Measurement and Evaluation of Fiscal Policy

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Introduction

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My comments in this note will mostly refer to the article by Rosted, Schaumann and Sørensen in this issue of Nationaløkonomisk Tidsskrift and to the set of working papers issued from the Secretariat in September, 1973 under the title "Measurement of the effects of fiscal policy".

I. Size of model

The Secretariat should be praised for setting up a model, which, despite its simple structure, makes it possible to calculate and present in a lucid way effects of changes in the most important fiscal instruments.

On the other hand, any simplification has its price, and in the present case an important deficiency is the disregard of the monetary sector, which renders the estimated coefficients subejct to specification errors.

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On the other hand, any simplification has its price, and in the present case an important deficiency is the disregard of the monetary sector, which renders the estimated coefficients subejct to specification errors.

(*a*) Thus to the extent that changes in fiscal and monetary variables are correlated, the estimated coefficients of the fiscal variables are biased, when monetary variables are not taken into consideration[1].

(*b*) Liquidity effects of the budget are not included. In SMEC II "the line" in the budget is drawn after real expenditures, transfers and taxes, and all items "below that line" (for instance public lending and other capital expenditures) are assigned the weight zero. Furthermore, the net liquidity effect of the budget is disregarded, probably resulting in underestimation of the separate instrument effects, but not necessarily of the total budget effect.[2]

The various sources of errors may of course cancel out, but may also be reinforcing. Furthermore, it should be noted that if the model is later to be disaggregated into a federal and a municipal sector, consideration of liquidity effects becomes even more important, as municipal and federal budget changes have widely different liquidity effects, whereas the "real" effects probably do not differ much.

II. Measure of budget effect

In discussing the measurement of budget effects the Secretariat strongly emphasises that only discretionary changes should be taken into account. The argument may be presented in the framework of the simple macromodel used for illustrative purposes by the Secretariat, and repeated here for easy reference:

$$C = C_0 + c(Y - T)$$
$$T = tY$$
$$Y = C + I + G$$

where

Y = gross national product
C = consumption
I = investment
T = net taxes
G = public real expenditures

Letting $A = C_0 + I$ denote private autonomous expenditures, equilibrium income can be written:

1. If for instance both taxes and exogenous money supply increase in a given year, the coefficient of taxes will be numerically underestimated, as the increase in the money supply has probably had an expansionary influence on total activity.

2. It should be noted that the Secretariat is fully aware of the above mentioned biases and deficiencies in the model.

$$Y = (A + G) / (1 - c(1 - t)) \text{ or as}$$
$$Y = (A + G - cT) / (1 - c)$$

Using the last expression, an equilibrium change in income can be written:

$$dY = (dA + dG - cdT) / (1 - c)$$

If there were no change at all in the budget, the income change would have been:

$$dY_P = dA / (1 - c)$$

and the effect of budget changes might therefore be defined as:

$$dY_G = dY - dY_P = (dG - cdT) / (1 - c) \qquad (1)$$

Alternatively only part of the tax change might be "assigned" to the public sector. If that part, which is due to changes in private autonomous expenditures, is excluded, we get:

$$dY'_P = (dA - ctdY'_P) / (1 - c)$$

and the effect of budget changes is now to be defined as:

$$dY'_G = dY - dY'_P = (dG - cY_0dt) / (1 - c(1 - t)) \qquad (2)$$

where Y_0 denotes the initial income.

Finally one might argue, that *all* automatic tax changes should be excluded from the budget effect. This results in:

$$dY''_P = (dA - cdY''_Pt - cdY''_Gt) / (1 - c)$$

and the budget effect will be:

$$dY''_G = dY - dY''_P = (dG - cY_0dt) / (1 - c) \qquad (3)$$

The measure suggested in SMEC II corresponds to (2) above, which is also recommended by Oakland[3]. (1) and (2) will of course give identical measures if $dA = 0$, whereas for $dA > 0$, (2) will yield a larger effect than (1), as the latter is negatively affected by the automatic tax increase due to $dA > 0$. As pointed out by Lotz (1974) (2) may therefore give an exaggerated impression of the expansionary effects of budget changes.

