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A Simple Method for the Measurement of Total Factor Productivity in Poorly Documented Economies

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SUMMARY: *Total factor productivity analysis in poorly documented economies, say pre 1850, is hampered by the fact that output and input data seldom can be estimated independently. A method that replaces shaky evidence on output with more robust data on wages and prices is developed and applied to an economy that exhibits the stylized facts of late 18th century Britain.*

The rate of total factor productivity increase stems from changes in knowledge and organization, in short technology, and it is thus the increase in output that cannot be allotted to increments in inputs of capital, land and labour. In historical studies and for Britain C. Feinstein (1981), N. F. R. Crafts (1987), D. McCloskey (1981), among others, have attempted to measure total factor productivity but with widely differing results.[1] The reason for this divergence is that such estimates are based on data which is, at least partly, conjectural in nature. Consider the formula (derived from a CES production function with constant returns to scale, in this particular case a Cobb-Douglas production function) for total factor productivity, denoted r as in residual

$$r = Q^* - \alpha L^* - (1 - \alpha)K^* \tag{1}$$

where a^* after a variable indicates the proportionate rate of increase in that variable. Q is aggregate production, L is labour, K is capital and α is a parameter measuring the elasticity of output with regard to labour, and $(1 - \alpha)$ is the elasticity of output with regard to capital, assuming two factors of production. As can be seen the information needed for a total factor productivity analysis is quite demanding and far greater than

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permitted by available data before 1850 also for the most researched of economies such as Britain.

There is also in existing studies a certain amount of question-begging because it is seldom possible to estimate both inputs, L and K, and output, Q, independently. Information on inputs are often used to infer changes in output, or the other way round. The main advantage with the new method is that the vague information on output is replaced by the more robust data on prices and wages.

Consider now this alternative approach in which we do not have to estimate output directly. Two sectors will be considered, agriculture and industry but the degree of disaggregation is limited only by the availability of data. The geographical entity can be a region or a nation.

Let subscript i be industry and we can define output per labourer per period as

$$q_i = Q_i / L_i \tag{2}$$

where Q_i is industrial output and L_i is industrial labour force. Assume now that income shares are constant and let w_i be wage per labourer. Then (2) can be expressed as

$$q_i = p_i Q_i / p_i L_i = L_i w_i (1 + s) / p_i L_i = w_i (1 + s) / p_i \tag{3}$$

In (3) s is a profit or surplus parameter. Assuming that income shares are constant amounts to saying that s is a constant, and we get a simple formula for the proportionate rate of increase of industrial labour productivity, q_i

$$q_i^* = w_i^* - p_i^* \tag{4}$$

and by analogy we get the rate of increase of output per agrarian labourer, subscript a for agriculture, in

$$q_a^* = w_a^* - p_a^* \tag{5}$$

We can now insert (4) in the expression for the rate of increase of total factor productivity in (1), collect terms and it becomes

$$r_i = w_i^* - p_i^* + (L_i^* - K_i^*) (1 - \alpha_i) \tag{6}$$

The corresponding expression for agriculture is somewhat more complicated since we have a third factor of production, land, T, and a new parameter, β, which is the elasticity of output with regard to land,

$$r_a = w_a^* - p_a^* + (L_a^* - K_a^*)\,(1 - \alpha_a) + \beta(K_a^* - T_a^*) \tag{7}$$

In the remaining part of this note I will mostly be concerned with the agricultural sector which is the dominant sector in terms of value added and employment in most poorly documented economies.

We can, roughly speaking, distinguish between three different regimes.

(1) The quantity of land was fixed but other factors of production were not. That simplifies (7) to

$$r_a = w_a^* - p_a^* + (L_a^* - K_a^*)\,(1 - \alpha_a) + \beta K_a^* \tag{7'}$$

(2) There was approximately equal growth rates of all factors of production which simplifies (7′) to

$$r_a = w_a^* - p_a^* \tag{7''}$$

(3) Land was in fixed supply but capital and labour had equal growth rates, which implies

$$r_a = w_a^* - p_a^* + \beta K_a^* \tag{7'''}$$

With industry, and discussing pre-industrial periods industry is broadly defined as non-agrarian or urban production, I think it is possible to make qualified guesses about the *difference* between K_i^* and L_i^* without actually knowing much about the precise value of either K_i^* or L_i^*. In most periods before the industrial revolution the difference was probably so small that it was negligible, which simplifies the estimation procedure to a formula analogous to (7′′). By and large the method propopsed here requires data on prices and wages when it is applied to industry in periods before the industrial era. When applied to agriculture there is the additional need for empirical basis generating qualified guesses about the growth of capital, labour and land, and income shares. But as can be seen from the equations above there are regimes in which the data requirement is limited to none or only one of the factors of production and the information on income shares may also be reduced accordingly. As pointed out above we do not have to rely on admittedly very uncertain information on aggregate or sectoral output since it is substituted by price and wage data. This simplification is not without a cost, of course. Two assumptions are required. First that income shares do not change (much) and second, that raw material and other intermediate goods, that do not figure at all in this accounting method, will grow at a rate equal to the income share weighted growth of labour and capital.

