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On Distributional Assumptions in Demand Theory

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SUMMARY: *In this paper it is shown that nice properties of the aggregated demand function can be obtained from assumptions on the distribution of consumers' characteristics.*

Introduction

In traditional micro economic theory the starting point is a description of the economy by a set of agents: consumers, producers, the state etc., and individual characteristics of these agents, e.g. preferences and production technologies. In order to make the economic analysis one makes assumptions on the individual characteristics such as to obtain that each of the agents is well behaved.

However, when looking at an economy it is not only important how well the individual agents behave, but also how the individual characteristics are distributed over the space of agents' characteristics. Even if all the individual agents did not behave well, the aggregated system i.e. the total economy might be well behaved, due to the way the individual characteristics were distributed.

To involve distrubutional assumptions is still more important since even when the individual agents behave well, it is not necessarily the case that the aggregated system is well behaved. A classical example is the weak Axiom of Revealed Preference. Of course, a consumer characterized by a consumption set and a total, strictly convex preordering on the consumption set will give rise to a demand functon, which satisfies the weak Axiom. However, the aggregated demand function will in general not satisfy the weak Axiom. In Grodal and Hildenbrand (1989) and Hildenbrand (1989) it is even proved that generically the weak Axiom will not be satisfied for the aggregated demand function, if the economy has a special structure.

The need for distributional assumptions was highlighted in the important Sonnenschein – Debreu – Mantel Theorem (see e.g. Mas – Colell (1985)) on the arbitrariness of the excess demand function of an exchange economy. It was shown that any continuous function which satisfies Walras law and which is homogeneous of degree zero can be obtained (except for the boundary) as excess demand function for a pure exchange economy with ℓ consumers, who satisfy even the most restrictive of the individual

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The need for distributional assumptions was highlighted in the important Sonnenschein – Debreu – Mantel Theorem (see e.g. Mas – Colell (1985)) on the arbitrariness of the excess demand function of an exchange economy. It was shown that any continuous function which satisfies Walras law and which is homogeneous of degree zero can be obtained (except for the boundary) as excess demand function for a pure exchange economy with ℓ consumers, who satisfy even the most restrictive of the individual assumptions of general equilibrium theory (smoothness, strict convexity, monotonicity, etc.). This especially implies that in general the Jacobian matrix of the total demand of an exchange economy will not have any other properties than the ones, which can be obtained for any continuous differentiable function F, which is homogeneous of degree zero and for which the mapping $pF(p)$ is linear.

The reason for this negative result is that the Jacobian matrix of the total demand function only gets nice properties on the *intersection* of the subspaces orthogonal to the individual demands. For each consumer it is on the subspace orthorgonal to his demand that the income effects disappear. By constructing the individual consumers nicely, but with different demand behavior, one obtains that the intersection of the subspaces is $\{0\}$, and all the nice properties of the individual Jacobian matrices, the part coming from the Slutsky substitution matrices, have disappeared.

It is the purpose of the present paper to point at some examples within demand theory, where results on the aggregated behavior have been obtained, not due to strong assumptions on the individual behavoir, but due to distributional assumptions on the consumption sector of the economy.

A Consumption Sector

We shall define a consumption sector as a measure on the space of consumers' characteristics. Let C be a separable metric space of consumers' characteristics. We could e.g. have $C=\mathcal{P}\times\mathbb{R}^\ell$, where $\mathbb{R}^\ell$ is the commodity space and $\mathcal{P}$ the set of continuous irreflexive preference relations with the topology of closed convergence, or $C=\mathcal{F}\times\mathbb{R}_{++}$ where $\mathcal{F}$ is a set of individual demand functions with some separable metric. In the first case a consumer characteristic would be a pair $(\succ, \omega)$, where ω is a vector of initial endowments and $\succ$ a preference relation. In the second case a consumer characteristic would be (f, x), where f is a demand function and x an expenditure level or income. In order not to complicate notation we shall always assume that the consumption set is the positive orthant of $\mathbb{R}^\ell$ i.e. $\mathbb{R}^\ell_+$. We shall also assume that individual demand functions $f\in\mathcal{F}$, f: $\mathbb{R}^\ell_{++}\times\mathbb{R}_+\rightarrow\mathbb{R}^\ell_+$, are continuous differentiable and that they satisfy $p\cdot f(p,w)=w$. We do not assume that the individual demand functions are obtained by maximizing a preference relation.

