Automatic Stabilization in Static and Dynamic Models

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Introduction

It is frequently argued that the growth of the public sector in relation to the private sector and the increased reliance on income dependent taxes - as opposed to for instance property taxes - have increased built-in flexibility[1] and subsequently also the dampening effect, which the public sector may have on autonomous changes in income and activity.

Some economists[2] have even argued that due to a considerable time lag[3] discretionary fiscal policy cannot be relied upon for stabilization and may in fact accentuate the cyclical fluctuations in income. Instead, fiscal policy

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2. See Friedman (1948).

3. For a discretionary fiscal tax change the following lags are usually encountered: (*a*) Recognition lag, (*b*) reaction lag, (*c*) revenue effect lag, (*d*) adjustment lag for private demand, (*e*) adjustment lag for private output. An automatic stabilizer depends upon (*c*), (*d*) and (*e*), but not of course on (*a*) and (*b*).

I am indebted to J. Gelting, S. Hylleberg, G. Thorlund Jepsen, J. Vibe-Pedersen, C. Vastrup and E. Yndgaard for comments on an earlier draft of this paper.

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Introduction

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should be confined to those automatic changes in the budget that follow from changes in income, possibly strengthened by automatic changes in the tax rates[4].

Other economists have contended that automatic fiscal stabilizers can only reduce the secondary effects of an autonomous change in income, but cannot by themselves bring the economy back to full employment. In addition, a high degree of built-in-flexibility is a "mixed blessing" in that not only does it reduce the contractive effects of a decrease in income, but it also makes it more difficult to get an expansion started. The latter aspect has in recent years been interpreted as "fiscal drag"[5], which implies that in order to secure full employment growth it is necessary to expand government expenditures or reduce tax rates, unless expansion in private consumption or private investment can be relied upon.

Finally, some economists have argued that a high degree of built-in flexibility may have a destabilizing influence if we consider other endogenous variables than total real income or if we apply a dynamic model.

Before discussing the various measures of automatic stabilization we should like to point out that some arbitrariness is bound to occur when a distinction is made between automatic and discretionary changes in the budget. On the revenue side a distinction is relatively easy, given the tax laws and the interpretation of the laws, and the same applies to transfer payments on the expenditure side. However, when it comes to other expenditures, a distinction is less clear. Thus, in many countries public expenditures increase automatically with the general price level, and even for real expenditures one might say that some of them tend to follow an increased demand for public goods automatically, thus leaving very little for discretionary policies[6].

In the following we shall be somewhat traditional in that mainly automatic changes in net taxes (tax revenue - transfer payments) are taken into account, but in the section on growth we shall also allow for automatic changes in real expenditures.

As a starting point for discussing some of the problems mentioned above we take the traditional Musgrave-Miller measure[7] which we subsequently

4. The socalled "formula flexibility". See Musgrave (1959, p. 512). For a specific formula see Tanzi (1966).

5. This term was first used in the *Economic Report of the President*, January, 1962, U.S. Government Printing Office, Washington 1962. It refers to the automatic tendency towards a budget surplus at full employment, when the economy grows. This effect was an important argument for the American tax change in 1964, and we shall discuss the concept further in section III. B.

6. See Bent Hansen (1969, pp. 18-19).

7. See Musgrave and Miller (1948).

extend to more complicated static models and to dynamic models in which both short run cyclical movements and long run growth are considered. The scope of the following is somewhat limited in that monetary effects of changes in the budget are not taken into account, and neither do we consider effects of revenue changes on the supply of labour.

I. Measures of automatic stabilization in a small static model

A. Musgrave and Miller

Let us consider the following models for a closed economy[8]:

$$(1)\quad \begin{aligned} C &= a + b(Y - T) \\ I &= \bar{I} \\ G &= \bar{G} \\ T &= \bar{T} \\ Y &= C + I + G \end{aligned} \qquad \text{and} \qquad (2)\quad \begin{aligned} C &= a + b(Y - T) \\ I &= \bar{I} \\ G &= \bar{G} \\ T &= f(Y, q) \\ Y &= C + I + G \end{aligned}$$

where: Y = income, C = consumption, I = investments, G = government expenditures, and T = net direct taxes. All variables are measured in real terms.

In (1) all variables except Y and C are exogenous, whereas in (2) T is also an endogenous variable.

Substituting into the equation for Y and differentiating totally we obtain:

$$dY_1 = \frac{da + d\bar{G} + d\bar{I} - bd\bar{T}}{1 - b}$$

and[9] (1.1)

$$dY_2 = \frac{da + d\bar{G} + d\bar{I} - bf'(q)\,dq}{1 - b(1 - f'(Y))}$$

According to Musgrave and Miller automatic stabilization can now be defined as the proportion of income changes which is prevented due to income dependent taxes:

$$\alpha = 1 - \frac{dY_2}{dY_1} = \frac{b \cdot f'(Y)}{1 - b(1 - f'(Y))} = \frac{b \cdot e_{T,Y} \cdot (T/Y)}{1 - b(1 - e_{T,Y} \cdot T/Y)} \qquad (1.2)$$

where $e_{T,Y}$ is the elasticity of tax revenue with respect to income.

8. Throughout the paper we disregard the error terms and the effects that various tax structures might have on the error terms; cf. Bent Hansen (1969, p. 23). Initially we also disregard wage- and price changes.

9. $f'(Y)$ is the partial derivative of $f(Y,q)$ w.r.t. Y and $f'(q)$ the partial derivative w.r.t. the shift parameter q. For the sake of convenience we let dT equal $f'(q)\,dq$.

Thus α is really a comparison of income multipliers, one based on an economic structure with endogenous taxes and a hypothetical one, where taxes are assumed exogenous[10].

It may be argued that the above measure does not correspond to the definition, since the structural coefficients - in the above model only b - may change with the change in taxes, and this effect has not been taken into account[11, 12]. Alternatively, one might therefore take as a basis of comparison the income multiplier for a system with proportional taxes ($T = t \cdot Y$), as this measure will be less affected by changes in the marginal propensity to consume. This is done in:

B. Lusher

Using the same notation as above, but substituting a system with proportional taxes for model (1), we obtain[13]:

$$\beta = 1 - \frac{dY_2}{dY_1} = 1 - \frac{1/(1 - b(1 - f'(Y))}{1/(1 - b(1 - t))} = \frac{bf'(Y) - b \cdot t}{1 - b(1 - f'(Y))}$$

$$= \frac{b \cdot T/Y \cdot (e_{T,Y} - 1)}{1 - b(1 - e_{T,Y} \cdot T/Y)} \qquad (I.3)$$

In a system with proportional taxes, the tax elasticity $e_{T,Y} = 1$, and the rate elasticity $e_{f'(Y),Y} = 0$. According to the measure in (I.3), a tax system will only be stabilizing if the revenue elasticity is > 1 or alternatively, if the rate elasticity is > 0. Lusher's (1956) measure has the advantage that the structural coefficients of the actual system are much closer to those of the hypothetical system than in the Musgrave-Miller measure.

Cassidy (1970) has also applied a proportional tax as a basis of comparison, but from an entirely different point of view. Thus he argues that if in a linear

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10. Rewriting the definition of α it is esay to see that it equals the relative reduction of the income multiplier.

11. In empirical estimates of automatic stabilization the usual procedure has been to first derive estimates for all structural coefficients, assuming taxes endogenous, and subsequently to estimate the same coefficients, assuming taxes exogenous. This procedure will of course solve the problem mentioned above, but a bias is introduced into the estimation when taxes incorretcly are assumed exogenous.

12. A similar problem may arise from the fact that the marginal propensity to consume varies cyclically, thus giving rise to asymmetric stabilization. See Eilbott (1966).

