On the Empirical Verification of the Purchasing Power Parity and the Uncovered Interest Rate Parity

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SUMMARY: The long-run foreign transmission effects are first analyzed in a system of Danish and German prices, exchange rates and interest rates. The results give empirical evidence of purchasing power parity between the countries as well as interest rate parity. The adjustment coefficients indicate that the prices do adjust although slowly but that the adjustment is primarily in the exchange rates. The deviations from these parities are used as determinants in the final econometric analysis of the determination of exchange rates, prices and interest rates in Denmark. The final model provides empirical evidence on the strong dependence of the Danish economy on the West German prices and interest rates in particular after 1983 with the deregulation of capital movements.

1. Introduction

The determination of real exchange rates has been in the focus of interest in a vast number of empirically oriented papers, see for instance Adler and Lehman (1983), Baillie and Selover (1987), Corbae and Ouliaris (1988) and Schotman (1989). In Denmark a recent contribution is Paldam (1988). A common feature of basically all empirical papers is the rejection of standard economic hypotheses. The final breakdown of the Bretton Woods system in 1972, with the suspension of the convertibility into gold, led to the float of the dollar and some other currencies. In Europe several countries (the group of 10) including Denmark decided to manage their exchange rates within the so-called snake which later was followed by the EMS. The new more flexible system was generally expected to solve previous policy conflicts between maintaining either the external or the internal balance of the economy. In reality most economists were taken by surprise by the great volatility of the exchange rates. Today, approximately two decades after Bretton Woods, one dare say that there is no real understanding why exchange rates behave as they do.

In this short paper we will discuss the possibility that previous shortcomings can to some degree be explained by "missing econometrics" and then illustrate the basic points using an analysis of the purchasing power parity (\textit{ppp}) and the uncovered interest rate parity (\textit{uip}) between Denmark and Germany.
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1. Introduction
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In this short paper we will discuss the possibility that previous shortcomings can to some degree be explained by “missing econometrics” and then illustrate the basic points using an analysis of the purchasing power parity (ppp) and the uncovered interest rate parity (uip) between Denmark and Germany.
2. Economic background and the choice of econometric method

In an open economy the international link between the goods and the capital markets can be found in the determination of the exchange rates. The international transmission effects are usually analyzed in the goods market by assuming adjustment to purchasing power parity and in the capital markets by assuming market clearing based on uncovered interest rate parity.

The theoretical motivation for the **PPP** is based on the assumption that internationally produced goods are perfect substitutes for domestic goods. As a description of reality this is clearly an oversimplification and the empirical verification of this hypothesis has generally been very poor (Dornbusch, 1988). There are several reasons for this, among the more important can be mentioned, differences in tastes, technology, the relative importance of the tradable and nontradable sector between countries. These arguments imply that if the **PPP** mechanism is functioning at all, it must only be in a long-run perspective, i.e. as a tendency of the two markets to adjust toward this parity. Empirical evidence on **PPP** is therefore most likely to be found between countries of similar technological development with strong trade associations. These conditions seems to be approximately fulfilled for Denmark and West Germany. In fact, Schotman (1989) investigated the **PPP** hypothesis for 10 major OECD countries and found that these could be clustered into three groups of trading countries within which evidence of the **PPP** seemed to be supported by the data.

The **UIP** hypothesis states that the interest rate differential between two countries is equal to the expected change in the exchange rates. The empirical verification of the uncovered interest rate parity as a market clearing mechanism has neither not been convincing. Because of this many authors have tried to explain the lack of fit with the existence of a risk premium. For instance Fisher et.al. (1990) found empirical support for the parity between UK treasury bill rate and Eurodollar rate when adding a risk premium proxied by the balance of payments deficit relative to GNP.

But another possible explanation to the poor empirical support for both the **UIP** and the **PPP** hypothesis might be found in "inadequate econometrics". It seems like most empirical works have failed to properly account for (i) the time-series properties of the data, (ii) the interaction between the goods and the capital markets and (iii) different short-and long-run effects. These three points will be discussed below.

