Perceived difficulty of numbers relates to self-reported achievement in adults


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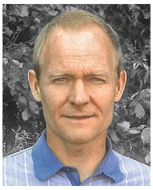
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Abstract: *Are some numbers perceived as more difficult than others, and if so, is this dependent on general mathematical ability? We analysed 689 adults' (age >18 years) responses to a questionnaire on perceived difficulty of and preference for different numbers' multiplications. Participants with low self-reported mathematical ability score (MAS) perceived difficult numbers, i.e., numbers including digits 7 and 8, as more difficult than high MAS participants did. This has implications for teaching as well as for research. If a task is perceived as difficult due to the presence of specific digits, this could influence performance and ability to engage in, for example, learning activities.*

Introduction

It is well known that humans have at all times assigned different attributes to different numbers (Major, 2017). For example, some consider the number 3 a lucky number, and the Chinese perception of 4 is death, therefore buildings may lack a fourth floor, equivalent to many western hotels lacking room number 13, as this number in a western culture is thought to bring bad luck. The idea that numbers have different attributes can also be found in textbooks for teaching mathematics in the early grades. For example, the Kontext textbook system from Alinea is accompanied by a story of the family of numbers targeting the learners in primary school. In this story, each member of the family (numbers from 0 to 9) is assigned different personalities, such as Elegant One, Thoughtful Two, Tired Three, Silly Four, Cool Seven (in Danish: Elegante Et, Tænksomme To, Trætte Tre, Fjollede Fire, Seje Syv), etc. How numbers are perceived and what characteristics are assigned to them differs from person to person. Some prefer even numbers, others prefer odd. Some like 7, as this is believed to be a lucky or magic number, while others dislike 7 because they find it a 'difficult' number.

Difficult numbers and arithmetic

When is a number or a task considered difficult? Research on task difficulty has investigated how individuals make choices based on different characteristics of a task, such as cognitive demands, which includes the number of digits or letters that need to be remembered, the time necessary to perform the tasks and the amount of physical work that is required to perform the task (see e.g. Feghhi et al, 2021).

In arithmetic some numbers appear to be more difficult than others. In a study on difficulty of simple multiplication facts, Taraghi et al. (2014) found that when analysing the proportion of correct answers, multiplications with certain numbers were more difficult than others. They found multiplications with 1, 2, 5 or 10 to be the easiest, multiplications with 3, 4, 6 and 9 to be of intermediate difficulty and those with 7 and 8 to be the most difficult multiplications. It will come as no surprise to teachers that children find single-digit multiplication with operands 7 and 8 more difficult than multiplications with other operands (Taraghi et al., 2014; van der Ven et al., 2015). These differences in difficulty levels have been explained according to several aspects (for an overview see van der Ven et al., 2015), for example problem-size effect, that is, problems with small operands are solved faster and more accurately than problems with large operands. However, existing research has primarily analysed the difficulty of specific numbers in, for example, multiplication through correctness of performed operations. How the difficulty of numbers is perceived irrespective of the individual's ability to perform a calculation correctly has to the best of our knowledge not been systematically investigated.

How a person feels about a number, or whether a person prefers or not to do calculations involving a specific number based, for example, on how the person perceives that number is important. Bandura (1997) writes about beliefs about personal efficacy as follows: "Beliefs of personal efficacy constitute the key factor of human agency. If people believe they have no power to produce results, they will not attempt to make things happen" (p. 3). Thus, there is a risk that (self-perceived) low performers will not engage in problem solving including numbers they perceive as difficult. As a result, they will have less experience and fewer potential successes in problem solving, further contributing to low self-efficacy.

The main aim of this first exploratory study was to investigate if it would be possible to measure differences in preferences for and perceived difficulty of different numbers. We therefore designed a simple questionnaire and collected data through social media. Of course, this has some implications when analysing data and interpreting results due to the self-selection bias (see e.g. Khazaal, et al., 2014). However, we find the nature of the results interesting and will therefore report the findings here. Our hope is that this can inspire further research into perceived difficulty of and preferences for engaging with mathematics tasks, as well as the implications for learning.

The research questions for the current study are the following: Are some numbers, and multiplications including these numbers, preferred more often than others? If so, is this dependent on self-perceived mathematical ability?

