An investigation of random walks in the Danish stock market

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1. Introduction

According to the random walk hypothesis, succesive stock market price changes are identically distributed and independent random variables. If that is so, then stock prices follow a random walk. In a stock market satisfying the random walk hypothesis, previous price changes provide no information about future price changes which can be utilized to construct profitable trading rules. By a profitable trading rule is meant here one yielding a return above what can be obtained through a naive buy-and-hold investment strategy. If the random walk hypothesis holds true, then that does not imply that trading on the stock exchange is necessarily unprofitable. However, if a profitable trading system is to be constructed, it must be founded on information other than past price changes.

Even if the random walk hypothesis does not hold true, there may nevertheless exist no profitable trading systems based on historical price changes. That is, the random walk hypothesis poses a sufficient, but not necessary, condition for the absence of such trading systems. The reason is, loosely speaking, that the deviations from randomness may be so small that it is not worthwhile to attempt to exploit them, given the transaction costs involved. Nevertheless, in investigating price formation in a particular stock market, it is customary to start out with an exploration of the validity of the random walk hypothesis.

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The random walk hypothesis became fashionable during the 1960's, largely due to the work of Fama (1965). Fama investigated stock price behavior on the New York Stock Exchange and found impressive evidence in favor of the random walk hypothesis. As a consequence, the random walk hypothesis became widely accepted for the US. For other capital markets, the situation is not so clear. Dryden (1970) replicated Fama's tests, using data from the London Stock Exchange. He reported certain deviations from randomness. Reiss (1974) rejected the random walk hypothesis for his sample of 50 German stocks. Jennergren and Korsvold (1974) investigated stock price behavior on the Oslo and Stockholm Stock Exchanges; they also found large deviations from randomness.

As a consequence of studies such as Reiss (1974), and Jennergren and Korsvold (1974), the random walk hypothesis seems to have suffered a decline in popularity. There seems to be some agreement, in the case of small capital markets at least, that the random walk hypothesis is not universally an accurate description of stock price formation.

This paper investigates the validity of the random walk hypothesis for the Danish stock market. The outline is as follows: The next section (Section 2) will give some information about the Copenhagen Stock Exchange and about the data set utilized. Section 3 considers briefly the empirical distributions of share price differences. Section 4 gives results from runs tests of the independence of successive share price changes. Apart from the runs tests, we also use spectral analysis to study the independence of successive price changes. Since the spectral analysis methodology may be unfamiliar to some readers, we present a little of the underlying theory in Sections 5 and 6. Section 7 discusses spectral analysis results. Section 8 contains concluding remarks.

2. The Data Set

The data set used for this investigation consists of daily closing prices for 15 stocks traded on the Copenhagen Stock Exchange, Jan. 1, 1973 – Dec. 31, 1975. This exchange is the only one in Denmark. It is small by international comparison. Total turnover in stocks was around 330 million Danish crowns in 1975 (nominal values). Actually, bond trading is considerably more important than stock trading on the Copenhagen Exchange (total nominal turnover in bonds was around 12 billion crowns in 1975). Approximately 300 stock and 1500 bond issues are listed on the exchange. Buyers and sellers on the Copenhagen Stock Exchange are representatives of around 25 brokerage firms. Trading is mainly by auctioning.

The fifteen stocks selected do not constitute a random sample. Rather, they were picked simply because they are among the most actively traded ones on the Copenhagen Stock Exchange. Their names are listed in Table 1, column A. Table 1 also gives the number of transaction days for each stock (column C). A transaction day is a day when there is at least one transaction in the given stock under consideration. Altogether, the Stock Exchange was open 750 days during 1973-1975. It is seen from Table 1, column C, that none of the fifteen stocks was traded every single day out of the 750. That is, in all of the fifteen series there are gaps due to a lack of transactions on certain days. This gap phenomenon also occurred for the Norwegian and Swedish stock price series used by Jennergren and Korsvold (1974). It is a reflection of the smallness of the Danish stock market.

TABLE 1. *Number of Transaction Days, Empirical Distributions, and Runs Tests.*

A Stock Name	B Stock Number	C Number of Transaction Days	D b_1	E Right Skew (RS) or Left Skew (LS)?	F b_2	G K
Privatbanken	1	503	2.48	RS	17.26	−7.42
Landmandsbanken	2	640	0.14	RS	9.62	−4.42
Handelsbanken	3	582	0.33	RS	9.77	−7.48
St. Nordiske Telegraf	4	643	0.005	RS	6.26	−4.92
Dansk Trælast Co.	5	569	0.018	LS	5.05	−4.06
Korn og Foderstof, gl	6	556	8.46	RS	39.01	−3.84
Wessel & Wett, præf	7	578	0.042	RS	5.81	−4.83
ØK	8	707	0.011	RS	9.28	−2.36
DFDS	9	617	0.149	LS	8.84	−5.85
Danske Sukkerfabrikker	10	702	0.151	LS	10.57	−6.51
Forenede Bryggerier, ord	11	505	0.002	LS	6.24	−3.68
Forenede Papirfabrikker	12	712	1.23	LS	13.16	−2.93
Nord. Kabel- og Traad	13	651	0.131	LS	6.65	−6.70
F. L. Smidt og Co., B	14	650	0.012	RS	5.48	−4.55
Superfos, ord	15	659	0.62	LS	11.99	−6.25

