Surface Movement and Mass Balance at the Hans Tausen Drill Site determined by use of GPS

By Kristian Keller, Christine S. Hvidberg, Niels Gundestrup and Peter Jonsson

Abstract

During the deep drilling at Hans Tausen Iskappe in Peary Land, North Greenland, the ice cap has been mapped with GPS. Kinematic GPS and ice penetrating radar measurements were done at the southeast dome extending to approximately 3 km from the drill site at the top of the dome, 82.5°N; 37.5°W (Gundestrup et al. 2001). Furthermore a strain net was established consisting of a center pole at the top of the dome and three rings, each of eight poles, at distances 0.3, 1.5, and 3.0 km from the center.

The bottom and surface topography, ice surface velocities from the strain net and precipitation data were used to calculate the present mass balance of the dome. The present mass balance of the dome was found to be +0.04±0.02 m of ice/year, i.e. the ice thickness at the dome increases with a rate of about one third of the annual accumulation. The result is highly consistent and independent on assumptions regarding the ice flow. Our result shows that the central part of the ice cap is far from steady state, implying that simple models cannot be used to determine the time scale of the ice core and the thinning of annual layers.

Keywords: GPS; strain net; ice flow model; mass balance; North Greenland.

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Introduction
Before the ice core drilling at the Hans Tausen Iskappe (Ice Cap) in 1995, an airborne radar survey was performed in 1993 in order to map the ice cap. The radar survey was supplemented with a surface survey in 1994 in order to select the drill site at the exact top of the southeastern dome (Gundestrup et al. 2001). This paper presents a height model of the ice surface topography around the dome. The 1994 survey found the exact
position of the dome and allowed a strain net to be erected and precisely positioned by relative GPS observations. The area covered by the strain net is approximately 6 km by 6 km. In 1995, GPS measurements of the strain net were repeated and ice surface velocities were computed. The strain net data enable us to determine the mass balance of the Hans Tausen Iskappe at the drill site.

The present mass balance of Hans Tausen Iskappe is important for understanding the history and state of the ice cap. In this context, the mass balance at the dome is a supplement to observations along the margins (Thomsen et al. 2001). Furthermore, changes over time in ice thickness in the central part will influence the ice flow pattern and the thinning of annual layers. The mass balance at the dome is therefore relevant for interpretation of data from the ice core drilled at the dome.

The mass balance is determined with three different glaciological approaches; two based on the continuity equation combined with results from a thermomechanically coupled ice flow model, one based on the surface principal strain rates. We obtain a consistent estimate of the mass balance at the drill site. The high quality of the result is partly due to the accurate and comprehensive data set, partly to the geometry of the ice cap with an horizontal bedrock around the drill site.

The surface survey
In June 1994, the exact top of the dome was identified (Gundestrup et al. 2001). At the top, an aluminum pole was erected as a local reference point. This point was tied into the geodetic benchmark KMS No. 1001 (81°36’01.43065”N; 16°39’19.84959”W; H (ellip)=72.883 m; WGS84) at Station Nord, 250 km east of the drill site, by multiple GPS sessions in both 1994 and 1995 with an accuracy of approximately 0.05 m.

During skidoo traverses in 1994, ice thickness and height measurements were collected simultaneously along north south going lines with an intended spacing of 300 m. The surface survey was extended by traverses to erect strain net poles, which supplemented the data; in total more than 8000 height measurements were obtained. Because of RF-interference from a satellite std-C telex, the reference GPS data had many gaps, and the static initializations were lost. Nevertheless, it was possible to process all the GPS data using Trimbles software: GPSurvey, ver. 2.0 using the ‘On-The-Fly’ mode. We obtained very accurate height measurements with an estimated overall rms-error of less than 0.05 m. This result is very fine especially considering, that the geometrical configuration of the GPS satellites causes GPS height measurements to be approximately three times less accurate than the horizontal positions.

A height model is presented in Fig. 1. It is composed by two data sets: The data from the surface survey in 1994, combined with data from the airborne radar survey of the Hans Tausen Iskappe in 1993 (Gundestrup et al. 2001). The model was made using the software...
Surfer (Golden Software) by detrending
the data manually by a fitted para-
boloid, then gridding the residuals by
kriging interpolation and finally adding
the gridded values onto the paraboloid.

The accuracy of the model is 0.05 m
within the strain net (3 km from the
dome), and 5-10 m outside the strain net
area. All heights are reduced from ellip-
soidal heights to mean sea level heights
by using the Greenland geoid ‘Geoid
94a’ (Forsberg 1996). The geoid height at
the Hans Tausen drill site is approxi-
mately 28.5 m.