13. The relation between α and β is: $\beta = \alpha - \frac{b \cdot T/Y}{1 - b\,(1 - e_{T,Y} \cdot T/Y)}$.

system we have a constant rate of growth of full employment real income, government expenditures proportional to real income and no autonomous elements in tax and expenditure functions, then a tax elasticity of 1 will be *neutral*, as the equilibrium growth will not be disturbed by the presence of the public sector. This definition, however, is relevant only under the rather strict conditions stated above, since for any other development in income or specification of functions, an elasticity of 1 is not desirable. Consequently, if we choose as reference base a tax system which either leaves a desirable position unchanged or moves the system from an undesirable position towards a desirable one, we shall have to change the basis, whenever there are changes in the economic conditions, the economic structure, or the targets. This procedure has the disadvantage that computation of automatic stabilization becomes rather laborious. On the other hand, it emphasizes the fact that automatic stabilization is a relative concept, which ought to be evaluated in relation to the prevailing economic situation.

C. Brown

Brown (1955) has used the Musgrave-Miller concept in a slightly different way, as he defines automatic stabilization as the difference between amounts "of autonomous shifts in demand necessary to secure a unit shift in output or income" (p. 430) under respectively endogenous and exogenous taxes. Letting dI denote autonomous changes in demand we then have:

$dI_1 = 1 - b$ and

$dI_2 = 1 - b(1 - f'(Y))$ obtaining for automatic stabilization:

$$\varrho = dI_2 - dI_1 = bf'(Y). \qquad (1.4)$$

As pointed out by Brown, (1.4) will often make calculations easier, and ϱ will obviously vary positively with b and $f'(Y)$. The limits are 1 and 0 as for the Musgrave-Miller concept.

D. Pearse

Pearse (1961-62) has defined a measure which takes into account that automatic stabilization can only affect the secondary effects of an autonomous change in income. This is done by subtracting the primary change from both the hypothetical and the actual income change, but otherwise relying on the Musgrave-Miller measure. Assuming that the autonomous change occurs in private investment, we thus obtain:

$$\lambda = 1 - \frac{dY_2 - dI}{dY_1 - dI} = 1 - \frac{b(1 - f'(Y)) \cdot dI/(1 - b(1 - f'(Y))}{bdI/(1 - b)}$$
$$= \frac{f'(Y)}{1 - b(1 - f'(Y))} \tag{1.5}$$

It is interesting to note that $\lambda =$ the automatic change in tax revenue, but this interpretation is only valid, when the marginal propensity to consume is assumed constant.

E. Bent Hansen

Bent Hansen (1959) has considered automatic stabilization in connection with defining a measure of the total budget effect during a certain period. Using the model explained above we obtain by subtracting dY_2 from dY_1:

$$(d\bar{I} + da + d\bar{G} - bd\bar{T})/(1 - b) - (d\bar{I} + da + d\bar{G} - bf'(q)dq)/$$
$$(1 - b(1 - f'(Y)) = bf'(Y)\, dY_2/(1 - b) \tag{1.6}$$

as an expression for automatic stabilization[14], [15]. This measure depends positively on $f'(Y)$, b, and dY_2, and it is important to note that the measure is independent of the source of the income change.

In (1969) Bent Hansen defines dY_1 and dY_2 exclusive of dI and da, and we then find the following expression for $dY_1 - dY_2$:

$$(d\bar{G} - bd\bar{T})/(1 - b) - (d\bar{G} - bf'(q)\, dq)/(1 - b(1 - f'(Y))$$
$$= bf'(Y)\,(dI + da)/(1 - b)\,(1 - b(1 - f'(Y)) \tag{1.7}$$

According to Bent Hansen (1969, p. 24) this measure depends positively on "how much extra-budgetary disturbance there has actually been to dampen". However, in view of the equality given in (1.7) this conclusion does not seem appropriate, and it furthermore seems more natural to *define* automatic stabilization independent of the source of the disturbance[16].

Before analyzing the extended model below we consider automatic stabilization in relation to a target level of income, Y^*. Thus if a certain Y^* is given exogenously, and if for model (2) above we assume proportional taxes above an exemption level ($T = c + tY$), we get the following expressions for the

14. We let $dT = f'(q)\, dq$ cf. footnote 9.

15. The same expression can be derived from Lars Mathiesen's (1961) definition of total and discretionary budget effects.

16. The economic structure may make the measure dependent upon the source of autonomous change, but this is an entirely different problem. For further discussion see II b.

required levels of the instrument (T and c)[17] in respectively model (1) and model (2):

$$T^* = (a + \bar{I} + \bar{G} - (1 - b)\, Y^*)/b \qquad \text{and}$$
$$c^* = (a + \bar{I} + \bar{G} - (1 - b(1 - t)\, Y^*)/b \qquad (1.8)$$

For a given change in private investments the deviations from Y^* will be respectively $dY_1 = dI/(1 - b)$ and $dY_2 = dI/(1 - b(1 - t))$, and if we consider the necessary changes in the instruments to bring Y back to Y^*, it may seem that more is required in model (1) than in model (2), since some change in tax revenue has already occurred in (2). However, in both cases the necessary change is dI/b. This result is merely a reflection of the fact, that automatic stabilization works both ways.

II. Extended static model

In general automatic stabilization depends upon three factors:

a. The endogenous variable being analyzed.
b. The exogenous variable, causing the changes in the endogenous variable.
c. The tax instrument considered.

Generalizing the Musgrave-Miller measure, we may write:

$$S_{ijk} = 1 - \frac{\partial y_i'/\partial x_j' = f_2(y_i, x_j, t_k)}{\partial y_i/\partial x_j = f_1(y_i, x_j)}$$

where: S_{ijk} = automatic stabilization of the i'th endogenous variable with respect to changes in the j'th exogenous variable, when t_k is the instrument used[18].
y_i = endogenous variable i
x_j = exogenous variable j
t_k = tax instrument k

f_1 and f_2 will of course depend upon the economic structure, and as an illustration we shall consider the following linear model:

$C = a + b(Y - RP - T_p)/(1 + t_3)$ $\qquad Y$ = income[19]
I[20] $= c + n(RP - T_c)$ $\qquad C$ = consumption

17. We have selected c as the instrument; alternatively we might have chosen t.
18. This implies that other taxes are exogenous.
19. All variables are measured in factor prices.
20. The formulation of this function implies that all production takes place in companies which are subject to the corporate tax.

$RP = e + fP$

$P = g + hY$

$T_p = t_1(Y - RP)$

$T_c = t_2 RP$

T_i[21] $= t_3 C$

$Im = k + mY$

$Ex = \overline{Ex}$

$G = \overline{G}$

$Y = C + I + G + Ex - Im$

I = investment
RP = retained profits
P = profits
T_p = personal income tax net of transfer payments
T_c = corporate income tax
T_i = indirect taxes
Im = imports
Ex = exports
G = government expenditures on goods and services.

A. Automatic stabilization of income (Y)

The reduced form equation for Y will now be:

$$Y_2 = \frac{a + c + \overline{G} + \overline{Ex} - k - (e + fg)\ (b(1 - t_1)\ (1 + t_3)^{-1} - n(1 - t_2))}{1 - b(1 - t_1)\ (1 - fh)\ (1 + t_3)^{-1} - nfh(1 - t_2) + m} \tag{II.1}$$

whereas if all taxes were exogenous we would get:

$$Y_1 = \frac{a + c + \overline{G} + \overline{Ex} - k - (e + fg)\ (b - n)}{1 - b(1 - fh) - nfh + m} \tag{II.2}$$

Applying the Musgrave-Miller measure we then obtain:

$$\alpha = 1 - \frac{dY_2}{dY_1} = \frac{b(t_3 + t_1)\ (1 - fh)\ (1 + t_3)^{-1} + nfh\, t_2}{1 - b(1 - fh)\ (1 - t_1)\ (1 + t_3)^{-1} - nfh(1 - t_2) + m} \tag{II.3}$$

As appears, α is positive and depends upon all three taxes, but if we want to analyze which tax is the more stabilizing, we can first define built-in flexibility and subsequently derive the effect of changes in built-in flexibility on α. Defining built-in flexibility as the increase in tax revenue due to an autonomous increase in income, we have:

$$\begin{aligned} F_1 &= dT_p/dY = t_1\,(1 - fh) \\ F_2 &= dT_c/dY = t_2 fh \\ F_3 &= dT_i/dY = (t_3/(1 + t_3))\ b\ (1 - fh)\ (1 - t_1). \end{aligned} \tag{II.4}$$

21. In formulating the consumption function, we have assumed no money illusion. If certain parts of consumption are exempt from the indirect tax, t_3 is a weighted average, which depends upon the proportion being exempt.