This study

In a survey conducted on Facebook, adults (age >18 years) were invited to complete a short questionnaire designed to capture perceived difficulty of and preference for different numbers. The questionnaire was made available on 20 March 2023 via the second author's network and was boosted on the following day to increase the outreach. The boost ran for three days. For the current analysis we included data from the first six days, from 20 to 26 March 2023. Participants were informed at the start of the questionnaire about the aim of the research, what type of questions would be asked, that data would be fully anonymised from the start and therefore it would not be possible to withdraw after finishing the questionnaire, however it was possible to quit during the questionnaire by simple exiting the questionnaire as information and answers would not be saved.

Questions on age, gender (female, male, other, prefer not to answer) and mathematical ability score (MAS) (self-evaluation on a scale from 1 to 5: 'How do you rate yourself in mathematics?' [Hvor god oplever du, at du er til matematik?]) were included at the start of the questionnaire. After answering the questions on perceived difficulty and preference for different numbers, participants were encouraged to share their experiences with numbers in general (open answer category). The sample includes 691 adults, of whom 515 reported their gender as female, 174 as male and two reported 'other' or did not want to report their gender. For analytical reasons, we only include data for participants that reported gender as male or female resulting in a dataset of 689 participants.

To capture the *perceived difficulty of specific numbers,* participants were asked to state on a scale from 1 (unlikely) to 5 (most likely) whether they believed they would be able to perform a specific multiplication (see example in Figure 1A). The participants were not asked to perform the calculation. The multiplications consisted of nine items with two single-digit operands, and nine items with single-digit operand multiplied by a two-digit operand (Table 1).

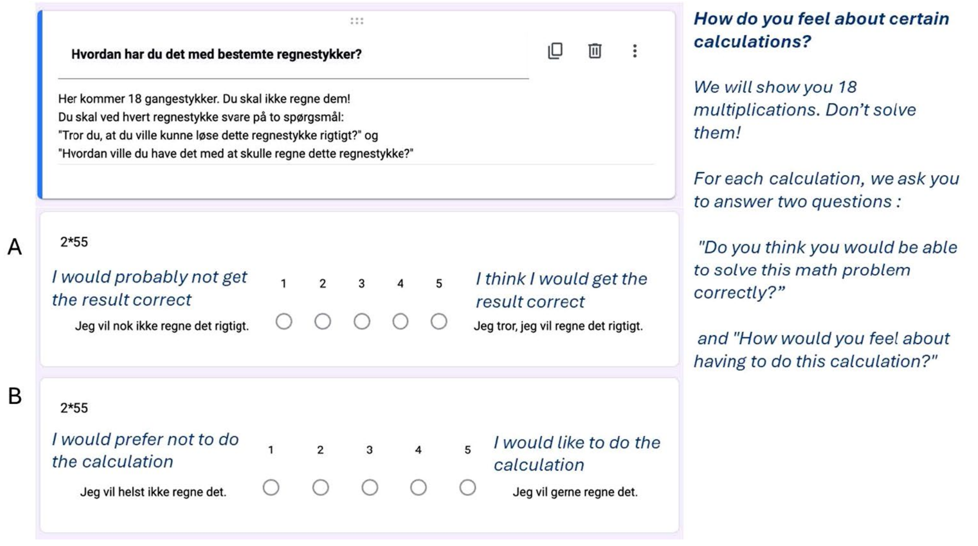

Figure 1. *Examples of questions on multiplication. Each multiplication has two questions. A: Whether the participant believes that he/she would get the calculation correct. B: To what extent the participant would prefer to perform the actual calculation.*

To capture *preferences for specific numbers*, participants were asked if they would prefer, on a scale from 1 (least) to 5 (most), to perform the same multiplication or not (Figure 1B), and additionally they were asked about their preferences (1 to 5) for specific numbers (see examples in Figure 2): nine two-digit and nine three-digit num-

