Production cost calculation with »oscillating« cost categories

Af Manfred Welscheid

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The classification of various cost categories is introduced to guarantee a differentiated analysis of the profit centers' income statements. In the chemical industry the problems of production cost determination with partially cyclic and partially coupled production processes (by-products) are handled satisfactorily through formulation of a matrix model. This model is iteratively applied to each single cost category according to a well defined number of them. Thus all different kinds of cost accounting such as full, direct, and standard costing are possible.

More detailed considerations within these objectives require refinements of the original model, which was based on the traditional Leontief theorem. Using as input (intermediate) products or recycled goods, the splitting of total cost into various cost categories does not necessarily remain constant: The cost categories may cross over into each other or partially aggregate at the separation and calculation of costs for contents and packaging – the cost categories are »oscillating«.

In the packaging area we find (intermediate) products, whose input may be classified as both »packaging« and »contents«, this depending on the characteristics of their respective input processes – moreover we find similar situations at the reuse of damaged goods in the recycling case. The solution of the problems will be an approach, that analyzes all process formulas, describes the relevant properties by 0-1-values, and keeps them in separate »packaging« matrices. Based on these results and on the former Leontief model a sequence of eliminations and transformations with the quantity coefficients of production will take place. Thus an additional set of rules within the refined framework of the reflection on production costs will be fulfilled.

The algorithms satisfy all requirements of a centralized input-output-oriented costing model for a multi-division corporation using complex loop, staged, and coupled production processes. The consequences for decentralized costing models with pre-/post-calculations considering the specific market constraints of individual business sectors are also taken into account.

1. Introduction

In the chemical industry the problems of production cost determination with partially cyclic and partially coupled production processes (»by-products«) are handled satisfactorily through formulation of a matrix model.

At the production of n different products in n respective cost centers the following system of linear equations (i = 1,..., n) has to be solved (notation in view of computer languages):

$$k(i) + a(i,1) \times x(1) + a(i,2) \times x(2) + \dots + a(i,n) \times x(n) = x(i)$$

The model parameters have the following meaning:

- k(i): Primary cost in cost center i per unit (product). Cost vector k is known.
- x(i): Total cost in cost center i per unit (product). Cost vector x has to be determined.
- a(i, j): For the production of product i in cost center i the quantity a(i, j) of product j is necessary.

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Three matrices are introduced:

- A(n, n) : »Basic« interlacing matrix of the a(i, j)
- I(n, n) : »Identity« matrix
- M(n, n) : »Balanced« interlacing matrix: $M = I - A$

Now the interrelations of costs can be described:

$x = k + A \times x$ or $(I - A) \times x = k$ or
$M \times x = k$

The calculation of U different cost categories is given by the solution of the equivalent number of systems of linear equations:

$M \times xu = ku \quad (u = 1, ..., U)$

For disjunct cost categories is true:

$x1 + x2 + ... + xU = xG$ (Total cost)

In general, the separation of total cost (xG) of intermediate products into variable cost (xH) and fixed cost (xL)

$xH(i) + xL(i) = xG(i)$

is transferred to a finished product, which has as components several intermediate products:

Product 1 (Plastic bottle):
$xH(1) + xL(1) = xG(1)$
Product 2 (1 kg glue):
$xH(2) + xL(2) = xG(2)$

For the finished product 3 (1 kg glue packaged in a plastic bottle) is true:

$$xH(3) = xH(1) + xH(2)$$
$$xL(3) = xL(1) + xL(2)$$
$$xG(3) = xG(1) + xG(2)$$

Two specific cost categories – variable and fixed – had been chosen for the above considerations. The results cannot be arbitrarily transferred to any set of selected cost categories. The demonstrated correlations are not valid any longer, if a framework of rules is formulated, that does not keep constant the cost categories as they were determined in a specific previous production process. The concept of »oscillating« cost categories will be illustrated by a practical example. Then the procedures and and algorithms for the solution of the problem will be developed.