The actual consumers in an economy $\mathcal{E}$ can the be dscribed by a measureable mapping $\mathcal{E}$: $(I, \mathcal{I}, \gamma)\rightarrow C$, where $(I, \mathcal{I}, \gamma)$ is a measure space of consumers and where for all $i\in I$, $\mathcal{E}(i)\in C$ is the individual characteristics of consumer i. In the case, where I is a finite set, we can take $\mathcal{I}$ to be all subsets of I and γ to be counting measure

$$\gamma(E) = \frac{\#\{i \in E\}}{\# I}$$

Given the mapping $\mathscr{E}$ we can define the corresponding distribution $\mu_{\mathscr{E}}$ on C by $\mu_{\mathscr{E}}(F) = \gamma\{i \in I \mid \mathscr{E}(i) \in F\}$, for all measurable sets $F \subset C$. $\mu_{\mathscr{E}}(F)$ is the relative number of consumers, who have characteristics in the set F.

We define a *consumption sector* as a measure μ on the space of consumers' characteristics.

Any description $\mathscr{E}$ of a set of consumers will as shown above give rise to a uniquely defined consumption sector. However, a given consumption sector can be generated by means of different sets of consumers; any consumption sector μ can e.g. be obtained as the distribution of an atomless measure space of consumers. Indeed let the atomless measure space of consumers be given by $I = C \times [0,1]$, with the product measure $\gamma = \mu \otimes \lambda$, where λ denote the Lebesque measure on $[0,1]$. Now we let $\mathscr{E}: I \to C$ be defined by $\mathscr{E}(c,\beta) = c$. Then we obtain that the atomless measure space of consumers $(I, \mathscr{I}, \gamma)$ with the discription $\mathscr{E}$ has exactly the distribution μ. Looking at a consumption sector μ we often have his standard representation with many consumers in mind.

We shall in the following consider consumption sectors μ defined on $C = \mathscr{P} \times \mathbb{R}_+$ or $\hat{\mathscr{P}} \times \mathbb{R}_+$. It should be noticed that in this setup the distribution of total expenditure (income), i.e. the marginal distribution μ_2 on $\mathbb{R}_+$ of μ, is by definition independent of the price system p.

We shall also assume that mean expenditure $\bar{x} = \int_{\mathbb{R}_+} x \, d\mu_2$ is finite.

Continuous and Differentiable Demand

We now consider a consumption sector μ defined on $\mathscr{P} \times \mathbb{R}_+$. Let μ_1 be the marginal distribution of μ on $\mathscr{P}$.

If we assume that for all preference relations $\succ$ in the support of μ_1, $\succ$ is either transitive or convex, then we can for each characteristic $(\succ, x)$ define the corresponding demand correspondence $\varphi(\succ, x, \cdot): \mathbb{R}^{\ell}_{++} \Rightarrow \mathbb{R}^{\ell}$ and we can define that the mean market demand for the consumption sector by $F(p, \mu) = \int_{\mathscr{P} \times \mathbb{R}_+} \varphi(\succ, x, p) \, d\mu$.

Under standard assumptions we shall have that the demand correspondence $F(\cdot, \mu): \mathbb{R}^{\ell}_{++} \Rightarrow \mathbb{R}^{\ell}$ is upper hemi-continuous, but a natural question to ask is, if there exist conditions on μ such that $F(\cdot, \mu)$ is in fact a *continuous function*. Of course, this will be the case if we assume that for every consumer characteristic $(\succ, x)$ in the support of μ, the preference relation $\succ$ corresponds to a strictly convex and total preordering. Then indeed $\varphi(\succ, x, \cdot)$ will be single valued and consequently $F(\cdot, \mu)$, will be single valued, and therefore a continuous function. However, the main question is, if there exist conditions on μ such that $F(\cdot, \mu)$ is single valued even when the individual demand correspondences do *not* have this property.

In the book W. Trockel (1984), which partly is based on papers by E. and H. Dierker and W. Trockel, is given a very profound discussion of the different ways to obtain into a situation where the consumption sector μ is a product measure and where preferences are smooth (but not strictly convex). The main idea is that if we look at a fixed preference relation, then (for most preference relations) the set of price systems for which there is not a unique maximal element in the budget set is a null set. This null set of prices will vary systematically when income of the consumers vary, but the preference relation stays fixed. By making the assumption that income is dispersed they obtain that for all preferences, except a residual subset in the space of C^{∞} utility functions, the demand correspondence after having just integrated over the wealth distribution, will be a continuous function.

