doi.org/10.5951/teacchilmath.20.8.0469

Boaler, J. & Selling, S. K. (2017). Psychological Imprisonment or Intellectual Freedom? A Longitudinal Study of Contrasting School Mathematics Approaches and Their Impact on Adults' Lives. *Journal for Research in Mathematics Education*, *48*(1), 78–105. https://doi.org/10.5951/jresematheduc.48.1.0078

California Commission on Teacher Credentialing. (2022). An Annual Report on Passing Rates of Commission Approved Examinations from 2016-17 to 2020-21. https://www.ctc.ca.gov/docs/default-source/commission/agendas/2022-08/2022-08-2f.pdf?sfvrsn=cc3227b1_3

Carey, R. L. (2014). A Cultural Analysis of the Achievement Gap Discourse: Challenging

the Language and Labels Used in the Work of School Reform. *Urban Education, 49*(4), 440-468. https://doi-org.ezproxy.cul.columbia.edu/10.1177/0042085913507459

Cole, M. (1996). Putting culture in the middle. In *Cultural Psychology: A Once and Future Discipline* (pp. 116-145). Cambridge, MA: Harvard University Press.

Cole, M., & Engeström, Y. (1993). A cultural historical approach to distributed cognition. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 1–46). Cambridge: Cambridge University Press.

Douglas, D. & Attewell, P. (2017). School Mathematics as Gatekeeper. *Sociological Quarterly*, *58*(4), 648–669. https://doi.org/10.1080/00380253.2017.1354733

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of education and work, 14*(1), 133-156.

Engeström, Y. & Sannino, A. (2021). From mediated actions to heterogenous coalitions: four generations of activity-theoretical studies of work and learning. *Mind, Culture and Activity*, *28*(1), 4–23. https://doi.org/10.1080/10749039.2020.1806328

Gresham. G. (2009). An Examination of Mathematics Teacher Efficacy and Mathematics Anxiety in Elementary Pre-service Teachers. *The Journal of Classroom Interaction*, *44*(2), 22–38.

Grimalt-Álvaro, C., & Ametller, J. (2021). A Cultural-Historical Activity Theory Approach for the Design of a Qualitative Methodology in Science Educational Research. *International Journal of Qualitative Methods*, *20,* 1-12. https://doi.org/10.1177/16094069211060664

Hernandez, M. (2014). The Relationship between Mathematics Achievement and Socio-Economic Status. JEP: Ejournal of Education Policy.

Hernandez-Martinez, P. & Vos, P. (2018). "Why do I have to learn this?" A case study on students' experiences of the relevance of mathematical modeling activities. *ZDM*, *50*(1-2), 245–257. https://doi.org/10.1007/s11858-017-0904-2

Kaasila. (2007). Using narrative inquiry for investigating the becoming of a mathematics teacher. *ZDM*, *39*(3), 205–213. https://doi.org/10.1007/s11858-007-0023-6

Lave, J., Murtaugh, M. and de la Rocha, O. (1984). The dialectic of arithmetic in grocery shopping. In B. Rogoff and J. Lave (Eds.). *Everyday Cognition: Its Development in Social Context* (p. 67-94). Harvard University Press.

Leont'ev, A.N. (1979). The problem of activity in psychology. In J.V. Wertsch (Ed.). *The Concept of Activity in Soviet Psychology* (pp. 37-71). M.E. Sharpe, Inc.

Lutovac, S., Kaasila, R. (2014). Pre-service teachers' future-oriented mathematical

identity work. *Educational Studies in Mathematics*, *85*(1), 129–142. https://doi.org/10.1007/s10649-013-9500-8

McCulloch, A.W., Marshall, P. L., DeCuir-Gunby, J. T., & Caldwell, T. S. (2013). Math Autobiographies: A Window into Teachers' Identities as Mathematics Learners. *School Science and Mathematics*, *113*(8), 380–389. https://doi.org/10.1111/ssm.12041

Miller, K. K. (2016). Closing the Achievement Gap in Mathematics and Science Education: Using Critical Discourses, Mathematics and Science Agency, and Hybrid Third Spaces to Teach Urban Students. *Education and Urban Society*, *48*(3), 288-294. https://doi.org/10.1177/0013124514530155

National Center for Education Statistics (n.d.). Fast Facts: Long-term trends in reading and mathematics achievement. https://nces.ed.gov/fastfacts/display.asp?id=38

National Center for Education Statistics (2019). Status and trends in the education of racial and ethnic groups. https://nces.ed.gov/programs/raceindicators/indicator_rcb.asp