F_i will of course be affected by both the tax rates and the coefficients of the structure, but if we confine ourselves to the former, we can derive the following relations between the degree of built-in flexibility and α:

$$\frac{\partial \alpha}{\partial F_1} = \frac{\partial \alpha}{\partial t_1} \cdot \frac{\partial t_1}{\partial F_1} = \frac{b \cdot (1 + t_3)^{-1} \cdot (1 - b(1 - fh) - nfh + m)}{D^2} > 0$$

$$\frac{\partial \alpha}{\partial F_2} = \frac{\partial \alpha}{\partial t_2} \cdot \frac{\partial t_2}{\partial F_2} = \frac{n(1 - b(1 - fh) - nfh + m)}{D^2} > 0$$

$$\frac{\partial \alpha}{\partial F_3} = \frac{\partial \alpha}{\partial t_3} \cdot \frac{\partial t_3}{\partial F_3} = \frac{1 - b(1 - fh) - nfh + m}{D^2} > 0 \quad \text{(II.5)}$$

where $D = 1 - b(1 - fh)(1 - t_1)(1 + t_3)^{-1} - nfh(1 - t_2) + m$

Thus in terms of changes in the degree of built-in flexibility the indirect tax has the strongest influence on α, whereas the relative importance of T_p and T_c depends upon $b(1 + t_3)^{-1}$ compared with n.

If we alternatively wish to analyze the effects on the degree of automatic stabilization of changes in the tax rates directly, we obtain:

$$\frac{\partial \alpha}{\partial t_1} = \frac{b(1 - fh)(1 + t_3)^{-1}(1 - b(1 - fh) - nfh + m)}{D^2} > 0$$

$$\frac{\partial \alpha}{\partial t_2} = \frac{nfh(1 - b(1 - fh) - nfh + m)}{D^2} > 0$$

$$\frac{\partial \alpha}{\partial t_3} = \frac{b(1 - fh)(1 - t_1)(1 + t_3)^{-2}(1 - b(1 - fh) - nfh + m)}{D^2} > 0 \quad \text{(II.6)}$$

For plausible values of the coefficients changes in t_3 are seen to have the smallest effect whereas the relative importance of t_1 and t_2 depends upon $b(1 - fh)(1 + t_3)^{-1}$ compared with nfh.

Speaking loosely the difference in ranking between (II.5) and (II.6) may be explained by the fact that it takes a substantial change in t_3 to change F_3 by "one unit"[22], and since α is positively affected by increases in t_3, a "one unit" change in F_3 will have a rather large effect on α.

It is, perhaps, more interesting to note that the results in (II.6) are the reverse of what is often found for discretionary fiscal policy. Thus, it is well known[23] that in order to obtain a certain reduction in total activity, t_1 and t_3 must be changed by equal amounts, whereas the required tax revenue changes are larger for T_1 than for T_2. Hence, for discretionary policy, changes in t_3 are

22. This is easily seen by partial differentiation of (II.4).

23. See for instance Peacock and Shaw (1971, pp. 89-90).

in a sense more effective than changes in t_1, but as seen from above the reverse holds for automatic stabilization.

For all taxes we find that automatic stabilization depends positively on built-in flexibility and marginal tax rates. In the above model we have assumed no money illusion, but if we alternatively formulate the consumption function as:

$$C(1 + t_3) = a + b(Y - RP - T_p)$$

it is easy to show that the stabilizing effect of the indirect tax will increase. On the other hand, if we have "positive" money illusion - i.e. C is a positive function of t_3 - the indirect tax will be destabilizing. This, however, does not seem very likely, so we shall not consider this case any further.

B. Automatic stabilization of Y for various exogenous variables

In the results derived above we have not specified which exogenous variable is changing, but it is easy to see that the results will hold for changes in a, c, G, Ex and k. This similarity, however, is due to the rather simple structure of the model. In large scale econometric models[24] it has been found that automatic stabilization very much depends on where the change takes place, and this again implies that α cannot be measured uniquely. In that case it seems most relevant to define α in relation to the exogenous component that has caused most of the changes and to evaluate the tax structure accordingly.

C. Automatic stabilization of other endogenous variables

Endogenous variables, which depend on income, normally will be stabilized, when income is stabilized. Thus, if imports are a positive function of income, the balance of payment will change less when autonomous demand for domestic products is shifting. If, on the other hand, the autonomous shifts occur in demand for foreign products or in exports, the changes in the balance of payment are accentuated, when taxes are endogenous, since income is now restrained in bringing the economy towards balance of payment equilibrium[25].

If prices are a positive function of nominal income, endogenous taxes will also reduce price movements, but this effect may be counterbalanced by the fact that private demand and government expenditures are influenced by

24. See Balopoulos (1967), Goldberger (1970), and Chalmers and Fischel (1967).
25. For derivation of a similar result from a somewhat different approach, see N. Kaldor (1960, p. 48).

price movements. Let us consider this problem in terms of the following simple model:

$$C = aP + b(Y - T)$$
$$I = \bar{I}$$
$$G = \bar{G} + cP$$
$$P = e + fY$$
$$T = tY$$
$$Y = C + I + G$$

where P denotes the level of prices and all variables are stated in nominal terms. We have assumed that investors are subject to money illusion - whereas consumers are not -, that the level of prices is determined partly autonomously and partly by income, and that the level of government expenditures is adjusted for price movements.

Solving for P we obtain:

$$P = (f(\bar{I} + \bar{G}) + e(1 - b(1 - t)))/(1 - b(1 - t) - f(a + c)). \qquad \text{(II.7)}$$

For exogenous taxes, we derive:

$$P_1 = (f(\bar{I} + \bar{G} - b\bar{T}) + e(1 - b))/(1 - b - f(a + c)), \qquad \text{(II.8)}$$

if G is adjusted for price changes, and

$$P_2 = (f(\bar{I} + \bar{G} - b\bar{T}) + e(1 - b))/(1 - b - fa) \qquad \text{(II.9)}$$

if G is not adjusted for price changes.

Computing the Musgrave-Miller measure for changes in e, we get respectively:

$$\alpha_1 = 1 - dP/dP_1 = btf(a + c)/(1 - b)(1 - b(1 - t) - f(a + c)) \qquad \text{(II.10)}$$

and

$$\alpha_2 = 1 - dP/dP_2 = f(tab - c(1 - b))/(1 - b)\ (1 - b(1 - t) - f(a + b)) \qquad \text{(II.11)}$$

α_1 is clearly positive, whereas α_2 may be negative - and the public sector thus destabilizing - if the adjustment factor for G is large and the tax rate small. With respect to the latter, two influences seem to be present under inflation. Thus most transfer payments vary directly with the level of prices causing a reduction in t, whereas the income tax schedule when based on nominal income produces an increase in t, if the tax schedule is progressive[26],[27].

26. For further evaluation of automatic stabilization under inflation see Gelting (1966, pp. 126-28).
27. A higher marginal tax rate, on the other hand, may give rise to demand for higher wages and subsequently higher prices. The effect of this destabilizing influence of a higher t can be worked out by making the autonomous element in the price equation a function of t.

Brown (1955) and Waldorf (1967) have analyzed built-in flexibility and the effect on price movements, using a stricter criterion than the one applied above. Thus they require, that the level of real taxes must increase, when prices rise, and this requirement leads - not surprisingly - to the conclusion that for autonomous changes in prices, the tax must be progressive in order to stabilize price movements.

III. Dynamic Models

When time is introduced into the model either through an explicit time variable or through lagged relationships, a number of problems arise, and as will appear from below, the interpretation of automatic stabilization becomes some what ambiguous. We shall analyze these problems in both short term models and under long term growth[28].