In random walk investigations, there is a preliminary question whether it is stock price differences ($p_t - p_{t-1}$) or differences in the logarithm of prices ($\log p_t - \log p_{t-1}$) that should be used. That is, is it stock prices or their logarithms that obey the random walk? In this study, we will be mainly concerned with differences in the logarithm of prices (henceforth called: log price differences). One reason for this is that $\log p_t - \log p_{t-1}$ represents the yield, with continuous compounding, from holding the stock during day t. Log price differences have, in fact, been used in most earlier studies (for instance, Fama (1965), Dryden (1970), Jennergren and Korsvold (1974), and Reiss (1974)). It may be remarked that for the runs tests reported in Section 4, it does not matter whether $p_t - p_{t-1}$ og $\log p_t - \log p_{t-1}$ is used, since only signs, not magnitudes, are involved. Prior to testing, the price series were adjusted for dividends, stock dividends, and rights issues. These adjustments were carried out in the standard fashion; see, e.g., Fama (1965, p. 46).

To illustrate the nature of the data set, Figure 1 a gives a plot of the daily closing prices of stock Nr. 9, DFDS, during 30.12.1974 – 26.12.1975. It may be remarked here that for this stock, there were no dividends, stock dividends, or rights issues during 1973–1975, so no adjustments were necessary. It should also be noted that gaps in the data series due to a lack of transactions during certain trading days or due to holidays during the week

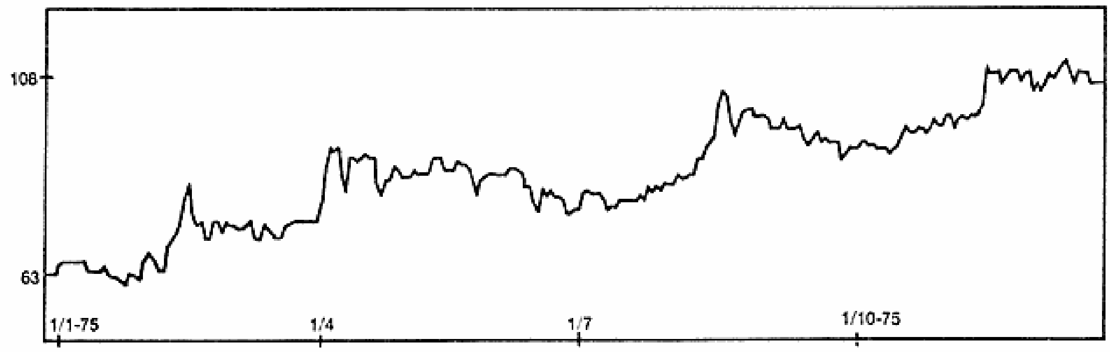


FIGURE 1*a*. *DFDS, time trace of daily closing prices. 30.12.1974 – 26.12.1975.*

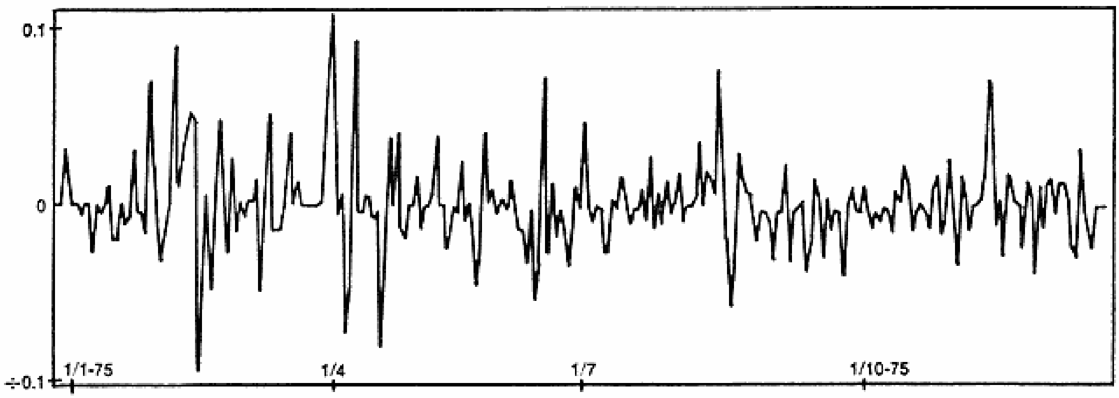


FIGURE 1*b*. *DFDS, time trace of log price differences of daily closing prices. 31.12.1974 – 26.12.1975.*

(Monday through Friday) have been filled by inserting the last previous actually observed closing prices. The reason for this gap filling will be discussed later (Section 6). Figure 1 b gives a plot of the successive log price differences of the series exhibited in Figure 1 a.