In order to obtain not only continuity of $F(\cdot,\mu)$ but also that $F(\cdot,\mu)$ is a continuous differentiable function, one needs to introduce assumptions also on the measure μ_1 on preferences.

The Weak Axiom and The Law of Demand

We shall go one step further. In order to make comparative statics and also to use the economic model for policy issues, one wants to have more information on the market demand. Indeed one should like to know that $F(\cdot,\mu)$ satisfies the Law of Demand or at least the weak Axiom of Revealed Preference. E.g. it is well known that the weak Axiom for the market demand function is closely related to uniqueness of equilibria.

The Law of Demand, i.e. the monotonicity of the market demand function F for the consumption sector, means that the price change and the demand change are in "opposite directions", and therefore especially that the market demand for a commodity is a non-increasing function of its own price. This is a useful property when e.g. studying stability of the equilibrium or making comparative statics.

Formally the market demand function $F(\cdot,\mu)$ satisfies *the weak Axiom* if for all price systems $p,q \in \mathbb{R}^{\ell}_{++}: qF(p,\mu) \leq \bar{x}$ implies $pF(q,\mu) \geq \bar{x}$.

Correspondingly, the market demand function $F(\cdot,\mu)$, satisfies *the Law of Demand* if for all price systems $p,q \in \mathbb{R}^{\ell}_{++}: (p-q)(F(p,\mu) - F(q,\mu)) \leq 0$.

We are interested in finding conditions on a consumption sector μ such that $F(\cdot,\mu)$ satisfies respectively the weak Axiom and the Law of Demand.

Let us assume that each consumer is described by a demand function f and a level of total expenditure $x \in \mathbb{R}_{+}$, and let us assume that the admissible demand functions are parametrized by a parameter $\alpha \in \mathscr{A}$. This is no restriction since the parameter space $\mathscr{A}$ can be any seperable metric space, for example, the space of all admissible demand functions.

The space of consumers' characteristics is then $\mathscr{A} \times \mathbb{R}_{+}$, and the market demand function $F(\cdot,\mu): \mathbb{R}^{\ell}_{++} \rightarrow \mathbb{R}^{\ell}_{+}$ is

$$F(p,\mu) = \int_{\mathscr{A}\times\mathbb{R}_+} f^{\alpha}(p,x)\, d\mu.$$

We shall assume that the marginal distribution μ_2 of expenditure has a density ρ and that the conditional distribution μ/x on $\mathscr{A}$ exists for every expenditure x in the support of ρ. We can then define the *cross-section demand function* $\bar{f}$: $\mathbb{R}^{\ell}_{++}\times\mathbb{R}_+\to\mathbb{R}^{\ell}_+$

$$\bar{f}\,(p,x) = \int_{\mathscr{A}} f^{\alpha}(p,x)\, d\mu|x$$

Using the cross-section demand function $\bar{f}$ we obtain

$$F(p,\mu) = \int_{\mathbf{R}_+} \bar{f}(p,x)\, \rho(x)\, dx.$$

Now it is well known that the Law of Demand as well as the weak Axiom for the market demand function $F(\cdot,\mu)$ can be expressed in terms of the $\ell\times\ell$ Jacobian matrix

$$\partial_p F(p,\mu) = (\partial_{p_k} F_j(p,\mu)^{\ell}_{k=1}{}^{\ell}_{j=1}$$

Indeed

(1) $F(\cdot,\mu)$ satisfies the weak Axiom, if and only if $\partial_p F(p,\mu)$ is negative semi-definite on

$$F(p,\mu)^{\perp}=\{v\in\mathbb{R}^{\ell}\,|\,F(p,\mu)\cdot y=0\}, (i.e. \sum_{k=1}^{\ell}\sum_{j=1}^{\ell} v_k v_j \partial_{p_k} F_j(p,\mu)=\leq 0 \text{ for all } v\in(F(\cdot,\mu))^{\perp};$$

and

(2) $F(\cdot,\mu)$ satisfies the Law of Demand (monotonicity), if and only if $\partial_p F(p,\mu)$ is negative

semi-definite $(i.e. \sum_{k=1}^{\ell}\sum_{j=1}^{\ell} v_k v_j \partial_{p_k} F_j(p,\mu)\leq 0$ for all $v\in\mathbb{R}^{\ell})$

Therefore in order to find out whether F satisfies the weak Axiom and the Law of Demand one has to investigate if the Jacobian matrix $\partial_p F(p,\mu)$ has the above properties.