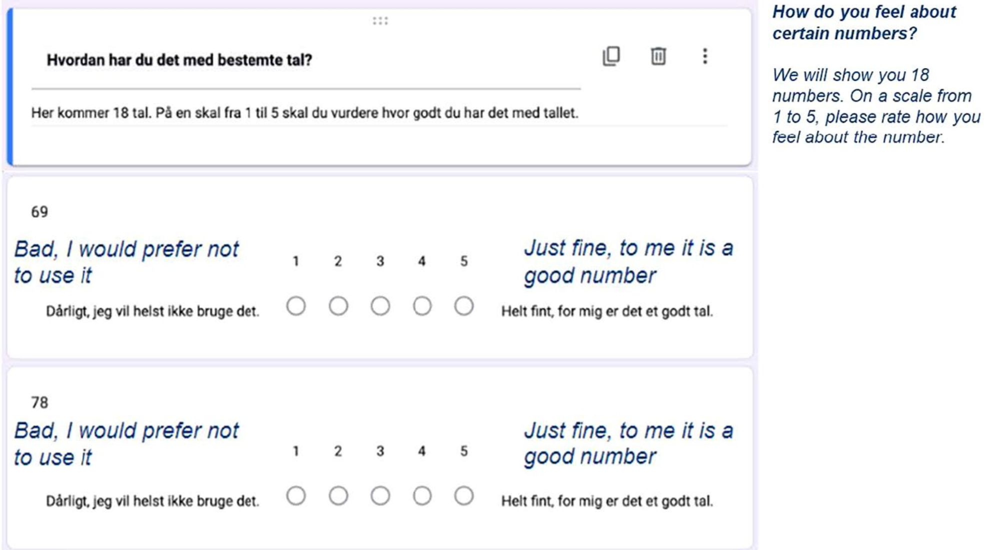

Figure 2. *Examples of questions related to multidigit numbers.*

Figure 3. *Examples of question on doing calculations in context.*

The price difference for diesel and petrol is the same in the two cities of Duckburg and Goosetown.

In Duckburg, a liter of diesel costs DKK 17.78 and a liter of petrol DKK 18.57.
In Goosetown, a liter of diesel costs DKK 15.43 and a liter of petrol DKK 16.22.

In which city would you prefer to calculate the price difference?

- ❑ Duckburg
- ❑ Goosetown
- ❑ I think it is equally easy or difficult

bers (Table 1). Furthermore, we included three items on calculations in context (see example in Figure 3). In this paper, we only analyse data on preference to perform a multiplication without everyday context (Figure 1B) and number preference (Figure 2).

Coding for item difficulty

The difficulty of the numbers in each item of the questionnaire was scored from 1 to 6 based on the presence of specific digits. Following the findings of Taraghi et al. (2014) regarding difficulty of operands in single-digit multiplication, we categorised the digits into three groups: the easiest (e) were 1, 2 and 5; intermediate difficult (i) were 3, 4, 6 and 9; and most difficult (d) were 7 and 8. We do not include 0 in this study. Difficulty level of 2- and 3-digit items was scored as shown in Table 1. The score of an item was based on the difficulty of included digits irrespective of the sequence of the digits.

Table 1. *Overview of coding for difficulty following the findings of Taraghi et al. (2014). Easiest numbers or digits (e): 1, 2 and 5. Intermediate difficulty (i): 3, 4, 6 and 9. Most difficult (d): 7 and 8. Sequence of digits or operands was not taken into account.*

Score	2-digit items	Numbers	Multiplications	3-digit items	Numbers	Multiplications
1	ee	25	2 * 5	eee	255	2 * 55
2	ei	23; 41	2 * 3; 4 * 5	eei	253; 451	2 * 53; 4 * 55
3	ed	27; 57	2 * 7; 5 * 7	eed	272; 572	5 * 72; 2* 72
4	ii	69	6 * 9	eii	692	6 * 92
5	id	38; 89	3 * 8; 8 * 9	eid	382; 892	3 * 82; 8 * 92
6	dd	78	7 * 8	edd	782	7 * 82

Analysis

In this paper we will only analyse data for preference, which is the score indicating whether the informant would prefer to solve the multiplication task or use the number. For every study subject we calculated the average preference score (APS) for each difficulty category. Hence every subject was represented by six average preference scores, one for each difficulty category, resulting in 4,134 APS values (6 categories multiplied by 689 subjects) in the dataset. We calculated APS separately for multiplication and multidigit number items. Correlation coefficients (Pearson's r) between the two measures (n = 4134) were r = 0.637 (r^2 = 0.409) for females and r = 0.499 (r^2 = 0.249) for males, showing that less than 41% (r^2) of the variation in one measure was explained by variation in the other measure. Therefore, we analysed and interpret variation in the two measures (multiplication and multidigit numbers) separately.