3. Empirical Distributions

This section discusses the empirical distributions of successive log stock price changes. Gaps in the original data series are ignored. That is, holidays (when the Stock Exchange was closed) and trading days when there were no transactions are simply discarded from the data set. One then obtains fifteen series of successive log price differences, each of a different length (since the number of transaction days during 1973–1975 for the fifteen stocks is different in each case).

The hypothesis was earlier advanced that successive log stock price changes would be normally distributed. This hypothesis has, however, been rejected by several authors; see, e.g., Fama (1965), Dryden (1970), and Jennergren and Korsvold (1974). The empirical distributions reported by these authors have typically been leptokurtic (i.e., too many observations near the mean and too fat tails). In order to permit a comparison of the Danish empirical distributions with the normal, the third and fourth normalized moments have been computed. The r^{th} (un-normalized) moment is defined as

$$m_r = \frac{1}{N} \Sigma_{i=1}^{N} (x_i - \bar{x})^r.$$

The third normalized moment b_1 indicates skewness. It is defined as

$$b_1 = (m_3)^2 / (m_2)^3.$$

For a normal distribution $N(0, 1)$, $b_1 = 0$. If $m_3 > 0$, the distribution is right skew. If $m_3 < 0$, it is left skew. The fourth normalized moment b_2 is called kurtosis and indicates peakedness. It is defined as

$$b_2 = (m_4) / (m_2)^2.$$

A normal distribution $N(0, 1)$ has $b_2 = 3$. A distribution with $b_2 > 3$ is hence peaked.

Results are displayed in Table 1, columns D–F. If successive log stock price differences are independent and drawn from the same normal distribution, then $\sqrt{(b_2 - 3)(N/24)}$ is approximately normally distributed $N(0, 1)$, (Praetz (1969), p. 127). By this comparison, it is seen that all fifteen stocks display substantial peakedness. Also, under the same assumptions, $m_3/m_2^{\frac{3}{2}}$ is distributed normally $N(0,1)$ (Kendall and Stuart (1969, p. 297)). One may then conclude from Table 1, column D that some of the Danish stocks exhibit noticeably skew distributions. There is no systematic right or left skewedness. In conclusion, we reject the idea that the Danish empirical distributions of succesive log stock price changes are drawn from an underlying normal process.

4. Runs Tests

The independence of successive price changes for the fifteen Danish stocks has been investigated by means of runs tests. A run is defined as a sequence of consecutive price changes of the same sign. For stock prices, there are three possible types of price changes: plus, zero, or minus. There are hence three types of runs. The runs tests here are non-parametric. They have been used in many earlier random walk studies. They are discussed in greater detail in Wallis and Roberts (1956, pp. 569–572).

The data series are the same as in Section 3, i.e., gaps due to holidays and a lack of transactions on some trading days have been ignored. Under the assumption of independence, the total expected number of runs of all three kinds for a stock can be calculated as

$$E(R) = [N(N+1) - \Sigma_{i=1}^{3} n_i^2] / N$$

where N is the total number of price changes, and the n_i are the numbers of price changes of each kind (plus, zero, or minus). R denotes the actual total number of runs observed. For large N, the sampling distribution of R is approximately normal. The standard error of R is

$$SD(R) = \left(\frac{\Sigma_{i=1}^{3} n_i^2 [\Sigma_{i=1}^{3} n_i^2 + N(N+1)] - 2N\Sigma_{i=1}^{3} n_i^3 - N^3}{N^2(N-1)}\right)^{\frac{1}{2}}$$

For each of the fifteen Danish stocks, the normalized variable

$$K = (R - E(R))/SD(R)$$

has been computed. Values of K are listed in Table 1, column G. It is seen that all fifteen stocks have negative K-values, and fairly large in absolute magnitude. The average K-value is −5.05. The conclusion is that none of the fifteen stocks can be considered to exhibit independence of successive price changes. This implies that the random walk hypothesis is probably not a suitable model of stock price formation in Denmark.

It may be remarked that the results in Table 1, column G, agree with earlier ones reported by Reiss (1974) and Jennergren and Korsvold (1974). All the German stocks in Reiss's sample had negative K-values. The average was -6.78. The Norwegian and Swedish stocks investigated by Jennergren and Korsvold also all had negative K-values. The Norwegian average (fifteen stocks) was -4.69. The Swedish average (30 stocks) was -4.64.

5. Introduction to Spectral Analysis

Let $x(t)$ be a time series of log price differences of a stock. In what follows, we will assume that $x(t)$ is (weakly) stationary, which means the following: For all t, it holds

$$E[x(t)] = \mu \tag{1}$$
$$E[(x(t)-\mu)^2] = \sigma^2 \tag{2}$$
$$E[(x(t)-\mu)(x(t+\tau)-\mu)] = C(\tau) \ (\tau = 1, 2, 3 \ldots) \tag{3}$$