Whether one should prefer (1), (2) or (3) seems rather arbitrary. It might be argued - as done by the Secretariat - that it is unrealistic to compare a given

3. See Oakland (1969) and Corrigan (1970) for an unweighted measure. The measure suggested in (1) corresponds to Musgrave's »change in fiscal leverage«, see Musgrave (1964), whereas (3) is discussed, but not recommended by Matthiessen (1961). See also Bent Hansen (1958 and 1969).

change in income with a hypothetical change, which assumes away the public sector. Furthermore, for ex ante planning of fiscal policy, it seems rational to consider only discretionary changes.

On the other hand, (1) is ex post much easier to calculate, as actual realized budget figures can be used, and the multiplier is independent of tax rate changes. In addition it is really a matter of choice whether existing tax rates should be regarded as exogenously given or might be interpreted as deriving from discretionary decisions over past years. Finally, it seems rather "asymmetrical" to argue so strongly that automatic changes due to changes in private expenditures are to be excluded from the budget effect, whereas automatic changes due to changes in fiscal instruments are to be included (compare (2) and (3)). Thus I fail to see any features that distinguish $dT = tdA$ from $dT = tdG$.

The above discussion also implies that the distinction between first and second round effects becomes somewhat arbitraty. If (1) is used, the first round effect (the multiplicand) becomes rather small, as the total induced tax change is taken into account in the first round, whereas the second round effects (the multiplier) become rather large. The opposite applies to (2) and (3), which from this point of view seem preferable. This argument, however, does not seem to have had any role in the Secretariat's choice of measure, as the budget effect is calculated as multiplier times actual change in each fiscal instrument.

Summing up the above discussion, it seems to me that none of the measures suggested is uniquely better than the others, and any choice should therefore be made with the actual application in mind. From that respect, use of only discretionary effects seems most appropriate for ex ante planning, whereas the total budget effect is most relevant and certainly much easier to calculate for ex post measures.

Before turning to the problems of application and evaluation let me point out that in deriving a measure of budget effects it seems natural to take the dynamic aspects of the multiplier into account and base the budget effect for a given period on both present and past changes. The mere fact that the fiscal year is different from the calendar year points in that direction. How many past periods should be included will depend on the time profile of the multipliers and the time horizon of the decision makers[4]. It is pointed out by the Secretariat that most of the effects will come in the first year, but this can only be true after the introduction of the pay-as-you-earn tax system and after the abolition of the right to deduct paid taxes in taxable income. Furthermore, one

4. For a measure along this line see Biehl et al (1973).

cannot exclude the possibility, that the ranking of the multipliers of the various fiscal instruments is changed somewhat, if several periods are considered.

III. Application and evaluation

When it comes to *application* of the measures derived for budget changes and to *evaluation* of fiscal policy, the ex ante planning and the ex post analysis must use a given target, which is determined by political decisions. Thus the interconnection between political and economic decisions is just as important for the former problem as it is for the latter, whereas from the reports of the Secretariat one neight get the impression that application is purely an economic problem, whereas evaluation is beyond economic theory.

