Aggregate Consumption and Investment Functions for the Household Sector considered in the Light of British Experience [1])

By RICHARD STONE [2]) and D. A. ROWE [3])

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1. Introduction

SINCE Kahn's original article on the multiplier [8], published in 1931, and early attempts to determine statistically the multiplier and the marginal propensity to consume, the examination of various forms of the consumption function has become one of the most intensive fields of econometric research. It seems plausible that aggregate consumption should be related in a comparatively simple way to aggregate income but experience shows that it is

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1. Introduction

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more difficult than might be supposed to find a satisfactory relationship of this kind. Early studies of the consumption function obtained an apparent success with very simple hypotheses such as that consumption is a linear function of income plus a disturbance term which expresses the effect of other influences not explicitly introduced. Numerous variants of this relationship were tried. Consumption and income were expressed in real terms. They were expressed per head of the population. A trend was introduced into the relationship. These and similar devices made it possible to find close relationships between annual time-series of consumers' purchases and income relating to the nineteen twenties and thirties and, in some instances, to longer periods as well [13].

Simple statistical investigations on such lines were not without their critics but for a time it could at least be said that the relationships seemed to 'work'. Various reasons were advanced why in theory they should not work but it was not until they demonstrably failed to do so that any substantial step forward was made.

The arrangement of this paper is as follows. Section 2 contains a brief account of the main improvements that have been suggested in the formulation of the aggregate consumption function. In section 3 it is shown how these improvements can, to a large extent, be accommodated within a system of ideas in which the aggregate consumption function is derived by adding up the equations for individual commodities which appear in a simple, dynamic, theory of demand. In this theory consumers' purchases are divided between consumption and net investment and a distinction is drawn between actual consumption and equilibrium consumption. In the case of durable goods, an excess of equilibrium over actual consumption involves an addition to stocks. Net investment takes place to reduce the gap between equilibrium and actual stocks and the consumption of each commodity comes to depend on income, prices, capital and the consumption of the previous period. This theory contains, in addition to the long-period income and price parameters, familiar in static theory, additional parameters associated with durability and with the rate at which adjustment to equilibrium is attempted in the case of different commodities.

Sections 4 through 8 are concerned with the further clarification and development of the basic theory. In section 4, a distinction is drawn between expenditure functions, which include net investment, and consumption functions. Section 5 is concerned with the use of index-numbers. In principle, the aggregate functions could be built up from empirical studies of individual commodities but this is not done in the present paper. Instead, the basic relationships are reformulated in terms of index-numbers of consumption, retail prices and the like and the parameters in these functions are estimated. Section 6 is concerned with the introduction of some approximation to capital into the analysis in the absence of any statistics on this subject. The method adopted follows Brumberg [2] in introducing last year's saving (less the amount of net investment in consumers' durables) into an equation for the year-to-year change in consumption. In section 7, the consequences are examined of assuming that consumers adjust not to current income but to some average of recent income experience. Section 8 is devoted to the subdivision of income. It is usually supposed that the immediate response to an income change is likely to be smaller for the rich than for the poor and a reason for this is given in terms of the basic theory. The empirical investigation shows that there is in fact a significant difference of this kind.

Sections 9, 10 and 11 contain the results of applying these ideas to British experience over the interwar period (1924—38) and over this and the postwar period (1947—54) combined. The equations used all involve the first or second differences of the dependent variable and yield, directly, estimates of year-to-year changes or rates of change.

Section 12 sets out the results obtained by combining certain interwar equations with postwar data on changes in the determining variables to estimate changes in saving in the postwar period. These results, which are encouraging, are compared with calculations based on equations fitted over the whole period.

Section 13 contains a summary of the conclusions reached in the paper and section 14 contains a list of works cited. A brief appendix sets out the sources of the basic data used.

2. The Development of the Consumption Function

This section contains an account of the development of the consumption function. It is in no sense complete and is concentrated on certain basic suggestions which are taken into account in later sections of the paper. It provides a background for the ideas set out below rather than an historical survey.

For the present purpose it is sufficient to consider only simple linear relationships and to ignore the effect of disturbing factors. The simple linear consumption function may be expressed in the form

$$v = \varphi \mu + \chi \tag{1}$$

where v and μ denote aggregate consumption and disposable personal income respectively expressed per head of the population. In this formulation φ denotes the marginal propensity to consume and χ denotes a constant. If actual behaviour could be represented by this relationship then a change in income of $\delta\mu$ would change consumption by $\varphi\delta\mu$ whether this change represented normal growth or cyclical fluctuation. With this form of the consumption function, the short- and long-period marginal propensities to consume are the same.

Modigliani's formulation [9], designed to make possible a distinction between the two propensities, consists in making χ in (1) depend simply on the previous highest level of income per head, μ_0 say. Thus (1) is changed to

$$v = \varphi\,\mu + \psi\,\mu_0 \tag{2}$$

In the case of (2) the short-period marginal propensity to consume is still measured by φ. A value for the long-period marginal propensity can be obtained by considering what would happen if μ were to change from μ_0 to μ_1 and then remain at μ_1. Before the previous peak of income changes from μ_0 to μ_1, the increase in consumption will equal $\varphi(\mu_1 - \mu_0)$ but thereafter it will be increased to $(\varphi + \psi)(\mu_1 - \mu_0)$. Similarly if income is assumed to grow at a steady rate, so that μ is related to μ_0 by the expression

$$\mu = (1 + \varkappa)\,\mu_0, \tag{3}$$

there results, on combining (2) and (3),

$$v = [\varphi + \psi/(1+\varkappa)]\,\mu \tag{4}$$

Thus, under conditions of steady growth, consumption is proportional to income and the average and marginal propensities to consume are given by the term in square brackets. This term reduces to $\varphi + \psi$ if $\varkappa = 0$; if $\varphi + \psi = 1$, (4) would imply the equality of consumption and income in a stationary state.

Duesenberry's formulation [6] of the consumption function is similar to (2) except that μ_0 is replaced by μ^2/μ_0. The symmetry of the two formulations is more apparent if the equations are written out not for consumption but for the consumption-income ratio, since the determining variable, μ_0/μ in this form of (2), would simply be replaced by its reciprocal in Duesenberry's formulation.

The argument for including the previous peak of income in the consumption function is that the continued growth of real income accustoms people to a rising standard of living which they are reluctant to abandon in the face of falls in income which may prove to be only temporary. It seems reasonable then to consider the hypothesis that it is the previous peak in consumption rather than in income which should be introduced. This is Davis' suggestion [5] and may be written as fortegnet på — men Stone begynder så at regne med symbo megen forsigtighed. Ikke alle regneregler kan anvendes

$$v = \varphi\,\mu + \psi\,v_0 \tag{5}$$

where v_0 denotes the previous peak in consumption. It should be noted that while both (2) and (5) have the merit of introducing past values they do this in a rather inflexible way. As a consequence they might be adequate to describe behaviour in a fairly steadily growing economy but they could hardly be expected to be of general validity. For example they would not be useful in describing the relationship of consumption to income over the life cycle.

The kind of spending and saving behaviour associated with (5) can be expressed in an especially simple form if it can be assumed that $\varphi + \psi = 1$. For in this case

$$\begin{aligned} v &= \varphi\,\mu + (1-\varphi)\,v_0 \\ &= v_0 + \varphi\,(\mu - v_0) \end{aligned} \tag{6}$$

and saving, σ, is given by

$$\sigma = (1-\varphi)\;(\mu - v_0) \tag{7}$$

Equation (7) shows that, with a low value of φ, the short-period marginal propensity to consume, the community will depart only slowly from its previous peak of consumption v_0. If $\mu < v_0$ it will reduce its current consumption below its previous peak by $\varphi\,(\mu - v_0)$; if $\mu > v_0$ it will increase its current consumption above its previous peak by $\varphi\,(\mu - v_0)$. In the second case it will raise the value of v_0 for future periods and so if income remains unchanged in the next period it will further increase its consumption. Similarly (7) shows that, with a small value of φ, the greater part of the excess of μ over v_0 will be saved or, if negative, dissaved.

Equations (2) and (5), permit the highest level of income or consumption reached in the past to influence the current level of consumption. An alternative hypothesis that seems worthy of consideration is that the actual development of income, mainly in the recent past, exerts a similar influence. In principle it would be possible to introduce past values of income explicitly but the formulation which will now be considered is that current consumption is influenced by current income and by last year's consumption. Under conditions of steady growth this approach would give the same results as (5) but in general it expresses a different form of behaviour.

The present hypothesis can be expressed in the form

$$v = \varphi\,\mu + \psi\,E^{-1}v \tag{8}$$

where E denotes an operator which retards or advances the variable to which it is applied so that $E^{-\theta}v_t = v_{t-\theta}$. Since the application of E^{-1} to (8) yields

$v_t = \varphi\mu_0 + [\alpha v_{t-1} + \alpha^2 v_{t-2} + \ldots]$

$$E^{-1} v = \varphi\, E^{-1}\, \mu + \psi\, E^{-2}\, v \tag{9}$$

it can be seen that continuous substitution for $E^{-\theta} v$ in the right-hand side of (8) will yield, with $\psi < 1$ and a stationary or increasing income stream, the convergent series

$$v = \varphi \sum_{\theta=0}^{\infty} \psi^{\theta} E^{-\theta} \mu \tag{10}$$

whence

$$\sigma = \left(1 - \varphi \sum_{\theta=0}^{\infty} \psi^{\theta} E^{-\theta}\right) \mu \tag{11}$$

Thus (8) is equivalent to the hypothesis that consumption is proportional to a weighted sum of incomes up to and including the present, the weight attached to the income of θ years ago being ψ^{θ}.