Condition (1) implies that there is no trend. This does not rule out that there could be a trend in the original price series. What is ruled out is that there is a trend in the price changes. Another reason for using $\log p_t - \log p_{t-1}$ rather than $p_t - p_{t-1}$ is to make condition (1) more plausible (cf. Fama (1965), pp. 45-46). Condition (2) states that the variance of the time series does not change over time, i.e., does not depend on t. Condition (3) states that the covariance $C(\tau)$ depends only on the lag τ but not on t. If $C(\tau) = 0$ for $\tau = 1, 2, 3 \ldots$, then the series is often called white noise. $R(\tau) = C(\tau)/\sigma^2$ is the autocorrelation for lag τ. It varies between $+1$ and -1. The ensemble $R(\tau)$, $\tau = 1, 2, 3 \ldots$, is called the correlogram of the series $x(t)$. If successive log stock price changes are independent (as called for by the random walk hypothesis), then $R(\tau)$ is zero for all τ. One way of testing the random walk hypothesis is hence to estimate the correlogram for an observed series of log price differences. This has, in fact, been done in many earlier random walk studies, including all the ones referred to in this article. We have also estimated correlograms for the fifteen Danish series of log stock price changes. However, rather than presenting the estimated autocorrelations directly, we have transformed them, through the use of spectral analysis. In Section 7, we will present results. This section and the next is devoted to a brief explanation of the spectral analysis technique. For more detailed discussions, see Granger and Hatanaka (1964), Jenkins and Watts (1968), or König and Wolters (1972). It may be remarked that spectral analysis has not been applied to a very large extent in earlier random walk studies, although Reiss (1974) is one exception. Spectral analysis is also used prominently in Granger and Morgenstern (1970).

The basic idea of spectral analysis is to decompose the total variance σ^2 of the series $x(t)$ into components. These components are supposed to be attributable to cyclical patterns inherent in the series $x(t)$.

That is, $x(t)$ is viewed as consisting of a set of cyclical patterns superimposed on each other. One such cyclical pattern could, for instance, be a weekly cycle. If it is found that a substantial part of σ^2 can be attributed to a weekly cycle, then that would contradict the random walk hypothesis. The reason is that according to the random walk hypothesis, successive log stock price differences are independent, and that specifically rules out the existence of cyclical patterns of duration one week (or any other duration).

Formally, the spectrum of the series $x(t)$ is defined as

$$g(\lambda) = \frac{1}{2\pi}\sigma^2 + \frac{1}{\pi}\Sigma_{\tau=1}^{\infty} C(\tau)\cos\lambda\tau \qquad (-\pi \leq \lambda \leq \pi). \quad (4)$$

For $0 < \lambda < \pi$, define $f(\lambda) = 2g(\lambda)$. It can be shown that

$$\sigma^2 = \int_0^{\pi} f(\lambda)\, d\lambda. \quad (5)$$

Equation (5) justifies interest in the spectrum. It represents the desired decomposition of the total variance σ^2 into parts, attributable to cyclical patterns in the time series $x(t)$. $f(\lambda)d\lambda$ indicates that part of the total variance that can be attributed to cycles in the frequency band from λ to $\lambda + d\lambda$.

This leaves the question what is to be meant by "the frequency band from λ to $\lambda + d\lambda$". Frequency is defined as the number of complete cycles in the time interval from 0 to 2π. A cycle of length k days hence corresponds to a frequency of $(1/k)2\pi$. Since the series $x(t)$ represents daily observations, the shortest cyclical component of $x(t)$ has a two-day period. This corresponds to the frequency $(1/2)2\pi = \pi$. This agrees with the fact that $f(\lambda)$ is defined for $0 < \lambda < \pi$; i.e., cycles shorter than two days cannot be detected using daily observations.

Independence of successive log price differences obviously implies the white noise property $C(\tau) = 0$ for $\tau = 1, 2, 3 \ldots$. In that case, from (4) it follows that $f(\lambda) = \sigma^2/\pi$ for $0 < \lambda < \pi$, so $f(\lambda)$ is a horizontal straight line. The way to test the random walk hypothesis using spectral analysis is hence to estimate $f(\lambda)$ for the series of successive log stock price differences. One then plots that estimate, denoted $\hat{f}(\lambda)$, over $0 < \lambda < \pi$ and compares it with the constant function $f(\lambda) = \sigma^2/\pi$. Peaks in $\hat{f}(\lambda)$ indicate cycles. That is what we have done for the 15 Danish stocks in our sample. Results are discussed in Section 7. In the next section, the precise methodology for comparing $\hat{f}(\lambda)$ with the constant function $f(\lambda) = \sigma^2/\pi$ is discussed in somewhat greater detail. For our computations, we have used a standard computer program UCLA – BMD02T. This program utilizes modified Dirichlet weights. The number of lags was set to 60.