Most empirical works have not given the time-series properties of the data sufficient attention. This means that valuable information in the data has not been used as efficiently as possible. For instance, recent advances in the theory of nonstationary time-series have provided a powerful tool to discriminate between on one hand stationary and on the other hand nonstationary processes of different order and to determine those directions in which the nonstationary variables can be made stationary by linear combinations. These stationary combinations determine the so-called cointegration relations,
among which hypotheses about hypothetical long-run relations can be tested. Failure to account for these important time-series properties of the data is likely to invalidate the statistical inference.

The two parties have usually been investigated as isolated phenomena either in the goods or the capital market, without paying enough attention to the interactions between the two. As a consequence, most empirical work on the ppp and the uip hypothesis has been done in a single equation framework and usually without reference to the rest of the economy. Under the assumption that the ppp is only valid in the long run, adjustment can a priori take place both in the prices of either country and in the exchange rates. In addition one can also expect information about ppp to be found in the interest rate equations. The motivation is the following. Under assumption of sticky prices deviations from ppp are likely to have some persistence. This in turn will result in deficits in the trade balance, which will affect the capital account and hence lead to a pressure on the interest rates. Therefore one cannot exclude the possibility that interest rates react to deviations from the ppp. This means that information about the ppp and the uip can be found in the price equations, interest equations as well as in the exchange rate equation. Therefore single equation estimates are likely to be both biased and inefficient, and the ppp and the uip should consequently be analyzed in a system of equations.

Many studies derive the empirical model directly from a very restrictive hypothetical economic model. If in reality there are costly arbitrage, imperfect information, costly adjustment, etc., the short-run behaviour can be very different from the long-run. In such cases one should be careful not to start with a two restrictive model, since that could easily deteriorate any inference on the how the markets work. In a macro-economic perspective prices and interest rates are determined by the interplay of forces driving the domestic and international markets. Both effects have to be accounted for in order to obtain a full understanding how these important variables react to different shocks in the economy. If the adjustment to ppp is very slow, as is indeed very likely, the relative weight of this term compared to other short-run effects is probably small. Moreover, if the short-run effects are basically different from the long-run effects, the explicit specification of the former becomes mandatory, even if the main empirical interest is in the long-run relations.

The methodological implication of all this is that one should be careful with the usual ceteris paribus assumptions. The fact that economic theory is seldom unambiguous, or never omnipotent w.r.t. the economic reality, is the econometric motivation for starting with a general model formulation. This leads us to formulate a model that does not directly impose the uip and/or the ppp restrictions but, more indirectly, assumes a tendency in the market to react to deviations from these relations. These considerations motivate a model specification of the error-correction type (for an extended discussion, see Juselius, 1991a):
Prices
\[ \Delta p_{1t} = f_{p1}(\Delta X) + \gamma_1(p_1-p_2-e_{12})_{t-1} + \gamma_2(i_1-i_2)_{t-1} + \epsilon_{p1t} \]  
\[ \Delta p_{2t} = f_{p2}(\Delta X) + \gamma_3(p_1-p_2-e_{12})_{t-1} + \gamma_4(i_1-i_2)_{t-1} + \epsilon_{p2t} \]  
(1) (2)

Exchange rates:
\[ \Delta e_{12t} = f_{e}(\Delta X) + \gamma_5(p_1-p_2-e_{12})_{t-1} + \gamma_6(i_1-i_2)_{t-1} + \epsilon_{e_{12t}} \] 
(3)