The short-period marginal propensities to consume associated with (5) and (8) are, as in previous cases, equal to φ. The corresponding long-period propensities can be calculated by making the assumption that income grows at a steady rate, so that

$$\mu = (1 + \varkappa)^{\theta} E^{-\theta} \mu \tag{12}$$

If a substitution is made for $E^{-\theta} \mu$ from (12) into (10) there results

$$\begin{aligned} v &= \varphi \sum_{\theta=0}^{\infty} \left(\frac{\psi}{1+\varkappa}\right)^{\theta} \mu \\ &= \frac{\varphi\,(1+\varkappa)}{1+\varkappa-\psi}\, \mu \end{aligned} \tag{13}$$

If (12) holds, $v_0 = E^{-1} v$ and (13) applies not only to (8) but also to (5).

The expressions considered up to now have all sought to explain consumption in terms of flow variables: the level of capital has not so far been introduced. In their studies of individual and community behaviour [10, 11], Modigliani and Brumberg proposed a linear function in which the constant term is replaced by a constant times total capital per head, A say. This variable may here be defined to exclude capital already sunk in consumers' durable goods. On this basis the consumption function becomes

$$v = \varphi\, \mu + \psi\, A \tag{14}$$

If changes in the size of the population and in the value of money can be neglected and if the fact of investment in consumers' durable goods is ignored, then

$$\Lambda = \sum_{\theta=1}^{\infty} E^{-\theta}\sigma \qquad (15)$$

If a substitution for Λ is made from (15) into (14) and if first differences are taken there results

$$\Delta v = \varphi \Delta \mu + \psi E^{-1} \sigma \qquad (16)$$

where $\Delta \equiv 1 - E^{-1}$.

Again, the short-period marginal propensity to consume is given by φ. The corresponding long-period value can be calculated as follows. Equation (16) can be written in the form

$$\begin{aligned} v &= \varphi \mu - (\varphi - \psi) E^{-1} \mu + (1 - \psi) E^{-1} v \\ &= \varphi \mu + \sum_{\theta=1}^{\infty} (1 - \varphi) \psi (1 - \psi)^{\theta-1} E^{-\theta} \mu \end{aligned} \qquad (17)$$

If (12) is assumed to hold and if a substitution for $E^{-\theta}\mu$ is made from it into (17) there results

$$\begin{aligned} v &= \left\{ \varphi + \frac{(1-\varphi)\psi}{(1+\varkappa)} \cdot \sum_{\theta=1}^{\infty} \left(\frac{1-\psi}{1+\varkappa} \right)^{\theta-1} \right\} \mu \\ &= \frac{\varphi \varkappa + \psi}{\varkappa + \psi} \mu \end{aligned} \qquad (18)$$

Up to this point, disposable income per head, μ, has been treated as a single, homogeneous flow. The hypothesis that the short-period marginal propensities are different for different types of income-recipient can be tested, as in Brown's study for Canada [1], by replacing $\varphi \mu$ by $\sum_i \varphi_i \mu_i$ where μ_i denotes a particular type of income payment per head of the total population. On this basis the consumption function becomes

$$v = \sum_i \varphi_i \mu_i + \psi x \qquad (19)$$

where x denotes χ/ψ, μ_0, v_0, $E^{-1}v$ or Λ according as (1), (2), (5), (8) or (14) represents the hypothesis to be considered.

3. A Formulation of the Aggregate Consumption Function

From the foregoing discussion of the work of previous writers, it would appear desirable to introduce into the consumption function the stock of capital and the flow of past consumption in addition to the flow of current income. A formulation that will permit this to be done can be derived from the simple dynamic theory of demand for individual commodities which has been set out elsewhere [15] by the present authors. This theory will now be summarised and extended to the present purpose.

The vector of commodity purchases by the household sector, q say, is divided into two parts, u and v say. The first part, u, represents consumption and the elements of u represent the amounts required to maintain the initial stock against the depletion arising from current use. The second part, v, represents net investment and the elements of v represent the net additions during the period to the opening stocks of the various commodities. Thus

$$q \equiv u + v \tag{20}$$

It is assumed that the amount of each commodity consumed in a period is a specific fixed proportion of the opening stock and the amount purchased in the period, which, for simplicity, is assumed to be concentrated at the outset. Thus.

$$u = \hat{n}^{-1} (s + q) \tag{21}$$

where s is the vector of initial stocks and $\hat{n}^{-1}$ is a diagonal matrix, denoted by the circumflex accent, with the consumption (or depreciation) rates in the diagonal.

The vector of closing stocks, denoted by Es, is simply the sum of opening stocks plus the net investments of the period. Thus

$$Es \equiv s + v \tag{22}$$

If a substitution for q is made from (20) into (21) there results

$$\begin{aligned} u &= \hat{n}^{-1} (s + v + u) \\ &= \hat{n}^{-1} (Es + u) \\ &= \hat{m}^{-1} Es \end{aligned} \tag{23}$$

from (22), where

$$\hat{m} = \hat{n} - I \tag{24}$$

If a substitution is made for u from (23) into (21), if the resulting equation is multiplied by $E^{-1}\hat{m}$ and if a continuous substitution is made for $E^{-\theta}s$, there results

$$s = \sum_{\theta=1}^{\infty} (\hat{m}\, \hat{n}^{-1})^{\theta}\, E^{-\theta} q \tag{25}$$

which shows how, on the assumptions made, s can readily be built up from past values of q.

The elements of n are greater than or equal to unity. If an element is equal to unity the corresponding commodity is a perfect perishable, stocks and net investment are always zero and consumption is equal to the amount purchased. If an element is greater than unity the corresponding commodity is, in greater or less degree, durable.

The vector u contains as elements the actual consumption of each commodity in a period. In contrast to this, consider a vector u^* the elements of which represent the equilibrium levels of consumption. The elements of u^* may be supposed to depend on income and the price structure. Since this paper is concerned with the aggregate consumption function and therefore with a system of demand relationships which can be added up to give this function it is convenient to assume that u^* is related to personal diposable income, μ, and the vector of the prices of consumers' goods and services, p, by a linear expenditure system [14]. On this basis

$$u^* = \hat{p}^{-1}\, [b\, \mu + (I - bi')\, \hat{c}\, p] \tag{26}$$

where b and c are vectors of parameters subject to the restriction that $i'b = 1$ and where i denotes the unit vector, the prime indicating transposition. This restriction ensures that, under equilibrium conditions, the consumption functions for individual commodities given by (26) have the following properties.

(*i*) They are additive in value terms and their sum is total income, that is

$$p'\, u^* \equiv \mu \tag{27}$$

as can be seen by premultiplying (26) by p'.

(*ii*) They are homogeneous, that is if μ and the elements of p all change in the same proportion then u^* will not change. In other words the sum of the elasticities of each element of u^* with respect to μ and p sum identically to zero, that is

$$(\hat{u}^*)^{-1}\, (g\, \mu + G\, p) \equiv 0 \tag{28}$$

where g and G denote the vector and matrix, respectively, of partial derivatives of the elements of u^* with respect to μ and the elements of p.

(*iii*) They possess a symmetric substitution matrix (Slutsky's condition), that is

$$S \equiv S' \tag{29}$$

where

$$S = (\hat{u}^*)^{-1} [gi' + G (\hat{u}^*)^{-1}] \mu \tag{30}$$

One objection to the simple formulation (26) is that it does not allow capital, as opposed to income and prices, to exercise an influence on the level or composition of equilibrium consumption. A part of the capital of the personal sector is absorbed by investment in consumers' durables but the remainder, equal to accumulated net lending at current values, A say, could be spent and might be expected to influence at least the composition of u^*. It is not clear how A might best be introduced into (26) and it is here assumed that it enters into the term in square brackets in as simple a manner as possible. On this assumption (26) becomes

$$u^* = \hat{p}^{-1} [b \mu + (I - bi') \hat{c} p + aA] \tag{31}$$

where a is a vector of parameters. If A is assumed to affect the composition of u^* but not the value total $p'u^*$, the elements of a are restricted by the relationship, $i'a = 0$. On the other hand (29) will only continue to hold if $a = \alpha b$ where α is a parameter. In this case, however, the total value of equilibrium consumption is $\mu + \alpha A$. There is thus some doubt about the way in which a term in A should be introduced into the equation for u^* and no finality can be claimed, even within the present simple system of ideas, for the actual formulation in (31).

Corresponding to u^*, the vector of equilibrium consumption levels, let Es^* denote the vector of equilibrium closing stocks which are related to the elements of u^* by the equation

$$Es^* = \hat{m} u^* \tag{32}$$

which may be compared with (23).

If consumption is not in equilibrium then consumers may be supposed to attempt to adjust towards the equilibrium levels. In order to do this they will have to approximate more closely the equilibrium stocks of durable goods. If their actual stocks are below the equilibrium stocks, this means that they will have to buy these goods in excess of the amounts needed to replace their consumption. It is assumed that their investment in each case is a certain fixed proportion of the excess of their equilibrium stock over their opening stock. Thus

$$\begin{aligned} v &= \hat{r} (Es^* - s) \\ &= \hat{r} \hat{m} (u^* - E^{-1}u) \end{aligned} \tag{33}$$

The elements of r are associated with the rates at which adjustment is attempted for the different commodities. In general they may be expected to be less than unity and they are defined for all commodities and not simply for durables.