6. A White Noise Test

Before attempting to detect cycles in the empirical series of log stock price differences by means of spectral analysis, a preliminary matter relating to the data set must be discussed. As mentioned in Section 2, there are gaps in the price series due to a lack of transactions on some trading days. Additionally, the Stock Exchange is closed certain weekdays over Easter, Christmas, etc. If there are regular patterns in the series of log price differences (weekly or bi-weekly, for instance), any such patterns would be destroyed if one simply disregards gaps due to holidays or missing transactions on some trading days. That is, if there is a five-day (weekly) cycle, for example, then one can only detect that by insuring that every fifth observation in the time series pertains to precisely the same weekday (Monday through Friday). This means that gaps due to holidays and missing transactions must somehow be rectified. That has been achieved as follows: Whenever there is a holiday during the week or a trading day with no transaction, the last previous closing price is substituted. In that way, one obtains for each of the fifteen stocks a price series consisting of 783 observations. The Copenhagen Stock Exchange was open 750 days in 1973–1975, as was mentioned in Section 2. The difference between 783 and 750 consists of 33 holidays falling during the week (i.e., Monday through Friday). From each such series, a series of 782 consecutive log price differences was constructed, hence consisting of 782 observations. For each stock, this series of 782 observations was used as the data input to the spectral analysis. Figure 1 b illustrates part of that series for stock Nr. 9, DFDS.

It was stated in Section 5 that a white noise sequence should teoretically have $f(\lambda) = \sigma^2/\pi$ for $0 < \lambda < \pi$. If the number of observations, N (in this case 782), is large, then it can be shown that the smoothed spectral estimator $\nu\hat{f}(\lambda)/f(\lambda)$ is approximately distributed as $\chi^2(\nu)$ (Jenkins and Watts (1968),

p. 253). v is called the "equivalent degrees of freedom" and given by $v = N/M$ for modified Dirichlet weights (Dhrymes (1970), p. 504), where M equals the number of lags (in this case 60).

To investigate whether a peak in $f(\lambda)$ is significantly higher than one would expect under the white noise assumption, the simplest intuitive test is to construct an approximate $100(1-\alpha)\%$ test band for $\hat{f}(\lambda)$ in the interval $0 < \lambda < \pi$ under the null hypothesis that $f(\lambda) = \sigma^2/\pi$ against the alternative that $f(\lambda) > \sigma^2/\pi$. In view of the preceding, the null hypothesis is rejected if

$$\frac{v\hat{f}(\lambda)}{\sigma^2/\pi} > c$$

for $0 < \lambda < \pi$, where c is the $100(1-\alpha)\%$ value of χ^2 with v degrees of freedom. Since σ^2 is unknown, it is replaced by $m_2 = \hat{\sigma}^2$.

In this case, $v = N/M = 782/60 \approx 13$. A 95% significance level is chosen. Taking logarithms, the white noise hypothesis is rejected if

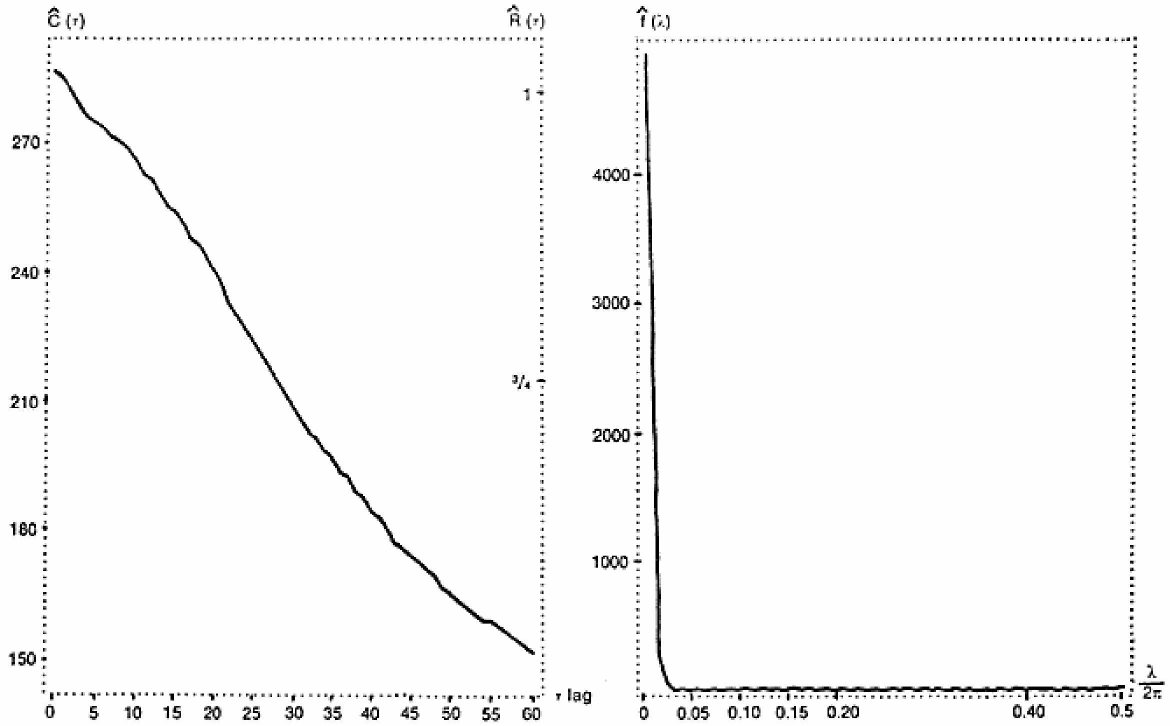
$$\log \hat{f}(\lambda) > \log \left\{\frac{\chi^2_{0.95}(13)}{13}\right\} + \log \frac{\hat{\sigma}^2}{\pi}$$

for $0 < \lambda < \pi$.

