A Restatement of the Problem of the Sufficient Economic Horizon.

By Odd Langholm*)

Forfatteren diskuterer i denne artikkelen problemet å finne fornuftige kriterier for å avgrense i tiden bedriftsökonomiske planleggingsmodeller. Han viser at problemet i mange tilfeller kan stilles som et såkalt sensitivitetsproblem, idet en gitt modellhorisont er tilstrekkelig dersom den optimale nåtidsbeslutning som finnes i modellen er insensitiv overfor variasjoner i modellens tilstand ved modellhorisonten.

1. In normative analysis, the relevant economic horizon[1]) at a given point of planning, as defined originally by Svennilson (L 13)[2]), is the locus of the first of those breaks in the intertemporal relations between economic variables which permit determination of an optimal decision at the point of planning without consideration of the future beyond the break. Relating the question of relevance to information about the

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1) This paper gives a preliminary presentation of a few of the ideas embodied in not yet published monograph treating some problems in the methodology or normative economics, centered on the interpretation and analysis of the economic horizon. Previous publications by this author on the same subject are L 7, which discusses the possibilty of interpreting the notion of the horizon in terms of the psychological theory of the aspiration level, and L 8, which covers the central topic of the monograph in popularized form.

2) In Svennilson's system, "intressehorisonten" is a future point of time beyond which any consequence of a plan is assigned a zero value at the point of planning (because of time preference and risk preference). Essentially, the horizon corresponds to that subsequently defined by Lange (L 6). Usually, however, a plan does not have to cover the entire interval up to "intressehorisonten". The location of "planrelevanshorisonten" in this interval depends on those "ekonomiska tidssamband" (intertemporal economic relations) which are the subject matter of Svennilson's theory of planning.

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values of specified parameters in a model describing a decision process beyond the horizon of complete information, Modigliani and Cohen (L 11, L 12) have developed a set of general criteria for irrelevance. The purpose of the present paper is to suggest an extension of the analytical principle introduced by these authors. The models employed in an important section of the dynamic theory of the firm are such that the horizon of complete information in a given model corresponds to the horizon of the model itself: Within this horizon, expectations are assumed to be single valued. As for the future beyond it, no description is included at all. In testing the sufficiency of a given horizon of this nature, i. e. the sufficiency of the length of a given model, the information irrelevance criteria of Modigliani and Cohen have no direct applicability. In the present paper the notion of the states of a decision system at the model horizon is introduced, and it is suggested that the relevance of any consideration of the future beyond it may be analysed in term of the sensitivity of the optimal decision at the point of planning to variations in the state at the horizon.

2. In the dynamic theory of production, the possibility of carrying inventory establishes exactly the type of intertemporal relations which lie at the core of the problem of the horizon. For this reason a simple inventory model is chosen to illustrate the line of reasoning followed here. The problem discussed is similar to one of Modigliani and Hohn (L 10). Contrary to theirs, however, the model employed is continuous over time, the optimization problem taking the form of a problem in the calculus of variations. This makes for greater simplicity in the mathematical formulation of the conclusions.

Consider a firm producing a single commodity at a rate of x units per unit of time. Let the inventory of finished units at time $t = o$, "the point of planning", be $I(o) = o$. Let $t = T$ be some future point of time "beyond which the entrepreneur does not envisage operations of any kind",[3]) and let $D(t)$ be the demand schedule for the commodity, defined over the interval from $t = o$ to $t = T$. Assume that the entrepreneur has to meet demand. It is possible, however, at any point of time in the interval considered, to carry a positive or a negative inventory. In the latter case, demand is met by recource to some non-specified basic inventory which has to be replenished before $t = T$. Whenever

[3]) L 9, p. 25. This is another variant of Lange's horizon and of Svennilson's "intresschorisont". A "liquidation of the enterprise" (see footnote 6), i. e. a zero inventory level, is planned for this point.

the firm's own inventory is negative, the entrepreneur has to pay some stockout charge. If he wants to end up mith a zero inventory at $t = T$, the problem af the entrepreneur is to chose a production schedule which satisfies

$$\int_o^T x(t)\ dt = \int_o^T D(t)\ dt$$

where

$$x(t) = I'(t) + D(t)$$

and which minimizes the total costs of production, inventory and stock-out.