2. Problem definition

The separation of total cost into costs for packaging and contents (contents as the essential component to be sold) cannot always be treated in the above described way. A set of calculation rules may cause problems, that are not solvable through the simultaneous approach:

$M \times x = k$ with $M \times xH = kH$ and
$M \times xL = kL$

The following considerations are made:

- Performance of separate calculation for intermediate and finished products for both of their components

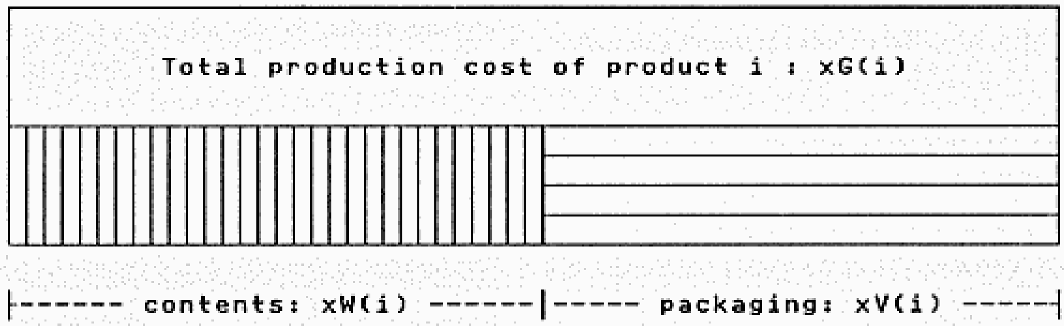
  »contents« and »packaging«

- Depending on an intermediate product's input processes its production cost components »contents« and »packaging« may be transferred to the produced products (as output of these processes) in three different ways:

  – separately as calculated for this input product
  – as the total cost with the character »contents«
  – as the total cost with the character »packaging«

- The aggregated cost of all production processes must be transferred to and shared by the finished products to be sold. Up to a certain degree damaged products may be recycled – i.e. parts of them become input again in previous production stages. Special arrangements for the calculation aspects have to be made: Not only the cost of the intact parts of such recycled products, but their total cost are to be considered.

The separation of total cost xG into two cost components »contents« (xW) and »packaging« (xV) is considered.
Each product i has the cost relation
$xG(i) = xW(i) + xV(i)$ or graphically:



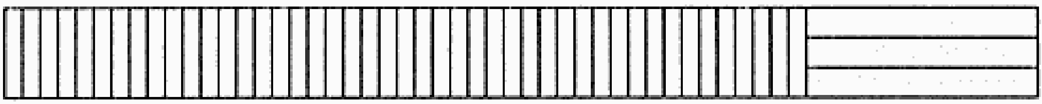
The theoretically formulated requirements to a cost calculation are now illustrated by three practical examples:

Example 1

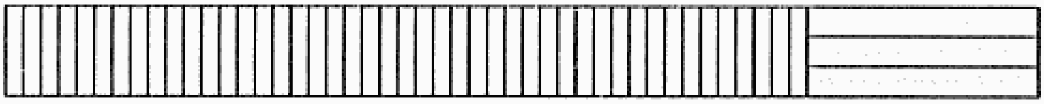
In the standard case, the separation of costs of a product consistently is respected, whenever this product serves as input in subsequent processes:

The separation of costs of a plastic bottle filled with glue is not changed, when this bottle is packed together with other glueing material for a do-it-yourself-kit (as a new product).

Calculated production costs of the bottle:



Costs are treated at the calculation of the kit:



Example 2

A glue (product q) was calculated with total cost xG(q) and the two cost components xV(q) and xW(q). This glue may have various input conditions in subsequent production processes:

- In a filling plant (packaging process) acid is filled into barrels. Glue is used to stick the labels on the barrels (glue as packaging material). For the production cost components of the glue is true (the symbol »&« will mean, that the cost components may change at the input of the product in particular subsequent processes – the characteristic is not necessarily constant):

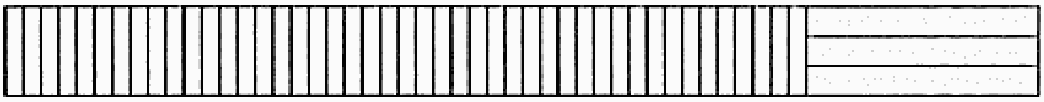
The total cost (xG) of the glue are regarded as packaging cost:

$xV\&(q) = xG(q).$

Glue as an input material brings no cost parts for the characteristic »contents«: $xW\&(q) = 0.$

Therefore the costs of the glue are regarded as follows:

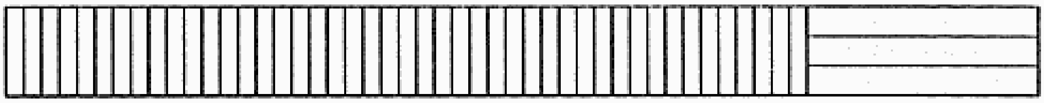
Calculated production costs of the glue:



Costs are treated at the calculation of the filled barrel:

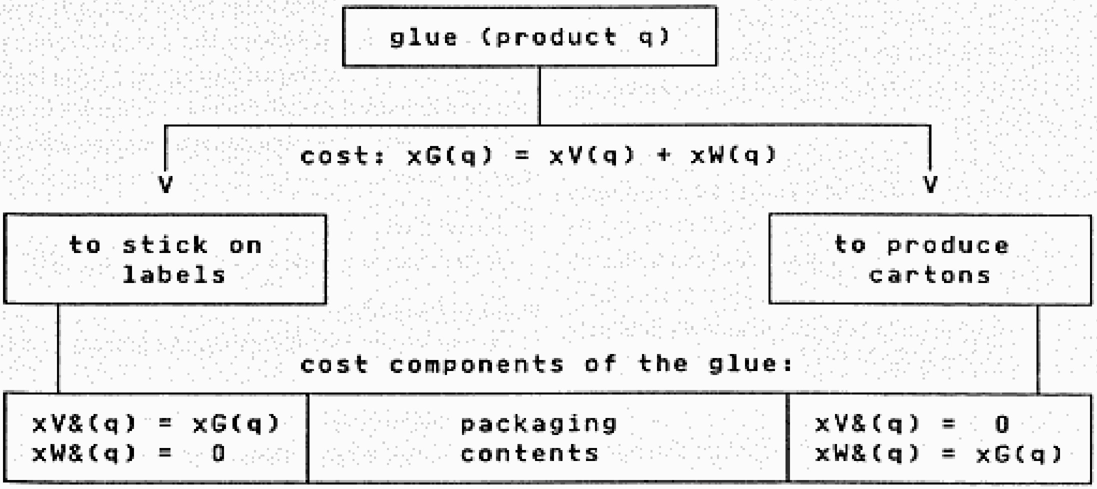
- The glue is used as raw material to produce cartons. Here the glue's total cost xG(q) are regarded as pure cost components for the contents; the glue does not cause packaging cost:
  xV&(q) = 0 and xW&(q) = xG(q).

Calculated production costs of the glue:



Costs are treated at the calculation of the carton:

It may be pointed out that the cartons themselves are pure packaging material in subsequent processes. The involved calculation problems are covered by the following considerations.

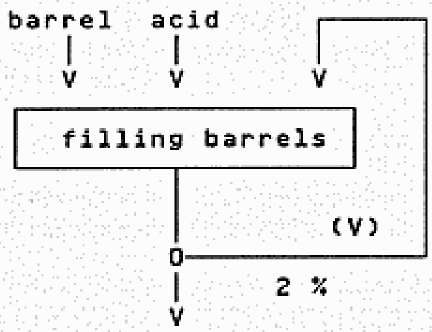


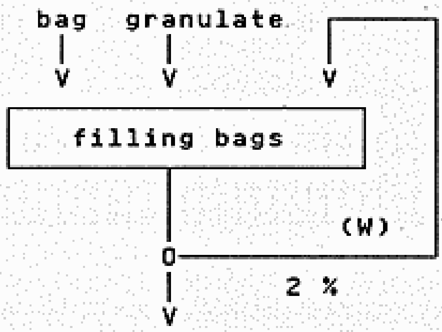
Example 3

A similar situation exists, if »retoured« goods must be taken into account. Parts of finished products, that were damaged in the chain of production(packaging – transport – stocking), become input again (»recycling«): The total cost of these products (even though only parts of them can be reused) are taken into account. From the different points of view this is illustrated by the discussion of two filling processes:

- *Filling of acid into barrels*
  2% of the filled acid – finished product a – get spoiled in storage or run out. The contents – acid of value $xW(a)$ – is lost. The barrel itself with value $xV(a)$ is intact and can be reused as packaging material with the cost: $xV\&(a) = xG(a)$ and $xW\&(a) = 0$.