Nicol, C., Gerofsky, S., Nolan, K., Francis, K., & Fritzlan, A. (2020). Teacher Professional Learning with/in Place: Storying the Work of Decolonizing Mathematics Education from within a Colonial Structure. *Canadian Journal of Science, Mathematics and Technology Education*, *20*(2), 190–204. https://doi.org/10.1007/s42330-020-00080-z

Orozco, S. (2022). Math Anxiety in Math Methods Courses: Self-exploration tools for healing during remote learning. In Raj, A. (Ed.) *Creativity as Progressive Pedagogy: Examinations into Culture, Performance, and Challenges* (296-315). IGI Global. https://doi.org/10.4018/978-1-7998-8287-9.ch014

Peacock, J. & Holland, D. C. (1993). The Narrated Self: Life Stories in Process. *Ethos*, *21*(4), 367–383. https://doi.org/10.1525/eth.1993.21.4.02a00010

Pizzie, R.G., McDermott, C. L., Salem, T. G., & Kraemer, D. J. M. (2020). Neural evidence for cognitive reappraisal as a strategy to alleviate the effects of math anxiety. *Social Cognitive and Affective Neuroscience*, *15*(12), 1271–1287. https://doi.org/10.1093/scan/nsaa161

Phillips, L.G., Bunda, T., & Quintero, E. P. (2018). *Research Through, With and As Storying* (1st ed.). Routledge. https://doi.org/10.4324/9781315109190

Quaye, J. & Pomeroy, D. (2022). Social class inequalities in attitudes towards mathematics and achievement in mathematics cross generations: a quantitative Bourdieusian analysis. *Educational Studies in Mathematics*, *109*(1), 155–175. https://doi.org/10.1007/s10649-021-10078-5

Raj, A. (2022). Anchoring Storying: A quest for Emotional Truth. In Raj & Ulanoff (Eds.)

(2022) *Storying a Reflexing Praxis for Pedagogy: Concept, method, and practices*. Palgrave Mcmillan.

Ramirez, G., Hooper, S. Y., Kersting, N. B., Ferguson, R., & Yeager, D. (2018). Teacher Math Anxiety Relates to Adolescent Students' Math Achievement. AERA Open, *4(*1). https://doi.org/10.1177/2332858418756052

Razfar, A. (2013). Multilingual Mathematics: Learning Through Contested Spaces of Meaning Making. *International Multilingual Research Journal*, *7*(3), 175–196. https://doi.org/10.1080/19313152.2012.665204

Razfar, A., & Rumenapp, J.C. (2014). Narratives: Living a narrated life. In Razfar & Rumenapp, *Applying Linguistics in the Classroom: A sociocultural approach* (pp. 247-268). Routledge.

Razfar, A., Brier, J., Musick, H., Lin, J., Wilhelmi, M., & Chisti, A. (2024, Feb.). Toward Empathetic Listening and Solidarity: A study of long COVID narratives in community health. Paper presentation at the Annual Innovations in Medical Education Conference, Los Angeles, CA. https://sites.usc.edu/ime-conference/ime-award-winners/2024-ime-conference-award-winners/

Rosas, C.E. & West, M. (2011). Pre-Service Teachers' Perception and Beliefs of Readiness to Teach Mathematics. *Current Issues in Education* (Tempe, Ariz.), *14*(1), 1-23.

Roth. MW. (2012). Cultural-historical activity theory: Vygotsky's forgotten and suppressed legacy and its implication for mathematics education. *Mathematics Education Research Journal*, *24*(1), 87–104. https://doi.org/10.1007/s13394-011-0032-1

Uffen, I., de Vries, S., Goei, S. L., van Veen, K., & Verhoef, N. (2022). Understanding teacher learning in lesson study through a cultural–historical activity theory lens. *Teaching and Teacher Education*, *119*, 103831-. https://doi.org/10.1016/j.tate.2022.103831

Wake, G., Swan, M., & Foster, C. (2016). Professional learning through the collaborative design of problem-solving lessons. *Journal of Mathematics Teacher Education*, *19*(2–3), 243–260. https://doi.org/10.1007/s10857-015-9332-9

Acknowledgements

We are grateful to the Cultural Historical Research Special Interest Group of the American Educational Research Association (AERA) for facilitating this special issue. We thank our respective institutions for providing the necessary resources to conduct our research and the space to implement innovative teacher preparation practices.

About the authors

Aria Razfar – Professor of Education & Linguistics at the University of Illinois Chicago. E-mail: arazfar@uic.edu

Socorro Orozco – Assistant Professor of K-12 Mathematics Education at the California State University, Los Angeles. E-mail: sorozco7@calstatela.edu