To make the analytical solution of the problem manageable, some simple cost functions are assumed. Let $bx(t)^2$ be the cost of production per unit of time. As for inventory and stock-out charges, assume symmetry, and put $aI(t)^2$ as the cost per unit of time of either a positive or a negative inventory. Then the decision problem may be stated mathematically as follows: Choose an inventory schedule such that $I(o) = I(T) = o$ and such that

$$\int_o^T [aI(t)^2 + b(I'(t) + D(t))^2]\ dt \qquad \text{(i)}$$

is minimized.

3. The problem of the relevant economic horizon in this model will be discussed in terms of incomplete knowledge about the demand schedule. Assume that the form of the function $D(t)$ is completely known in the interval from $t = o$ to $t = H$, where $H < T$, while nothing at all is known about the demand schedule in the remaining interval. Then $t = H$ is the horizon of complete information, and the question is whether this horizon is or is not sufficiently far removed from the point of planning, i. e. whether an optimal decision at this point, henceforth called an "initial decision", can or can not be reached on the basis only of knowledge about the demand schedule within the horizon.

The decision problem at the point of planning is the problem of choosing an optimal rate of production at this point. Since

$$x(o) = I'(o) + D(o)$$

where $D(o)$ is known, the problem of the sufficiency of the existing information relates only to the choice of an optimal rate of increase in inventory at the point of planning.

To approach this problem, consider the following restatement of the minimization problem: Choose an inventory level $I(H) = S$ at the economic horizon, and choose inventory schedules in the intervals from $t = o$ to $t = H$ and from $t = H$ to $t = T$ such that

$$I(o) = o$$
$$I(H) = S$$
$$I(T) = o$$

and such that

$$\int_o^H [aI(t)^2 + b(I'(t) + D(t))^2]\ dt + \int_H^T [aI(t)^2 + b(I'(t) + D(t))^2]\ dt \qquad \text{(ii)}$$

is minimized. Clearly, (i) and (ii) must give the same optimum condition. But the second statement of the problem shows explicitly the fundamental role played in the problem of the horizon by the parameter S.

4. In a manner of speaking, the inventory level at the economic horizon constitutes the only link between the two optimization problems within and beyond the horizon. Let $I(H) = S(\text{opt})$ be the optimal inventory level determined by a solution to (ii). If $S(\text{opt})$ were attained at $t = H$, the optimal solution to the problem of choosing an inventory schedule beyond the horizon would be independent of the initial decision. Hence, in this case, the optimal initial decision could be found by minimizing

$$\int_o^H [aI(t)^2 + b(I'(t) + D(t))^2]\ dt \qquad \text{(iii)}$$

for $I(o) = o, \quad I(H) = S(\text{opt})$.

Let us write out the solution to (iii) for any S.

Euler's differantial equation takes the form

$$aI(t) = bI''(t) + bD'(t).$$

A general solution is given by

$$I(t) = A_1 e^{ct} + A_2 e^{-ct} + Y(t) \qquad \text{(iv)}$$

where $c = \sqrt{\frac{a}{b}}$, and the form of the function $Y(t)$ depends upon the known demand schedule. Using $I(o) = o$ and $I(H) = S$ to determine the remaining parameters, we find

$$A_1 = \frac{S + Y(o)e^{-cH} - Y(H)}{e^{cH} - e^{-cH}}$$
$$A_2 = \frac{Y(H) - S - Y(o)e^{cH}}{e^{cH} - e^{-cH}} \qquad \text{(v)}$$

Differentiation (iv), substituting by means of (v), and putting $t = o$, we get the following condition for an optimal initial decision:

$$I'(o) = \frac{c}{e^{cH} - e^{-cH}}\ (2S + Y(o)\ (e^{cH} + e^{-cH}) - 2Y(H)) + Y'(o). \qquad \text{(vi)}$$

Except for S, all terms of this formula are known.

Clearly, the optimal inventory level at the horizon is a function of the unknown demand schedule beyond the horizon. However, if the optimal initial decision is independent of the inventory level, all information about the further demand schedule is completely irrelevant. Thus, by means of (vi), the relevance of such information may be analysed in terms of the sensitivity of $I'(o)$ to variations in S. The existence of absolute insensitivity, i. e. the vanishing of the fraction $\frac{2c}{e^{cH}-e^{-cH}}$, constitutes a criterion for irrelevance in the sense of Modigliani and Cohen.