If (22) is multiplied by $\hat{m}^{-1}$ there results

$$\begin{aligned} u &= E^{-1}u + \hat{m}^{-1}v \\ &= E^{-1}u + \hat{r}\,(u^* - E^{-1}u) \\ &= \hat{r}\;u^* + (I - \hat{r})\;E^{-1}u \end{aligned} \tag{34}$$

Total consumption at current prices is the vector product $p'u$ which, from (34) and (31) is given by

$$p'u = r'b\,\mu + r'\,(I - bi')\,\hat{c}\,p + r'a\,A + p'\,(I - \hat{r})\,E^{-1}u \tag{35}$$

Equation (35) is thus the general form of the consumption function which emerges from the system of ideas put forward in this paper. It can be seen that total consumption at current prices depends on four variables. The first of these is disposable income, μ, and the short-period marginal propensity to consume, $r'b$, is the vector product of the adjustment rates, r, and the long-period marginal propensities for individual commodities, b. The second variable is the price structure, p. The third is the current value of accumulated lending, A. The fourth is the sum of the current values of consumption in the previous period each weighted by the complement of its adjustment rate with respect to unity.

An examination of (35) shows that it is not a relationship between aggregate variables since it contains a term in each price and in last year's consumption of each commodity. If it is to be transformed into an aggregate relationship it is necessary to assume that r takes an average value, ϱ say, which applies to each commodity so that

$$r = \varrho\,i \tag{36}$$

This is not a realistic assumption since it is known that the elements of r vary considerably and are larger, for example, for food and clothing than they are for household durables and motor cars. Nevertheless it must be made if an aggregative analysis is to be possible on the basis of (35) and the experience of later sections suggests that for this purpose it is probably not seriously misleading.

By combining (35) and (36) it can be seen that

$$p'u = \varrho\,\mu + \varrho\,i'a\,A + (1 - \varrho)\,p'\,E^{-1}u \tag{37}$$

Thus the term in p in (35) has disappeared, since $i'\,(I - bi') = [0,\ldots,0]$, and the term in A would also disappear if $i'a = 0$.

The saving function is easily derived from the consumption function since saving, σ, is defined by the equation

$$\begin{aligned}\sigma &\equiv \mu - p'u \\ &= (1 - r'b)\,\mu - r'\,(I - bi')\,\hat{c}\,p - r'a\,A - p'\,(I - \hat{r})\,E^{-1}u \\ &= (1 - \varrho)\,\mu - \varrho\,i'a\,A - (1 - \varrho)\,p'\,E^{-1}u \qquad (38)\end{aligned}$$

given (35), and (35) together with (36), respectively.

This section contains the basic ideas that will be examined empirically in this paper. Before this is done, however, there are a number of subsidiary issues to which attention will first be given.

4. The Aggregate Expenditure Function

As has already been remarked, consumers' expenditure is made up of two parts, the value of consumption and the value of net investment in consumers' durable goods. It was seen in (34) that the consumption of each good was a weighted average of equilibrium consumption and the consumption of the previous period. If (33) and (34) are added together then

$$\begin{aligned}q &\equiv u + v \\ &= \hat{r}\,\hat{n}\,u^* + (I - \hat{r}\,\hat{n})\;E^{-1}u \qquad (39)\end{aligned}$$

Thus the elements of q are also weighted averages of the corresponding elements of u^* and $E^{-1}u$ but the weights depend on the elements of n as well as on those of r.

The aggregate expenditure function, expressed in current money terms, is obtained by premultiplying (39) by p'. But, as will shortly be seen, this operation will only lead to a relationship between aggregate variables if, in addition to (36), it is assumed that the elements of n also take an average value, ν say, so that

$$n = \nu\,i \qquad (40)$$

Thus if u^* takes the form of (31), then

$$\begin{aligned}p'q &= r'\,\hat{n}\,b\,\mu + r'\,\hat{n}\,(I - bi')\,\hat{c}\,p + r'\,\hat{n}\,a\,A + p'\,(I - \hat{r}\,\hat{n})\,E^{-1}u \\ &= \varrho\,\nu\,\mu + \varrho\,\nu\,i'a\,A + (1 - \varrho\,\nu)\,p'\,E^{-1}u \qquad (41)\end{aligned}$$

if (36) and (40) hold.

Equation (40) represents a further drastic simplification which puts a severe restriction on aggregate equations derived from its use. Thus if $i'a = 0$, the term in Λ will disappear from (41). In fact, however, if $i'a = 0$ it is to be expected that the true value of $n'a > 0$ because it is likely that the elements of a are positive for durable goods. Thus Λ may well have a significant effect on aggregate expenditure even if it does not have a significant effect on aggregate consumption but this result cannot emerge from an aggregative analysis based on (41).

It remains only to write down the equations for the value of net investment in consumers' durables and for net lending which can readily be derived from the foregoing. Thus if (31) holds the premultiplication of (33) by p' yields

$$\begin{aligned} p'v &= r'\,\hat{m}\,b\;\mu + r'\,\hat{m}\,(I - bi')\;\hat{c}\;p + r'\,\hat{m}\,a\;\Lambda - p'\;\hat{r}\;\hat{m}\;E^{-1}u \\ &= \varrho\;(\nu - 1)\;\mu + \varrho(\nu - 1)\;\;i'a\,\Lambda - \varrho\;(\nu - 1)\;p'\;E^{-1}u \qquad (42) \end{aligned}$$

if (36) and (40) hold. Similarly net lending, λ say, is defined by the relationship

$$\begin{aligned} \lambda &= \sigma - p'v \\ &= (1 - r'\hat{n}b)\,\mu - r'\;\hat{n}\;(I - bi')\;\hat{c}p - r'\hat{n}a\;\Lambda - p'\;\;(I - \hat{r}\;\hat{n})\;\;E^{-1}u \\ &= (1 - \varrho\;\nu)\;\mu - \varrho\,\nu\,i'a\,\Lambda - (1 - \varrho\,\nu)\;p'\;E^{-1}u \qquad (43) \end{aligned}$$

if (36) and (40) hold.

5. The Use of Index-numbers

The equations just referred to relate to aggregates expressed in current money terms. Thus, for example, (37) expresses the current value of consumption in terms of certain variables also expressed in current values. At times when prices are highly variable, (37) is unlikely to be appropriate for estimation purposes since, if an error term is introduced, it is likely that its variance will increase roughly in proportion to the level of prices. It would be more appropriate therefore to work with deflated values.

This means that the variables in (37) should each be divided by the consumption price index. A further simplification is introduced if $p'E^{-1}u$ is represented by the current price index multiplied by the volume index of consumption for the preceding year. If, for the sake of simplicity, it is assumed that $i'a = 0$ so that the term in Λ disappears, then (37) could be modified to the expression

$$(p'u)/\pi = v$$
$$= \varrho\ (\mu/\pi) + (1 - \varrho)\ E^{-1}v \qquad (44)$$

where v, a scalar, denotes the volume index of consumption and π denotes the price index of consumption. In a similar way the investment and expenditure equations can be simplified to

$$(p'v)/\pi = \varrho\ (\nu - 1)\ [(\mu/\pi) - E^{-1}v] \qquad (45)$$

and

$$(p'q)/\pi = \varrho\ \nu\ (\mu/\pi) + (1 - \varrho\ \nu)\ E^{-1}v \qquad (46)$$

Additional simplifications of this sort will be introduced throughout the applications given in this paper.

6. An Approximation to Λ/π

As already explained Λ denotes the value at the beginning of the period of the accumulated net lending of the personal sector. If there were no change over time in the value of the net claims held by the personal sector then Λ would be simply the sum over all past periods of λ. In fact, of course, these values change considerably and so Λ may be defined as the accumulation of λ adjusted to current values, that is

$$\Lambda = \tau \sum_{\theta=1}^{\infty} E^{-\theta}\ (\lambda/\tau) \qquad (47)$$

where τ is an index of the average value of claims. If it is assumed that $\tau = \pi$ then

$$\Lambda/\pi = \sum_{\theta=1}^{\infty} E^{-\theta}\ (\lambda/\pi) \qquad (48)$$

and

$$\Delta\ (\Lambda/\pi) = E^{-1}\ (\lambda/\pi) \qquad (49)$$

The values of λ/π can readily be calculated and with this approximation the influence of Λ can be estimated by writing the equations in first-difference form. This has the additional advantage that in this form it will probably be more reasonable to assume that the disturbance term is serially independent.

7. Current Income and Income Time Lags

It is implied in (31) that consumers are continually attempting to adjust to current income levels since the equilibrium consumption vector depends, in part, on μ. It is worthwhile to examine the hypothesis that adjustment is made not to current income but to some average of recent income experience. This can be done by defining a new variable, μ^*, as

$$\mu^* = \eta \, \mu + (1 - \eta) \, E^{-1}\mu \tag{50}$$

This variable can then replace μ in the consumption function. In a similar way it would be possible to operate with an average of real rather than money income but this possibility need not be separately exemplified.

If $\eta = 1$ then $\mu^* = \mu$ and the community adjusts to current income. If $0 < \eta < 1$ then the community adjusts to an average with positive weights of current income and the income of the previous period. If $\eta > 1$ then the community adjusts to a linear extrapolation of recent income experience.

8. The Subdivision of Income

A complication is introduced by the subdivision of income and this will now be considered. It is usually thought that the poor will respond to income changes to at larger extent in the short run than will the rich. The analysis of section 3 provides one reason why this should be so. From (35) it can be seen that the short-period marginal propensity to consume is $r'b$. In a milieu in which only perishable goods are considered there is no obvious reason why adjustment should not take place immediately, in which case the short-period marginal propensity to consume would be equal to the long-period marginal propensity. As durable goods come more and more into the consumer's horizon the possibility of immediate adjustment will recede and this will be particularly so in the case of durable goods themselves. The net result will be a fall in $r'b$, that is in the short-period marginal propensity to consume. This effect will be reinforced if, in fact, there is a tendency for the elements of b to fall as the level of income rises.

