Average Inventory

By Peter Kierkegaard-Hansen*)

If, during a period, we have an invariable rate of inventory decrease and a specific number of equivalent, but not necessarily equidistant, inventory increases, the minimum average inventory can be calculated without determining the distribution of increase.

In the placing of orders or manufacture it will usually be profitable to over-order or to over-produce for a few days, and then store the lot and consume for the rest of the period.
An optimum solution to the problem of how often we have to give input to a store may be found by introducing an input frequency, φ, defined by

$$\varphi = \frac{\text{number of input days}}{\text{number of days in the period}} = \frac{n}{m} \quad .$$

The number of input days, is, for instance, the days of manufacture or receipt of orders.
If the store receives x items on input days, the average input will be φx per day. If the cost function to acquire x items is $f(x) + C$, where C is a cost independant of x, then the average cost per day to get an average of φx items per day is

$$\varphi[f(x) + C].$$

It is possible to minimize this function with respect to the variables x and φ subject to a constraint equation, say

$$\varphi x = K,$$

which expresses the fact that the daily output or average input must be K. The above expressions assume an inventory cost equal to zero.
If the inventory cost is non-zero, an optimum distribution of input days may be found by a graphical method, cf. example 1.

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EXAMPLE 1

If the input frequency is, say, $\varphi = 0.71$, it is reasonable to choose a 7-day period with 5 input days. Further, if the output of items takes place at a constant rate, say, $K = 1000$ per day, we must have an input of

$$x = \frac{K}{\varphi} = \frac{1000}{5/7} = 1400$$

items on input days. The output distribution is a discrete uniform distribution as shown in figure 1,

Figure 1.

and the input distribution is a discrete distribution fulfilling the condition that the cumulative amount of input must be greater than or equal to the cumulative amount of output. The cumulative input- and output- functions are shown in figure 2. The amount in store is the vertical difference between the two step functions, or the area between

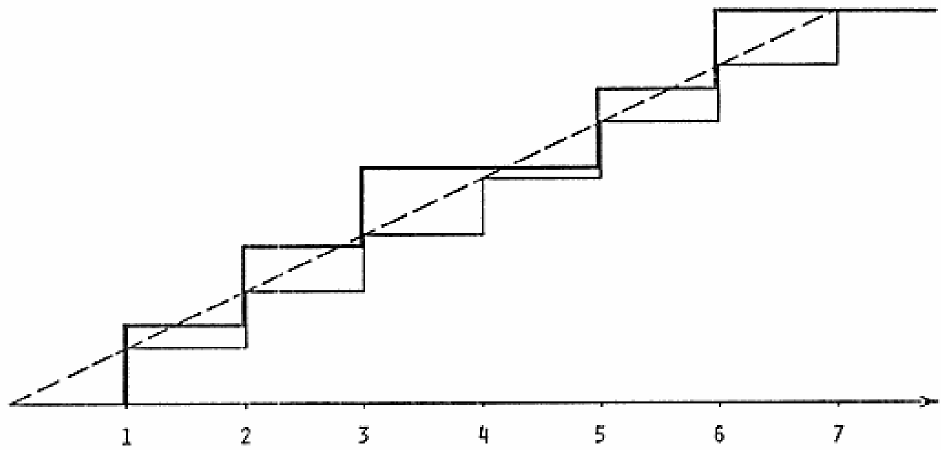

Figure 2.

the functions. When we move the cumulative input function one step to the left, we get the function shown in figure 3. The amount in store may then be expressed as the area between the moved step-function and the oblique line minus the seven small congruent triangles (hatched).

We then have to find the area between the moved step-function and the oblique line.