As a starting point let it be assumed that there is only one target (full em-

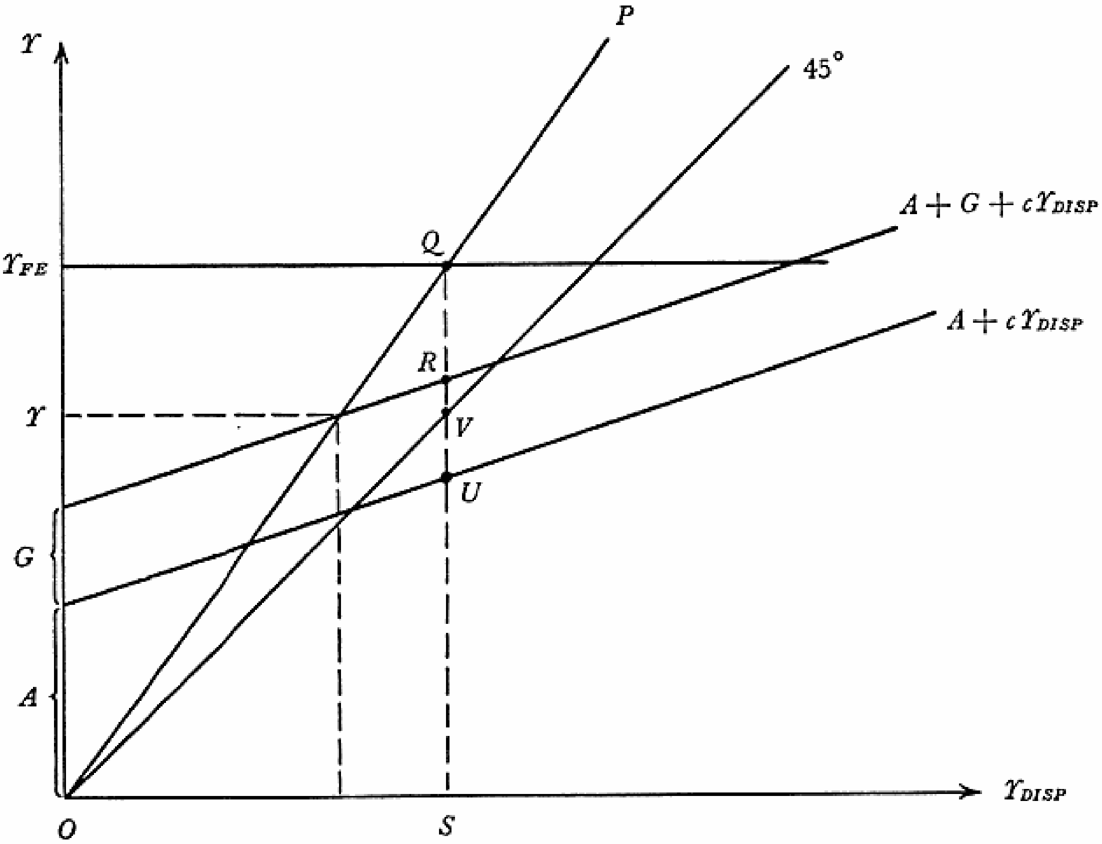

FIG. 1[5]

5. For further discussion of this graph see Gurley (1952). *OP* is the tax function, with tax revenue being measured in relation to the 45°-line. Thus *OP* maps total income into disposable income, and equilibrium income occurs where the total expenditure curve intersects *OP*. In the graph taxes equal government expenditure in the initial equilibrium, but we could have started out with any budget balance.

ployment income), but two available instruments, government expenditures and income taxes, expressed as revenue at full employment income. Given expectations with respect to private expenditures, the problem at hand can be illustrated in figure 1 where the simple macromodel used above is retained.

At present levels of the instruments, equilibrium income will be $\bar{Y}$, and the income "gap" is $Y_{FE} - \bar{Y}$, which corresponds to an effective demand "gap" $= QR$. The latter can be partitioned into a private saving surplus $= VU$ and a government full employment surplus $= QV - RU$[6].

Y_{FE} may be reached in infinitely many ways, as there are two instruments and only one target. On the other hand, G and T_{FE} have proportional effects on Y, and they can therefore be combined linearly, giving $G - cT_{FE}$ as a single instrument. The required change in fiscal policy is $\Delta(G - cT_{FE}) = (1 - c)(Y_{FE} - \bar{Y})$. Or in other words the required change in the instrument has to equal the induced increase in total saving.

Under certainty and with only one target and one instrument, no evaluation problem will arise, as the required adjustment in fiscal policy would be made[7], but how is fiscal policy to be evaluated, if for instance unexpected changes in private demand occur? Is fiscal policy to take the blame for this? According to the "adequacy" measure suggested by Musgrave (1964) and later modified by Gramlich (1966), the answer seems to be yes. Thus in Gramlich's version "adequacy" is measured as:

$$\alpha = (G - cT_{FE}) / ((1 - c)\, Y_{FE} - A).$$

That is, as the ex post ratio between "full employment weighted public deficit" and total private saving surplus. Any unexpected change in the economy will show up in the denominator and thus be "debited" to fiscal policy performance. This may be justified by the argument that with existing indicators of future prospects, fiscal policy "ought" to be adjusted soon enough to counterbalance any changes.