The question of how the subdivision of income affects the analysis can be studied by supposing that a simple equation of the form of (44) holds for two separate classes of household distinguished by the subscripts 1 and 2. It is also assumed that a single price index, π, may be applied to each group. Then the system of equations becomes

$$v_1 = \varrho_1 \, (\mu_1/\pi) + (1 - \varrho_1) \; E^{-1}v_1 \tag{51}$$

$$v_2 = \varrho_2 \, (\mu_2/\pi) + (1 - \varrho_2) \; E^{-1}v_2 \tag{52}$$

In practice it usually happens that series are not available for separate types of household but only for separate types of income payment and that separate consumption series are not available at all. For this reason, (51) and (52) cannot be used directly for estimating ϱ_1 and ϱ_2. Various possibilities suggest themselves.

One possibility is to substitute continuously in (51) for $E^{-\theta}v_1$ so that v_1 is expressed in terms of a weighted sum of all values of (μ_1/π) up to the present time. If (52) is treated similarly and if the two equations are added together, there results

$$v = \varrho_1 \sum_{\theta=0}^{\infty} (1-\varrho_1)^{\theta} E^{-\theta} (\mu_1/\pi) + \varrho_2 \sum_{\theta=0}^{\infty} (1-\varrho_2)^{\theta} E^{-\theta} (\mu_2/\pi) \tag{53}$$

In principle it would be possible by trial and error, to find values of ϱ_1 and ϱ_2 which would minimize the sum of squares of the differences between the actual and calculated values of v. This methods would be laborious and would not provide a basis for estimating the reliability of the resulting estimates.

A second possibility is to assume that v_1 is related to v, in the simplest case by a factor of proportionality, ω_1 say. This assumption permits the terms in $E^{-1}v_1$ and $E^{-1}v_2$ in (51) and (52) to be replaced by terms in $E^{-1}v$. The addition of the two equations then yields

$$v = \varrho_1 (\mu_1/\pi) + \varrho_2 (\mu_2/\pi) + [1 - \omega_1 \varrho_1 - (1-\omega_1) \varrho_2] E^{-1}v \tag{54}$$

This method is used in a subsequent section. Two variants may be considered. In the first, the term in square brackets is estimated by regression analysis and this yields an estimate of ω_1. In the second, an independent estimate is made of ω_1 which makes possible estimates of v_1 and v_2 so that ϱ_1 and ϱ_2 can be separately estimated from suitable transformations of (51) and (52).

An obvious objection to the preceding assumption, which would not, however, necessarily distort greatly the estimates of ϱ_1 and ϱ_2 from (54), is that if the shares of the two classes of income payment in total income vary then ω_1 is not likely to be constant. This suggests a third possibility, namely that the ratio μ_1/μ should enter into the relationship connecting v_1 and v. For example it might be assumed that

$$v_1 = \omega_1^* (\mu_1/\mu)\, v \tag{55}$$

or, in other words that the consumption-income ratio for type 1 incomes is proportional to the consumption-income ratio for all incomes. The same assumption can, however, only be made for type 2 incomes as well if $\omega_1^* = \omega_2^* = 1$. If this condition is not satisfied then the assumption can be made only with respect to one type of income payment and different results will follow according to which type is chosen.

9. Empirical Results: the Consumption Function with One Income Variable

The first question examined in this section is the estimation of the parameters in various versions of the consumption function when no subdivision of income is made. Estimates obtained from eight different models are brought together in table 1. In each case the results are presented on the basis of interwar experience alone and on the basis of interwar and postwar experience combined. The estimating equations actually used are shown in the table. The forms adopted are needed partly to ensure that the estimates in each equation are consistent with the other estimates in that equation and partly to give effect to the assumption that the disturbance term is more likely to be serially independent when the original equation is expressed in first-difference form.

The assumptions which lead to the eight models are as follows.

(1) It is assumed that the long-period marginal propensity to consume is unity; that the effect of accumulated net lending can be neglected; and that consumers attempt to adjust to the current level of income. As a consequence there is only one parameter to be estimated, namely the short-period marginal propensity to consume which is equal to the average rate of adjustment, ϱ.

(2) This model is similar to (1) except that it is assumed that the long-period marginal propensity, β, is not necessarily equal to unity but that nevertheless the influence of prices on the aggregate consumption function can be neglected. The short-period marginal propensity is now $\varrho\,\beta$ where ϱ is the average rate of adjustment.

(3) This model is similar to (1) except that it is assumed that consumers attempt to adjust to the current value of some average of the money incomes received in this and the preceding period. The weight attaching to this year's money income is η while that attaching to the money income of the preceding year is $(1 - \eta)$.

(4) This model is similar to (1) except that it is assumed that accumulated net lending cannot necessarily be neglected.

(5) This model involves a combination of the complications considered in (2) and (3) and so involves three parameters, ϱ, β and η.

(6) This model involves a combination of the complications considered in (2) and (4) and so involves three parameters, ϱ, β, and α.

(7) This model involves a combination of the complications considered in (3) and (4) and so involves three parameters, ϱ, η, and α.

(8) This model involves all the complications considered and so involves four parameters, ϱ, β, η and α.

The first eight columns of figures in table 1 show the estimated values of the parameters and their standard errors, in brackets. In all the models except the first, one or more of the parameters are estimated in combination with one or more others. These combined values are given first and are followed by the estimates of the components which, in all cases can be obtained by simple division. The final three columns in the table show: R^2, the square of the multiple correlation coefficient; s, the estimated standard deviation of the residual expressed in £s of 1938 purchasing power per head of the population; and d which denotes Durbin and Watson's [7] statistic for testing serial correlation in the residuals. While in no case is there clear evidence of serial correlation at the 5 per cent level of significance, a single asterisk in this column denotes that the test is inconclusive at that level.

A number of conclusions are suggested by a study of table 1.

In the first place, the table lends considerable support to the view that the short-period marginal propensity to consume in Britain lies in the neighbourhood of one-half. The estimates of this parameter, which is equal to ϱ in models (1) and (4), to $\varrho\beta$ in models (2) and (6), to $\varrho\eta$ in models (3) and (7) and to $\varrho\beta\eta$ in models (5) and (8), vary between 0.48 and 0.52 in the interwar analyses and between 0.48 and 0.56 when the two periods are combined. The standard errors of these estimates lie between 0.06 and 0.08. The difference to the estimate made by the introduction of the postwar years is reduced still more when β, the long-period marginal propensity to consume, is freely determined instead of being assumed equal to unity.

Second, when β is freely determined its value appears unexpectedly low though allowance must of course be made for the considerable standard errors that attach to the estimates $\bar{\beta}$ of β in models (2), (5), (6) and (8). It seems plausible that β should be a little less than unity but not that it should be as low as one-half or even two-thirds. The low values obtained may, however, be associated with the fact that there is no subdivision of income in the models and that a low value of β appears in some measure as a compensation for a comparatively high value of ϱ, the average rate of adjustment, which is not really applicable to high-income households which cannot adjust very quickly because of the importance of durable goods in their budgets. In models (1), (3), (4) and (7) in which, by assumption, $\beta = 1$, the values of ϱ tend to be lower than in the remaining models in which β is freely determined.

Third, when account is taken of the standard errors attaching to the estimates of η in models (3), (5), (7) and (8) it cannot be said that the table