We first notice that the Slutsky decomposition of the cross-section demand function $\bar{f}$ leads to

$$\partial_p \bar{f}(p,x) = S\bar{f}(p,x) - \partial_x \bar{f}(p,x)\ \bar{f}(p,x)^T$$

where $S\bar{f}(p,x)$ is the Slutsky substitution matrix of the cross-section demand function $\bar{f}$.

Hence we obtain

$$\partial_p F(p,\mu) = \int_{\mathbb{R}_+} S\bar{f}(p,x)\ \rho(x)\ dx - \int_{\mathbb{R}_+} \partial_x \bar{f}(p,x)\, \bar{f}(p,x)^T \rho(x)\ dx = \bar{S} - \bar{A}, \text{ where}$$

$$\bar{S} = \int_{\mathbb{R}_+} S\bar{f}(p,x)\ \rho(x)\ dx \text{ and } \bar{A} = \int_{\mathbb{R}_+} \partial_x \bar{f}(p,x)\, \bar{f}(p,x)^T \rho(x)\ dx.$$

Therefore, if $\bar{S}$ is negative semi-definite and $\bar{A}$ is positive semi-definite, we have that $\partial_p F(p,\mu)$ is negative semi-definite.

The matrix $\bar{S}$ is studied in Härdle (1991) and Hildenbrand (1991). They show that if the individual demand functions satisfy the weak Axiom, and the dispersion of the distribution of the demands of the consumers with total expenditure x would increase if they have had the expenditure $x+\Delta$, then $S\bar{f}(p,x)$, and therefore $\bar{S}$, is negative semi-definite.

The other matrix $\bar{A} = \int_{\mathbb{R}_+} \partial_x \bar{f}(p,x)\, \bar{f}(p,x)^T \rho(x)\ dx$, which is the most delicate one, is analyzed in details in the paper by Grodal and Hildenbrand (1991). We want to find conditions such that $\bar{A}$ is positive semi-definite (on the hyperplan determined by mean demand).

First we remark that the matrix $\bar{A}$ is positive semi-definite (on the hyperplan determined by mean demand) if and only if the symmetrized matrix

$$B(\bar{f},\rho) = \int_{\mathbb{R}_+} \partial_x(\bar{f}(p,x)\, \bar{f}(p,x)^T)\ \rho(x)\ dx.$$

is positive semi-definite (on the hyperplan determined by mean demand). Let us denote mean demand with a $(\bar{f},\rho)$ i.e. $a(\bar{f},\rho) = \int_{\mathbb{R}_+} \bar{f}(p,x)\ \rho(x)\ dx$.

Looking at $B(\bar{f},\rho)$ we see by partial integration that if the density ρ has the support $\mathbb{R}_+$ and is differentiable, then

$$B(\bar{f},\rho) = -\int_{\mathbb{R}_+} (\bar{f}(p,x)\, \bar{f}(p,x)^T)\ \rho'(x)\ dx.$$

Now we notice that the matrix $(\bar{f}(p,x)\, \bar{f}(p,x)^T)$ is a positive semi-definite for all $x \in \mathbb{R}_+$ (it has rank 1 and the only non-zero eigenvalue is $|\bar{f}(p,x)|^2$). Therefore if the density is non-increasing, then $B(\bar{f},\rho)$ is indeed positive semi-definite. This gives essentially the example in Hildenbrand (1983).

In the general case we have $B(\bar{f},\rho) = B_1 - B_2$, where

$$B_1(\bar{f},\rho) = \int_{\{x|\rho'(x)\leq 0\}} \bar{f}(p,x)\, \bar{f}(p,x)^T(-\rho'(x))\ dx, \quad \text{and}$$

$$B_2(\bar{f},\rho) = \int_{\{x|\rho'(x)>0\}} \bar{f}(p,x)\, \bar{f}(p,x)^T \rho'(x)\ dx$$

B_1 and B_2 are positive semi-definite matrixes, and we get that $B(\bar{f},\mu)$ will be positive semi-definite if every eigenvalue of B_1 is larger or equal to every eigenvalue of B_2. However, if B_1 has eigenvalue 0, e.g. since $\{\bar{f}(p,x) | x: \rho'(x) \leq 0\}$ is contained in a lower dimensional subspace of $\mathbb{R}^{\ell}$, then even small eigenvalue of B_2 can prevent the positive semi-definiteness of B.