Personally I would not subscribe to this argument, but the other alternative of measuring the ex post budget in relation to the ex ante planned budget, would be rather uninteresting, as this ratio only indicates possible institutional difficulties or lack of control over the instrument[8].

6. From the graph it is easily seen that $QR = VU + QV - RU$.

7. If the required change in the fiscal instrument violates certain boundary conditions, complete adjustment will not be made, but then we are really faced with a two target case.

8. For an interesting discussion of this point see Okun (1972). Another version of the »adequacy« measure is given in Snyder (1970).

However, instead of pursuing this rather unfruitful discussion of fairness, let me turn to other problems, which have implications for both ex ante planning and ex post evaluation.

A. Several targets and instruments

In any modern society the authorities will have several targets and instruments, but very rarely does there exist a one-to-one correspondence between targets and instruments. At the planning stage this does not cause any problems as long as the model can be solved with respect to the instruments after the target values have been inserted - or the welfare function maximized in case of relative targets. However, at the ex post evaluation stage the task now seems more impossible than in the simple case considered above. If the performance of a single instrument is measured in relation to all the targets, "adequacy" will not only be affected by unexpected changes in private demand, but also by inadequacies of the other instruments. Mundell's (1962) proposal of assigning one instrument to one target according to the "Principle of Effective Market Classification" does not solve the problem, as - because of the interdependence-even the assigned targets will be influenced by other factors, notably inadequacies of other instruments. The only remaining alternative will then be to measure the ex post instrument value in realtion to the ex ante planned value, but this again is rather uninteresting.

B. Instrument Instability

The question of stability in an economic model has been analyzed for a number of years, but not until recently has the problem of instrument stability been studied. Both problems derive from the existence of lags, but as pointed out by Holbrook (1972), they are to some extent independent of each other.

Let:

$$AY_t + BP_t + CY_{t-1} + DP_{t-1} + EZ_t = 0$$

where

Y = endogenous (target) variables

P = instrument variables

Z = exogenous variables

A, B, C, D, E = matrices of parameters.

Solution with respect to Y_t yields:[9]

$$Y_t = -A^{-1}BP_t - A^{-1}CY_{t-1} - A^{-1}DP_{t-1} - A^{-1}EZ_t$$

and whether the model is stable or unstable is seen to depend on $-A^{-1}C$, as the characteristic roots are derived from that matrix. If, on the other hand, the target values are inserted, the solution with respect to P is:

$$P_t = -B^{-1}AY^*_t - B^{-1}CY^*_{t-1} - B^{-1}DP_{t-1} - B^{-1}EZ_t$$

and the stability of P will depend on the matrix $-B^{-1}D$.

As pointed out by Poole (1971), the combination of model- and instrument instability will give the following four cases:

		Model	
		Stable	Unstable
Instrument	Stable	1	2
	Unstable	3	4

1 and 3 would justify Friedman's proposal of simply setting rules for policy instruments[10], as efforts to "fine-tune" the economy are unnecessary and in 3 even harmful. Cases 2 and 4 give the widest scope for active economic policy, as the target variables are unstable, but only in 2 can the instruments be used without risk of increasing changes, whereas in 4 it seems necessary to reach some sort of compromise.

For illustration the latter case is discussed with only one lag in the instrument:

$$Y_t = aP_t + bP_{t-1} + cZ_t \text{ where } a + b = 1 \quad \text{and} \quad 0 < a < b$$ [11]

If Y_t deviates from Y^* because of an unexpected change in Z, an adjustment of the instrument is required, but because of the lag structure any effort to reach the target value in period t by means of changes in the instrument will lead to an even larger instrumental change in $t+1$, $t+2$ etc. Does this imply that the instrument cannot be used at all and that economic policy must be based on rules? In answer to this the following options seem available:

9. It is here assumed that the instruments have been set according to certain rules. Poole in discussing the same case and the optimal policy seems to assume that P is chosen according to »optimal control theory«, but this can hardly be the case, since it would then by definition be unnecessary to search for the optimal policy.

10. Another way of explaining this, is to say that there is no feed-back from the development of the target variable to the values chosen for the instruments.