Table 1. Consumption Functions with a Single Income Variable

		Period	$\overline{\varrho\beta\eta}$	$\overline{\varrho\beta}$	$\overline{\varrho\eta}$	$\overline{\varrho\alpha}$	$\bar{\varrho}$	$\bar{\beta}$	$\bar{\eta}$	$\bar{\alpha}$	R^2	s	d
1.	$\Delta^2 v = \varrho\,\Delta\,[(\mu/\pi) - E^{-1}v] + \varepsilon$	Interwar	—	—	—	—	0.51 (0.08)	—	—	—	0.77	0.64	2.23
		Combined	—	—	—	—	0.56 (0.07)	—	—	—	0.78	0.77	2.14
2.	$\Delta^2 v = \varrho\,\beta\,\Delta\,(\mu/\pi) - \varrho\,\Delta\,E^{-1}v + \varepsilon$	Interwar	—	0.48 (0.07)	—	—	0.70 (0.12)	0.68 (0.18)	—	—	0.83	0.59	2.35
		Combined	—	0.51 (0.06)	—	—	0.75 (0.11)	0.68 (0.15)	—	—	0.82	0.70	2.05
3.	$\Delta^2 v = \varrho\eta\Delta[(\Delta\mu)/\pi] + \varrho\Delta[(E^{-1}\mu)/\pi - E^{-1}v] + \varepsilon$	Interwar	—	—	0.52 (0.08)	—	0.39 (0.12)	—	1.34 (0.36)	—	0.79	0.63	2.74*
		Combined	—	—	0.56 (0.08)	—	0.55 (0.08)	—	1.03 (0.16)	—	0.78	0.79	2.14
4.	$\Delta^2 v = \varrho\,\Delta\,[(\mu/\pi) - E^{-1}v] + \varrho\,\alpha\,E^{-1}(\lambda/\pi) + \varepsilon$	Interwar	—	—	—	−0.01 (0.07)	0.51 (0.08)	—	—	−0.03 (0.14)	0.77	0.67	2.13
		Combined	—	—	—	−0.02 (0.06)	0.56 (0.07)	—	—	−0.04 (0.11)	0.78	0.78	2.07
5.	$\Delta^2 v = \varrho\beta\eta\Delta[(\Delta\mu)/\pi] + \varrho\beta\Delta[(E^{-1}\mu)/\pi] - \varrho\Delta\,E^{-1}v + \varepsilon$	Interwar	0.49 (0.07)	0.40 (0.11)	—	—	0.60 (0.15)	0.66 (0.13)	1.24 (0.30)	—	0.85	0.57	2.73*
		Combined	0.51 (0.08)	0.51 (0.08)	—	—	0.75 (0.12)	0.68 (0.11)	1.00 (0.16)	—	0.82	0.72	2.05
6.	$\Delta^2 v = \varrho\beta\,\Delta\,(\mu/\pi) + \varrho\,\alpha\,E^{-1}(\lambda/\pi) - \varrho\Delta\,E^{-1}v + \varepsilon$	Interwar	—	0.48 (0.06)	—	0.14 (0.07)	0.86 (0.13)	0.56 (0.09)	—	0.16 (0.07)	0.87	0.52	2.31
		Combined	—	0.48 (0.07)	—	0.10 (0.07)	0.84 (0.12)	0.57 (0.12)	—	0.12 (0.08)	0.84	0.69	1.97
7.	$\Delta^2 v = \varrho\eta\,\Delta\,[(\Delta\,\mu)/\pi] + \varrho\,\alpha\,E^{-1}(\lambda/\pi) + \varrho\,\Delta\,[(E^{-1}\mu)/\pi - E^{-1}v] + \varepsilon$	Interwar	—	—	0.52 (0.08)	0 (0.07)	0.39 (0.13)	—	1.34 (0.38)	0 (0.18)	0.79	0.66	2.65*
		Combined	—	—	0.56 (0.08)	−0.02 (0.07)	0.56 (0.09)	—	1.00 (0.18)	−0.04 (0.12)	0.78	0.81	2.13
8.	$\Delta^2 v = \varrho\,\beta\,\eta\,\Delta\,[(\Delta\mu)/\pi] + \varrho\alpha\,E^{-1}(\lambda/\pi) + \varrho\beta\Delta\,[(E^{-1}\mu)/\pi] - \varrho\Delta E^{-1}v + \varepsilon$	Interwar	0.50 (0.06)	0.40 (0.10)	—	0.14 (0.07)	0.76 (0.16)	0.53 (0.10)	1.24 (0.26)	0.18 (0.08)	0.89	0.51	2.75*
		Combined	0.50 (0.08)	0.44 (0.09)	—	0.12 (0.08)	0.83 (0.13)	0.53 (0.13)	1.12 (0.21)	0.14 (0.09)	0.84	0.70	2.01

1*

provides any strong evidence that the community attempts to adjust to some level of income other than the current level. In all cases, however, $\eta \geqq 1$ so that any departure from the presumption of adjustment to the current level of income would be in the direction of adjustment to an extrapolation of recent income experience and not to some value of income intermediate between this year's and last year's level.

Fourth, there is again relatively little evidence in support of the view that accumulated net lending, as it can be reflected with available data, has much influence on the level of consumption. The only cases in which comparatively large values af $\bar{\alpha}$ appear are in models (6) and (8) in which a freely-determined β is also included. It must be remembered, however, that it is likely to be difficult to find a significant value of $\bar{\alpha}$ from a small sample of observations since α can hardly be very large.

Fifth, for reasons which have already been discussed it is difficult to determine the value of ϱ with much precision. If it is assumed that $\beta = 1$ then it would appear that ϱ lies in the neighbourhood of one-half but with the lower estimates which appear if β is freely determined, it would appear that ϱ lies in the neighbourhood of threequarters or four-fifths.

10. Empirical Results: the Consumption Function with Two Income Variables

The conclusions of the preceding section will now be examined in the light of the hypothesis that there is a significant difference between the short-period marginal propensities to consume from large and from small incomes. For this purpose, personal disposable income, μ, is divided into two parts: μ_1 which comprises wages and salaries, the pay of the armed forces and small transfers; and μ_2 which comprises all income from property including public debt interest.

Estimates have been made in respect of five models, the last of which is capable of three possible interpretations. The results are set out in table 2. Alternative versions of these models which contain a term in E^{-1} (λ/π) were also tried out but it did not appear that this term was of any importance.

All the models considered in this section can be written in the general form

$$v = \varrho_1\beta_1 (\mu_1/\pi) + \varrho_2 \beta_2 (\mu_2/\pi) + (1 - \varrho_2) E^{-1}v + (\varrho_2 - \varrho_1) \left\{\omega_1 E^{-1} [(\mu_1/\mu)^{\delta} v] + \xi_1\right\} \tag{56}$$

where ξ_1 is a constant term in the equation assumed to connect v_1 and v, together with a complementary expression in which the subscripts 1 and 2 are interchanged. As before, the equations in table 2 show the forms used for estimation purposes. The reasons for these forms are the same as were set out in the first paragraph of section 9 above. When first differences are taken the constant term, $(\varrho_2 - \varrho_1)\ \xi_1$ in (56), disappears.

In terms of (56) the assumptions which lead to the seven variants of the five models can be expressed as follows.

(1) It is assumed that the long-period propensities to consume, β_1 and β_2, are each equal to unity and that $\delta = 0$. This model is therefore the same as (54) except for the inclusion of a constant term which disappears when first differences are taken.
(2) It is assumed that $\beta_1 = \beta_2 = 1$ and that $\delta = 1$. In this case ω_1 is equal to ω_1^* in (55). It is assumed that $\omega_1 = 1$.
(3) It is assumed that $\beta_1 = \beta_2 = 1$; $\delta = 1$; ω_2 is undefined; ω_1 is determined from the observations.
(4) It is assumed that $\beta_1 = \beta_2 = 1$; $\delta = 1$; ω_1 is undefined; ω_2 is determined from the observations.
(5) It is assumed that $\beta_1 = \beta_2 = \beta$, to be determined from the observations; $\delta = 1$; ω_2 is undefined.
(6) It is assumed that $\beta_1 = \beta_2 = \beta$; $\delta = 1$, ω_1 is undefined.
(7) It is assumed that $\delta = 1$; $\omega_1 = 1$; β_1 and β_2 are to be separately determined from the observations.

The last three variants represent alternative interpretations of a single model and the different results follow from different interpretations of the parameters as embodied in the above assumptions. Further variants could be obtained if β were given or if $\varrho = 1$ but these possibilities are not considered in this paper.

An examination of table 2 suggests the following conclusions.

In the first place, whatever assumption is made, the estimate of the short-period marginal propensity to consume out of small incomes, $\bar{\varrho}_1$ in models (1) through (4) and $\overline{\varrho_1\beta_1}$ in models (5) through (7) is, as expected, distinctly larger than the corresponding propensity for large incomes. The difference is least marked in models (2) and (3) in which, however, the value of R^2, especially for the interwar period, is markedly lower than for the other models.

Another reason can be given for rejecting model (3) in favour of model (4). Models (3) and (5) on the one hand and models (4) and (6) on the other hand differ in the following respect only. In the first of each pair it is assumed that $\beta_1 = \beta_2 = 1$ whereas in the second it is assumed that $\beta_1 = \beta_2 = \beta$ which is to be determined from the observations. It can be seen that when β, the long-period marginal propensity to consume, is determined freely, the values obtained in model (6), being equal to or a

