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# Unequal Pay for Equal Work

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SUMMARY: *Frequently one can observe unequal pay for equal work. Equally productive workers are paid different wage rates and those with relatively high wage rates have relatively uncertain employment (viz. consultants, journalists, stevedore workers). We show that this phenomenon can be explained by profit maximizing behaviour of a monopsonistic firm confronting equally productive workers with different, privately observed reservation wages.*

## 1. Introduction

In some labour markets it can be observed that workers, who apparently are equally productive, are employed under different conditions. Some have guaranteed employment at a relatively low wage, while others have much higher wages but also more volatile employment. This phenomenon can be observed for instance in the markets for consultants, journalists, other professionals and also for stevedore workers in harbours around the world. At first glance one might think that this phenomenon is easily explained by demand or revenue uncertainty. Firms take in extra workers at high wages in peak periods. However, this explanation implies that workers bear a considerable amount of income uncertainty and that contradicts the received wisdom of contract theory. According to this, risk neutral firms will offer optimal contracts which insure risk averse workers against income or wage fluctuations, see e.g. the surveys in Hart (1983), Hart and Holmström (1986), Blanchard and Fisher (1989), or Theorem 1 in Jacobsen and Schultz (1991). One therefore suspects that something else must be around. In this paper, we show that the phenomenon can be explained as successful wage discrimination by profit maximizing firms facing workers with equal productivity but different, privately observed reservation wages.

In order to make things simple we study a model with one firm having a non-stochastic, linear revenue function[1]. The firm hires some out of many workers in a labour market. There are two types of workers, some with a high and some with a low reservation wage. All workers are risk averse and equally productive. Each worker knows his own reservation wage, but the firm only knows the distribution of workers over reservation wages. So, there is *asymmetric information* with respect to reservation wages.

1. A stochastic revenue function would be interesting to analyze, but it complicates matters considerably and it turns out that the basic phenomenon at play also turns up in a deterministic setting. In Jacobsen and Schultz (1991) the case of a stochastic revenue function is dealt with.

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In order to make things simple we study a model with one firm having a non-stochastic, linear revenue function[1]. The firm hires some out of many workers in a labour market. There are two types of workers, some with a high and some with a low reservation wage. All workers are risk averse and equally productive. Each worker knows his own reservation wage, but the firm only knows the distribution of workers over reservation wages. So, there is *asymmetric information* with respect to reservation wages.

1. A stochastic revenue function would be interesting to analyze, but it complicates matters considerably and it turns out that the basic phenomenon at play also turns up in a deterministic setting. In Jacobsen and Schultz (1991) the case of a stochastic revenue function is dealt with.

The firm decides the conditions under which workers are hired. It may choose to act as a normal monopsonist and just offer employment at one wage. Workers will then decide whether they want to supply labour at this wage or not. But the firm may also simultaneously offer two contracts, one with a low wage and certain employment and one with a high wage but uncertain employment. Workers then decide under which condition, if any, they want to supply labour. We show that such a contract pair may serve as a *self selection device* such that high reservation wage workers choose the high wage with uncertain unemployment, and low reservation wage workers choose the low wage with certain employment. Low reservation wage workers are scared by the prospect of ending up as unemployed if they choose the high wage, and therefore they choose the low. We give necessary and sufficient conditions under which it is indeed profit maximizing for the firm simultaneously to offer such a pair of contracts. In this case there is wage discrimination and unemployment or *unequal pay for equal work.*

The *welfare consequences* of unequal pay for equal work are ambiguous. There are cases where unequal pay for equal work weakly Pareto dominates equal pay for equal work and there are cases where this is not true.

The outline of the paper is as follows. Section 2 describes the labour market considered. Section 3 deals with the normal monopsonist offering equal pay for equal work. In section 4 it is explained how two contracts can work as a self selection device for workers. Section 5 gives the conditions under which the firm actually chooses to wage discriminate and studies the welfare implications. Section 6 concludes.

## 2. The labour market

There is one firm with a revenue function $T \cdot \ell$, where $\ell \geq 0$ is labour input, and $T > 0$ is the marginal revenue. The simple linear form of the revenue function is chosen for analytical convenience. The revenue function can be thought of as $p \cdot g \cdot \ell$, where $g$ is the marginal productivity of labour and $p$ is the price of output. The firm maximizes profit, which is revenue less labour costs.

