The Empty Core or the Empty Theorem

Karl Vind
Institute of Economics, University of Copenhagen

SUMMARY: *An example given by Aivazian and Callen of an empty core does not show that the Coase theorem is incorrect, it shows that the theorem may be empty in the sense that its conclusion is about elements in an empty set.*

It is well known that the core of a game may be empty. The implication of this fact for the Coase theorem is discussed in two papers, Aivazian & Callen (1981) and Coase (1981).

Aivazian and Callen give an example with three firms, where the profits of the possible coalitions of firms are such that the core is empty so for all feasible distributions of profits there will exist a coalition (consisting of one, two, or three firms) who can get a higher profit.

From this they conclude »that the Coase theorem cannot be proved if the core of the game is empty« (pp. 175-176).

In his »Comment« Coase states that the example by Aivazian and Callen »has not led me to modify my views« (p. 183).

The part of the Coase theorem under discussion is the trivial observation made already by Wicksell in the context of competitive equilibrium in his review of Pareto's *Manuel d'Economie Politique*, that if the coalition of all firms (players, agents, consumers etc.) without transactions cost can make any joint decision on the distribution of profits (on strategies of a game, on reallocation of initial resources, etc.), then any equilibrium will be optimal.

It is an error in logic, that Aivazian and Callen from their example conclude, that the Coase theorem is incorrect. What their example shows is that the set of equilibria may be empty. The Coase theorem for the cases, where the set of equilibria is empty, is trivially true, simply because the elements of the empty set have all properties, they are also optimal.

The »Comment« shows in my opinion, that the theoretical world in which Coase is living, is a world without existence of equilibria. The set of actions available to the

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firms and the coalitions of firms in his discussion of the example is so large that no equilibria can exist.

Only under very special assumptions will an economy, a game, or a social system, where all the agents and all the subsets of agents – coalitions – can choose any preferred action, have an equilibrium. A precise formulation and precise results can be found in Vind (1983) – especially Theorems 1 and 2 – and Keiding (1985).

If one wants an economy, a game, or a social system for which both the Coase theorem applies and equilibria exist one should, except for a few special cases, have to prevent individuals and »coalitions« of two from doing what is best for them, unless it is improving for everybody influenced by the action. Such assumptions are not realistic as a description of economic reality, and would be very difficult to enforce through a legal system.

References

Aivazian, V.A. & J.L. Callen. 1981. »The Coase Theorem and the Empty Core«. Journal of Law and Economics 24, pp. 175-81.

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