Table 2. Consumption Functions with Two Income Variables

		Period	$\overline{\varrho_1\beta_1}$	$\overline{\varrho_2\beta_2}$	$(\bar\varrho_2-\bar\varrho_1)\bar\omega_1$ or $(\bar\varrho_1-\bar\varrho_2)\bar\omega_2$	$\bar\varrho_1$	$\bar\varrho_2$	$\bar\beta_1$	$\bar\beta_2$	$\bar\omega_1$	$\bar\omega_2$	R^2	s	d
1.	$\Delta^2 v = \varrho_1\Delta(\mu_1/\pi) + \varrho_2\Delta[(\mu_2/\pi) - E^{-1}v] + (\varrho_2-\varrho_1)\omega_1\Delta E^{-1}v + \varepsilon$	Interwar	—	—	—0.44 (0.09)	0.64 (0.06)	0.24 (0.08)	—	—	1.11 (0.23)	—	0.93	0.38	2.19
		Combined	—	—	—0.44 (0.14)	0.62 (0.08)	0.32 (0.11)	—	—	1.50 (0.54)	—	0.85	0.66	1.81
2.	$\Delta^2 v = \varrho_1\Delta[(\mu_1/\pi) - E^{-1}(\mu_1 v/\mu)] + \varrho_2\Delta[(\mu_2/\pi) - E^{-1}v + E^{-1}(\mu_1 v/\mu)] + \varepsilon$	Interwar	—	—	—	0.57 (0.08)	0.39 (0.10)	—	—	—	—	0.82	0.59	1.62
		Combined	—	—	—	0.62 (0.08)	0.44 (0.10)	—	—	—	—	0.80	0.75	1.80
3.	$\Delta^2 v = \varrho_1\Delta(\mu_1/\pi) + \varrho_2\Delta[(\mu_2/\pi) - E^{-1}v] + (\varrho_2-\varrho_1)\omega_1\Delta E^{-1}[(\mu_1/\mu)v] + \varepsilon$	Interwar	—	—	—0.20 (0.09)	0.55 (0.08)	0.40 (0.08)	—	—	1.28 (0.89)	—	0.82	0.47	1.87
		Combined	—	—	—0.25 (0.13)	0.57 (0.09)	0.48 (0.10)	—	—	2.54 (3.11)	—	0.81	0.73	1.86
4.	$\Delta^2 v = \varrho_1\Delta[(\mu_1/\pi) - E^{-1}v] + \varrho_2\Delta(\mu_2/\pi) + (\varrho_1-\varrho_2)\omega_2\Delta E^{-1}[(\mu_2/\mu)v] + \varepsilon$	Interwar	—	—	—0.13 (0.09)	0.65 (0.05)	0.24 (0.07)	—	—	—	—0.30 (0.21)	0.94	0.35	2.44*
		Combined	—	—	—0.06 (0.15)	0.66 (0.07)	0.29 (0.11)	—	—	—	—0.15 (0.41)	0.84	0.68	1.82
5.	$\Delta^2 v = \varrho_1\beta\Delta(\mu_1/\pi) + \varrho_2\beta\Delta(\mu_2/\pi) - \varrho_2\Delta E^{-1}v + (\varrho_2-\varrho_1)\omega_1\Delta E^{-1}[(\mu_1/\mu)v] + \varepsilon$	Interwar	0.66 (0.06)	0.23 (0.07)	0.13 (0.10)	2.20 (0.80)	0.78 (0.11)	0.30 (0.10)		—0.09 (0.08)	—	0.94	0.37	2.65*
		Combined	0.62 (0.08)	0.32 (0.12)	—0.02 (0.16)	1.42 (0.66)	0.75 (0.16)	0.43 (0.18)		0.02 (0.24)	—	0.85	0.67	1.84
6.	$\Delta^2 v = \varrho_1\beta\Delta(\mu_1/\pi) + \varrho_2\beta\Delta(\mu_2/\pi) - \varrho_1\Delta E^{-1}v + (\varrho_1-\varrho_2)\omega_2\Delta E^{-1}[(\mu_2/\mu)v] + \varepsilon$	Interwar	0.66 (0.06)	0.23 (0.07)	—0.13 (0.10)	0.65 (0.08)	0.23 (0.09)	1.02 (0.14)		—	—0.31 (0.25)	0.94	0.37	2.65*
		Combined	0.62 (0.08)	0.32 (0.12)	0.02 (0.16)	0.77 (0.11)	0.40 (0.17)	0.80 (0.14)		—	0.04 (0.44)	0.85	0.67	1.84
7.	$\Delta^2 v = \varrho_1\beta_1\Delta(\mu_1/\pi) + \varrho_2\beta_2\Delta(\mu_2/\pi) - \varrho_2\Delta E^{-1}v + (\varrho_2-\varrho_1)\Delta E^{-1}[(\mu_1/\mu)v] + \varepsilon$	Interwar	0.66 (0.06)	0.23 (0.07)	—0.13 (0.10)	0.65 (0.08)	0.78 (0.11)	1.02 (0.14)	0.30 (0.10)	—	—	0.94	0.37	2.65*
		Combined	0.62 (0.08)	0.32 (0.12)	—0.02 (0.16)	0.77 (0.11)	0.75 (0.16)	0.80 (0.14)	0.43 (0.18)	—	—	0.85	0.67	1.84

little less than unity, are plausible whereas those obtained in model (5) are much too low to be accepted. It would appear, therefore, that of the two assumptions, ω_2 undefined or ω_1 undefined, it is better to make the second. The similarly unplausible results that are obtained if the final estimating equation is interpreted as model (7) suggests that the assumption that $\omega_1 = \omega_2 = 1$, is also not a good one. If this line of thought is accepted then of the models involving the assumption that $\delta = 1$, models (4) and (6), which give rather similar results, are to be preferred to models (2), (3), (5) and (7). Models (4) and (6), again give, in the main, similar results to those obtained from the simpler model (1) in which it is assumed that $\delta = 0$.

If the results in the two tables are compared it would appear that the most important improvement on the simplest model, (1) of table 1, is to introduce a subdivision of income. If that model is compared with model (1) of table 2 it can be seen that the latter shows a higher value of R^2 and a lower value of s in both periods. Of the two-parameter models in table 1, model (2), in which β is freely determined, gives the best results. The introduction of a freely-determined η into some of the models of table 1 produces relatively little improvement and the same may be said of the introduction of α. On the whole it would seem that model (6) of table 2 is the most acceptable but this does not differ substantially from model (1) of the same table. Accordingly, on the interpretation given here the magnitude of the short-period marginal propensity to consume is associated mainly with the average rate of adjustment to equilibrium and the difference between the two marginal propensities is traced, basically, to the same source. This interpretation, which is but one among several possibilities, rests on the belief that the long-period marginal propensity to consume is unlikely to be substantially less then unity for any large class of income receivers, rich or poor, whereas the difference in the importance of durable goods in the budgets of the two classes provides a reason for expecting that the average rate of adjustment is likely to be lower for the rich than for the poor.

11. Empirical Results: Expenditure and Other Functions

In this section estimates are given of the parameters in the investment, expenditure, saving and net lending functions associated with the consumption functions set out as model (1) in tables 1 and 2. If no subdivision of income is made the relationships, expressed in terms of year-to-year changes, are as follows.

$$\Delta\,[(p'u)/\pi] = \Delta\,v = \varrho\,\Delta\,(\mu/\pi) + (1-\varrho)\,\Delta E^{-1}v \tag{57}$$

$$\Delta\,[(p'v)/\pi] = \varrho\,(\nu-1)\,\Delta\,(\mu/\pi) - \varrho\,(\nu-1)\,\Delta\,E^{-1}v \tag{58}$$

$$\Delta\,[(p'q)/\pi] = \varrho\,\nu\,\Delta\,(\mu/\pi) + (1-\varrho\,\nu)\,\Delta\,E^{-1}v \tag{59}$$

$$\Delta\,(\sigma/\pi) = (1-\varrho)\,\Delta\,(\mu/\pi) - (1-\varrho)\,\Delta\,E^{-1}v \tag{60}$$

$$\Delta\,(\lambda/\pi) = (1-\varrho\,\nu)\,\Delta\,(\mu/\pi) - (1-\varrho\,\nu)\,\Delta\,E^{-1}v \tag{61}$$

The average value of n can be estimated by taking the regression of $\Delta\,[(p'v)/\pi]$ on $\Delta\,[(\mu/\pi) - E^{-1}v]$ in (58) and combining this estimate of $\varrho\,(\nu-1)$ with the estimate of ϱ obtained in model (1) of table 1. The estimate of $\varrho\,(\nu-1)$ obtained in this way from interwar data alone is 0.06 ± 0.03 and accordingly $\bar{\nu} = 1.12$: if postwar data are also included the estimates are 0.08 ± 0.03 and 1.16 respectively. The estimates of ν are thus greater than unity, as they must be, but rather smaller than might have been expected in view of the importance of durable goods in household budgets.

For the interwar period the five equations, corresponding to (57) through (61), are

$$\Delta\,v = 0.51\,\Delta\,(\mu/\pi) + 0.49\,\Delta\,E^{-1}v \tag{62}$$

$$\Delta\,[(p'v)/\pi] = 0.06\,\Delta\,(\mu/\pi) - 0.06\,\Delta\,E^{-1}v \tag{63}$$

$$\Delta\,[(p'q)/\pi] = 0.57\,\Delta\,(\mu/\pi) + 0.43\,\Delta\,E^{-1}v \tag{64}$$

$$\Delta\,(\sigma/\pi) = 0.49\,\Delta\,[(\mu/\pi) - E^{-1}v] \tag{65}$$

$$\Delta\,(\lambda/\pi) = 0.43\,\Delta\,[(\mu/\pi) - E^{-1}v] \tag{66}$$

while for the combined period they are

$$\Delta\,v = 0.56\,\Delta\,(\mu/\pi) + 0.44\,\Delta\,E^{-1}v \tag{67}$$

$$\Delta\,[(p'v)/\pi] = 0.08\,\Delta\,(\mu/\pi) - 0.08\,\Delta\,E^{-1}v \tag{68}$$

$$\Delta\,[(p'q)/\pi] = 0.64\,\Delta\,(\mu/\pi) + 0.36\,\Delta\,E^{-1}v \tag{69}$$

$$\Delta\,(\sigma/\pi) = 0.44\,\Delta\,[(\mu/\pi) - E^{-1}v] \tag{70}$$

$$\Delta\,(\lambda/\pi) = 0.36\,\Delta\,[(\mu/\pi) - E^{-1}v] \tag{71}$$

The aggregate investment equations, (63) and (68), do not fit very well. This should probably be attributed to the simplifications introduced to obtain an aggregate relationship since, for individual groups of durable goods, investment equations based on a variant of the same model fit comparatively well [15]. Thus better results may be expected from a disaggregative approach but this will not be undertaken in the present paper.