A. Short Term Models

Some of the problems mentioned above can be shown in the following very simple model for a closed economy, where for the sake of simplicity we only consider a personal income tax:

$$C_t = a + b(Y_t - T_t) + c(Y_{t-1} - T_{t-1}) \qquad c < b < 1$$
$$I_t = v Y_{t-1}$$
$$T_t = r Y_t$$
$$G_t = \overline{G}$$
$$Y_t = C_t + I_t + G_t$$

By substitution we get a first order difference equation in Y:

$$Y_t(1 - b(1 - r)) - Y_{t-1}(c(1 - r) + v) = a + \overline{G} \qquad \text{(III.1)}$$

The particular solution will be:

$$Y_p = (a + \overline{G})/(1 - b(1 - r) - c(1 - r) - v) \qquad \text{(III.2)}$$

and the homogenous solution is:

$$Y_h = A((c(1 - r) + v)/(1 - b(1 - r))^t, \qquad \text{(III.3)}$$

yielding a general solution:

$$Y_g = Y_h + Y_p.$$

28. The analysis is incomplete, as we mainly study lags from the point of view of automatic stabilization on the revenue side. Thus we do not at all deal with lags connected with discretionary changes in expenditures and revenues.

In order to determine the coefficient A we need an initial condition, which we may specify as $Y_{t=0} = Y_0$, and the complete solution, will then be:

$$Y_t = \left(Y_0 - \frac{a + \bar{G}}{1 - (b + c)(1 - r) - v}\right) \cdot \left(\frac{c(1 - r) + v}{1 - b(1 - r)}\right)^t + \frac{a + \bar{G}}{1 - (b + c)(1 - r) - v} \quad \text{(III.4)}$$

If, on the other hand, we assume taxes exogenous, we get the following complete solution:

$$Y_t = \left(Y_0 - \frac{a + \bar{G} - (b + c)\bar{T}}{1 - b - c - v}\right)\left(\frac{c + v}{1 - b}\right)^t + \frac{a + \bar{G} - (b + c)\bar{T}}{1 - b - c - v} \quad \text{(III.5)}$$

The last term in the above equations is the long term equilibrium for Y, and if we apply the Musgrave-Miller measure to the long run situation, we obtain:

$$\alpha = 1 - \frac{1/(1 - (b + c)(1 - r) - v)}{1/(1 - b - c - v)} = \frac{r(b + c)}{1 - (b + c)(1 - r) - v} \quad \text{(III.6)}$$

This expression can be interpreted as the relative reduction in the long run multiplier. α is positive, if the denominator is positive, and this again depends upon the stability of the long run equilibrium[29]. However, as appears, the endogenous income tax will increase the likelihood of having the stability condition satisfied.

For a short run analysis, however, a study of the long run equilibrium is not particularly interesting, as it may never be reached, and we therefore turn to the first term in equation (III.4), which will determine the time path of the system if an equilibrium situation is disturbed. This term consists of two parts, of which the first is the initial deviation from equilibrium and can be interpreted as a scale factor. The second part determines both the actual time path and the time required for reaching the new equilibrium. When comparing equations (III.4) and (III.5), we find that endogenous taxes have reduced both the initial disturbance[30] - and thus the scale factor - and the adjustment time, the latter because r will reduce the numerator and increase the denominator in the exponential part. In addition, endogenous taxes will increase the

29. In general the measures derived for automatic stabilization in the static model will correspond to long run automatic stabilization in a dynamic model. Cf. Balopoulos (1967, p. 243).

30. Thus the static Musgrave-Miller measure is applicable for evaluating the "scale effect", as it is determined by the multipliers. See also Thalberg (1971, p. 307).

likelihood of obtaining a stable solution, for which the condition is that the base of the exponential expression is absolutely < 1.

For other models, however, the situation is less clear. Thus, when it comes to higher order difference equations, it is not unlikely that we may have to compare systems, of which one is characterized by initially rather violent fluctuations, but a rapid adjustment time, and the other by a smooth movement towards the new equilibrium, but a rather long adjustment time. A possible way out may be to integrate over time the absolute deviations from equilibrium, but a priori it is difficult to judge, which system is the more stabilizing.

In the following we shall analyze a few examples where the emphasis will be on various tax structures, whereas the specification of the other relations is constrained by the fact that difference equations of order higher than two are rather difficult to evaluate without simulations or empirical measures[31].

1. Let us initially consider a model consisting of the following equations:

$$C_t = a + b(Y_t - T_t)$$
$$I_t = v\,Y_t$$
$$T_t{}^{32} = r(Y_{t-1} - T_{t-1})$$
$$G_t = \overline{G}$$
$$Y_t = C_t + I_t + G_t$$

Solving the system for Y, we get the solution[33]:

$$Y_t = \left(Y_o - \frac{(a+\overline{G})(1+r)}{1-b-v+r(1-v)}\right)\left(-\frac{r(1-v)}{1-b-v}\right)^t + \frac{(a+\overline{G})(1+r)}{1-b-v+r(1-v)} \qquad \text{(III.7)}$$

As appears, a high tax rate[34] will reduce the scale effect - and thus reduce the amplitude of the oscillations - but it will increase the adjustment time and

31. The method used is similar to that applied by Smyth (1963) and Gørtz (1971).

32. This formulation resembles a tax system, which Denmark had until 1967.

33. See appendix II.

34. When tax revenue in period t is a function of income in period $t-1$ we have to define built-in flexibility in terms of a certain time period, If t and $t-1$ refer to years, and the time period is 1 year, built-in flexibility in the above model will be 0, whereas it is r for a time period of 2 years. If the time period is 3 years built-in flexibility is $< r$, since we now have to take T_{t-1} into account, and in the long run the built-in flexibility will be $r/(1+r)$, which is less than r.
In the following we shall use built-in flexibility and marginal tax rate interchangeably.

the likelihood of getting antidamped oscillations, which occur when $r(1 - v)/(1 - b - v) > 1$[35].

If, on the other hand, the tax function had been $T_t = rY_{t-1}$ the complete solution of Y would have been:

$$Y_t = \left(Y_o - \frac{a + \bar{G}}{1 - b - v + br}\right)\left(-\frac{br}{1 - b - v}\right)^t + \frac{a + \bar{G}}{1 - b - v + br} \qquad \text{(III.8)}$$

Thus a high tax rate will reduce the scale effect, but it will still have a destabilizing influence on the adjustment path[36], although the influence of r is somewhat reduced, as b is in most cases less than $1 - v$. These results, however, very much depends on the lag structure specified for the other equations.

2. Next, let us consider the following model:

$$\begin{aligned} C_t &= a + b(Y_t - T_t) + c(Y_{t-1} - T_{t-1}) \\ I_t &= v(Y_{t-1} - Y_{t-2}) \\ T_t &= rY_{t-1} \\ G_t &= \bar{G} \\ Y_t &= C_t + I_t + G_t \end{aligned}$$

which yields the following equation for Y:

$$Y_t(1 - b) - (c + v - br)Y_{t-1} + (v + cr)Y_{t-2} = a + \bar{G} \qquad \text{(III.9)}$$

As we now have a second order difference equation, we shall confine ourselves to analyzing the roots, as analysis of the complete solution gets rather complicated.

Solving the above equation we find complex roots and fluctuations if $(c + v - br)^2 < 4(v + cr)(1 - b)$, and the fluctuations are damped for $(v + cr)/(1 - b) < 1$. If the roots are real, a stable solution is obtained[37] for $(c - r(b + c))/(1 - b) < 1$. Hence we find, that a high degree of built-in flexibility will increase the likelihood of getting a fluctuating adjustment path and of getting antidamped fluctuations, implying that a new equilibrium may not be reached. On the other hand, when the roots are real and the adjustment path smooth, a high degree of built-in flexibility will make the system converge faster to a new equilibrium.

35. It is interesting to note that Thalberg (1971, p. 309) comes to the same conclusion; viz. that there tends to be explosive fluctuations in Y, when the lag in the tax function is longer than the lag in the consumption function.