- *Filling of granulated glue into bags*
  2% of the bags filled with granulate – finished product b – burst during transportation activities. The packaging material with value $xV(b)$ is lost. The contents can be recollected and packed again. These 2% of the granulated glue are recycled with the cost:
  $xW\&(b) = xG(b)$ und $xV\&(b) = 0$.





It is not very difficult to cover the linear part of the correlations from above by mathematical procedures. Some influences, however, make the algorithms more complicate: Multi-loop, coupled production, and the fact, that the cost components of selected products do not get standardized treatment. Depending on the respective input process their two cost categories are not kept constant, they are »oscillating«, this depending on their particular use at their input. Therefore three input characteristics must be distinguished:

- (N) – Standard: Costs separated into contents and packaging
- (W) – Contents: Input with total cost as contents
- (V) – Packaging: Input with total cost as packaging

Due to this definition of »oscillating« cost categories the simultaneous approach via the »balanced« (Leontief) interlacing matrix M operating with the two submodels

$$M \times xV = kV \text{ and } M \times xW = kW$$

cannot be successful. The necessary »manipulations« on the input-output-matrix-model are developed.

Additional problems arise, when certain specifications are fixed before the calculations can be started. Those requirements for intermediate and finished products may result form current marketing constraints; three types of »cost fixing« exist:

- Fixed cost:
  A product is given a fixed cost value K0
- Cost bounds:
  K1 as lower and K2 as upper bounds for the cost
- Cost relations:
  Cost of product a must be lower or higher than cost of product b

In all these cases the direct approach

$M \times x = k$

fails, too.

Models are needed with features that allow and support interactive manipulations in the calculation procedures and yet fulfill the complex framework of calculation rules.

3. Solution and algorithms

Dedicated mathematical procedures must be developed to guarantee a cost calculation with the background of above described rules of calculation.

Separate calculations for the cost components »contents« and »packaging« must be performed. They have to respect the cost components of input products as calculated in previous stages of their production. Each intermediate product transfers its components to the (output) products relative to its own (input) processes. It was pointed out that according to the use of the input product the cost categories may »oscillate« – in the extreme case the same input product is to be treated in three different ways.

We discuss an intermediate product q, which is input for the three production processes R, S, T, that have the output products r, s, t respectively. One of the three basic input characteristics is found in each process:

Input process	Calculation character.	Value of input contents	packaging
R	(N)	xW(q)	xV(q)
S	(W)	xV(q) + xW(q)	0
T	(V)	0	xV(q) + xW(q)

This table illustrates again (»xV(q) + xW(q)«), that multi-loop production cycles and »retoured« goods (recycling) do not allow the standard approach. A stepwise calculation, that determines in the first step

- Cost components »contents«:
  $M \times xW = kW$

and in the second step

- Cost components »packaging«:
  $M \times xV = kV$

cannot be successful.

When calculating the cost components »contents« (see process S) the cost component »packaging« is to be used, although not yet determined at this point. Starting the calculation with »packaging« the same problem arises – vice versa.

The essential starting point for the solution is the interpretation of the consequences of the two new input »characteristics«

- (W): Input with total cost as »contents«
- (V) : Input with total cost as »packaging«

The cost separation into components of contents and packaging has to be neglected for several intermediate products in selected processes. The first requirement (»Calculate separately contents and packaging !«) seems to be in conflict with the second one (»Use the sum of the two components as input value for either the contents or the packaging component !«). And this applies not uniformly for all processes: For the same product q we find both characteristics (W) and (V) – and in addition the standard case (N), of course.