It was shown in the preceding section that the consumption function was improved considerably by a subdivision of income. If this complication is introduced while the remaining simplifying assumptions are made, then the set of equations corresponding to (57) through (61), but built round the consumption function given by model (1) of table 2 instead of model (1) of table 1, is as follows

$$\begin{aligned}\Delta\,[(p'u)/\pi] &= \Delta\, v \\ &= \varrho_1 \Delta(\mu_1/\pi) + \varrho_2 \Delta(\mu_2/\pi) + [1-\omega_1\,\varrho_1 - (1-\omega_1)\,\varrho_2] E^{-1} \Delta\, v \end{aligned} \quad (72)$$

$$\begin{aligned}\Delta\,[(p'v)/\pi] = {} & \varrho_1(\nu_1-1)\,\Delta(\mu_1/\pi) + \varrho_2(\nu_2-1)\,\Delta(\mu_2/\pi) - [\omega_1\varrho_1(\nu_1-1) + \\ & + (1-\omega_1)\,\varrho_2\,(\nu_2-1)]\,E^{-1}\Delta\, v \end{aligned} \quad (73)$$

$$\begin{aligned}\Delta\,[(p'q)/\pi] = {} & \varrho_1\,\nu_1\,\Delta(\mu_1/\pi) + \varrho_2\,\nu_2\,\Delta\,(\mu_2/\pi) + [1-\omega_1\,\varrho_1\,\nu_1 - \\ & -(1-\omega_1)\,\varrho_2\,\nu_2]\,E^{-1}\,\Delta\, v \end{aligned} \quad (74)$$

$$\begin{aligned}\Delta\,(\sigma/\pi) = {} & (1-\varrho_1)\,\Delta\,(\mu_1/\pi) + (1-\varrho_2)\,\Delta\,(\mu_2/\pi) - [1-\omega_1\,\varrho_1 - \\ & -(1-\omega_1)\,\varrho_2]\,E^{-1}\Delta\, v \end{aligned} \quad (75)$$

$$\begin{aligned}\Delta\,(\lambda/\pi) = {} & (1-\varrho_1\,\nu_1)\,\Delta\,(\mu_1/\pi) + (1-\varrho_2\,\nu_2)\,\Delta\,(\mu_2/\pi) - \\ & -[1-\omega_1\,\varrho_1\,\nu_1 - (1-\omega_1)\,\varrho_2\,\nu_2]\,E^{-1}\,\Delta\, v \end{aligned} \quad (76)$$

If (72) and (73) are compared it will be seen that they involve six coefficients but only five independent parameters. If estimates of ϱ_1, ϱ_2 and ω_1 are obtained from (72), then (73) will yield estimates of ν_1 and ν_2 and also a second estimate of ω_1. The simultaneous solution of the two equations would hardly be feasible but it would be possible, for example, to fit the investment equation subject to a particular value of ω_1 as given by the consumption equation. This refinement has been ignored and (73) has been fitted separately and taken in conjunction with model (1) of table 2. In this way the following estimates were obtained

	$\overline{\varrho_1(\nu-1)}$	$\overline{\varrho_2(\nu-1)}$	$\overline{\varrho}_1$	$\overline{\varrho}_2$	$\overline{\nu}_1$	$\overline{\nu}_2$	$\overline{\omega}_1$	R^2	s
Interwar	0.062 (0.037)	0.029 (0.045)	0.64	0.24	1.10	1.12	2.81	0.43	0.23
Combined	0.088 (0.041)	0.039 (0.058)	0.62	0.32	1.14	1.12	1.58	0.31	0.39

A noteworthy feature of these results is that the values of ν_1 and ν_2 are virtually the same, do not change substantially when the postwar years are included and do not differ substantially from the values obtained when no subdivision of income is made. The equations do not fit very well and the values of $\bar{\omega}_1$ have little significance. While it is to be expected that a disaggregative analysis would show that the average value of n_2 is larger than the average value of n_1, reflection will show that, with the model adopted here, expenditure on durable goods may play a very different part in the budgets of the two types of income recipient in spite of the apparent similarity of $\bar{\nu}_1$ and $\bar{\nu}_2$.

For the interwar period the five equations, corresponding to (72) through (76), are

$$\Delta\, v = 0.64\, \Delta\, (\mu_1/\pi) + 0.24\, \Delta\, (\mu_2/\pi) + 0.32\, \Delta\, E^{-1}v \tag{77}$$

$$\Delta\, [(p'v)/\pi] = 0.06\, \Delta\, (\mu_1/\pi) + 0.03\, \Delta\, (\mu_2/\pi) - 0.12\, \Delta\, E^{-1}v \tag{78}$$

$$\Delta\, [(p'q)/\pi] = 0.70\, \Delta\, (\mu_1/\pi) + 0.27\, \Delta\, (\mu_2/\pi) + 0.20\, \Delta\, E^{-1}v \tag{79}$$

$$\Delta\, (\sigma/\pi) = 0.36\, \Delta\, (\mu_1/\pi) + 0.76\, \Delta\, (\mu_2/\pi) - 0.32\, \Delta\, E^{-1}v \tag{80}$$

$$\Delta\, (\lambda/\pi) = 0.30\, \Delta\, (\mu_1/\pi) + 0.73\, \Delta\, (\mu_2/\pi) - 0.20\, \Delta\, E^{-1}v \tag{81}$$

while for the combined period they are

$$\Delta\, v = 0.62\, \Delta\, (\mu_1/\pi) + 0.32\, \Delta\, (\mu_2/\pi) + 0.24\, \Delta\, E^{-1}v \tag{82}$$

$$\Delta\, [(p'v)/\pi] = 0.09\, \Delta\, (\mu_1/\pi) + 0.04\, \Delta\, (\mu_2/\pi) - 0.12\, \Delta\, E^{-1}v \tag{83}$$

$$\Delta\, [(p'q)/\pi] = 0.71\, \Delta\, (\mu_1/\pi) + 0.36\, \Delta\, (\mu_2/\pi) + 0.12\, \Delta\, E^{-1}v \tag{84}$$

$$\Delta\, (\sigma/\pi) = 0.38\, \Delta\, (\mu_1/\pi) + 0.68\, \Delta\, (\mu_2/\pi) - 0.24\, \Delta\, E^{-1}v \tag{85}$$

$$\Delta\, (\lambda/\pi) = 0.29\, \Delta\, (\mu_1/\pi) + 0.64\, \Delta\, (\mu_2/\pi) - 0.12\, \Delta\, E^{-1}v \tag{86}$$

12. Empirical Results: Estimates and Projections

The equations set out at the end of the preceding section all relate to year-to-year changes. Calculated values of this year's levels of the variables can therefore be obtained by adding the calculated changes from last year to this year on to the actual levels of last year. Estimates made in this way are shown in table 3.

These estimates are based on (77) through (86). Thus for each of the five dependent variables, the table shows: (*i*) the actual value; (*ii*) the value calculated from the interwar equation; and (*iii*) the value calculated from the combined equation.

Table 3. Actual and Calculated Values of the Five Variables: £ (1938) per head

	Consumption			Net investment			Expenditure			Saving			Net lending		
	(i)	*(ii)*	*(iii)*	*(i)*	*(ii)*	*(iii)*	*(i)*	*(ii)*	*(iii)*	*(i)*	*(ii)*	*(iii)*	*(i)*	*(ii)*	*(iii)*
1925......	77.33	76.83	76.73	1.06	0.93	0.96	78.39	77.76	77.69	−0.59	−0.09	0.01	−1.65	−1.02	−0.95
1926......	77.05	77.31	77.40	0.87	0.85	0.84	77.92	78.16	78.24	0.31	0.05	−0.04	−0.56	−0.80	−0.88
1927......	79.57	80.07	80.08	1.12	1.21	1.34	80.69	81.28	81.42	3.28	2.79	2.78	2.16	1.58	1.44
1928......	80.65	80.55	80.39	0.98	0.83	0.85	81.63	81.38	81.24	2.85	2.94	3.10	1.87	2.11	2.25
1929......	82.26	81.97	81.85	0.95	0.94	0.98	83.21	82.91	82.83	2.77	3.06	3.18	1.82	2.12	2.20
1930......	83.30	83.71	83.67	0.70	0.85	0.89	84.00	84.56	84.56	4.18	3.77	3.81	3.48	2.92	2.92
1931......	83.87	83.96	83.88	0.63	0.60	0.63	84.50	84.56	84.51	4.21	4.12	4.20	3.58	3.52	3.57
1932......	83.00	83.56	83.24	0.52	0.50	0.48	83.52	84.06	83.72	2.19	1.63	1.95	1.67	1.13	1.47
1933......	84.50	84.04	84.12	0.77	0.76	0.80	85.27	84.80	84.92	3.16	3.62	3.54	2.39	2.86	2.74
1934......	86.47	86.10	85.86	1.02	0.69	0.74	87.49	86.79	86.60	2.40	2.77	3.01	1.38	2.08	2.27
1935......	88.15	88.42	88.40	1.20	0.93	0.99	89.35	89.35	89.39	4.16	3.89	3.91	2.96	2.96	2.92
1936......	90.35	90.22	90.10	1.11	1.15	1.21	91.46	91.37	91.31	4.91	5.04	5.16	3.80	3.89	3.95
1937......	91.96	91.84	91.70	0.66	0.93	0.96	92.62	92.77	92.66	5.07	5.20	5.34	4.41	4.27	4.38
1938......	92.79	92.78	92.60	0	0.49	0.51	92.79	93.27	93.11	4.49	4.50	4.68	4.49	4.01	4.17
1948......	85.39	86.09	85.77	1.68	1.49	1.47	87.07	87.58	87.24	1.99	1.30	1.62	0.31	−0.19	0.15
1949......	86.86	86.42	86.52	2.02	1.94	1.99	88.88	88.36	88.51	3.06	3.50	3.40	1.04	1.56	1.41
1950......	87.80	87.79	87.63	2.12	1.88	1.91	89.92	89.67	89.54	2.67	2.68	2.84	0.55	0.80	0.93
1951......	88.15	89.05	88.88	1.26	2.09	2.13	89.41	91.14	91.01	3.39	2.50	2.67	2.13	0.41	0.54
1952......	86.91	89.15	89.12	0.91	1.31	1.35	87.82	90.46	90.47	6.19	3.94	3.97	5.28	2.63	2.62
1953......	89.30	88.94	89.01	1.73	1.30	1.39	91.03	90.24	90.40	7.95	8.32	8.25	6.22	7.02	6.86
1954......	92.37	92.38	92.15	2.34	1.66	1.77	94.71	94.04	93.92	8.84	8.81	9.04	6.50	7.15	7.27

The calculated level in any year is obtained by adding the calculated change from the preceding year to the actual level in the preceding year. Thus, for example, the calculated change in saving between 1953 and 1954 is 8.81—7.95 = 0.86 from the interwar equation, column *(ii)*, and 9.04—7.95 = 1.09 from the combined equation, column *(iii)*. The actual change was 8.84—7.95 = 0.89, column *(i)*.