The ice surface topography around
the dome is very regular with smoothly
varying surface contour lines (Fig. 1).
This is a result of the relatively flat and
smooth bedrock in the area (Gundestrup
et al. 2001). The overall structure of the
dome is, however, rather complicated.
Three ridges radiate from the dome with
troughs in between, where particularly
the northern ridge is very narrow and
lengthy.

The ice thickness in the measured pro-
files varied between 300 and 393 m with
an estimated accuracy of ±5 m. The
mean thickness is 350 m. In general the
thicknes varies between 340 and 360 m.
When converting the measured times to
depths, a velocity of 171 m/µs was used,
and correction for the firm layer was
assumed to be constant within the strain
net. The radar measured thickness is
tied to the logged depth by a profile
measured in close proximity to the drill
hole (Jonsson 2001).

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Table 1. GPS data at the
24 surface points shown
at Fig. 1: Pole number,
height, horizontal posi-
tion and surface move-
ment over 337 days
(1994-1995). The posi-
tion of the reference
pole is 82°29′30.6″N;
37°28′20.6″W;
H=1270.54 m a.s.l.
Strain net

The strain net was put up during the surface survey in 1994. It consists of three concentric rings with eight poles in each ring, around the reference pole in the center. The distances to the rings from the center are 0.3 km, 1.5 km and 3.0 km, corresponding to one, five and ten ice thicknesses. In 1995, GPS observations of the strain net were collected again, giving the movement of the poles relative to the reference pole over a period of 337 days. The horizontal movement of the strain net poles relative to the reference pole are listed in table 1, and illustrated at the map in Fig. 1.

The movement of the reference pole was found to be few centimeters (3.5 cm North, 1.4 cm East), i.e. within the accuracy of the location of the pole. This confirms that the reference pole is placed at the top of the dome. We also estimated the location of the top of the dome from the surface velocities with a simple model: For each pole in the inner ring of the strain net, we calculated an estimate of the center point assuming that the direction to the center is in the opposite direction of the velocity, and that the ice flow is simple axi-symmetric flow (model parameters as in the circular mass balance model below). The average of these points is found to be within few meters of the reference pole, again confirming that the reference pole is at the top of the dome. In the mass balance calculation below, we therefore assume that the reference pole is located at the ice flow center, i.e. the velocity of the reference pole is vanishing.

The overall ice flow pattern (Fig. 1) shows highly divergent flow along the ridges, while the flow is plane or convergent in the troughs. The velocities increase with distance from the center, with high velocities in the troughs, low along the ridges. In general, the surface velocity vectors point in the steepest downward direction.

Mass balance at the drill site

The mass balance is calculated in the following way: Flow lines from the dome to each strain net pole are determined. The equation of continuity is solved along each flow line to give the mean rate of ice thickness change ($\partial H/\partial t$), along that flow line. These ice thickness changes are assumed to be representative for the geographic direction from the dome to the strain net pole. The rate of ice thickness change at the dome is calculated as a weighted average of the mean rates along the flow lines at the surface.

In order to solve the continuity equation along a flow line, we assume that the direction of the flow does not change with depth, and that the flow lines are perpendicular to the surface contour lines, and we neglect shear stress transverse to the flow line (more details in Hvidberg et al. 1997; Reeh 1988). The equation of continuity is here written,

$$\frac{\partial q}{\partial x} + \frac{q}{R} = a - \frac{\partial H}{\partial t},$$

where $x$ is the horizontal distance along the flow line, $q=q(x)$ is the depth integrated ice volume flux per unit width, $H$ is the ice thickness, $t$ is time, and $a=a(x)$ is the net mass balance, and $R=R(x)$ is the radius of curvature of the surface contour lines at intersections with the flow line. The flux may be written

$q=u_w H=f u_s H$, where $u_w$ is the depth average of the horizontal velocity, $u_s$ is the horizontal surface velocity, and

$f=u_w/u_s$ is the shape factor of the horizontal velocity profile. In order to calculate the mean rate of ice thickness change along the flow line, we need to know the following parameters: The course of the flow lines, $a$ and $R$ along the flow lines, and $H$ and $f$ at the strain net poles.

The ice thicknesses around the drill site are determined from maps of surface movement and mass balance.
face and bedrock topography (Gunde-
strup et al. 2001; Jonsson 2001). The
uncertainty of the ice thickness is ±5 m.