The starting point for the development of the algorithms is based on the demand, that the

sum of the two components is used as input: The procedure starts with the determination of the total cost (xG). The second step will then arbitrarily calculate one of the two cost categories, for example the packaging parts. Choosing this succession the cost component »contents« will then be the balance:
xW = xG – xV.

It was explained, that the packaging cost cannot be calculated via the direct approach
M × xV = kV.

The calculations only can be continued after certain »manipulations« have been performed in the matrix model. Nature and scope of the necessary operations will be developed from the framework of cost calculations as it is applied to product q in the processes R, S, T.

It will be evident, that the algorithms must be based on the »basic« interlacing matrix A. The »balanced« Leontief matrix M contains already a »mixture« of the influences and does not allow the necessary separation of costs. The arithmetic interrelations will be transformed into generalized expressions, which allow easy handling by electronic data processing.

Process R – Characteristic (N)

At the calculation of product r the two cost components xV(q) und xW(q) of intermediate product q are »added« to the components xV(r) and xW(r) respectively.

xV(r) is an unknown quantity in the calculation step »packaging«: a(r, q) × xV(r) is part of the calculation.

Process S – Characteristic (W)

At the production of products the intermediate product q is pure raw material; q does not influence the calculation of the packaging components.

During this step of calculation: a(s, q) = 0.

Process T – Characteristic (V)

At the production of product t the intermediate product q is used as pure packaging material, this with the cost value xV&(q) = xG(q) from the first calculation step »total cost«.

Thus a(t, q) × xV(q) = a(t, q) × xV&(q) is a known quantity and from the calculation's point of view it only does represent pseudo-secondary cost. This cost component is balanced with the corresponding primary cost kV(t) – the also known quantity of the right hand side of the linear equations' system. Then the calculation is continued, as now a(t, q) = 0, because of the correction of kV(t) via a(t,q) × xV&(q).

The three typical production processes R, S, T show, what kind of manipulations are performed in the step »packaging«, when the input of product q takes place. This discussion allows the development of the formalism, which is the precondition for an algorithmic treatment of the allied input-output-problem.

The bilateral input-output-relations have to be analyzed for all (intermediate) products being involved in the production processes and in the coherent cost calculation. The results are described by 0-1-relations and mapped by incidence matrices:

For any possible relation (i, j = 1, ..., n)

»Input of product j at the production of product i«

two incidence coefficients d(i, j) and e(i,j) together with their related incidence matrices D(n, n) and E(n, n) have to be established. Their values are derived from the following table:

Character.	d(i,j)	e(i,j)
(N)	1	0
(W)	0	0
(V)	0	1
No input	0	0

For each element a(i,j) of the interlacing matrix A(n, n) two multiplications with these incidence coefficients are defined: At first a(i,j) is to be multiplied by d(i,j), then separately by e(i,j). Two matrices describe the results:

$B(n,n) = (a(i,j) \times d(i,j))$ and
$C(n,n) = (a(i,j) \times e(i,j))$

Now the following transformations must be performed:

Interlacing matrix A	→ »Packaging matrix« B
Primary cost kV	→ $kV + C \times xG$
$(I - A) \times xV = kV$	→ $(I - B) \times xV = kV + C \times xG$

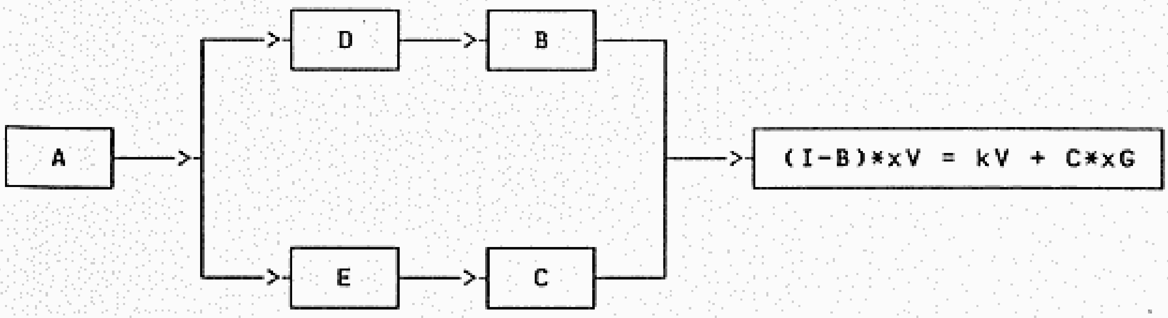
The respective deductions from the interlacing matrix A together with their resulting linear equations' systems realize the cost considerations from above. Within the step of packaging calculation each product is guaranteed to be treated in the following way:

- No cost contribution at its input as pure component of contents (coefficient = 0).
- The known quantity »total cost« from the previous calculation step is the cost contribution at its input as pure packaging material. Then the respective coefficient is modified.
- In the standard case no interference into the system of linear equations takes place.

The calculation of production costs – according to the cost components »contents«, »packaging«, »total« – is performed in these steps:

- Transformation of the interlacing matrix A: For each product its input relations at the production of other products are analyzed. Based on the results of this analysis the matrix A is transformed into the two matrices B and C.
- Calculation of the total cost xG: $M \times xG = kG$
- Calculation of the packaging cost xV: $(I - B) \times xV = kV + C \times xG$
- Calculation of the cost of contents xW: $xW = xG - xV$

Scheme of proceeding:



4. Example

An example will show the consequences of the defined cost calculation framework; moreover it will verify the derived procedures.

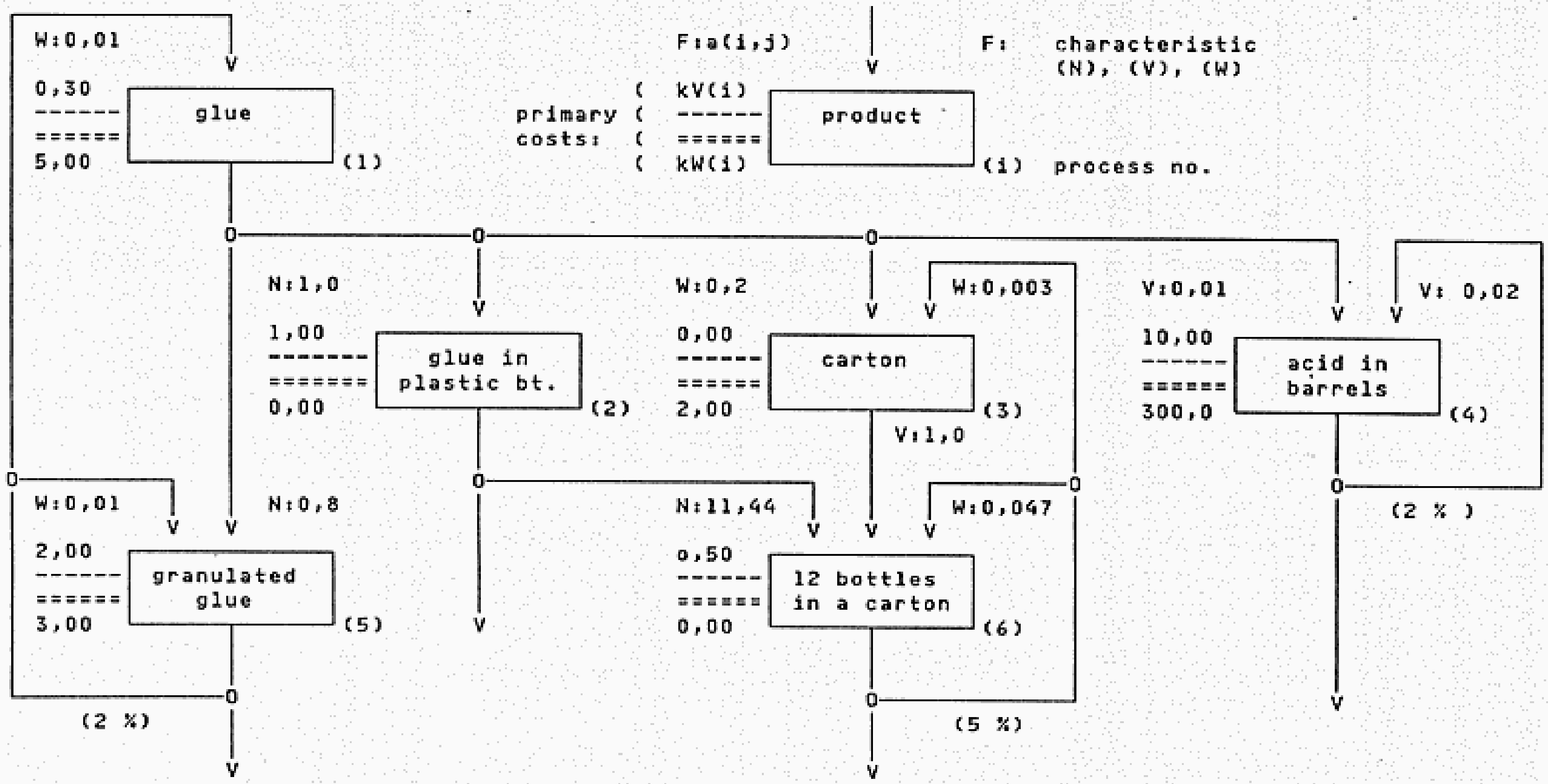
Six production processes (n = 6) with cyclic interlacings are chosen. Their interrelations are described in a table. In addition a chart will make the tracing of the costs much easier.