36. Thus a high r will increase the likelihood of getting antidamped oscillations.

37. We can assume both roots positive, since for most cases $br - c - v < 0$, $v + cr > 0$ and $1 - b > 0$. For further explanation of the method used for deriving the above conditions see app. 1.

3. Let us now change the tax function to[38]

$$T_t = r_1 Y_t + r_2 Y_{t-1} \qquad r_1 > r_2$$

whereas the other functions from case 2. are retained.

Solving for Y we obtain:

$$Y_t(1 - b(1 - r_1)) - Y_{t-1}(c(1 - r_1) - br_2 + v) + Y_{t-2}(cr_2 + v) = a + \overline{G} \qquad \text{(III.10)}$$

which will yield complex roots if $(c(1 - r_1) - br_2 + v)^2 < 4(cr_2 + v)(1 - b(1 - r_1))$. The roots will furthemore yield antidamped fluctuations for $(cr_2 + v)/(1 - b(1 - r_1)) > 1$. From these conditions we observe that r_1 will have a stabilizing influence, whereas a high r_2 will be destabilizing. Thus, lags in the tax structure will tend to accentuate autonomous disturbances, and this effect would be even stronger, if we also took monetary effects into account.

When the roots are real, and we have a smooth adjustment path, the stability condition is[39] $1 - c - b > -(b + c)(r_1 + r_2)$, and we find that a high degree of built-in flexibility will increase the probability of convergence, regardless of whether taxes are lagged or unlagged.

4. Let us finally consider a case where consumption is assumed a distributed lag function of disposable income:

$$C_t = a + b(\lambda(Y_t - T_t) + \lambda^2(Y_{t-1} - T_{t-1}) + \ldots$$
$$\ldots + \lambda^n(Y_{t-n+1} - T_{t-n+1}) + \ldots)$$

where $0 < \lambda < 1$ and $\sum_{i=1}^{\infty} \lambda_i = 1$. By a Koyck-transformation this function can be reduced to:

$$C_t = a(1 - \lambda) + b\lambda(Y_t - T_t) + \lambda C_{t-1}$$

In order to avoid third order equations we assume the following equation for respectively investment and taxes:

$$I_t{}^{40} = v\,Y_{t-1}$$
$$T_t = r\,Y_t$$

38. The argument behind this formulation is that the tax payments occuring in year t for institutional reasons are determined by both present and past income. Consequently we also have to assume that consumption is determined by tax payments and not by tax liabilities. Another implication of the formulation is that r_1 and r_2 do not refer to actual tax rates.

39. We can again assume both roots positive, cf. app. 1.

40. An investment function based on the capital stock adjustment principle can be dealt with as the distributed lag consumption function, if the model is supplemented with a function for the capital stock ($K_t = I_t + K_{t-1}$).

Solving for Y we then get[41]:

$$Y_t(1 - \lambda b(1 - r)) - Y_{t-1}(\lambda + v) + Y_{t-2}v\lambda = (a + G)(1 - \lambda) \quad \text{(III.11)}$$

and the roots are complex if $(\lambda + v)^2 < v\lambda\,(1 - \lambda b\,(1 - r)$, but yield damped fluctuations for $v\lambda/(1 - b\lambda\,(1 - r)) < 1$. Hence, a high tax rate will increase the likelihood of a fluctuating but damped adjustment path.

For real roots and a smooth adjustment path, we get convergence for $(1 - \lambda)\,(1 - v) - b\lambda(1 - r) > 0$, and a high tax rate is seen to stabilize the adjustment.

The above four cases should only be considered illustrations of the appropriate analysis of automatic stabilization in dynamic short term models, and consequently the conclusions we have arrived at are very tentative. However, under the present Danish personal income tax system, a high degree of built-in flexibility seems to have a stabilizing influence on the adjustment path, especially if there are no lags in tax payments. It should be kept in mind, however, that the models used are extremely simple, and that any realistic model would lead to difference equations of order higher than two, which are too complicated to discuss when the structural coefficients are unknown.

B. Long term growth

A number of economists[42] have discussed the effects of discretionary fiscal policy on the equilibrium rate of growth and on deviations from this equilibrium. This discussion, however, is not the subject of the following sections, where we merely analyze the stabilizing or destabilizing effects of a given tax and expenditure structure.

These effects are often thought of in terms of "fiscal drag", which conceptually corresponds to the automatic budget reaction. Thus, if government expenditures are constant, "fiscal drag" will correspond to the automatic increase in tax revenue, when full employment income increases, and it is therefore equivalent to Pearse's static measure of automatic stabilization[43, 44].

41. See appendix II.

42. See for instance: Kurihara (1956), Gurley (1953) and Musgrave (1959, ch. 20).

43. "Fiscal drag" should be distinguised from "fiscal leverage" (cf. Musgrave (1964)), which refers to $dG\text{-}bf'(q)\ dq$ — i.e. the impact of a discretionary change in the budget — and from "changes in full employment budget surplus", which most often refers to $dG—f'(q)\,dq$; i.e. the "unweighted" budget change at full employment, when discretionary changes occur. $f(Y,q)$ denotes the tax function (cf. p. 3) and b the marginal propensity to spend.

44. Note that "fiscal drag" is independent of the source of the income change; cf. Bent Hansen (1969, pp. 23-24).

Despite this resemblance, it does not seem appropriate to interpret automatic stabilization in a long run growth model exclusively as "fiscal drag". Whether a "fiscal drag" occurs will depend on the tax function compared with the government expenditure function[45], and whether the postulated contractive effect occurs depends upon the development in private expenditures compared with the government budget. In addition, however, it is of interest to analyze to what extent the existence of endogenous taxes may dampen possible deviations from the equilibrium path, and this problem is entirely different from that of "fiscal drag". Both questions will be dealt with in terms of a modified Harrod-Domar model and a neoclassical model.

Harrod-Domar model. The former has also been used by Peacock and Shaw[46] and is specified as follows:

$$C_t = b(Y_{t-1} - T_{t-1})$$
$$I_t = c\,Y_t$$
$$G_t = g\,Y_t$$
$$T_t = r\,Y_t$$

Assuming that p % of G is invested, and that the capital output ratio is $1/q$ the annual increase in productive capacity is qdY_{t-1} where $d = c + pg$, and the equilibrium rate of growth is[47]:

$$dY/Y = b(1-r)/(1-(c+g)) - 1 = qd \qquad \text{(III.12)}$$

For given values of b, c, g, p and q it is possible to find a tax rate that will secure full capacity growth. Thus for $b = 0.9$, $c = 0.15$, $g = 0.1$, $p = 0.5$, and $q = 0.40$, the equilibrium tax rate is 0.1, and if that rate is adopted, the development of income will be as shown in the second column of table 1, where column 1 indicates growth in productive capacity. If $r < 0.1$, income will grow faster than productive capacity, and if $r > 0.1$ income will grow less than capacity, implying that we have a "fiscal drag".

As the concept of automatic stabilization deals with the response of the

45. Thus for the Danish economy there does not seem to be any problem of insufficient increase in government expenditures!

46. See Peacock and Shaw (1971, pp. 125 ff).

47. We have $Y_t = b(1-r)Y_{t-1} + cY_t + gY_t$ or

$$Y_t = b(1-r)Y_{t-1}/(1-c-g) \quad \text{or}$$

$$dY_t/Y_{t-1} = b(1-r)/(1-c-g) - 1$$

which in equilibrium must equal the growth in capacity.

economy to autonomous changes, we shall next turn to the question of how the existence of a public sector will affect this response. Let us assume that in period 1 c increases to 0.2 the other parameters remaining unchanged. This change will increase the capacity rate of growth from 8 % to 10 %, and we find a development in capacity as shown in column 3. If the tax rate remains unchanged at 0.1, income will grow as shown in column 4, and we notice an increasing discrepancy between capacity and income.