We analysed the variation in APS with a general linear mixed model (PROC MIXED in SAS 9.4) with gender (M, F), self-reported score of mathematical ability (MAS) and item difficulty score (DIFF) as categorical fixed effects variables. We stated subject ID as random intercept. Residual plots indicated reasonably normal distributed error terms, leading us to believe that the model's predictions are trustworthy.

We evaluated the statistical significance of the different predictors from type-1 (sequence of fixed effects: gender, MAS, DIFF, MAS*DIFF interaction term) and type-3 sum-of-square F-statistics. Since the type-1 analysis revealed that the initially highly significant effect of gender was reduced to borderline significant (B) or not significant at all (N), to estimate the raw difference between M and F we also ran models with gender as the only fixed effect. We derived predicted mean values for the different categorical variables as least square means predictions.

Results

Descriptive analyses on gender and age distribution of participants are presented in Table 2.

In general, women scored their mathematical ability (MAS) 0.32 points lower than did men (F; mean = 3.92 [95%CI: 3.83-4.01]; M mean = 4.25 [4.12-4.37], median = 4 95%; Kruskall–Wallis test: P = 0.0013). The distribution of self-reported mathematical ability divided by gender can be seen in Figure 4.

Men reported higher preference scores for number items (mixed model: B/SE = 0.209/0.085, t_{3440} = 2.47, P = 0.014) as well as for multiplication items (B/SE = 0.209/0.055, t_{3440} = 3.77, P = 0.0002) than women did (Fig. 5). For both item types most or all of this apparent gender difference disappeared when included in models that also accounted for MAS in interaction with item difficulty score (remaining gender differences when

Table 2. *Distribution of the different age groups divided by gender. For legal reasons concerning the remote possibility of identifying individual participants, groups comprising five or fewer observations are not shown.*

Age group	Women (F)	Male (M)	Total
Below 20	*	*	11
20-29	50	18	68
30-39	123	43	167
40-49	176	47	224
50-59	104	35	139
60-69	40	21	61
70-79	14	6	20
80 or above	*	*	*
Total	515	174	691

adjusted for MAS in interaction with difficulty score: number items: P = 0.49, difference = 0.05 points lower for F than for M; multiplication items: P = 0.046, difference = 0.09 points lower for F than for M), which in turn were highly statistically significant for both types of items (P < 0.0001; Fig. 6).

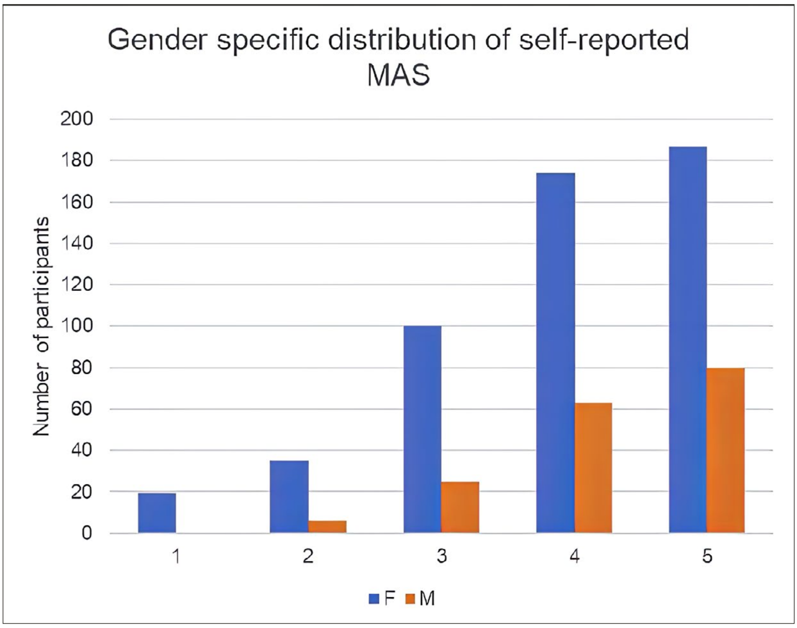

Figure 4. *Distribution of self-reported mathematical ability score (MAS; 1: lowest) divided by gender (F: female; M: male).*