When we in the general case want to investigate the matrix $B(\bar{f},\rho)$, we have two different ingredients, namely the cross-section demand function for a fixed p i.e. the cross- section Engelfunction $\bar{f}(p,\cdot)$, and the distribution ρ of consumers' total expenditure. It is the *shape* of the various cross-section Engelfunctions as well as the *shape* of the expenditure distribution, which play an important role in deriving the positive semi-definiteness (on the hyperplan determined by mean demand) of the matrix $B(\bar{f},\rho)$. In Grodal and Hildenbrand (1991) it is shown that $B(\bar{f},\rho)$, and therefore the mean income effect matrix, tends to be positive semi-definite if the cross-section Engelfunctions and the expenditure distribution have the following characteristics:

(1) The cross-section Engelfunctions "bend slowly", in the sense that higher derivatives can be neglected.

(2) There is sufficient "dispersion" in the distribution of total expenditure.

Grodal and Hildenbrand consider cross-section Engelfunctions, which can be generated by given family of base function $b_1(\cdot,\cdot)$, ..., $b_S(\cdot,\cdot)$ from $\mathbb{R}^{\ell}_{++} \times \mathbb{R}_+$ into $\mathbb{R}$. I.e. it is assumed that the cross-section Engelfunctions have the form

$$\bar{f}_k(p,\cdot) = \sum_{s=1}^{S} \alpha_s^k(p)\ b_s(p,\cdot)\ ,\ k=1,\ldots,\ell.$$

Of course, any cross-section demand function $\bar{f}$ can be written in the above form by choosing $S=\ell$ and choosing the base function appropriately. However, the idea is that the number of base functions is small compared to the number of commodities, and that the base function are chosen a priori. Let $\mathcal{L}(b)$ be the linear function space spanned by the functions $b_1(p,\cdot),\ldots,b_S(p,\cdot)$. Since we only consider a fixed price system, we can drop p as a variable.

When looking at a family of base functions b we can for this family and any density ρ on $\mathbb{R}_+$, define the $S \times S$ matrix $B(b,\rho)$, by

$$(B(b,\rho))_{h,j} = \int_{\mathbb{R}_+} (b_h(x) b_j(x))'\ \rho(x) dx$$

and the vector $a(b,\rho) \in \mathbb{R}^S$ by

$$(a(b,\rho))_s = \int_{\mathbf{R}_+} (b_s(x)\ \rho(x)dx$$

In Grodal and Hildenbrand (1991) the following proposition is proved:

PROPOSITION. The rank of the matrix $B(\bar{f},\rho)$ is not larger than the number S of base functions, and the $\ell \times \ell$ matrix $B(\bar{f},\rho)$ is positive semi-definite (on $a(\bar{f},\rho)^{\perp}$) for every function $\bar{f} \epsilon \pounds(b)$ if and only if the $S \times S$ matrix $B(b,\rho)$ is positive semi-definite (on $a(b,\rho)^{\perp}$).

There are two different ways of looking at this Proposition. From a theoretical point of view one may hope to specify families of base functions which imply that $B(b,\rho)$ is positive semi-definite (or, at least, positive semi-definite on $a(b,\rho))^{\perp}$) for a reasonable large class of expenditure distributions. However, the Proposition also shows that if, for a given data set, one wants to analyze whether the matrix $B(\bar{f},\rho)$ is positive semi-definite, then one should not estimate the cross-section demand function, provided one assumes that it is generated by a given family of base functions b. Indeed, one should investigate directly whether the matrix $B(b,\rho)$ is positive semi-definite for the estimated expenditure distribution.

As an example of how the Proposition can be used, look at the base functions $b_s(x)=x^s$, $s=1,...,S$, and the corresponding class of cross-section Engelfunctions $\pounds(b)$. In this case the matrix $B(b,\rho)$ will reduce to a "matrix of moments", $M(S,\rho)$, of the expenditure distributions, and $a(b,\rho)$ to a vector $m(S,\rho)$ of moments. The matrix $M(S,\rho)$ involves the first $2S$-1 moments. Using the proposition above we get that $B(\bar{f},\rho)$ will be positive semi-definite (on the hyperplan $a(\bar{f},\rho)^{\perp}$) if the matrix of moments $M(S,\rho)$ is positive semi-definite (on the hyperplan $m(S,\rho)^{\perp}$). Therefore the problem reduces to the question: For which kinds of income distributions ρ is the "matrix of moments" positive semi-definite?

First we notice that if S=2 then the matrix $M(S,\rho)$ is positive semi-definite on $(m(S,\rho))^{\perp}$ for any density ρ. For higher S the question can be approached either empirically or theoretically.