7. Spectral Analysis Results

To begin our presentation of spectral analysis results, we will consider stock Nr. 9, DFDS, in some detail. For illustrative purposes, we have performed some analyses using the series of DFDS prices (rather than log price changes). Gaps due to holidays and lack of transactions have been handled by substituting the last previous closing prices, as explained in Section 6. That is, a series of 783 daily price observations has been used.

Figure 2 a gives a plot of the estimated autocovariance function for daily price observations of DFDS. It is represented as a continuous graph but should ideally be represented as a sequence of discrete points. The estimated variance is 286. The estimated autocovariances are falling with increasing lag size. Since $\hat{R}(\tau) = \hat{C}(\tau)/\hat{\sigma}^2$, Figure 2 a may also be interpreted as a plot of the estimated correlogram. This is indicated by the scale on the right. Figure 2 b gives a plot of $\hat{f}(\lambda)$ for the same series of 783 daily price observations. Evidently, most of the variance of 286 can be attributed to a very long cycle (or a trend, since a trend cannot be distinguished from the beginning of a very long cycle). Altogether, 91% of the variance can be explained by a very long cycle or a trend[1]. The spectral estimate, $\hat{f}(\lambda)$, in Figure 2 b is constructed from 61 equidistant points and should, strictly speaking, be represented as a histogram with 61 bars. For practical reasons, all the points are connected with straight lines. This remark also applies to the spectral estimates presented in the following figures.



a. Estimated autocovariance function for DFDS. Price series, weekdays.

b. $\hat{f}(\lambda)$ *for DFDS. Price series, weekdays.*

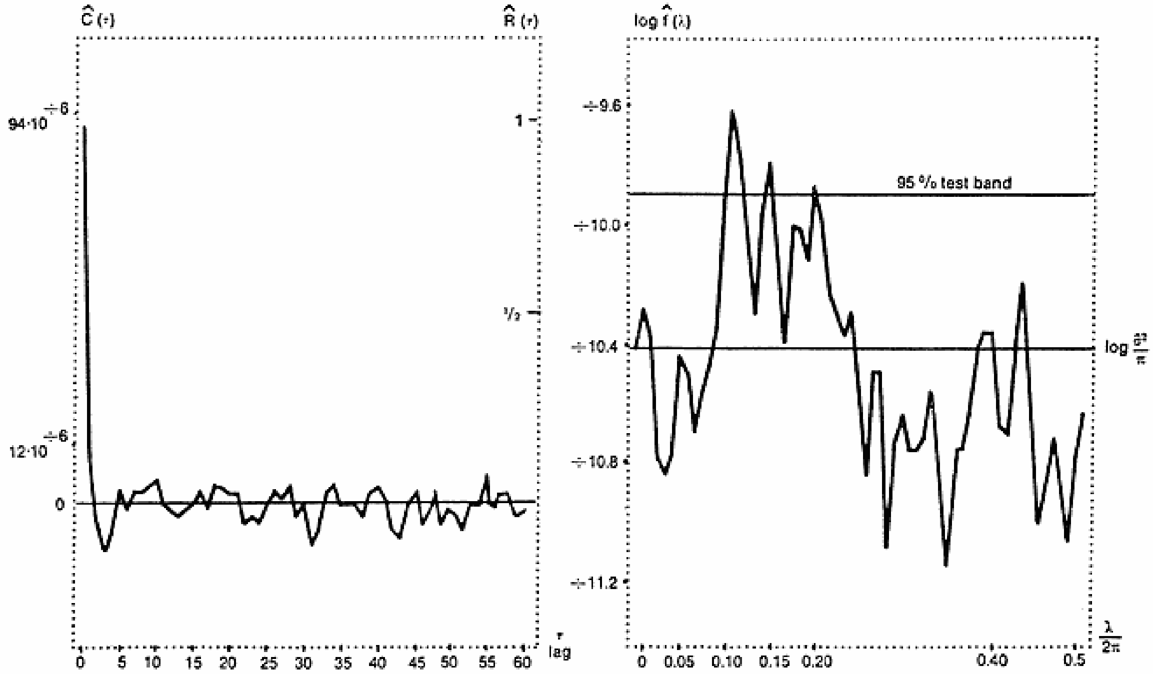
FIGURE 2.

Obviously, spectral analysis of the price series (as opposed to the log price difference series) does not give very much information about shorter cycles, such as weekly ones. Any peak in $\hat{f}(\lambda)$ pertaining to a weekly cycle will be swamped by that part of the total variance attributable to a trend or a very long cycle. In fact, Danish stock market prices in general fell from the beginning of 1973 to a low at the end of 1974 and then rose somewhat

1. The 91% figure is calculated as the relative share of the total variance attributed to the two lowest frequencies.

during 1975. This means that $f(\lambda)$ for most of the 15 price series may be expected to look about the same as Figure 2 b.