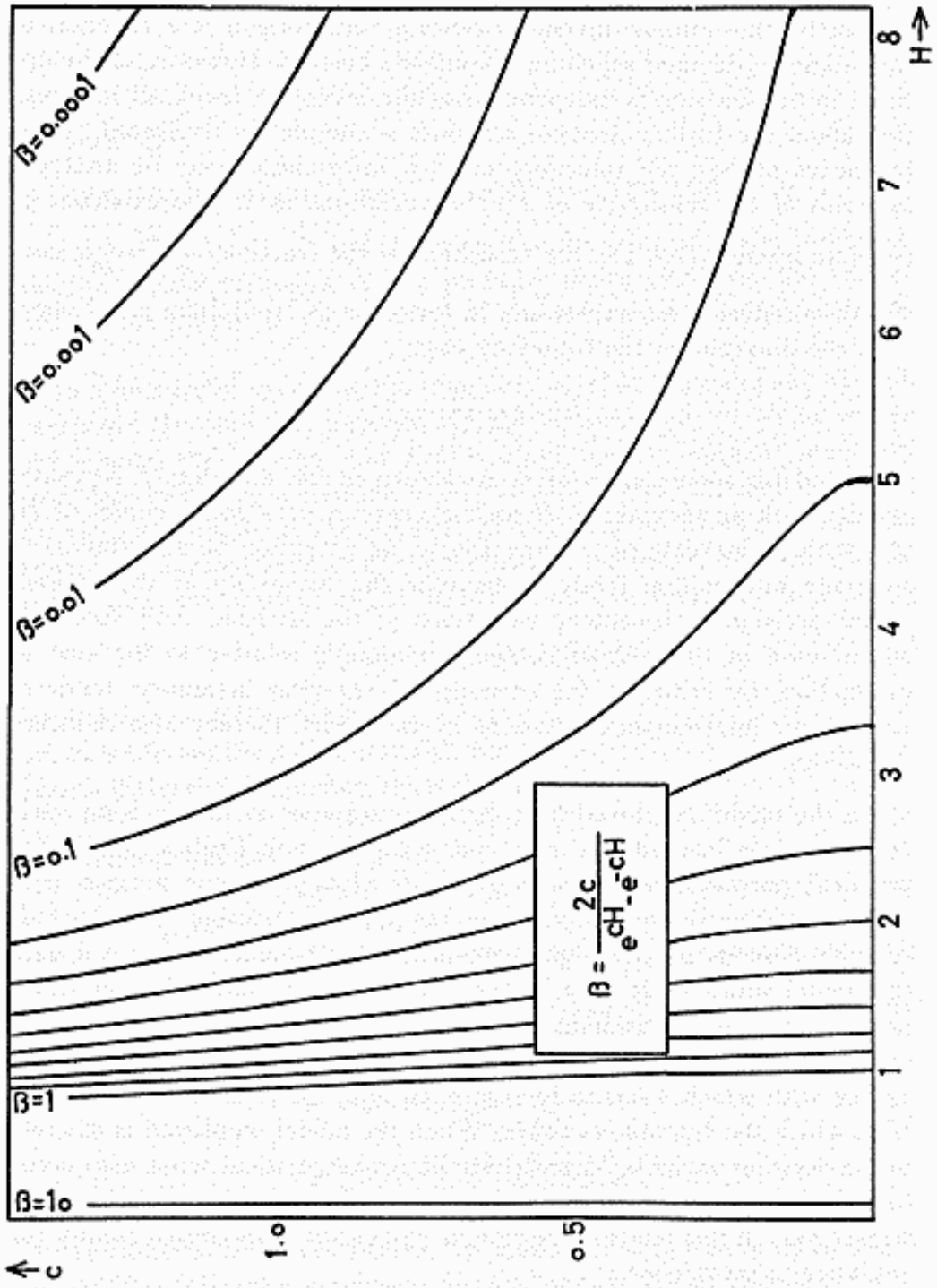
In the diagram on the following page

$$\beta = \frac{2c}{e^{cH}-e^{-cH}}$$

is plotted for some values of β. As shown in the diagram, β decreases rapidly with an increase in H and, except for very small values of H, also with an increase in c. Thus, firstly, an extension of the interval of complete information tends to decrease the sensitivity of the optimal initial decision to inventory variations at the horizon. And secondly, an increase in the cost of carrying inventory relative to the cost of production, by reducing the economy of carrying inventory, tends to weaken the intertemporal relations in the model, thereby also reducing sensitivity.