11. It should be noted that Y, P and Z now refer to single variables.

(*a*) Economic policy can aim at stabilizing the target variable over several periods and not for each single period. If in the above example the time horizon is enlarged to two periods, all that is needed in period t is a change in P equal to $Y^* - Y_t$, and the target value will be reached without further instrument adjustments in period $t + 1$. This rule can be adapted to any lag structure, but of course becomes less satisfactory the longer the lags.

(*b*) The authorities may also adopt the decision rule that for each single period P is only to be changed by a fraction of the required change to close the "gap". As pointed out by Gramlich (1971) this will often be an optimal policy, if the instrument is included in the aggregate welfare function. Thus if a quadratic welfare function is assumed, we have:

$$W_t = -((Y^* - Y_t)^2 + b(P_t - P_{t-1}(1 + r))^2)$$

where r is the desired rate of change of the instrument. As appears a "penalty" is imposed on too large changes in the instrument[12], and this "penalty" - expressed by the coefficient b - will determine the fraction of the "gap", which it is optimal to close in each period[13]. In the extreme case where $b = \infty$, Friedman's rules for economic policy will be the optimal policy.

Whether instrument instability exists has not been established for very many macroeconomic models[14], but even if it is a common problem, the consequences for economic policy are not that serious. Thus, as shown in the two options above, active economic policy does not have to be abandoned, but a compromise can be reached, where some short run stability in the target variable is sacrificed for stability in the instrument.

C. Uncertainty

It has so far been assumed that certainty prevails with respect to both forecasting and effects of the instruments. However, as pointed out by Brainard

12. The »penalty« will equally apply to »too small« changes in the instruments. This does not seem very realistic, but is imposed by the quadratic structure of the welfare function.

13. b may, apart from political views with respect to appropriate values for the instrument, be interpreted to reflect costs of changing the instrument. As pointed out by Okun (1972), changes in fiscal instruments can be quite costly, whereas monetary instruments are much »cheaper«. As the welfare function has been normalized with respect to $(Y^* - Y_t)^2$, b will further reflect the trade-off between instrument- and target variations. Allowing for several target variables and instruments the welfare function can be written $Y'KY$, where Y is a vector containing both target variables and instruments and K is a diagonal matrix, the diagonal elements reflecting all assumed trade-offs. Changes in K with the economic structure remaining unchanged can then produce rather substantial changes in the optimal policy.

14. Some results are reported in Holbrook (1972), Gramlich (1971) and Poole (1971).

(1967) the decision rules are changed considerably, when uncertainty exists. Consider the relation:

$$Y_t = aP_t + u_t$$

where as usual Y is the target variable and P the instrument, whereas u denotes exogenous effects or stochastic disturbances. Uncertainty with respect to u is not very serious, as according to Theil's (1964) "Certainty Equivalence Theorem", one merely takes the expected value of u and then maximizes expected utility, which is a quadratic function in Y[15]. If uncertainty also exists with respect to a - either because a is estimated or is a stochastic variable - the theorem no longer applies. This can be seen by taking the expected value of the aggregate welfare function:

$$\begin{aligned} E(W) &= -E(Y - Y^*)^2 \\ &= -E(\bar{Y} - Y^* + Y - \bar{Y})^2 \\ &= -[(\bar{Y} - Y^*)^2 + s_{Y}^2] \end{aligned} \qquad (1)$$

Taking uncertainty for both a and u into consideration the variance of Y can be written:

$$s_Y^2 = s_a^2\, p^2 + s_u^2 + 2\, s_a s_u\, r P \qquad (2)$$

where r is the correlation coefficient between P and u. Inserting this in (1) and substituting for $\bar{Y}$, we have:

$$E(W) = -[(\bar{a}P + \bar{u} - Y^*)^2 + s_a^2\, P^2 + s_u^2 + 2\, s_a s_u\, r P] \qquad (3)$$

and maximization with respect to P yields:

$$P^* = \frac{(\bar{a}Y^* - \bar{u}) - r s_a s_u}{\bar{a}^2 + s_a{}^2} \qquad (4)$$

Thus, if uncertainty exists, information about both variances and covariances is required to determine the optimal policy.