Interest rates:
\[ \Delta i_{1t} = f_{i1}(\Delta X) + \gamma_7(p_1-p_2-e_{12})_{t-1} + \gamma_8(i_1-i_2)_{t-1} + \epsilon_{i1t} \]  
\[ \Delta i_{2t} = f_{i2}(\Delta X) + \gamma_9(p_1-p_2-e_{12})_{t-1} + \gamma_{10}(i_1-i_2)_{t-1} + \epsilon_{i2t} \]  
(4) (5)

where \( p \) denotes prices, \( e \) denotes exchange rates, \( i \) denotes interest rates, \( a \) subscript '1' and '2' denotes Denmark and West Germany respectively, \( f_m(\Delta X) \) indicates a linear function of \( \Delta X \), \( \Delta X_{m1} \), ..., where \( \Delta X_m \) is excluded and \( m = p_1, p_2, e_{12}, i_1, i_2 \).

In the above formulation the backward adjustment to deviations from the ppp is described by the ppp error-correction mechanism:
\[ ecm_{ppp} = (p_1-p_2-e_{12}) \]  
(6)

whereas the adjustment to the uncovered interest rate parity is not as straightforward in the above formulation and therefore requires some comments. If this parity holds exactly we would have that \((i_1-i_2)_{t} = (\Delta e_{12})_{t+1} \), where superscript \( e \) denotes expectation. If future exchange rates are difficult to predict, one would expect the market to adjust in a backward looking manner to the realized deviation from this parity, namely to the uip error-correction mechanism:
\[ (ecm_{uip})_{t-1} = (i_1-i_2)_{t-1} - (\Delta e_{12})_{t}. \]  
(7)

In the above model formulation no restrictions on the coefficients of the short-run dynamics, \( f_m(\Delta X) \), are made. In particular, the coefficient of \( \Delta e_{12} \), is not restricted according to the \( ecm_{uip} \) hypothesis. The motivation for this is that the inclusion of \( \Delta e_{12} \), unrestrictedly in \( f_m(\Delta X) \) allows exchange rates to affect interest rates and prices also through other channels than the uip. Thus in the above form the model allows for other explanations in the short run but presumes ppp and uip to determine the long-run steady-state solution.
3. The statistical model

It was shown in Granger (1983) and Engle and Granger (1987) that an error-correction model implies cointegration and vice versa. The multivariate cointegration model seems therefore to be a natural choice of statistical model for our empirical problem. This means that we can utilize the statistical distinction between stationary and nonstationary processes to analyze whether there exist stationary linear relations between the levels of the nonstationary variables (Johansen (1988), Johansen & Juselius (1990)), and if this is the case whether the unrestricted relations are consistent with the hypothetical economic long-run relations. The empirical multivariate cointegration model discussed below is given by:

\[ \Delta z_i = \Gamma_1 \Delta z_{i-1} + \Pi z_{i-1} + \mu + \psi D_t + \epsilon_i, \; t = 1973:1,...,1987:3 \]  

(8)

where \(z_i = [p_1, p_2, e_{12}, i_1, i_2] \) is defined above, \(z_0\) and \(z_0\) are fixed, \(\epsilon_1,...,\epsilon_T\) are \(i.i.d\) \(N_p(0, \Sigma)\) and \(D_t\) are centered seasonal dummies. All long-run information is contained in the levels component \(\Pi z_{i-1}\). Under the hypothesis that \(z_i\) is \(I(1)\) and \(\epsilon_i\) is stationary it follows that the matrix \(\Pi\) cannot be of full rank, since a stationary process \(\Delta z_i\) cannot equal something nonstationary. If on the other hand \(\Pi = 0\) the model is empirically consistent but there is no long-run information in the data and the model would be reduced to a standard \(VAR\)-model in differences. The third possibility is that \(\Pi\) is of \textit{reduced rank} which is the hypothesis of cointegration formulated as \(\Pi = \alpha \beta'\) where \(\alpha\) and \(\beta\) are \(p \times r\) matrices. In this case \(\beta'z_i\) defines the cointegration relations, which often can be given an interpretation as economic long-run relations, whereas \(\alpha\) defines the weights with which these enter each of the equations \(\Delta z_i\). The properties of the \(I(1)\) model can be summarized by noting that \(\Delta z_i\) is stationary, \(z_i\) is nonstationary but \(\beta'z_i\) is stationary.