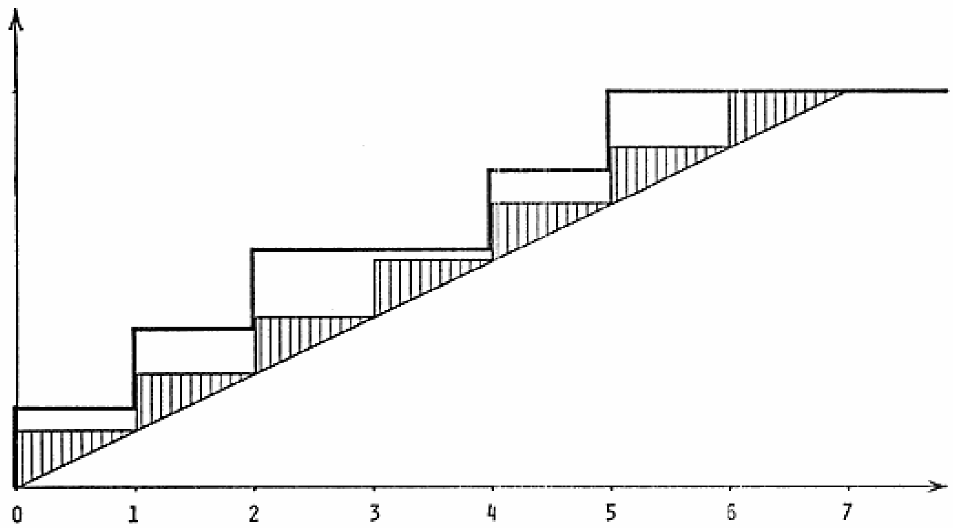

Figure 3.

If we take the moved step-function and the oblique line mentioned in the example as statistical distributions, the former is a max-mean distribution enclosed by the latter (the uniform distribution). Therefore, the area sought is the difference between the mean of the uniform distribution and the mean of the max-mean distribution. The area between the two statistical distributions will be,

$$\frac{1}{2}\ m - \frac{1}{2n}\ [m(n-1) - n + 1] = \frac{1}{2n}\ [m + n - 1].$$

Because of the affinity, which transfers the statistical distributions to the moved step-function and the oblique line, the area between these original distributions is

$$\frac{mK}{2n}\ [m + n - 1],$$

the coefficient of affinity being mK. Subtracting the small congruent triangles, we find that the total amount in store is

$$\frac{mK}{2n}\ [m + n - 1] - \frac{1}{2}\ mK = \frac{mK}{2n}\ [m - 1].$$

Therefore, the average inveentory is

$$\frac{K}{2n}\ [m - 1] = \frac{K}{2\varphi m}\ [m - 1]. \qquad (1)$$

Thus, given the length m of the period, the input frequency φ, and the daily inventory output K, the minimum average inventory is given by the explicit expression above.

If the business has to have the whole amount of the output ready at the very start of a day, this quantity must be stored from the day before. Therefore the moved step-function, cf. figure 3, will be the real input distribution. Then the amount in store will be increased by the total amount of output mK during a period, i.e.

$$\frac{mK}{2n}\ [m - 1] + mK = \frac{mK}{2n}\ [2n + m - 1].$$

Thus, if the business wants to meet the demand at any time of the day, the minimum average inventory is

$$\frac{K}{2\varphi m}\ [2\varphi m + m - 1]. \qquad (2)$$

Formula (1) or (2) is applicable in economic models considering optimum input distribution during a period of length m, when the input frequency is φ and the daily output K.

EXAMPLE 2

If the inventory output is $K = 1000$ per day, and, in a period of 7 days, we get input on 5 days, then the input frequency, φ, is equal to $5/7 = 0.71$, and the minimum average inventory may be calculated by means of (1).

$$\frac{1000}{2 \times 5}\ [7 - 1] = 600 \text{ items per day.}$$

Observe that the average inventory is determined without knowledge of the input distribution.

EXAMPLE 3

In a factory the cost of preparating production is 735 Dkr. when we start a machine, and the cost of production is 10 Dkr. per unit. The inventory cost is 5.60 Dkr. per item, and we need 100 units a day. How do we plan the production for a 7-day week with a view to minimizing the total cost?

The average cost of production is $\varphi[10x + 735]$, and the minimum average inventory cost is, cf. (1), $5.6 \times 100[7 - 1]/2\varphi 7$. Therefore, the total average cost amounts to

$$10\varphi x + 735\varphi + 5.6\ \frac{100}{2\varphi 7}\ 6.$$

Minimizing this function with respect to the variables x and φ subject to the constraint $\varphi x = 100$, i.e. the average input must be 100 units, we get the solution $\varphi = 0.57$ and $x = 175$.

Thus, in a period of 7 days we have to produce 175 units per day in $0.57 \times 7 = 4$ days. The distribution of input days may be determined graphically.

References:

Peter Kierkegaard-Hansen: "*The max-mean distribution enclosed by a uniform distribution*". (Internal paper).

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