The conclusions suggested by table 3 may be summarised as follows.

(1) Since the consumption relationship shows a relatively small residual variance, the same is also true of the saving relationship. Saving, it will be remembered, is here defined so as to include the value of net investment in consumers' durable goods.

(2) Very little difference is made to the values calculated from these relationships by the inclusion of postwar experience; that is the calculated values in columns (*ii*) and (*iii*) for consumption and saving are very alike.

(3) These two relationships go most seriously wrong in estimating the change from 1951 to 1952 and, to a less extent, from 1950 to 1951. It will be interesting to discover, at a later stage in this whole investigation, whether this failure is attributable to the simplifications inherent in aggregation or whether there is something unusual about the recorded events of those years which places them outside the scope of the simple system of ideas described in this paper.

(4) The relationships for net investment in durables do not fit well and so the expenditure function is not quite as satisfactory as the consumption function and the net lending function is not as satisfactory as the saving function. It seems likely that these relationships are particularly sensitive to the simplifications involved in aggregation.

13. Summary and Conclusions

This paper and its conclusions may be summarised as follows.

(1) After a brief survey of some recent developments in the theory of the consumption function, (section 2), a formulation is given, (sections 3 and 4), based on a dynamic theory of demand for individual commodities. In this theory, expenditure is divided between consumption and net investment, a distinction is drawn between actual consumption and equilibrium consumption and, in the case of durable goods, net investment is assumed to take place when equilibrium stocks are higher than actual stocks. Thus parameters reflecting durability and the rate at which adjustment to equilibrium is attempted enter into each expenditure equation in addition to the price and income parameters which appear in static theory.

(2) The aggregate functions are obtained by summing the functions for individual commodities. This process introduces some inevitable simplifications into the relationships connecting aggregate variables. It seems likely that a distinct improvement, especially in the investment function, could be obtained by following a disaggregative, or at least partially disaggregative, approach. This will be attempted in due course.

(3) Various further problems are discussed in sections 5 through 8. These

are: the use of index-numbers (section 5); the device used in an attempt to introduce capital into the basic relationships (section 6); a means of testing whether consumers attempt to adjust to some simple function of recent income experience rather than to current income (section 7); the problems introduced by a subdivision of income (section 8).

(4) A variety of models of the consumption function, based on the foregoing theory are examined in the light of experience in the United Kingdom. Calculations are made for the interwar years (1924 to 1938) alone and for these combined with the postwar years (1947 to 1954).

(5) Section 9 contains various versions of the consumption function using a single income variable. From this section it would appear that: (*i*) the short-period marginal propensity to consume is about one-half; (*ii*) there is little evidence that the substitution of a simple average of recent income experience for current income leads to any improvement in the consumption function; (*iii*) it does not appear that capital, as it can be reflected with available data, has any significant influence in the aggregate consumption function; (*iv*) it is difficult to decide, on the basis of the data, on the correct pair of values for the long-period marginal propensity to consume and the average rate of adjustment. On the assumption that the long-period marginal propensity to consume is never far short of unity for any large class in the community, then the average rate of adjustment would appear to be about one-half or a little more.

(6) Section 10 contains various versions of the consumption function using two income variables. The main conclusion from this section is that the short-period marginal propensity to consume for small incomes is significantly higher than the corresponding propensity for large incomes. This result is attributed to different rates of adjustment due to the different importance of durable goods in the budgets of the rich and the poor. It would also be compatible with the hypothesis that the long-period marginal propensity to consume of the rich was markedly less than that of the poor or with a combination of this and the preceding hypothesis.

(7) Sections 9 and 10 are concerned, explicitly, only with the consumption function and, since saving is equal to income minus consumption, implicitly, with the saving function also. In section 11 various estimates are made of the net investment function for consumers' durable goods. This makes possible the derivation of expenditure and net lending functions. The aggregate investment functions are not very satisfactory but, in part at least, this is attributable to the fact that unrealistic assumptions are needed to obtain such a function with constant coefficients.

(8) Section 12 contains the results of applying the functions involving two income variables set out in section 11. The consumption function, in particular, yields on the whole, rather accurate estimates except in 1952

and, to a lesser extent, in 1951. The relationship fitted to data for the interwar period alone yields results in the postwar period which are very similar to those obtained from a relationship fitted to interwar and postwar data combined.

(9) Two general conclusions may perhaps be singled out for emphasis from the numerous results presented in this paper. In the first place, of the numerous refinements introduced into the consumption function two appear to be important; first, that it should be formulated in dynamic terms and, second, that a distinction should be made between responses to changes in high and in low incomes. The other complications considered do not seem of much importance in the present context. In the second place, if the aggregate consumption function is considered in relation to the component functions for individual commodities, as it undoubtedly should be, then it is evident that the process of aggregation involves highly unrealistic assumptions. This conclusion holds even more strongly in the case of the investment equation. It may be surmised that future developments in this area of research will depend largely on the use of a disaggregative approach, a basis for which is provided in this paper.

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APPENDIX. THE BASIC DATA

The basic data used in the analyses of sections 9 to 12 of this paper are shown in the accompanying table 4. Values per head of the population at constant (1938) prices are given. The figures of income and expenditure for the postwar years 1946—54 are taken from the official estimates [3]. The methods used to compile the remaining series and the definitions employed may be briefly described.

The income figures shown represent personal disposable income, excluding both income tax paid and national insurance contributions. No adjustments for stock appreciation have been made but the amounts involved are relatively small in this sector. Total income has been subdivided into two categories: the first comprising wages and salaries, including employers' contributions, pay in cash and kind of the Armed Forces, and current grants to persons from public authorities; and the remaining professional and property incomes being included in the second category.

For the interwar period the estimates of wages and salaries are taken from the work of Chapman [4]. Estimates for the second category of income were obtained by making use of the records of income tax assessment, unpublished estimates of the transactions of the government sector made by Utting, and a series of sample data relating to net corporate income and dividends. Tax payments were roughly allocated between the different sectors and between the two categories of income to derive estimates of disposable income.

Total consumers' expenditure is divided between consumption and net investment in durable goods. Net lending is obtained as a residual, being the difference between income and expenditure. Saving is the sum of net lending and net investment.

The estimates of total consumers' expenditure and purchases of durable goods for the interwar years will be treated elswhere [16] in considerable detail and need not be discussed here. For the present purpose three classes of durable goods were distinguished: clothing, household durable goods, and motor vehicles. Annual depreciation rates of 2/3, 1/5 and 1/5 respectively were assumed. With these depreciation rates and an initial level of stocks estimates of consumption in each year could be derived by means of the relationships set out in equations (20) to (22) of section 3. The estimates made by Prest [12] for the period 1900—1919 were used in obtaining the initial levels of stocks.

Figures at constant prices were obtained by using a price index appropriate to deflating consumers' expenditure.

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Figures at constant prices were obtained by using a price index appropriate to deflating consumers' expenditure.

Table 4. The Basic Series: £ (1938) per head

Year	Wages, etc.	Non-wage incomes	Total income	Con-sumption	Net in-vestment	Total ex-penditure	Saving	Net lending
1923...	...	...	...	74.69	1.10	75.79	...	...
1924...	49.25	26.44	75.69	75.88	1.02	76.90	—0.19	—1.21
1925...	50.05	26.69	76.74	77.33	1.06	78.39	—0.59	—1.65
1926...	48.41	28.95	77.36	77.05	0.87	77.92	0.31	—0.56
1927...	52.92	29.94	82.86	79.57	1.12	80.69	3.28	2.16
1928...	52.97	30.53	83.50	80.65	0.98	81.63	2.85	1.87
1929...	54.49	30.54	85.03	82.26	0.95	83.21	2.77	1.82
1930...	55.32	32.16	87.48	83.30	0.70	84.00	4.18	3.48
1931..	55.78	32.31	88.08	83.87	0.63	84.50	4.21	3.58
1932...	56.35	28.84	85.19	83.00	0.52	83.52	2.19	1.67
1933...	58.19	29.47	87.66	84.50	0.77	85.27	3.16	2.39
1934...	60.29	28.58	88.87	86.47	1.02	87.49	2.40	1.38
1935...	61.50	30.82	92.31	88.15	1.20	89.35	4.16	2.96
1936...	63.57	31.69	95.26	90.35	1.11	91.46	4.91	3.80
1937...	64.47	32.56	97.04	91.96	0.66	92.62	5.07	4.41
1938...	65.07	32.22	97.29	92.79	0	92.79	4.49	4.49
1946...	68.14	21.90	90.04	84.69	1.20	85.89	5.35	4.15
1947...	68.34	21.77	90.11	86.39	1.80	88.19	3.73	1.93
1948...	67.87	19.51	87.38	85.39	1.68	87.07	1.99	0.31
1949...	69.74	20.18	89.92	86.86	2.02	88.88	3.06	1.04
1950...	70.55	19.92	90.47	87.80	2.12	89.92	2.67	0.55
1951...	72.31	19.24	91.55	88.15	1.26	89.41	3.39	2.13
1952...	73.62	19.48	93.10	86.91	0.91	87.82	6.19	5.28
1953...	77.22	20.04	97.26	89.30	1.73	91.03	7.95	6.22
1954...	80.67	20.53	101.20	92.37	2.34	94.71	8.84	6.50