Using the U. K. family expenditure survey for the years 1969-83 estimations show that for S less or equal to 4 the estimated matrix turned out to be positive definite. For larger values of S, say S=7, some eigenvalues turned out to be negative but they were all very small. (These estimations and calculations have been made by H. P. Schmitz, SFB 303, Bonn).

As an example we also directly calculated the matrix $M(S,\rho)$ for a log normal distribution. In this case it turns out that for all S there exists a constant $k(S)$ such that if the coefficient of variation of ρ is larger than $k(S)$ then $M(S,\rho)$ is positive definite. For example, we calculated that $k(2)$=0,36, $k(3)$=0,50 and $k(4)$=0,59.

Empirical estimates of the coefficient of variation of income distributions show that the coefficient of variation is typically around 0,7, i.e. the empirical coefficient of variation is larger than the critical value 0,59, which is obtained for S=4.

In conclusion it is seen that there exists reasonable conditions on the distribution of total expenditure, which together with the assumption that cross-section Engelfunctions do not bend to quickly (low number of base functions) imply that the delicate matrix of mean income effects for a consumption sector is well behaved. The assumption that a low number of base functions can be used is of course closely related to the number of commodities i.e. to the definition of a commodity (commodity group), but is in the paper Grodal and Hildenbrand (1991) not directly related to the distribution of agents' characteristics. However, empirical investigations show that Engelfunctions for commodity aggregates are indeed bending rather slowly, see Hildenbrand and Hildenbrand(1986).

The above results for the mean income effect matrix $\bar{A}$ together with the results on the mean Slutsky substitution matrix $\bar{S}$ indeed show that there exist conditions on the distribution of consumers' characteristics, conditions which partly are supported by empirical evidence, and which give the mean demand function more structure than what can be obtained from assumptions on the individual characteristics.

Diagonal Dominance

In a recent paper, J. M. Grandmont (1991), is given a very beautiful example, which gives even more properties of market demand. As in the papers by E. and H. Dierker and Trockel, Grandmont considers certain transformations of preference relations and distributional assumptions on these transformations.

Let the space of consumers' characteristics be the set of pairs (f, x), where f is a demand function and x an expenditure level. The basis of Grandmont's example is that the consumption sector μ can be decomposed in a measure on a set A of types, where a type corresponds to a demand function f_a and an expenditure level x_a, and for each type a, a conditional distribution over the space of all transformations of the demand function f_a. A transformation corresponds to a change of units of measurement for the commodities.

Grandmont investigates conditions on the conditional distributions over the transformations, which imply nice properties for the market demand function. Among others he shows that if the integral of the absolute value of the derivatives of the conditional densities are sufficiently small, then the market demand for commodity h is a decreasing function of its own price, and moreover the Jacobian matrix has approximately a dominant diagonal for a large set of price systems.

One major difference between the previous results and the result by Grandmont in its present form is that the distributional assumptions cannot be tested directly. However, the descripton of the consumption sector used by Grandmont implies that for each type (f_a, x_a) there is, for each price system $p \in \mathbb{R}^{\ell}_{++}$, a specific distribution of the demands on the budget hyperplan determined by p and x_a. For each expenditure level and the actual price system p one has also information on the empirical distribution of demands on this budget hyperplan. If one assumes that all individuals with a given total expenditure x have demand functions, which are transformations of each other, then the consequences of distributional assumptions on the transformations can be compared to the empirical distribution on the budget hyperplan.

It will be a subject of further research to link assumptions on the distribution of transformations of a demand function and assumptions on the distributions of demand choices, and to find nice conditions on these last distributions, which will imply the beautiful conclusions in Grandmont's paper.

Distributions and Profit Functions

It is not only of importance to demand theory itself to investigate the properties of the aggregated demand function. If one e.g. looks at models with imperfect competition, it is necessary in order to prove existence of equilibria (in pure strategies) that the profit functions are quasi concave. Again, only in trivial examples one obtains this property if the theory is based on properties of the individual characteristics. In the recent papers E. Dierker (1991), and Caplin and Nalebuff (1991) it is shown that with distributional assumptions on the consumption sector it is possible to obtain quasi concave profit functions for the firms and thereby existence of equilibria.

Conclusion

As shown above distributional assumptions might throw more light on some of the areas where it has not been possible to obtain positive results from only individual assumptions. It might also be the case that distributional assumptions can replace some of the assumptions on individuals, of which we are not totally comfortable.

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