The firm is confronted with many workers, the number of which we normalize to one (i.e., $B$ workers means the fraction $B$ of all workers). Each worker sells one or zero units of labour. A worker with reservation wage $R$ gets the von Neuman-Morgenstern utility $U(w-R)$ from selling one unit of labour at wage rate $w$. The utility function $U$ is the same for all workers and fulfills $U(0) = 0$ = the utility from not working, $U$ is twice differentiable, $U'(y) > 0$ and $U''(y) < 0$ for all $y \in \mathbf{R}$. That $U$ is strictly concave means that all workers are strictly risk averse. The reservation wage $R$ of a worker can be thought of as representing unemployment benefits or the income equivalent of the value of leisure or work in his allotment garden.

There are two different types of workers, those with a low reservation wage $R_1$, and those with a high reservation wage $R_2$ $(R_1 < R_2)$. Reservation wages are normalized to lie between zero and one. The number (fraction) of workers with a high reservation wage is $A$, $0 < A < 1$, and the number of workers with a low reservation wage is $1-A$. Each worker knows his own reservation wage, but the firm cannot distinguish among workers. It only knows the distribution of workers over reservation wages. Hence, there is *asymmetric information* with respect to reservation wages. All workers are equally productive. In order to ensure that it is possible for the firm to employ high reservation wage workers at a wage rate that both parties can accept, we assume that $T > R_2$.

The firm chooses the conditions under which it offers employment. The point is to allow, a priori, that employment can be offered under different conditions. The normal case would be that of *one wage, w*. The firm offers a wage and workers choose to work or not. However, another possibility for the firm is to offer employment under two different conditions simultaneously. It can offer *guaranteed employment at a low wage* $w_1$ and *uncertain employment at a high wage* $w_2$. Here the firm choose a vector $s = (w_1, w_2, \lambda) \in S \equiv \{[0,1]^3 \mid w_1 \leq w_2\}$, where $\lambda$ is the unemployment or rationing probability attached to the high rate $w_2$. Each worker then the chooses under which conditions, if any, he wants to sell labour. There is no limitation in the constraint $w_1 \leq w_2$. No worker would sell labour at the uncertain wage $w_2$, if it was less than $w_1$. Similarly, there is no point in allowing the firm to offer uncertain employment at the low wage. This will only make the low wage even less attractive for workers. The point of making the high wage uncertain is to make risk averse workers with low reservation wages prefer the low wage. If a worker decides to go for the high wage and ends up unemployed, then he does not have the possibility afterwards to sell labour at the low wage. Were this the case, then all workers would, of course, first try their luck at the high wage, and the unlucky would then go for the low wage. Such an arrangement could never be better for the firm than offering just one wage rate.

One could conceive of different implementations of the two wage scheme. One possibility is that the firm at a date one, before production takes place, offers two different contracts to be signed at date one. When production takes place, at date two, it is not possible to sign up for contracts anymore. Another possibility is that the firm at date one only offers the low wage, sure employment contract. At date two remaining workers are offered the high wage, uncertain employment contract. The last institution resembles what can be observed in many harbours around the world.

If one dislikes the idea of the firm offering a rationing *probability* one can think of the firm simply choosing a given number of workers that it will employ at the high wage rate. If the number of workers seeking employment at the high wage rate is greater, then only a fraction $1-\lambda$, of these workers will become employed. If the determination of who will actually become employed is random, then the individual worker offering labour at $w_2$ will have a probability $1-\lambda$ of becoming employed.

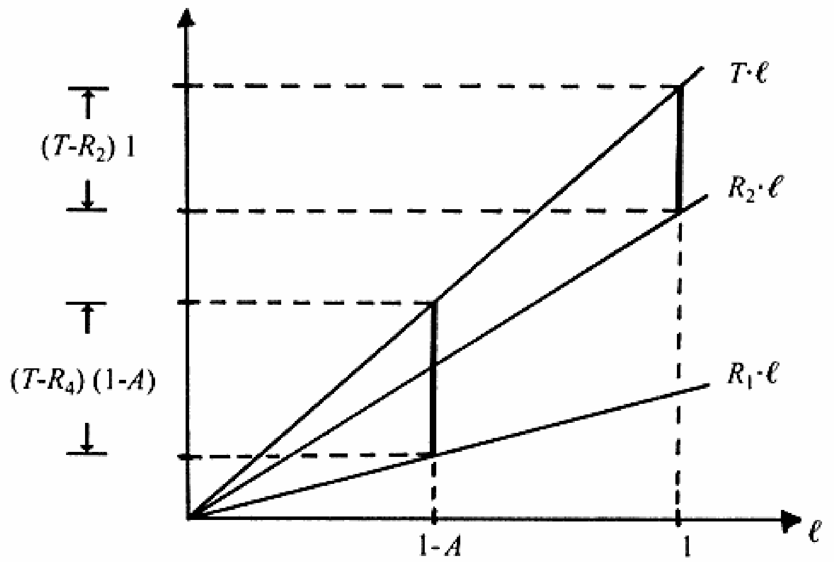



*Figure 1.*

The utility of a worker with reservation wage $R$ of accepting the low wage with sure employment is $U(w_1\text{-}R)$; the expected utility of accepting the high wage with uncertain unemployment is $\lambda U(0) + (1-\lambda)U(w_2-R) = (1-\lambda)U(w_2-R)$.