Table 1. Intervals of values of variables and parameters for the estimation of the rate of growth of total factor productivity.

	Percent per year (variables only).
w_a^*	1.75;1.85
p_a^*	1.65;1.75
K_a^*	0.2;0.55
L_a^*	0.1;0.35
α_a	0.4;0.6
β	0.2;0.3

Given the nature of data, more specifically the uncertainty attached to most numbers for this period, we must always treat it with considerable care. Take the wage series for example. There are stochastic disturbances in the admittedly vague information we have access to, and in addition to that the interpretation of the wage data in the accounting model assumes constant income shares. To the extent that income shares are not constant the inference drawn by using wage data will be misleading. For these reasons it would be wise to use intervals of values rather than discrete numbers.

Below in Table 1 I have constructed an example of an estimate of total factor productivity growth for an economy with some similarities with late 18th century England.[2] Land is in fixed supply but capital and labour have unequal growth rates so the relevant accounting equation will be (7′).

The deviation on w^* used in the calculations permits a change in income shares of 5 per cent points.

What can we do with these intervals of values? Combining values from the intervals can produce a wide variety of results when we feed them in equation (7′). The maximum value of r_a is 0.32 and the minimum value is – 0.15%. This is not a result of great interest. We cannot even decide whether growth is positive or negative. I suggest, however, a more rewarding strategy. Consider the intervals as a set of values where the true values are actually located. If that is all what is known the way to proceed is to draw values randomly from the intervals and enter them into the relevant equation. This procedure presupposes that values taken by variables are independent. I have experimented with two types of probability-distributions. The first supposed that the values in the

2. For an extended analysis of this method to Britain see Persson 1991.

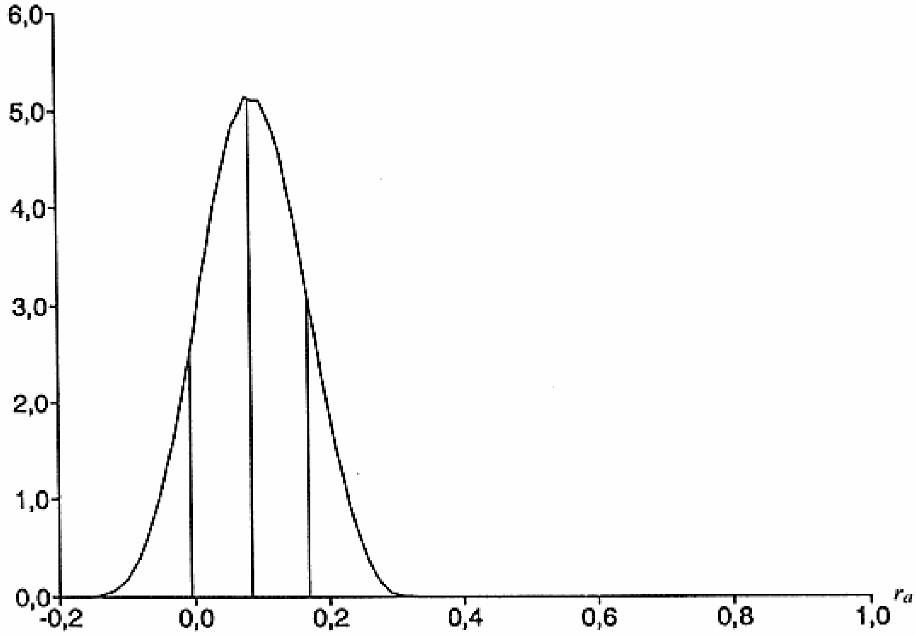

Figure 1. Probability density of yearly total factor productivity, r_a, in the agricultural sector. (See Table 1 for values on variables and parameters).

intervals were normally distributed around the mean of the interval. The second claimed that each value in the interval (ten values were ascribed to each interval) had an equal probability. The first argument takes account of the fact that any number must be adjusted for stochastic disturbances. The second argument can be defended as follows. When doing estimates of this kind you normally rely on estimates of the variables done by others and if you do these estimates yourself you try to determine upperbound and lowerbound limits. If so you can suspect that there is an optimistic and pessimistic bias, respectively, in two "limit" estimates.

Next step was to run a large number, around 200.000, of calculations of the equation with a random draw of values from each interval. It turned out to be of no importance whether the values on variables and parameters were normally distributed or if they had an equal probability to be drawn, so the actual calculations presented here stem from the latter type of experiment, which is less time-consuming to perform.

What we got from the repeated calculations based on random draws were almost normally distributed density functions of r_a.

Figure 1 shows the distribution of r_a for the period under consideration with the median and the left and right hand lines delimiting 75% of the surface under the curve plotted in the figure.

This procedure then gives a rather high prohability to a narrow estimate of total factor productivity. For example, it seems as if we can almost rule out a negative productivity growth. In the face of the great uncertainty attached to all numbers in the period up to, say, 1850, I find it more appropriate to present results as a prohability density function rather than the precise figures (to the third decimal) which you normally see in the economic history texts today.

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