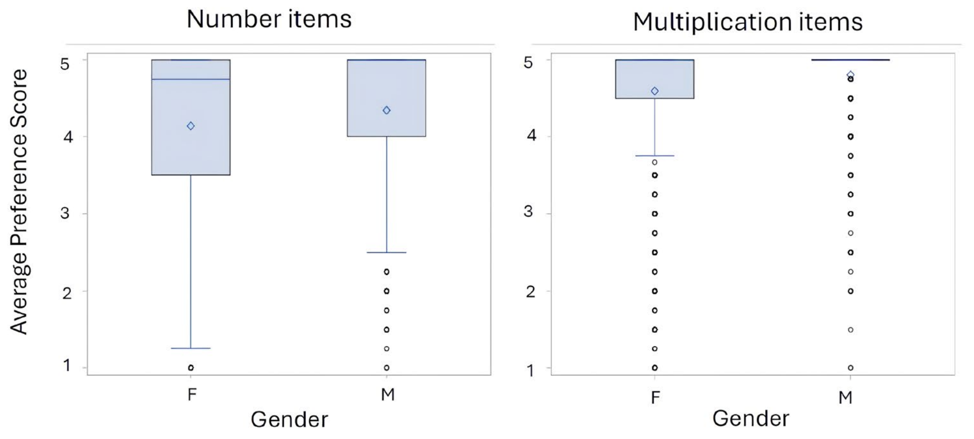

Figure 5. *Boxplot showing central tendency of average preference score for number items and multiplication items divided by 515 females (F) and 174 males (M): mean (dot inside box), median (horizontal line inside box), lower to upper quartile (greyish box), 1.5 times inter-quartile range (whiskers) and outlying observations (dots outside box).*

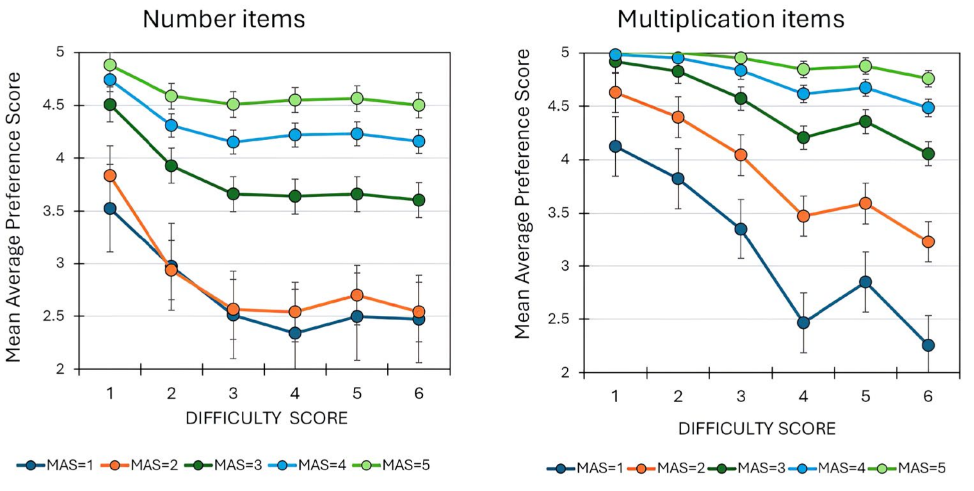

Figure 6. *Least square mean estimates with 95% confidence error bars of average preference scores (1 [least] – 5 [most]) for number items and multiplication items as function of difficulty score and self-reported mathematical ability score (MAS: 1 [low] – 5 [high]). For both types of items the interaction between difficulty score and MAS was highly statistically significant ($P < 0.0001$), whereas the partial effect of gender (after having accounted for difficulty of item and MAS) was negligibly small and is therefore not shown.*

For both types of items preference scores increased with increasing MAS score and was higher for items with the lowest difficulty score compared to those with the highest difficulty score. However, as can be seen in Figure 6, these overall correlations were neither linear nor additive between difficulty and MAS score.