### 3. Equal pay for equal work

For a moment we restrict attention to the case where the firm sets only one wage rate. We will then say that the firm behaves as a *normal monopsonist.* Clearly, since the firm maximizes profit, this wage should either be equal to $R_1$, in which case labour supply is $1-A$, or equal to $R_2$, in which case labour supply is 1. What is most profitable for the firm depends on whether $(1-A)(T-R_1) \gtreqless T-R_2$ or equivalently, whether

$$1-A \gtreqless \frac{T-R_2}{T-R_1}. \qquad (1)$$

Notice that the linearity of the revenue function implies that the firm will either employ all $1-A$ or all 1 workers. This is an important simplification that a linear revenue function gives us. Figure 1 illustrates a case where $(1-A)(T-R_1) > T-R_2$, and the firm employs the $1-A$ low reservation workers.

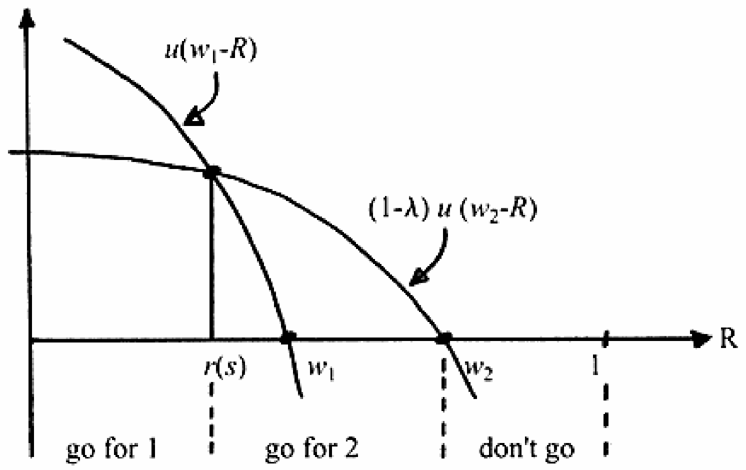



*Figure 2.*

The basic question of this paper is: Could the firm do better than offering just one wage? I.e., are there instances where it will be profitable for the firm to offer simultaneously two different wage rates and ration workers at the high wage? If the firm does so, we say that the firm wage discriminates or offer unequal pay for equal work. In order to answer the basic question we must first study workers' self selection.

**4. Workers' self selection**

Now we allow the firm to offer both the low, sure employment wage rate and the high, uncertain employment wage rate, i.e., to offer $s = (w_1, w_2, \lambda) \in S$. Consider a worker with reservation wage $R$. His expected utility from accepting the low wage with sure employment is $U(w_1-R)$, while his expected utility from the high wage with uncertain unemployment is $(1-\lambda)U(w_2-R)$. He is indifferent between the two if

$$U(w_1-R) = (1-\lambda)U(w_2-R). \tag{2}$$

The solution in $R$ to (2), we call $r(s)$. In Figure 2 we have depicted the utility a worker gets from the low wage, sure employment and the high wage, uncertain employment as a function of his reservation wage.

From Figure 2 it is clear that (2) has a unique solution whenever $\lambda > 0$. This solution is always less than $w_1$ if $w_2 > w_1$, but it may be negative. However, it will be argued below that from profit maximization considerations we can restrict attention to $r(s) > 0$. A worker with a reservation wage $R < r(s)$ will prefer the low wage with sure employment while a worker with a reservation wage $R > r(s)$ will prefer the high wage with uncertain employment. A worker with $R = r(s)$ is assumed to go for the low wage. Therefore, even though the firm cannot distinguish among workers, it can induce the workers themselves to split into two groups by adjusting $w_1$, $w_2$, and $\lambda$ appropriately. The firm induces the workers to perform *self selection.* As is clear from Figure 2, self selection is not dependent on workers being risk averse. If $U$ were linear the curves of Figure 2 would still intersect at a positive value of $R$ provided that $w_2 > w_1$ and $\lambda$ sufficiently high. Workers would then split up as discussed above.