If $r = 0$, P^* can be written:

$$P^* = \frac{Y - \bar{u}}{a} / \left(1 + \left(\frac{s_a}{\bar{a}}\right)^2\right) \qquad (5)$$

15. Certainty equivalence is not invalidated, if other welfare functions apply. If aggregate welfare depends on the absolute difference $Y^* - Y$, $E(Y)$ should be replaced by the median of Y, and the theorem can be used. For further discussion, see Okun (1972).

where the numerator is the required instrumental value, if certainty prevails, and the denominator is 1 + the coefficient of variation for a. In other words, uncertainty with respect to the instrumental effect implies that only a fraction of the "gap" is to be closed, the fraction varying inversely with the degree of uncertainty, as expressed in the coefficient of variation.

It is worth noting that uncertainty results in a one period decision rule which is similar to the one derived under B above, where uncertainty was absent, but lags were taken into account. In addition the existence of uncertainty makes instrument instability an even more serious problem, as apart from the undesirability of widely fluctuating instrumental values, the variance of the target variable is enlarged, when the instrument takes on greater and greater values. On the other hand, the existence of uncertainty may corroborate the above suggestion of increasing the time horizon, as the standard deviation attached to for instance one quarter multipliers is usually greater than the standard deviation of one year multipliers.

The implication for ex ante planning is obvious, and with respect to ex post evaluation, it is now no longer a question of fairness whether a single instrument should take the whole blame for an existing "gap", as closing the "gap" is inoptimal. The only alternative therefore is a comparison of the ex post with the ex ante planned value, and as already mentioned this is not a very interesting proposition.

D. Several targets and instruments

So far uncertainty and instrument instability have been discussed with only one target and one instrument. The problems do not change much if several but an equal number of instruments and targets exist. If, however, there are more instruments than targets, the obvious solution to instrument instability would be to drop the least stable instrument or to fix it a level, which insures the stability of the remaining ones[16].

Under uncertainty an entirely different outcome results. Without uncertainty one target value can be reached with one instrument, and any remaining instruments would be redundant. Under uncertainty, however, welfare will depend both on the average value and the variance of the target variable (cf. (3) above). Increasing the value of the instrument will bring the expected value

16. The income tax rate may for instance be an unstable instrument whereas government expenditures are stable for some values of the income tax rate, and unstable for others. The solution would then be to fix the income tax rate in the interval, which makes government expenditures stable, unless this will violate boundary conditions for the tax rate.

closer to the target, but will simultaneously increase the variance[17], and the optimal value for P in (4) and (5) occurs where the two effects offset each other. If two instruments are available for one target, they may be combined in such a way that the variance and thereby the uncertainty is reduced, the net result being an increase in aggregate welfare. How much the variance can be reduced by combining the instruments depends i.a. on the correlation between the coefficients of the two instruments, and in the extreme case where the correlation coefficient numerically equals 1, the variance becomes 0[18].

IV. Possible solutions

This note has really been rather negative, as most of the space has been used for pointing out problems in connection with planning economic policy and the impossibility of evaluating past performance in a relevant and interesting way. Not much has been said in the way of constructive suggestions.

Study of the existing literature does not give much comfort, as the results obtained are sparse and somewhat conflicting[19]. Let me, however, close this note by citing two recent results, which may entail some possibilities for future economic policy.

(*a*) Using a quadratic loss function with a time horizon of four quarters, Okun (1972) applies the St. Louis model to compute welfare losses of various policy responses to an unexpected change in the exogenous variables. Taking both uncertainty and lagged instrumental effects into account, he finds that "no response" gives the greatest loss, whereas "expected full adjustment" - i.e. a policy which aims at closing the "gap" completely in every quarter - gives the second highest loss, as the variance of the target variable becomes very large. A third alternative is "horizon adjustment", where the initial adjustment in the instrument aims at bringing the target variable back on the right track by the end of the last quarter. This policy response, which is very similar to the partial adjustment suggested by Holbrook and Gramlich (cf. above) results in a much smaller welfare loss, as the variance is considerably reduced. Finally, an optimal adjustment can be derived by minimizing the loss function with respect to the instrument values chosen for each of the four quarters considered. It is

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17. From (3) the variance of the target variable is seen to be proportional to the squared value of the instrument. This would seem to indicate that if an increase in the target is called for, one should use instruments that have a negative influence on the target, and vice versa for reductions in the target variable. Apparently this result has been excluded by the authors dealing with this problem. See for instance Brainard (1967, p.417, footnote 5).