An expression for automatic stabilization corresponding to the static measure might be obtained by deriving the effect on the demand determined rate of growth with and without the public sector. This will yield:

$$\alpha = 1 - \frac{dn_2/dc}{dn/dc} = 1 - \frac{b(1-r)/(1-c-g)^2}{b(1-c)^2}$$
$$= \frac{(1-c-g)^2 - (1-r)(1-c)^2}{(1-c-g)^2} \qquad \text{(III.13)}$$

where n, and n_2 denote the demand determined rates of growth. If this expression is negative, the public sector will have a destabilizing influence, and whether this occurs will clearly depend upon the tax rate in relation to g, whereas the proportion of G that is spent on public investments does not have any influence.

Alternatively - and in our opinion more satisfactory - we can make the tax function more realistic by introducing a progressive rate structure and compare the results of this change to either a system with proportional taxes or a system with no public sector at all. Following Peacock and Shaw we shall introduce a progressive structure by making the tax rate a function of the rate of growth of income:

(1) $r_t = r_{t-1} + s(Y_t - Y_{t-1})$
(2) $r_t = r_{t-1} + s(Y_{t-1} - Y_{t-2})$

These formulations do not correspond completely to the actual determination of a progressive rate structure, but if we take lags into consideration, the above formulations are quite realistic if T refers to tax payments and not to tax liabilities, and they furthermore simplify the calculations[48].

Starting with $r_{t-1} = 0.1$ and assuming $s = 0.0003$ the two formulations yield the results shown in respectively columns 5 and 6 in table 1. Peacock and

48. We might alternatively let tax *revenue* be a function of the rate of change in income. This is done by Musgrave (1959, p. 512) to illustrate "formula flexibility".

Shaw only consider the second formulation of the progressive rate structure and conclude that a progressive structure is destabilizing compared with a proportional structure as it will overshoot the new equilibrium growth rate, when changes in the economic structure occur.

This conclusion, however, does not seem warranted, since a proportional rate, which is an equilibrium rate before the change in c, overshoots to a much greater extent than the progressive rate. In other words, with a proportional tax rate a discretionary policy is required to bring the economy back to equilibrium growth, whereas under a progressive structure, the deviations from equilibrium growth are damped, although the system will not automatically be brought to the new equilibrium growth.

The first formulation of the progressive rate structure yields a desirable dampening influence immediately after the change in c, but later on the dampening is too strong, the deviations of $Y_3{}^m$ from $Y_2{}^c$ becoming absolutely larger than the deviations of $Y_2{}^m$.

TABEL 1. *Development in Capacity and Income*

	1	2	3	4	5	6
Year	$Y_1{}^c$	$Y_1{}^m$	$Y_2{}^c$	$Y_2{}^m$	$Y_3{}^m$	$Y_4{}^m$
1	1000	1000	1000	1000	1000	1000
2	1080	1080	1100	1157	1113	1157
3	1166	1166	1210	1339	1080	1269
4	1259	1259	1331	1549	1104	1336
5	1360	1360	1464	1792	1095	1373

NOTE: Y^c = productive capacity; Y^m = demand determined income.

The two formulations of the progressive rate structure have the common feature that they sooner or later will produce a "fiscal drag", unless g also changes with the rate of growth of income. In that sense a progressive rate structure may be undesirable and destabilizing. If, however, we consider stabilization in relation to the system's response to autonomous changes, a "lagged" progressive tax is more stabilizing than a proportional rate structure and consequently also more stabilizing than a system with no public sector. As appears from the table the progressive tax will damp the deviations from equilibrium growth and thus give the government more time for introducing the appropriate discretionary changes. An "unlagged" progressive structure, on the other hand, is destabilizing except for the first year, and it does not leave much time for the authorities to introduce changes. Therefore, from the point of view of stabilizing economic growth, a with-holding principle combined with a strongly progressive structure seems unfortunate.

Neoclassical growth model. As pointed out by several authors there are many possibilities for affecting growth rates in a Harrod-Domar model, but when it comes to neoclassical models, the influence of fiscal policy is rather limited, as the equilibrium growth rate is determined exogeneously. However, with a given rate of growth, it is still possible to affect the level of consumption through fiscal interventions, and furthermore discretionary fiscal policy may shorten the adjustment time[49].

Furthermore, fiscal policy may still have an important influence on the *actual* rate of growth, which probably is a much more relevant variable from a policy point of view. Consequently, the practical use of the following section may seem very limited, as we have only analyzed automatic stabilization within a neoclassical equilibrium framework. The model used is as follows:

$$\begin{aligned} Y_t &= e^{pt} K_t^a L_t^b \qquad \text{where } a + b = 1 \\ C_t &= (1 - s)(Y_t - T_t) \\ G_t &= r Y_t \\ T_t &= r Y_t \\ G_t^I &= mr Y_t \\ I_t &= K_{t+1} - K_t = c Y_t - d K_t, \text{ where } c = s(1 - r) + rm \\ L_t &= e^{nt} L_0 \end{aligned}$$

L and K denote respectively labour and capital, and I denotes net private and public investment. The rest of the notation is explained in earlier sections. As appears, we assume disembodied technical changes and a "natural budget reaction", since total tax revenue is spent on either consumption or investment[50].

The rates of growth of income and capital are respectively

$$\mathring{Y}/Y = p + a\mathring{K}/K + bn \qquad \text{and} \tag{III.14}$$

$$\mathring{K}/K = cq - d, \tag{III.15}$$

where q = the output/capital ratio, and d = the rate of depreciation.

49. See Peacock and Shaw (1971), and Cornwall (1963 and 1965).

50. Alternatively we might assume G constant or a positive function of Y. It makes no difference for the results derived below, as long as the model has to satisfy the condition that total investments = total savings.

We therefore have $\mathring{Y}/Y = p + a(cq - d) + bn$ and for $\mathring{Y}/Y = \mathring{K}/K$

$$\mathring{Y}/Y = p + a\,\mathring{Y}/Y + bn = p/b + n. \qquad \text{(III.16)}$$

Thus, the equilibrium rate of growth is given exogenously and cannot be influenced by fiscal policy. If, however, an equilibrium situation is disturbed, the fact that we have endogenous taxes may influence the time required for returning to the equilibrium movement.

If we assume that the private propensity to save increases from say s_o to s_1[51], the actual rate of growth will deviate positively from the equilibrium rate of growth, but after some time the system will return to the equilibrium, where the output/capital ratio is now lower and the per capita income higher.

As pointed out by R. Sato (1963), the adjustment time may be considerable, and we shall follow his method in analyzing the factors influencing the adjustment time. It is natural to consider the adjustment path for q, since q is constant in equilibrium. Consequently one can derive a difference equation for q, and on the basis of this equation it is possible to compute the following expression[52] for the time required to obtain k % of the total adjustment:

$$t_k = \frac{\log(1 + (s_o(1-r) + rm)\,k/s_1(1-r) + rm)\,(1-k))}{p + b(n+d)} \qquad \text{(III.17)}$$

Clearly, endogenous taxes affect adjustment time through the second term in the numerator, and differentiating this term with respect to r, we obtain:

$$\partial t_k/\partial r = \frac{k(1-r)\,(s_1 - s_o)\,m}{(s_1(1-r) + rm)^2(1-k)} > 0 \qquad \text{(III.18)}$$

Hence, we find that endogenous taxes will prolong the adjustment time, and a high degree of built-in flexibility will in that sense be destabilizing. This result may seem surprising, but can probably be explained by the fact that the existence of endogenous taxes will increase total investments and consequently the rate of growth of capital during the disequilibrium period. Since the latter is characterized by $\mathring{K}/K > \mathring{Y}/Y$, the relative increase in $\mathring{K}/K$ implies that it takes longer before $\mathring{K}/K$ returns to the same level as $\mathring{Y}/Y$.

Concluding this section we note, that in a neoclassical growth model the problem of "fiscal drag" is excluded by definition, as total savings always

51. We have also analyzed disturbances caused by changes in p, n and b, but in no case did the tax rate have any influence on the adjustment time.

52. Derivation of this result has been omitted due to lack of space, but is available upon request from the author. The derivation follows very closely the method applied by R. Sato.

equal total investments. But, of course, this observation is not very helpful in a situation, where "fiscal drag" *is* a problem.