The effect of the transformation $\log p_t - \log p_{t-1}$ is to filter out the trend (or most of it, in any case), as was indicated in Section 5. Figure 3 a gives the estimated autocovariance function for the DFDS series of log price differences (782 observations). The total variance has now been reduced to $93.9 \cdot 10^{-6}$. Figure 3 b shows $\log f(\lambda)$ for the same series. A trend (or a long cycle) no longer dominates. More precisely, only 2.7% of the total variance can now be attributed to a trend or long cycle. *A priori*, we supposed that there would be a weekly cycle (if any at all), i.e., one with a length of five weekdays, corresponding to a frequency of $0.2 \cdot 2\pi$. It can be seen from Figure 3 b that there is a significant peak around that frequency point. In addition one should also look for so-called harmonics. The first harmonic of a weekly cycle is a half-weekly one. The first harmonic of a four-week cycle is a two-week one; the second harmonic of a four-week cycle is one of $6\frac{2}{3}$ days' duration, etc. A "true" cycle will often spill over into its harmonics (cf. Granger



a. Estimated autocovariance function for DFDS, log price difference series, weekdays.

b. $\log f(\lambda)$ for DFDS, log price difference series, weekdays.

FIGURE 3.

and Hatanaka (1964, p. 63)). In Figure 3 b, there is also a peak centered on one half week ($0.4 \cdot 2\pi$). This may again be considered as supporting the notion of a weekly cycle.

In figure 3 b, there are also marked peaks corresponding to the frequencies $0.10 \cdot 2\pi$ (two weeks) and $0.15 \cdot 2\pi$ ($6\frac{2}{3}$ weekdays). The last one of these cycles cannot be given any reasonable economic interpretation. It must therefore be interpreted as the second harmonic of a four-week cycle. The two-week cycle may then be interpreted as the first harmonic of such a cycle. There is then some evidence of a four-week cycle, even if that cycle by itself is not very prominent in Figure 3 b.

In the same manner as in Figure 3 b, Figures 4 and 5 give spectral estimates, $\log \hat{f}(\lambda)$, for the series of log price differences for stocks Nr. 3 and 13 (Handelsbanken and Nordisk Kabel- og Traad). For Handelsbanken, the two-week and four-week cycles at frequencies $0.10 \cdot 2\pi$ and $0.05 \cdot 2\pi$ are found significant. The peak at $0.15 \cdot 2\pi$ (the second harmonic) may be regarded as support of a four-week cycle. For Nordisk Kabel- og Traad, the two-week

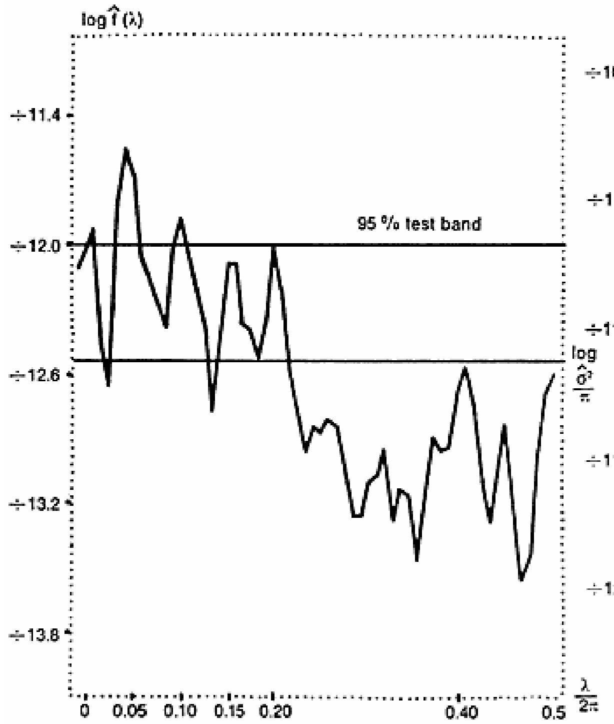


FIGURE 4. *log $\hat{f}(\lambda)$ for HANDELSBANKEN, log price difference series, weekdays.*

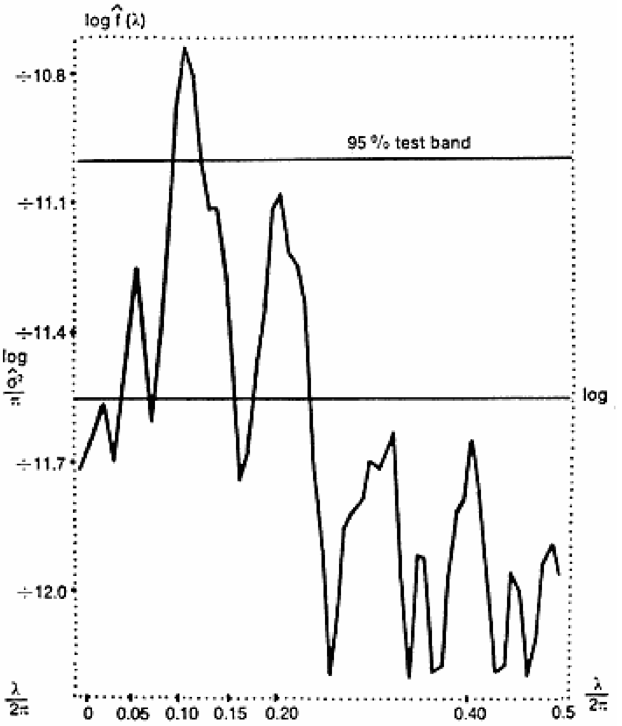


FIGURE 5. *log $\hat{f}(\lambda)$ for NORDISK KABEL- og TRAAD, log price difference series, weekdays.*

cycle is found significant, and there is also evidence of a one-week cycle.