18. See Brainard (1967) and the appendix for a more detailed discussion of this case.

19. See Holbrook (1972) and Gramlich (1971).

worth noting that this policy results in much smaller changes than "expected full adjustment", but greater changes than "horizon adjustment". The characteristic feature of this solution is therefore, that instrumental values are chosen for all four quarters simultaneously, and that for all four quarters the target variable is brought as close to the desired value as variance due to uncertainties and lag structure permits.

(*b*) Chow (1973) has analyzed the problems discussed above on the basis of "Optimal Control Theory" and derives a decision rule, entailing the same characteristics as Okun's "optimal adjustment": "It uses a dynamic model to take into account the effects of a decision in one period on the outcome of later periods. It treats the decisions of one period in conjunction with decisions of other periods." (Chow, 1973, p. 835). Due to limitations of space and my own knowledge of the subject, I shall not go any further into the area of "Optimal Control Theory", but merely point out that it seems to give the answer to many of the problems encountered in planning and evaluating economic policy.

Conclusion

This note has initially pointed out certain possible errors arising when using small models, where important instruments are left out. It has also dealt briefly with the problem of finding a relevant measure for the effects of budget changes. Most of the note has been devoted to the question of policy planning and evaluation. Even in a simple one target-one instrument model with no uncertainty on the instrumental effect a relevant and interesting evaluation measure is difficult to find. When time lags and uncertainty with respect to the instrument are introduced, optimal policy has to be planned for several periods, and any evaluation would therefore have to take the long-term character into account and should not be confined to a single period. Exactly how a relevant measure should be defined, I am, however, unable to see.

APPENDIX I

With two instruments available, the economic structure can be formulated as:

$$Y = a_1P_1 + a_2P_2 + u$$

where

Y = target variable

P_i = instruments

u = exogenous variables and/or stochastic elements.

correlation (numerically) between a_1 and a_2 the smaller the variance on Y. For $r_{12} = 0$ s_Y^2 becomes:

$$s_Y^2 = \frac{s_1^2 s_2^2}{s_1^2 + s_2^2} \cdot (P_1 + P_2)^2$$

If only a single instrument - say P_1 - had been used and the impact is equal to that of the combined policy, the variance on Y would be:

$$s_Y^2 = s_1^2 P_1^2 = s_1^2 (P_1 + P_2)^2$$

Comparing this expression with the one derived above for the combined policy, it is easy to see that even when the instruments are uncorrelated, there is a gain in efficiency by combining the instruments.

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worth noting that this policy results in much smaller changes than "expected full adjustment", but greater changes than "horizon adjustment". The characteristic feature of this solution is therefore, that instrumental values are chosen for all four quarters simultaneously, and that for all four quarters the target variable is brought as close to the desired value as variance due to uncertainties and lag structure permits.

(*b*) Chow (1973) has analyzed the problems discussed above on the basis of "Optimal Control Theory" and derives a decision rule, entailing the same characteristics as Okun's "optimal adjustment": "It uses a dynamic model to take into account the effects of a decision in one period on the outcome of later periods. It treats the decisions of one period in conjunction with decisions of other periods." (Chow, 1973, p. 835). Due to limitations of space and my own knowledge of the subject, I shall not go any further into the area of "Optimal Control Theory", but merely point out that it seems to give the answer to many of the problems encountered in planning and evaluating economic policy.

Conclusion

This note has initially pointed out certain possible errors arising when using small models, where important instruments are left out. It has also dealt briefly with the problem of finding a relevant measure for the effects of budget changes. Most of the note has been devoted to the question of policy planning and evaluation. Even in a simple one target-one instrument model with no uncertainty on the instrumental effect a relevant and interesting evaluation measure is difficult to find. When time lags and uncertainty with respect to the instrument are introduced, optimal policy has to be planned for several periods, and any evaluation would therefore have to take the long-term character into account and should not be confined to a single period. Exactly how a relevant measure should be defined, I am, however, unable to see.