Conclusion

In this paper we have endeavoured to analyze how automatic stabilization is defined and determined in various models with particular emphasis on the influence of the tax structure. The static conclusion that automatic stabilization is a positive function of marginal tax rates - or built-in flexibility - holds for some of the dynamic formulations but not for others, and it becomes particularly doubtful when we turn to long run growth in a neoclassical setting.

Since any realistic empirical model will contain lagged relationships, the static measure is inappropriate, except for long run equilibrium changes. On the other hand, the dynamic measures are not very useful either, as long as the lag structure and the parameters of the model are unknown. Similarly, it should be borne in mind that both the static and the dynamic measures are relative in the sense, that whether they are "high enough" depends upon how much there is to stabilize. This, as well as a more exact evaluation of the automatic stabilization in a dynamic model, is only possible on the basis of empirical measurements, which, however, are beyond the scope of this article.

APPENDIX I

In setting up the stability conditions for second order difference equations we have made use of figure 1[53] below, which is derived from the following expression for the difference equation:

$$Y_t + b\,Y_{t-1} + c\,Y_{t-2} = 0.$$

In all the cases we have considered $c > 0$, implying that the roots have the same sign, and b is either positive or negative. In the complex roots case the modulus $= \sqrt{c}$, implying that $c < 1$ is a stability condition. Therefore, any combination of c and b in III and IV will yield a stable solution. In the real roots case we are to the left of the parabola, but as $c > 0$ a stable solution is confined to the areas I and II. The condition for being in one of those areas *given* that we have a real solution can be stated in one equation; viz.

$b > -1 - c$ for $b < 0$ and

$b < 1 + c$ for $b > 0$

53. See Baumol (1970, pp. 219-24).

equal total investments. But, of course, this observation is not very helpful in a situation, where "fiscal drag" *is* a problem.

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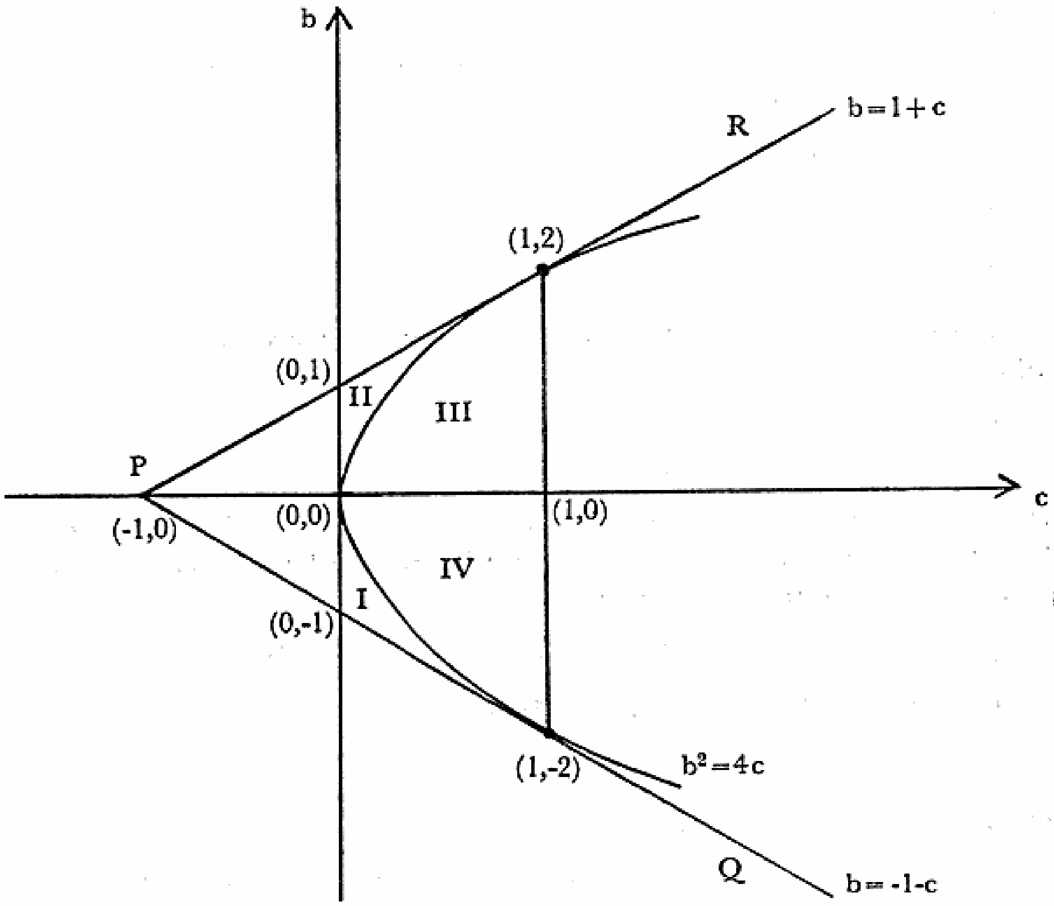

FIG. I

This is due to the fact that the tangency between *PQ* and *PR* and the parabola occurs in respectively $(-2,1)$ and $(2,1)$. Therefore, if we know that we have a real solution and that we are below *PR* or above *PQ*, the condition $c < 1$ is automatically satisfied.

APPENDIX II

In cases 1, and 4 the derivation of the equation for Y_t cannot be obtained by simply inserting in $Y_t = C_t + I_t + G_t$, as several of the endogenous variables are lagged. We shall, therefore, show how the second order equation is derived for case 4. For easy reference we repeat the model being used:

$$C_t = a(1-\lambda) + b\lambda(Y_t - T_t) + \lambda C_{t-1} \qquad (1)$$

$$I_t = v Y_{t-1} \qquad (2)$$

$$T_t = r Y_t \qquad (3)$$

equal total investments. But, of course, this observation is not very helpful in a situation, where "fiscal drag" *is* a problem.

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$$b > -1 - c \quad \text{for} \quad b < 0 \quad \text{and}$$
$$b < 1 + c \quad \text{for} \quad b > 0$$