### 5. Unequal pay for equal work

What is the optimal choice of $s = (w_1, w_2, \lambda)$ for the firm? First notice that if the firm decides to offer two different wages and employ workers on both, then it should choose $s$ such that

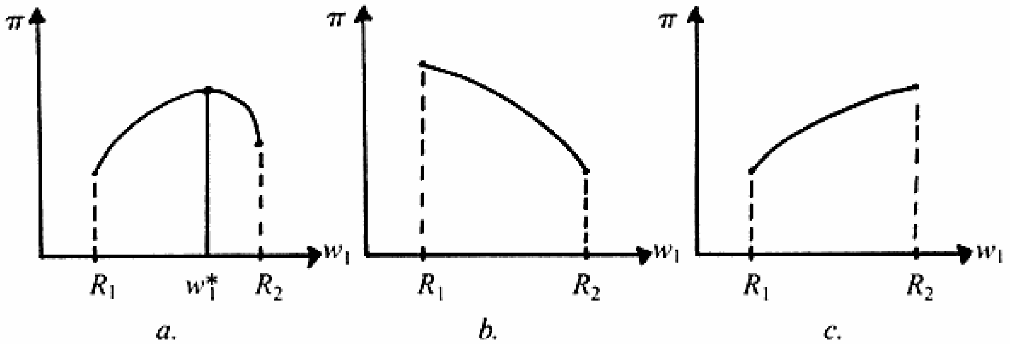
$$r(s) = R_1 \text{ and } w_2 = R_2. \tag{3}$$

This is so since if e.g. $R_2 > r(s) > R_1$, then the labour supply at the low wage is $1-A$. From Figure 2 it is clear that the same labour supply can be achieved also if $w_1$ is lowered a bit. So, $r(s) > R_1$ cannot be profit maximizing. Similarly, if $w_2 > R_2$, then the high and the low wage can be lowered slightly such that $r(s)$ is unchanged and such that the new high wage $w_2' \geq R_2$ and labour supply is unchanged. If either $r(s) < R_1$ or $w_2 < R_2$, then there is at most employment at one wage. Consequently, when we look for profit maximizing $s$, it suffices to consider $s \in S$ such that (3) is fulfilled. Using (2) we see that for such $s$, $(1-\lambda) = \frac{U(w_1-R_1)}{U(R_1-R_2)}$, and therefore for such $s$ we can write the profit as a function of $w_1$ alone $(w_1 \in [R_1, R_2])$ as follows:

$$\pi(w_1) \equiv T \cdot ((1-A) + \frac{U(w_1-R_1)}{U(R_1-R_2)}A) - w_1(1-A) - R_2 \frac{U(w_1-R_1)}{U(R_1-R_2)}A \tag{4}$$

$$= (T-w_1)(1-A) + (T-R_2) \frac{U(w_1-R_1)}{U(R_1-R_2)}A.$$

If $w_1 = R_1$, then $1-\lambda = \frac{U(R_1-R_1)}{U(R_1-R_2)} = 0$, so the firm buys no labour at the high wage $R_2$, and $\pi(R_1) = T(1-A) - R_1(1-A)$ which is equal to the profit the firm gets if it sets only one wage equal to $w_1$. Correspondingly, if $w_1 = R_2$, then $w_1 = w_2 = R_2$ and $\pi(R_2) = T - R_2$, the profit when the firm sets one wage equal to $R_2$. In these two




*Figur 3.a-c.*

cases the firm behaves as a *normal monopsonist.* In all other cases, i.e., when $R_1 = r(s) < w_1 < w_2 = R_2$, and therefore $0 < \lambda < 1$, the firm wage discriminates and offers unequal pay for equal work.

When $w_1$ equals $R_1$ or $R_2$ the profit function $\pi$ gives the profit of the normal monopsonist, and $\pi$'s maximum over $[R_1, R_2]$ coinsides with the profit of the optimal $s$. Therefore, to find the optimal $s$, call it $s^*$, we just need to find $w_1{}^*$ that maximizes $\pi$ and then let $s^* = (w_1{}^*, w_2{}^*, \lambda^*) = (w_1{}^*, R_2, 1 - \frac{U(w_1-R_1)}{U(R_1-R_2)})$. If $w_1$ equals $R_1$ or $R_2$, then the firm behaves as a normal monopsonist, otherwise it wage discriminates.