If however the (the appropriately transformed) matrix \(\Gamma_1\) is also of reduced rank the vector process \(\{z_i\}\) is second order nonstationary, implying that some (or all) of the variables \(z_i\) are integrated of order two (see Johansen, 1992b). In this case \(\beta'z_i\) is generally integrated of order one, although under certain conditions it is possible to divide the \(r\) cointegrating relations into \(r-s\) stationary relations and \(s\) first order nonstationary relations, where \(s\) is the number of common trends integrated of order two in the system (for applications see Johansen, 1992c and Juselius, 1992b).

4. An empirical illustration

We have chosen to illustrate the ideas discussed above with a data set that describes how Danish consumer prices, interest rates and exchange rates are affected by what happens to German prices and interest rates. The empirical analysis can be seen as a complement and an extension to Juselius (1991a) where the data are explained in more
detail. Similar analyses have also been performed on other data sets, for instance on UK data in Johansen and Juselius (1992) and Johansen (1992c).

The econometric procedure takes place in two steps: In the first step the long-run structure of the system is estimated and a priori hypotheses concerning long-run steady-states are tested. In the second step the deviations from these steady-states are used as determinants in the analysis of the short-run structure.

4.1 The cointegration analysis

The multivariate cointegration analysis of this data set is thoroughly discussed in Juselius (1991a) and we will here only discuss the \( I(2) \) extension. The statistical analysis of (8) indicated that there are three cointegration relations and consequently two common trends. When applying the recently developed \( I(2) \) procedure (Johansen, 1992b) it was found that the process was integrated of order 2 and consequently that some of the variables are approximately integrated of order two. An extended discussion of the \( I(2) \) analysis is however not possible within the present page restrictions. Therefore we will only informally discuss the additional information given by the \( I(2) \) analysis. The interested reader is referred to Johansen (1992c) and Juselius (1992b).

The analysis revealed that the Danish and German consumer price variables could be classified as second order nonstationary processes, whereas the exchange rates and the interest rates were first order nonstationary. However the price differential \( p_1 - p_2 \) could be considered a first order nonstationary process, implying that prices are cointegrating from \( I(2) \) to \( I(1) \). To illustrate the time series properties of the individual series they have been arranged below on a scale measuring the degree of integratedness:

\[
\begin{align*}
I(2): & \{p_1\}, \{p_1\} \\
I(1): & \{e_{i1}\}, \{p_1 - p_2\}, \{i_1\}, \{i_2\}, \{\Delta p_1\}, \{\Delta p_2\} \\
I(0): & \{\Delta e_{i1}\}, \{\Delta i_1\}, \{\Delta i_2\}, \{\Delta^2 p_1\}, \{\Delta^2 p_2\}
\end{align*}
\]

where \( \{ \cdot \} \) denotes a time-series process. Since only the two price series are \( I(2) \) one could of course do the cointegration analysis by replacing price levels with inflation rates in the vector \( z_t \). This however would imply that some valuable information contained in the levels would be lost and is therefore not a recommendable procedure. In table 1 below the three cointegration vectors from Juselius (1991a) are partitioned into the \( I(0) \) and \( I(1) \) space.

The two stationary vectors \( \hat{\beta}^0_i, i=1,2 \), suggests that the real exchange rate, \( \{p_1 - p_2 - e_{i2}\} \) and the interest rate differential, \( \{\hat{i}_1 - i_2\} \) are stationary processes. This can be seen by taking appropriate linear combinations of the first two vectors, using the result that linear transformations of stationary processes preserve the stationarity property. The third vector, which is first order nonstationary, is essentially a weighted ave-
Table 1. The cointegration vectors decomposed into the I(0) and I(1) directions.