APPENDIX I

With two instruments available, the economic structure can be formulated as:

$$Y = a_1P_1 + a_2P_2 + u$$

where

Y = target variable

P_i = instruments

u = exogenous variables and/or stochastic elements.

It is further assumed that the units are such that $a_1 = a_2 = 1$. If the aggregate welfare function is quadratic in Y, we get by taking expected values:

$$\begin{aligned} E(W) &= -E(Y^* - Y)^2 \\ &= -(\bar{Y} - Y^*)^2 - s_Y^2 \end{aligned}$$

where Y^* is the desired value for Y and s_Y^2 is the variance of Y. If there is no correlation between the instruments and u, the variance is:

$$s_Y^2 = s_1^2 P_1^2 + s_2^2 P_2^2 + 2 s_1 s_2 r_{12} P_1 P_2 + s_u^2$$

where s_i is the standard deviation of a_i and r_{12} is the correlation between a_1 and a_2. Inserting this in the expression for expected aggregate welfare and differentiating partially with respect to P_1 and P_2, we obtain as optimal conditions:

$$\begin{aligned} \partial E(W)/\partial P_1 &= -2(\bar{P}_1 + \bar{P}_2 + \bar{u} - Y^*) - 2s_1^2 P_1 - 2 s_1 s_2 r_{12} P_2 = 0 \\ \partial E(W)/\partial P_2 &= -2(\bar{P}_1 + \bar{P}_2 + \bar{u} - Y^*) - 2s_2^2 P_2 - 2 s_1 s_2 r_{12} P_1 = 0 \end{aligned}$$

By subtracting and solving with respect to P_2/P_1, we get:

$$P_2/P_1 = (s_1^2 - s_1 s_2 r_{12})/(s_2^2 - s_1 s_2 r_{12})$$

Finally by adding 1 on both sides, the optimal ratio of P_1 to the total policy impact is found as:

$$P_1/(P_1 + P_2) = (s_2^2 - s_1 s_2 r_{12})/(s_1^2 + s_2^2 - 2 s_1 s_2 r_{12})$$

According to Brainard (1967) this ratio can be interpreted as that policy combination which minimizes the coefficient of variation of the combined impact, and the optimal combined impact can subsequently be determined by the rule derived in the text for one instrument.

In order to compare the variance of the combined policy with the variance of using only a single instrument and to find the importance of the correlation between the two instruments, we return to the expression derived for s_4^2. Dividing by $(P_1 + P_2)^2$ on both sides and disregarding the variance on u, we have:

$$s_Y^2/(P_1 + P_2)^2 = \frac{(s_2^2 - r_{12}s_1s_2)^2}{(s_1^2 - 2r_{12}s_1s_2 + s_2^2)^2} s_1^2 + \frac{(s_1^2 - r_{12}s_1s_2)^2}{(s_1^2 - 2r_{12}s_1s_2 + s_2^2)^2} s_2^2 + \frac{2r_{12}s_1s_2(s_2^2 - r_{12}s_1s_2)(s_1^2 - r_{12}s_1s_2)}{(s_1^2 - 2r_{12}s_1s_2 + s_2^2)^2}$$

After some manipulations this expression reduces to:

$$s_Y^2/(P_1 + P_2)^2 = \frac{(1 - r_{12}^2)\, s_2^2\, s_1^2}{s_2^2 - 2r_{12}s_1s_2 + s_1^2}$$

It is easy to see that for $r_{12} = 1$, the variance on Y becomes 0, and that the higher the correlation (numerically) between a_1 and a_2 the smaller the variance on Y. For $r_{12} = 0$ s_Y^2 becomes:

$$s_Y^2 = \frac{s_1^2 s_2^2}{s_1^2 + s_2^2} \cdot (P_1 + P_2)^2$$

If only a single instrument - say P_1 - had been used and the impact is equal to that of the combined policy, the variance on Y would be:

$$s_Y^2 = s_1^2 P_1^2 = s_1^2 (P_1 + P_2)^2$$

Comparing this expression with the one derived above for the combined policy, it is easy to see that even when the instruments are uncorrelated, there is a gain in efficiency by combining the instruments.

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