Results of the spectral analysis of the series of log price differences for all 15 stocks (782 observations per series) are summarized in Table 2. Visible peaks are denoted by a cross x in the appropriate columns. Significant peaks (according to the test described in Section 6) are denoted by *. It can be seen from Table 2 that for almost all stocks there is evidence of a weekly cycle. Also, two- and four-week cycles are noticeable in many cases. It may be remarked that there is a slight tendency for $\log f(\lambda)$ to fall with increasing values of λ for many of the stocks. This can be noted in Figures 4 and 5. That may be interpreted to mean that the transformation $\log p_t - \log p_{t-1}$ does not entirely filter out the trend.

TABLE 2. *Summary of Spectral Analysis.*

	Stock name	Some fundamental cycles and their harmonics				
		$0.05 \cdot 2\pi$ 4 weeks	$0.10 \cdot 2\pi$ 2 weeks	$0.15 \cdot 2\pi$ $6\frac{2}{3}$ days	$0.20 \cdot 2\pi$ 1 week	$0.40 \cdot 2\pi$ $\frac{1}{2}$ week
1	Privatbanken	*	x	*	x	x
2	Landmandsbanken	*	x	*	x	x
3	Handelsbanken	*	*	x	x	x
4	St. Nordiske Telegraf		*			
5	Dansk Trælast Co	x			x	x
6	Korn og Foderstof, gl		x		x	x
7	Wessel & Wett, præf	x	*		x	x
8	ØK		*	x		x
9	DFDS		*	*	*	x
10	Danske Sukkerfabrikker		*		x	x
11	Forenede Bryggerier, ord		x	x		
12	Forenede Papirfabrikker		*		*	x
13	Nord. Kabel- og Traad	x	*		x	x
14	F. L. Smidt og Co, B	x	x	x	x	x
15	Superfos, ord		x	*	x	

NOTE: * significant at 5% level. x visible peak.

8. Conclusion

At the close of this investigation, the essential results may be summarized as follows:

1. The empirical distributions of log price differences for the fifteen Danish stocks exhibit systematic deviations from normality in the direction of leptokurtosis (Section 3). This agrees with results obtained in previous studies of the US, UK, German, Norwegian, and Swedish stock markets.

2. On the basis of runs tests (Section 4), one may conclude that none of the fifteen stocks conforms to the random walk hypothesis. The deviations from randomness are systematic (all stocks display too few runs). The general order of magnitude of deviations from randomness agrees with earlier results obtained for the German, Norwegian, and Swedish stock markets.
3. Through spectral analysis (Section 7), it has been demonstrated that there are regular patterns, for instance weekly ones, in the series of log price differences for all the stocks. This contradicts the random walk hypothesis.

One the basis of runs tests and spectral analysis we hence conclude that the random walk hypothesis is not an appropriate one for the Danish stock market.

It was pointed out at the outset of this article that the random walk hypothesis represents a sufficient, but not necessary, condition for the absence of mechanical trading rules, based on historical price changes, yielding higher rates of return than naive buy-and-hold portfolios. It now follows from the present investigation that it cannot be excluded that such trading rules do exist in the Danish stock market. In particular, the existence of weekly and other regular patterns seems to open up intriguing possibilities in this respect. Whether such trading rules can in fact be found, and, if so, what they look like, can unfortunately not be determined here. That can only be settled on the basis of further investigations.

In any case, our study has indicated the existence of certain cyclical regularities in successive stock price changes on the Copenhagen Stock Exchange. Irrespective of whether these cycles are pronounced enough to produce profitable mechanical trading rules (taking into account the associated transaction costs), the mere existence of such cycles is puzzling, and one is led to speculate about the underlying mechanisms. The following is one mechanism which could possibly contribute to the existence of a one-month cycle:

Banks face government regulations as to the total amount of money they can lend. These regulations must be met at one point every month (the end of the month). This means that the banks may demand repayment of certain amounts loaned at the end of each month, so as to avoid violations of the regulations. Certain borrowers may hence have to repay part of their loans. To the extent that the loans are utilized to finance stock portfolios, there will be a downward pressure on the stock market at the end of each month, as the bank customers sell off part of their portfolios to obtain cash to repay the banks.

Our study gives no justification for this type of speculation. However, we have in mind to continue our investigation of the Danish stock market, with a view towards uncovering the types of mechanisms that could result in cycles in the sequences of price differences.

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