Discussion and conclusion

The current study has shown that not only are some numbers perceived as more difficult than others, but that this is also related to self-reported mathematical ability. Groups with low self-reported MAS preferred to do multiplication or use difficult numbers, namely numbers including the digits 7 and 8, to a lesser extent than groups with high self-reported MAS. The differences in preference scores indicate that groups with low MAS perceived difficult numbers significantly more difficult than groups with high MAS did (Figure 6). This was illustrated by all (number items) or nearly all (multiplication problems) of the differences between male and female in item preference being attributable to females scoring themselves lower in self-reported mathematical ability. Hence, even though males overall scored significantly higher than females in item preference, there were no real differences in item preference between males and females with similar mathematical self-assessment scores.

Whether or not numbers are perceived as difficult by some achievement groups has implications for teaching as well as for research. If a person perceives a task as difficult due to the presence of specific digits, this could influence performance and ability to engage in, for example, learning activities. As one participant stated: "I am a math teacher, but even so it feels like a shock through my body if I have to do arithmetic with 7 and 8 and the result exceeds 10." These aspects of individual differences in perceived difficulty are relevant in teaching as well as research.

In relation to teaching, our findings could suggest that teachers and textbook authors should carefully consider their choices of numbers in the tasks. Based on our results in this study we would suggest that numbers that could be perceived as difficult by especially the lower performing students should be avoided when introducing new topics and when consolidating new knowledge. Especially for the low performing or insecure students, the use of the difficult numbers like 7 and 8 should be avoided until the students have gained a certain level of conceptual and procedural knowledge of the specific mathematical content. In our own experience, we and teachers alike often make easy tasks more difficult by 'adding a couple of 7s and 8s' to the numbers in the task, or reversibly, exchange the 7s and 8s for easier digits if we believe the task to be too difficult. Our current findings could indicate that this needs to be done with care as this might refrain some students from engaging in the tasks – not because the task is more difficult, but because the student is less convinced that they will be able to perform the task. If this is the case, this has implications for how the items for tests are designed, whether this is for evaluating students' performance, final exams or for research purposes. There might be a layer of difficulty that we inadvertently add to the test items that disproportionately disadvantages the lower achieving students.

Limitations

While analyses based on samples obtained by self-selection (as in this study) may reveal useful results about relationships between different predictor and response variables, inferences to populations should be treated with extreme caution as self-selected samples cannot be considered representative. Before repeated on a systematically obtained sample, our results should therefore be considered indicative rather than conclusive regarding the overall population of Danish citizens. This also includes overall population differences between males and females.

Only 61 participants out of the 691 scored themselves as low ability in mathematics (scores 1 and 2), approximately 9%, which is slightly lower than what is believed to be the prevalence of mathematical difficulties in the population (Lindenskov and Lindhardt, 2023; Mikkelsen et al., 2023). However, we did see a relatively good distribution of age, with a majority of participants in the age range of 30 to 60 years. With respect to gender, women were overrepresented (3:1). Considering the total number of participants (691) we believe the findings to be representative with respect to gender.

Concluding remarks

What we have presented here is a first exploratory pilot study that, despite the limitations inherent in the methodological approach, nevertheless indicates that numbers matter. People who perceive themselves as lower performing in mathematics tend to have stronger negative feelings towards engaging with difficult numbers, for example numbers including the digits 7 and 8. As we have outlined above, this can have important implications for teaching, examinations and research. Our hope is that this pilot study can inspire further research into perceived difficulty of and preferences for engaging with mathematics tasks, as well as the implications for learning for low performing students.

Acknowledgement

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Dansk abstract

Opfattes nogle tal som sværere end andre, og er det i så fald afhængigt af personens generelle matematiske evner? Vi analyserede 689 voksnes (alder>18) svar på et kort spørgeskema om talpræferencer og lyst til at udføre multiplikationer. Deltagere med lav selvrapporteret score for matematiske evner (MAS) opfattede svære tal, dvs. tal indeholdende cifrene 7 og 8, som sværere end deltagere med høj MAS. Dette har konsekvenser for både undervisning og forskning. Hvis en opgave opfattes som svær på grund af tilstedeværelsen af bestemte cifre, kan det påvirke præstationen og evnen til at engagere sig i f.eks. læringsaktiviteter.