The accumulation rate is determined
at the drill site by identified volcanic
horizons in the electric conductivity
record of the ice core (Clausen et al.
2001). We use a long term average of the
accumulation rate from 1912 AD (Kat-
mai eruption) to 1995 AD (year of
drilling) determined at the dome, and at
two previous drill sites in the area (1975
and 1976, situated at about 13 km SW
and 5 km S of the dome, at 125 m and
150 m lower elevations, respectively;
(Clausen et al. 2001) in order to set up an
elevation dependent accumulation rate,
a(z) = (–2.63·10^-4 z + 0.448) m of ice equiv./
year (z in meters), which we have used
here. This gives a=0.114 m/yr at the drill
site. Accumulation rates based on the
1783 AD horizon (Laki eruption) give
accumulation rates at about 0.08 m of
ice/year. The uncertainty of the accumu-
lation rate is set to ±0.02 m of ice/year.
The accumulation rate is assumed to
vary linearly along the flow lines from
the value at the drill site to the calcu-
lated value at each strain net pole.

Glaciological studies in the northern
part of the Hans Tausen Iskappe show
that the accumulation rate at the north
dome (about 35 km north of the 1995
drill site) is 0.42 m of ice/year (Thomsen
et al. 2001). This indicates large accumu-
lation gradients, probably strongly
related to the local topography. We do
not take accumulation data from the
northern part of the ice cap into account
here, as we do not know the local varia-
tion of the accumulation rate, e.g. the NS
ridge radiating from the southeast dome
may well introduce a strong local varia-
tion.

The shape factor of the horizontal
velocity, f is calculated for Hans Tausen
conditions. We have used a steady state,
thermo-mechanically coupled ice flow
model based on Glen’s flow law (Hvid-
berg 1996; Hvidberg et al. 1997). f has
been calculated for an ideal circular ice
cap with ice divide thickness and tem-
perature conditions as at the drill site.
The temperature difference between sur-
face and bedrock of only about 5% (per-
sonal communication with S. J. Johnsen)
results in values of f from 0.58 at the
dome to 0.8 in the outer ring, 3 km from
the center; which is only slightly higher
than calculated theoretically for iso-
thermal conditions (giving 0.5 at the
divide (Reeh 1988), 0.8 away from the
divide (Paterson 1994)). The uncertainty
of f is set to ±0.15.

As a first and simple approach, it is
assumed that the flow lines are straight
lines radiating from the dome in the
center of the strain net to each of the
poles. It is further assumed that the flow
is axi-symmetrical, i.e. R=x. The uncer-
tainty of the distance along the flow line
is set to ±15 m. The calculation only con-
sider the radial component of the hori-
zontal surface velocity at the strain net
poles. We call this the circular model. This
approach is naturally used here, where
the strain net poles are placed in (or
close to) three concentric circles around
the drill site at the center. For each strain
net pole, the mean rate of ice thickness
change along the corresponding flow
line is calculated (Fig. 2). For each of
the three strain net rings an estimate of the
regional mass balance is calculated. Each
of these estimates equals the mass bal-
ance found as the difference between the
total accumulation within the circle and
the mass flux across the perimeter of the
circle. The average mass balance at the
dome found as the mean of the results of
the three rings, and its accuracy (2 stan-
dard deviations) is calculated to be

<\partial H/\partial t> = 0.035 ± 0.015 m of ice equivalent /
year. (circular model)

To the uncertainty, a, f, and u_s all con-
tribute equally with about 1/3, while the
contributions from H and R are negli-
gible.
In a second approach, we utilize the mapped surface topography (Fig. 1) to derive information about the ice flow pattern and the flow lines. The flow lines are determined from the map by assuming that they radiate from the dome, and that they are perpendicular to the surface contour lines, i.e., that the ice flows in direction of the steepest surface slope. Because of the complex topography around the dome, the flow lines are curved in this approach, not straight lines as in the circular model above. The horizontal distance $x$ is measured along the flow line from the dome to the corresponding strain net pole, and the uncertainty of $x$ is set to $\pm 10\%$. $R(x)$ is derived from the map with the uncertainty set to $\pm 50\%$, which is relatively high partly due to uncertainties in the map introduced by the interpolation routine, but most importantly due to uncertainties in measuring the curvature. As seen at Fig. 1, the measured surface velocities are in the direction of the steepest surface slope, and therefore parallel to the flow lines in this approach. We call this the topographic model. As for the circular model, the results are shown at Fig. 2. The average mass balance found as the mean of the results from the three rings, and its accuracy (2 standard deviations) is calculated to be

$$<\frac{\partial H}{\partial t}> = 0.039 \pm 0.017 \text{ m of ice equivalent / year. (topographic model)}$$

The uncertainty is not much higher than in the circular model despite the low accuracy of $R$ and $x$. The difference is, however, caused by the contribution from $R$, which constitute about $1/3$ of the uncertainty, while $a$, $f$, and $u_s$ all contribute with about $1/6$.