By differentiating $\pi$ we get

$$\pi'(w_1) = (T-R_2)\,\frac{U'(w_1-R_1)}{U(R_1-R_2)} - (1-A), \tag{5}$$

and differentiating once more gives

$$\pi''(w) = (T-R_2)\,\frac{U''(w_1-R_1)}{U(R_1-R_2)} < 0. \tag{6}$$

Due to the strict risk aversion, $\pi$ is strictly concave, and therefore a necessary and sufficient condition for unequal pay for equal work is that the profit increases if we move $w_1$ slightly away from $R_1$ and $R_2$ respectively, that is if

$$\pi'(R_1) > 0 > \pi'(R_2). \qquad (7)$$

This is also clear from Figure 3 a-c above. Figure 3 a depicts the unequal pay for equal work case, Figures 3 b-c depict either of the two "normal" cases.

From (5) and (7) we now get

THEOREM 1. *The following is a necessary and sufficient condition for unequal pay for equal work*

$$\frac{U'(R_2-R_1)(R_2-R_1)}{U(R_2-R_1)} < \frac{1-A}{A}\,\frac{R_2-R_1}{T-R_2} < \frac{U'(0)(R_2-R_1)}{U(R_2-R_1)}. \qquad (8)$$

Since $U$ is strictly concave, the left hand side of (8) is less than one while the right hand side is greater than one. For the middle term sufficiently close to one (8) is therefore fulfilled, and unequal pay for equal work is optimal for the firm. The middle term equal to one is equivalent to $1-A = \frac{T-R_2}{T-R_1}$, or that (1) is fulfilled with equality. In this case the normal monopsonist is indifferent between posing $w = R_1$ or $w = R_2$. That unequal pay for equal work is optimal in this case can also be seen from Figure 3 taking the concavity of $\pi$ into consideration.

Notice that (8) cannot be fulfilled if $U$ is not strictly concave, i.e., if there is not strict risk aversion. In the absence of strict risk aversion, it will never be strictly profit improving for the firm to wage discriminate. This is so even though the basic self selection mechanism also works when workers are risk neutral as was demonstrated in Section 4. The wage discrimination increases profits when the firm can exploit the risk aversion of the workers. In this case the firm manages profitably to employ workers at a low wage and a high wage simultaneously because the prospect of getting unemployed is too frightening for the low reservation wage workers.

When there is unequal pay for equal work $r(s) = R_1 < w_1 < w_2 = R_2$ and $0 < \lambda < 1$. Employment is $(1-A)+(1-\lambda)A$, which is greater than $(1-A)$ and less than 1. The condition for unequal pay for equal work (8) is fulfilled when the middle term is *sufficiently* close to one. Thus (8) may be fulfilled both when $(1-A) > \frac{T-R_2}{T-R_1}$ and when $(1-A) < \frac{T-R_1}{T-R_2}$. In the first case the normal monopsonist employs $1-A$ workers at wage $R_1$. Hence, in this case unequal pay for equal work weakly Pareto dominates equal pay for equal work. Low reservation wage workers get higher wage, high reservation wage workers are indifferent, and the firm earns more profit. Further, employment is higher. In the second case the normal monopsonist employs all workers at $R_2$. In this case unequal pay for equal work is worse for low reservation wage workers, equally good for high reservation wage workers, better for the firm and gives less employment than equal pay for equal work. So, for workers alone it is weakly Pareto inferior. To summarize

THEOREM 2. *There are cases where unequal pay for equal work weakly Pareto dominates, and gives higher employment than equal pay for equal work. There are also cases where unequal pay for equal work is weakly Pareto inferior for workers, better for the firm and gives less employment than equal pay for equal work.*

In conclusion, the welfare effects of unequal pay for equal work depends on the exact configuration of the parameters of the model. Nothing can be said in general. It is interesting, though, that unequal pay for equal work may very well be Pareto improving.

### 6. Conclusion

In this paper we have explained the simultaneous existence of low paid workers with guaranteed employment and high paid workers with uncertain employment, even though the workers are equally productive. The basic explanation offered is that such a pair of contracts enable firms to wage discriminate even though only workers know their reservation wages. The trick is that low reservation wage workers are too scared by the possibility of unemployment to go for the high wage, they select the low wage themselves. The welfare consequences of such wage discrimination is, as we saw, ambiguous, so there is no clear cut case for government intervention.

As also touched upon in the introduction, it would be interesting to study these matters in a model with revenue uncertainty. This is done in Jacobsen and Schultz (1991). The most important new result of that analysis compared to the one performed here is that uncertainty with respect to revenue implies that the firm always - regardless of parameter values - will want to use two contracts. Furthermore, the contract with high wages always entails uncertainty for workers either in the form of income uncertainty of employment uncertainty.

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