Relations between the six processes:

Process	Input	Output - Process description
1	kV(1)= 0,30 kW(1)= 5,00 W: 0,01*x(5)	1 kg glue
2	kV(2)= 1,00 kW(2)= 0,00 N: 1*x(1)	filling of 1 kg glue into plastic bottles
3	kV(3)= 0,00 kW(3)= 2,00 W: 0,2*x(1) W:0,003*x(6)	1 carton
4	kV(4)= 10,00 kW(4)=300,00 V: 0,01*x(1) V: 0,02*x(4)	acid is filled into barrels, labels are sticked on with glue from process 1. 2% of the acid get spoiled in storehouse, the barrel is reused according to (V)
5	kV(5)= 2,00 kW(5)= 3,00 N: 0,8*x(1) W: 0,01*x(5)	granulated glue is packed in bags. 2% of the bags burst: 1% is packed again here (process 5) 1% is recycled into process 1
6	kV(6)= 0,50 kW(6)= 0,00 N:11,44*x(2) V: 1*x(3) W:o,o47*x(6)	12 bottles with 1 kg glue are packed in a carton. 5% of these cartons are demaged 94% of the process' value, i.e. 4,7% is getting a new carton. The demaged carton (o,3%) is recycled as raw material in p.3

The following chart will show the interlacing of the processes:

Production processes (Chart)



That means for the figures of the example:

1. Transformation of the interlacing matrix A:

A(6,6): Interlacing matrix of a(i,j):

0	0	0	0	0,01	0
1	0	0	0	0	0
0,2	0	0	0	0	0,003
0,01	0	0	0,02	0	0
0,8	0	0	0	0,01	0
0	11,44	1	0	0	0,047

As a result of the analysis of the input correlations of all products we find the two incidence matrices D(6,6) and E(6,6):

D(6,6): Incidence matrix of d(i,j):

0	0	0	0	0	0
1	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0

E(6,6): Incidence matrix of e(i,j):

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
1	0	0	1	0	0
0	0	0	0	0	0
0	0	1	0	0	0

Derived from the basic interlacing matrix A and the incidence matrices D and E we obtain the two matrices B and C:

B(6,6): Matrix (a(i,j) × d(i,j)):

0	0	0	0	0	0
1	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0,8	0	0	0	0	0
0	11,44	0	0	0	0

C(6,6): Matrix (a(i,j) × e(i,j)):

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0,01	0	0	0,02	0	0
0	0	0	0	0	0
0	0	1	0	0	0