<table>
<thead>
<tr>
<th>I(0) space</th>
<th>I(1) space</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1^0$</td>
<td>$\beta_2^0$</td>
</tr>
<tr>
<td>-3.60</td>
<td>1.000</td>
</tr>
<tr>
<td>.171</td>
<td>-1.188</td>
</tr>
<tr>
<td>.630</td>
<td>-9.35</td>
</tr>
<tr>
<td>1.080</td>
<td>.382</td>
</tr>
<tr>
<td>-1.435</td>
<td>-2.83</td>
</tr>
</tbody>
</table>

rage of the Danish and the German interest rate level, which indeed should be I(1). As shown in Johansen (1992b) the nonstationary $\beta$-vector can be brought down to stationarity by a linear combination, $\omega$, of the differenced process, i.e. $\{\beta_1'z_t + \omega^t\Delta z_t\} \sim I(0)$. As can be seen from Table 1 column $\omega$, the two inflation rates are primarily needed to obtain stationarity of $\beta_1'z_t$. This is intuitively an appealing result since it says that although nominal interest rates behave in nonstationary manner, real interest rates are in fact stationary.

Before moving to the analysis of the short-run effects the hypothesis that the real exchange rate and the interest rate differential are contained in the cointegration space will be formally tested as the hypothesis:

$$\beta = \{H\rho, \psi\} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} & \psi_1 \\ -\rho_{11} & -\rho_{12} & \psi_2 \\ -\rho_{11} & -\rho_{12} & \psi_3 \\ \rho_{21} & \rho_{22} & \psi_4 \\ -\rho_{21} & -\rho_{22} & \psi_5 \end{bmatrix},$$

where $\rho$ is a $2 \times 2$ matrix and $\psi$ is a $5 \times 1$ vector of unrestricted coefficients. This hypothesis can be tested by a likelihood ratio test procedure described in Johansen and Juselius (1992). The test statistic became 1.70 and is asymptotically distributed as $\chi^2(4)$. Therefore the hypothesis was clearly accepted. In table 2 the restricted estimates are given.

The first vector is essentially describing the interest rate differential, the second vector the real exchange rate combined with the interest rate differential, and the third vector the nominal interest rate level given by the sum of two interest rates.

In the next section we will use the cointegration results from the restricted analysis to estimate the short-run structure of the equation system (1)-(5) discussed in section 2. But since the German price and interest rates can be considered weakly exogenous
Table 2. The estimated cointegration vectors under restriction $\beta = \{H\rho, \psi\}$.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1 = H\rho_1$</th>
<th>$\beta_2 = H\rho_2$</th>
<th>$\beta_3 = \psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>0.022</td>
<td>1.000</td>
<td>0.019</td>
</tr>
<tr>
<td>$p_2$</td>
<td>-0.22</td>
<td>-1.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>$\epsilon_{12}$</td>
<td>-0.22</td>
<td>-1.000</td>
<td>-0.018</td>
</tr>
<tr>
<td>$i_1$</td>
<td>1.000</td>
<td>-1.304</td>
<td>1.000</td>
</tr>
<tr>
<td>$i_2$</td>
<td>-1.000</td>
<td>1.304</td>
<td>1.000</td>
</tr>
</tbody>
</table>

w.r.t. the Danish economy (for a definition of weak exogeneity see Engle, Hendry and Richard, 1983 and Johansen, 1992a) we will only present the results for the three Danish equations.

4.2 The short-run structure
The procedure we have followed here is to use the deviations from the long-run parities as determinants to the short-run behaviour of exchange rates, interest rates and price inflation. The short-run effects are estimated using a “general to specific” data based procedure. In the first place the unrestricted short-run structure as given by (1) – (5) was estimated. By restricting insignificant coefficients to zero and imposing other data consistent restrictions we arrived at the empirical models presented in Table 3.

We will now comment on the three equations in turn, starting with the model for the exchange rates which is considered to be the most interesting of this study.