The results for the strain net poles of the two models are compared at Fig. 2. The circular model shows high rates of ice thickness change along the ridges where the surface velocities are low, and vice versa in the troughs. As expected, the results of the circular model oscillates around the results of the topographic model, which are more constant, since the topographic variations are taken into account. The average mass balance of the two models equals, showing that all directions (ridges, troughs) are well represented in the circular model, and furthermore that the topographic model does describe the flow in a satisfactory way. An important reason is the relatively horizontal bedrock within the strain net. Two points in the inner circle in the NE and E direction from the center have exceptionally low values in both models. The relative uncertainty is large at these points, because the velocities are only about double the uncertainty, so the relatively large velocities (as seen at Fig. 1) may be overestimated.

Finally, as a third approach, the average mass balance is calculated from the principal strain rates at the surface, independent of the ice flow geometry. The sum of the surface principal strain rates at the dome is found to be $\dot{\varepsilon}_1 + \dot{\varepsilon}_2 = (3.72 \pm 1.8) \times 10^{-4} \text{ yr}^{-1}$ based on all the strain net stakes. In steady state this would balance the vertical strain rate. The rate of ice thickness change at the
dome is found from: \( \frac{\partial H}{\partial t} = \alpha H f (\varepsilon_1 + \varepsilon_2) \) (derived from Paterson, 1994, equation 33, p. 257). We assume \( \alpha = 0.114(0.02 \text{ m/yr}, \ H = 330(5 \text{ m}, \text{ and} \ f = 0.58(0.08} \) at the dome, and calculate the average mass balance to be
\[
\frac{\partial H}{\partial t} = 0.04 \ (0.04 \text{ m of ice equivalent / year. (from principal strain rates)
}

With three different approaches, we reach the same result for the mass balance at the southeast dome of the Hans Tausen Iskappe. The result depends on two critical parameters, which occur in all three models: the accumulation rate \( \alpha \) and the shape factor \( f \). As mentioned above, a longer time average of \( \alpha \) would reduce the result with 0.03 m/yr, and \( \alpha \) may vary locally in a way which could influence the result. The shape factor \( f \) is a modeled parameter without any direct bounds from data. However, the fact that we reach similar results for points at different distances from the center (Fig. 2), and that the calculation of \( \frac{\partial H}{\partial t} \) based on the principal strain rates gives the same result, does indicate consistency in the variation of \( f \). An additional reservation is that the three results are based on the same strain net data set with its possible deficiencies.

Discussion and conclusions

This paper presents a surface topography map around the drill site at the southeast dome of the Hans Tausen Iskappe based on a GPS survey. Furthermore, the surface movement of the ice cap around the drill site is mapped with GPS in a strain net consisting of 3 rings with 8 poles in each ring.

The data show that the ice thickness at the dome is increasing with a present rate of +0.04±0.02 m/year. The result is highly consistent and independent on assumptions regarding the ice flow. Important reasons for the successful calculation of the mass balance are the relatively high precision of the surface velocity data, and also the horizontal bedrock in the area, which secures a regular and smooth flow pattern.

Other glaciological investigations show that the Hans Tausen Iskappe disappeared during earlier changes in climate, and later emerged again (Landvik and Hansen 2001; Thomsen et al. 2001; Hammer et al. 2001). Our calculation of the mass balance show that the ice cap has not yet reached a steady state, but is still thickening. We calculate a thickening rate of about one third the present accumulation rate. Identified volcanic horizons in the electric conductivity measurements (ECM) record of the ice core show that the annual layer thickness is almost constant with depth down to about 100 m above the bed (Clausen et al. 2001). This supports that the ice cap is thickening, but the thickening rate may be as high as the accumulation rate, or lower if the accumulation rate is decreasing with time. Non-vanishing surface velocities show that the annual layers must be thinning, and studies of the crystal structure indicate that some deformation of the layers have occurred (Madsen et al. 2001); therefore, constant annual layer thickness with depth indicate that the accumulation rate must have been higher back in time, possibly as a result of the lower surface elevations back in time. A review of the available data from the Hans Tausen Iskappe suggests realistic scenarios with increasing ice thicknesses and decreasing accumulation rates (D. Dahl-Jensen, personal communication), where our calculated thickening rate is in agreement with the results from ice core analyses.

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References