2. Calculation of the total cost xG:

M × xG = kG

Matrix M = I − A : Vector kG:

1	0	0	0	−0,01	0	5,30
−1	1	0	0	0	0	1,00
−0,2	0	1	0	0	−0,003	2,00
−0,01	0	0	0,98	0	0	310,00
−0,8	0	0	0	0,99	0	5,00
0	−11,44	−1	0	0	0,953	0,50

Solution: xG(1) = 5,39 xG(2) = 6,39 xG(3) = 3,32
xG(4) = 316,38 xG(5) = 9,41 xG(6) = 80,77

3. Calculation of the packaging cost xV:

(I – B) × xV = kV + C × xG

Matrix (I – B): Vector kV + C × xG:

1	0	0	0	0	0	0,30
−1	1	0	0	0	0	1,00
0	0	1	0	0	0	0,00
0	0	0	1	0	0	16,38 (× 1)
−0,8	0	0	0	1	0	2,00
0	−11,44	0	0	0	1	3,82 (× 2)

(× 1): 16,38 = 10 + 0,01 × xG(1) + 0,02 × xG(4)
(× 2): 3,82 = 0,5 + 1 × xG(3)

Solution: xV(1) = 0,30 xV(2) = 1,30 xV(3) = 0,00
xV(4) = 16,38 xV(5) = 2,24 xV(6) = 18,69

4. Calculation of the cost of contents xW:

xW(i) = xG(i) – xV(i)

Solution:

xW(1)	=	5,39	–	0,30	=	5,09
xW(2)	=	6,39	–	1,30	=	5,09
xW(3)	=	3,32	–	0,00	=	3,32
xW(4)	=	316,38	–	16,38	=	300,00
xW(5)	=	9,41	–	2,24	=	7,17
xW(6)	=	80,77	–	18,69	=	62,08

5. Application and experience

The demand for solutions of the described problems comes from an existing production and calculation world in the chemical field.

These procedures were developed and successfully implemented on a large data processing unit.

The specific production conditions such as

- coupled production
- production loops
- alternative processes

together with a product range of about 8000 items lead to an input-output-model of high dimension (10.000).

The realization and the application of this model do not cause performance problems on the computer. This was achieved by decomposition and iteration algorithms in combination with powerful routines checking the consistency of all production processes according to their quantity and value figures.

This calculation instrument is used regularly – global cost calculations for all divisions of the enterprise are normally performed in each quarter of the year. In addition there are separate calculations in the planning or inventory periods; moreover specific divisions may get extra treatment.

Besides this centralized calculation system several decentralized calculation models were developed. They take care of the very specific conditions of selected divisions. These systems allow flexible reactions to changing marketing situations of the corresponding business sectors.

Decentralized calculations supply the centralized system with preliminary information and pre-calculated values. Based on this information the model parameters and the interlacing coefficients can be fixed – this is necessary for the consistency of the formulation of the centralized »overall calculation«, that covers the whole product range of the company.

6. References

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4. Calculation of the cost of contents xW:

xW(i) = xG(i) − xV(i)

Solution:

xW(1)	=	5,39	−	0,30	=	5,09
xW(2)	=	6,39	−	1,30	=	5,09
xW(3)	=	3,32	−	0,00	=	3,32
xW(4)	=	316,38	−	16,38	=	300,00
xW(5)	=	9,41	−	2,24	=	7,17
xW(6)	=	80,77	−	18,69	=	62,08

5. Application and experience

The demand for solutions of the described problems comes from an existing production and calculation world in the chemical field.

These procedures were developed and successfully implemented on a large data processing unit.

The specific production conditions such as

- coupled production
- production loops
- alternative processes

together with a product range of about 8000 items lead to an input-output-model of high dimension (10.000).

The realization and the application of this model do not cause performance problems on the computer. This was achieved by decomposition and iteration algorithms in combination with powerful routines checking the consistency of all production processes according to their quantity and value figures.

This calculation instrument is used regularly – global cost calculations for all divisions of the enterprise are normally performed in each quarter of the year. In addition there are separate calculations in the planning or inventory periods; moreover specific divisions may get extra treatment.

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