The exchange rate equation:
The estimated coefficients to the interest rate differential and the real exchange rate do seem to indicate that nominal exchange rates adjust to the assumed equilibrium states in both the capital and the goods market. As expected the speed of adjustment is much faster to the interest rate differential than to the ppp. The interrelatedness of the two markets can be noticed from the fact that the adjustment to the ppp is prolonged when the interest rate differential is positive. This could be consistent with the fact that membership of the snake and presently the EMS has placed restrictions on the magnitude of changes in the exchange rates. If prices are sticky, as is usually assumed, and the adjustment in them is not fast enough, interest rates have to perform some of the “work” to restore the external balance.

In addition to the two steady-state adjustment terms, nominal exchange rates seem to react quite strongly to short-run changes in the basic determinants. It is interesting to notice that a positive change in German real interest rate and a positive rate of change in the domestic interest rate both seem to lead to a depreciation in the short run. The estimated coefficients indicate however that the short-run effects are quite different from the long-run effects, and therefore illustrate the third point discussed in section 2.
Table 3. The estimated short-run structure for $\Delta e_{12t} \cdot \Delta i_{1t} \cdot \Delta p_{1t}$.

$\Delta e_{12t} = 2.62 \Delta e_{12t-1} + 2.29 \Delta^2 i_{1t} + 1.69 \Delta i_{2t-1} - 1.59 \Delta^2 p_{2t} + .70 (i_{1t-2}/ i_{t-1}) + .11 (p_{1t-2}/ p_{2t}) - 1.30 (i_{1t-2}/ i_{t-1}) + \mu + \phi D_t + \varepsilon_{e_t}$

$\sigma^2_e = .013 \quad Q(21) = 15.9$

$\Delta i_{1t} = .05 \Delta e_{12t} + .46 \Delta i_{1t-1} + .19 \Delta i_{2t} + .49 \Delta^2 p_{2t} - 10 (i_{1t-2}/ i_{t-1}) + \mu + \phi D_t + \varepsilon_{i_t}$

$\sigma^2_e = .0020 \quad Q(21) = 19.2$

$\Delta p_{1t} = 1.27 \Delta i_{1t} + .43 \Delta p_{2t} - .72 (p_{1t-2}/ p_{2t}) + .36 \Delta i_{1t-1} + \mu + \phi D_t + \varepsilon_{p_t}$

$\sigma^2_e = .0085 \quad Q(21) = 28.1$

$\hat{\sigma}$ is the estimated residual standard deviation, $Q(21)$ is the Box-Pierce test for uncorrelated residuals distributed as $\chi^2(21)$.

namely the importance of allowing for a general specification, especially when short-run and long-run effects are very different.

The interest rate equation:

In this case only the adjustment coefficient to the interest rate differential became significant. Combined with the significant coefficient to the current change in the exchange rates, this is clearly an indication that interest rates adjust to a perceived discrepancy of the uncovered interest rate parity. This can easily been seen by noting that $0.05 \Delta e_{12t} - 10 (i_{1t-2}/ i_{t-1}) = - 0.05 \Delta e_{12t} - 10 (i_{1t-2}/ i_{t-1} - \Delta e_{12t})$. Also in this case there are evidence of different short-run effects. The strong dependence of the German economy can be inferred from the fact that the Danish bond rate reacts primarily to changes in German interest rate and inflation rate.

The inflation equations:

Consistent with the assumption of sticky prices, the coefficient to the $ppp$ is quite small but anyway significant, indicating that prices do adjust although slowly. The two interest rates in levels were included unrestrictedly in the first general model, but only the Danish rate was significant. On the whole, Danish inflation rate seems to follow the German inflation rate and reacts quite strongly to changes in the German interest rate. It adjusts to the $ppp$ and is positively related to the level of the Danish bond rate. This is broadly consistent with the findings in Juselius (1992a) where a larger set of determinants were used in a more elaborate analysis of price inflation.
References