53. See Baumol (1970, pp. 219-24).

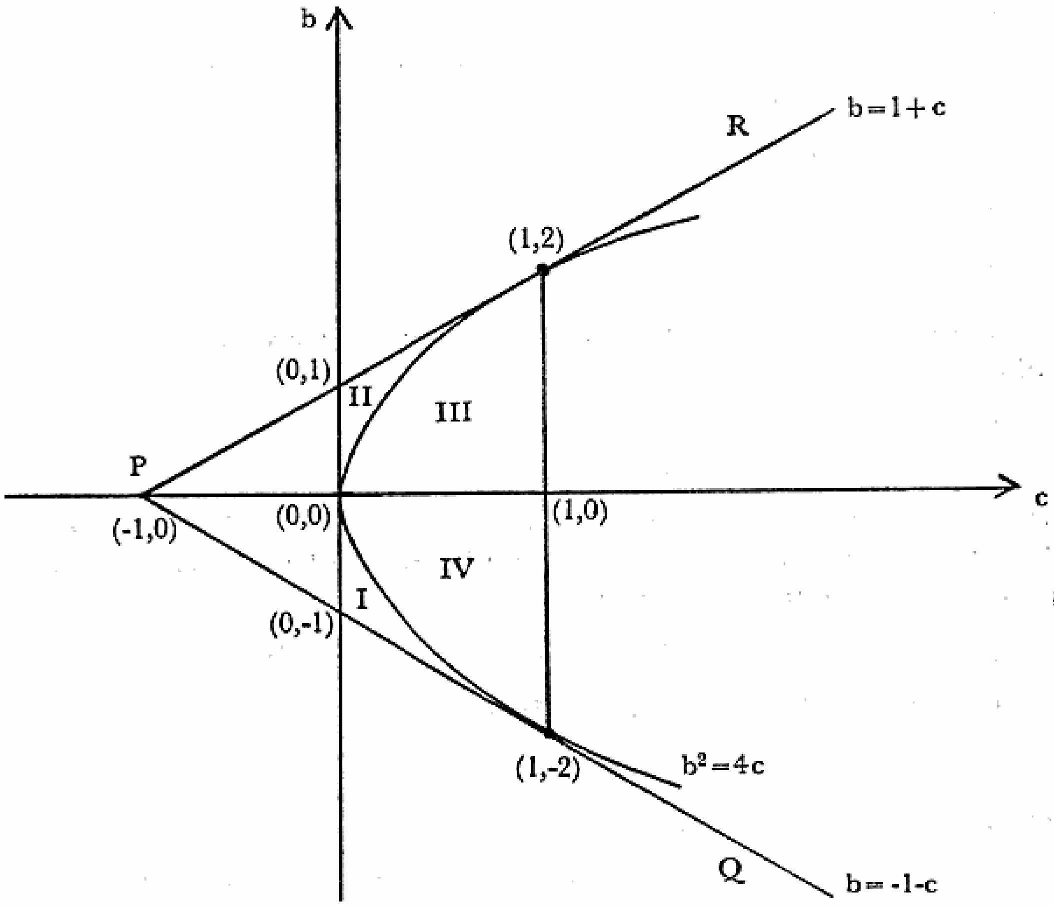

FIG. 1

This is due to the fact that the tangency between PQ and PR and the parabola occurs in respectively (—2,1) and (2,1). Therefore, if we know that we have a real solution and that we are below PR or above PQ, the condition $c < 1$ is automatically satisfied.

APPENDIX II

In cases 1, and 4 the derivation of the equation for Y_t cannot be obtained by simply inserting in $Y_t = C_t + I_t + G_t$, as several of the endogenous variables are lagged. We shall, therefore, show how the second order equation is derived for case 4. For easy reference we repeat the model being used:

$$C_t = a(1 - \lambda) + b\,\lambda\,(Y_t - T_t) + \lambda\,C_{t-1} \tag{1}$$

$$I_t = v\,Y_{t-1} \tag{2}$$

$$T_t = r\,Y_t \tag{3}$$

$$G_t = \bar{G} \tag{4}$$
$$Y_t = C_t + I_t + G_t \tag{5}$$

Inserting (2) and (4) into (5) we get:

$$Y_t = C_t + v\,Y_{t-1} + \bar{G} \tag{a}$$

Inserting (3) into (1) we further have:

$$C_t = a(1-\lambda) + b\lambda\,(1-r)\,Y_t + \lambda C_{t-1} \tag{b}$$

Introducing the lag operator E and using matrix notation, (a) and (b) can be written:

$$\begin{bmatrix} 1 - Ev & , -1 \\ -b\lambda(1-r), & 1 - E\lambda \end{bmatrix} \cdot \begin{bmatrix} Y_t \\ C_t \end{bmatrix} = \begin{bmatrix} \bar{G} \\ a(1-\lambda) \end{bmatrix}$$

We can then find Y_t as D_y/D, where D = the determinant of the coefficient matrix, and D_y = the determinant of the coefficient matrix with the first column replaced by $[\bar{G}, a(1-\lambda)]$. Alternatively we can write $D \cdot Y_t = D_y$, obtaining

$$(1 - Ev - E\lambda + E^2\lambda\,v - b\lambda(1-r))\,Y_t = (a + \bar{G})\,(1-\lambda) \qquad \text{or}$$
$$(1 - b(1-r))\,Y_t - (v+\lambda)\,Y_{t-1} + \lambda v Y_{t-2} = (a + \bar{G})\,(1-\lambda)$$

APPENDIX III: *List of Literature*

BALOPOULOS, E. T. 1967. *Fiscal policy models of the British economy*. Amsterdam.

BAUMOL, W. J. 1970. *Economic dynamics*. 3. ed. New York.

BROWN, E. CARY. 1955. The static theory of automatic fiscal stabilization. *Journal of Political Economy* 63: 427-40.

CASSIDY, HENRY J. 1970. Is a progressive tax stabilizing?. *National Tax Journal* 23: 194-205.

CHALMERS, JAMES A. and WILLIAM A. FISCHEL. 1967. Analysis of automatic stabilizers in a small econometric model. *National Tax Journal* 20: 432-43.

CORNWALL, J. 1963. Three paths to full employment growth. *Quarterly Journal of Economics* 77: 1-25.

CORNWALL, J. 1965. The structure of fiscal models. *Quarterly Journal of Economics* 79: 608-22.

EILBOTT, PETER. 1966. The effectiveness of automatic stabilizers. *American Economic Review* 56: 450-64.

FRIEDMAN, MILTON. 1948. A monetary and fiscal framework for economic stability. *American Economic Review* 38: 245-64.

GELTING, JØRGEN. 1966. Finanspolitikken under inflation. In *Inflationens Aarsager*. Betænkning No. 421. København.

GOLDBERGER, A. S. 1970. *Impact multipliers and dynamic properties of the Klein-Goldberger model*. Amsterdam.

GURLEY, JOHN G. 1953. Fiscal policy in a growing economy. *Journal of Political Economy* 61: 523-35.

GØRTZ, ERIK. 1971. *Stabilization policy,* *adjustment and income distribution.* Aarhus.

HANSEN, BENT. 1959. Statsbudgetens verkningar. *Ekonomisk Tidskrift* 61: 128-46.

HANSEN, BENT. 1969. *Fiscal policy in seven countries 1955-65.* Publ. by OECD. Paris.

KALDOR, NICHOLAS. 1960. *Essays on economic stability and growth.* London.

KULLMER, LORE. 1965. Die Praktische Bedeutung der Steuerprogression für die Grösse der Aufkommenselasticität einer Steuer. *Public Finance* 20: 137-49.

KULLMER, LORE. 1965. Zur Aufkommenselasticität einer Progressiven Einkommensteuer in einer Wachsender Wirtschaft. *Public Finance* 20: 411-19.

KURIHARA, K. 1956. Growth models and fiscal policy parameters. *Public Finance* 11: 148-60.

LUSHER, DAVID. 1956. The stabilizing effectiveness of budget flexibility. In *Policies to Combat Depression,* publ. by National Bureau of Economic Research, pp. 77-89.

MATTHIESSEN, LARS. 1961. En not om statsbudgetens verkningar. *Ekonomisk Tidskrift* 63: 231-46.

MILLER, M. H. and RICHARD A. MUSGRAVE. 1948. Built-in flexibility. *American Economic Review* 38: 122-28.

MUSGRAVE, RICHARD A. 1959. *The theory of public finance.* New York.

MUSGRAVE, RICHARD A. 1964. On measuring fiscal performance. *Review of Economics and Statistics* 46: 213-20.

PEACOCK, A. and G. K. SHAW. 1971. *The economic theory of fiscal policy.* London.

PEARSE, P. H. 1961/62. Automatic stabilization and the British taxes on income. *Review of Economic Studies* 29: 124-39.

SATO, R. 1963. Fiscal policy in a neoclassical growth model: An analysis of time required for equilibrium adjustment. *Review of Economic Studies* 30: 16-23.

SATO, K. 1966. On the adjustment time in neoclassical growth models. *Review of Economic Studies* 33: 263-68.

SMITH, PAUL E. 1963. A note on the built-in flexibility of the individual income tax. *Econometrica* 31: 704-11.

SMYTH, D. J. 1963. Can automatic stabilizers be destabilizing. *Public Finance* 18: 357-64.

SMYTH, D. J. 1966. Built-in flexibility of taxation and automatic stabilization. *Journal of Political Economy* 74: 396-400.

SMYTH, D. J. 1970. Tax changes linked to government expenditure changes and the magnitude of fluctuations in national income. *Journal of Political Economy* 78: 60-67.

TANZI, VITO. 1966. A dynamic self-adjusting income tax. *Public Finance* 21: 507-22.

THALBERG, B. 1971. Stabilization policy and the non-linear theory of the trade cycle. *Swedish Journal of Economics* 73: 294-311.

WALDORF, WILLIAM H. 1967. The responsiveness of federal income taxes to income change. *Survey of Current Business,* December 1967, Vol. 47, pp. 32-45.

YATES, JUDITH. 1971. *Optimal policies for stabilization and growth.* Rotterdam.