5. In the model employed here, being continuous in the decision variable, absolute insensitivity does not occur for any finite c or H. For practical purposes, however, a relatively close economic horizon may still be sufficiently far removed from the point of planning. This depends on how close its practical implementation can be made to accord with the model solution. It may happen, for instance, that only variations in the first couple of decimal places in $I'(o)$ correspond to actual changes in the decision carried out. This fact then determines the necessary accuracy with which β has to be computed and, for each finite c, a finite H at which the fraction vanishes. When the model omployed is discrete in the decision variable, insensitivity in a mathematical sense may occur for relatively close horizons. Numerous simple examples of this suggest themselves. In this paper a continuous model has been chosen simply for analytical convenience.

6. The inventory levels at the economic horizon represent in the problem dicussed above the states of the decision system at the horizon. This is a notion capable of generalization to any dynamic decision model. Essential



to a general definition of the states of a decision system at a given horizon is that the influence of a given state upon the formulation of any optimization problem beyond the horizon is independent of the manner

in which the state is reached. Whenever a complete set of states of this nature may be constructed, they represent the only link between the optimization problem within the horizon and any optimization problem that may be formulated beyond it. Consequently, the problem of the relevance of any consideration of the future not included in a given model within the horizon may be approached in terms of the sensitivity of the optimal initial decision to variations in the state at the horizon.

In the analytical system of Modigliani and Cohen, the criteria developed relate to the relevance of information about the actual values of well-defined parameters in a fully constructed model describing a decision process beyond the horizon of complete information. In an important section of the dynamic theory of the firm the models employed are such that these criteria have no direct applicability. Typically, these models include only a short section of the future, the assumption being that expectations within this section are single valued. The remaining part of the future, however, is seldom treated in the way suggested by Modigliani and Cohen, i. e. described by a model where expectional parameters vary within given limits. More often, no consideration of the future is taken at all. As far as this theory is concerned, the most pressing aspects of the complex of problems involved in the notion of the economic horizon, is not the problem of finding out whether expectational parameters are worth estimating[4]), but whether a given model, limited as to the time period it covers, is sufficiently long. In a sense, however, the following suggestions may be looked upon as an attempt to extend the analytical principle introduced by Modigliani and Cohen to cover also this aspect of the problem.

One should of course realize that an elimination of the formal description of the decision process beyond the horizon of complete information necessitates a fundamental change in the interpretation placed upon the terms of the analytical system. It may be argued that it is impossible to construct a complete set of states of the above definition without any knowledge of the types and conditions of the activities starting in these states. From the point of view of the methodology of normative economic, however, the problem of constructing such a set for the purpose of the sensitivity analysis is not fundamentally different from those facing an economic adviser who is to limit a decision model in other dimensions. Logically, the problem of isolating all those factors and relations which contribute to determining the locus of optimal de-

[4]) Which is, essentially, the problem analysed in L 12.

cisions at a given point of planning, is insoluble in the absence of a universal model covering the entire future and the entire economy at any moment. Quite apart from the prohibitive nature of the task of building a universal model of this type as a basis, the assumption of its existence makes the problem of limiting the model building activity itself meaningless. Thus whenever a problem is solved on the basis of a limited model, the question of the relevance of non-included factors and relations is either to some extent begged, or the assumption is that the problem stated is a suboptimum problem.

7. If this point of view is accepted, the construction of a set of different states at the economic horizon, even on the basis only of very rough estimates of future developments beyond it, may still be looked upon as a mean of testing the sufficiency of a limited model, and the state sensitivity approach leads to an explicit treatment of one of the most controversial problems of the normative dynamic theory of the firm. In this theory, the notion of the states at the economic horizon very often has a direct economic interpretation. The objective of the entrepreneur is usually taken as that of maximizing some stream of profits or minimizing some stream of costs in a operational system where the states at any moment are described by physical quantities, i. e. inventory levels, type, age, and quantity of durable equipment, etc. Whenever such models include only a short section of the future, the state at the horizon enters as an important parameter, the question being in what state the decision process described should end up.

Essentially, the horizon theory propounded by Lange (L 6) and subscribed to by such authors as Hicks (L 5) and Brems (L 2) consists in an attempt to evade this problem by introducing a particular type of assumption about the horizon state. In the opinion of the present author, this attempt fails, not because it necessarily produces, as Friedman puts it[5]), a "fantastic picture" of economic behavior, but because the length of the model necessary to make the assumption reasonable, deprives the theory of its practical usefulness. Even if risk preference and time preference make the entrepreneur behave as though he "plans to liquidate his enterprise"[6]) at some point in the future, this point lies far beyond that which constitutes the horizon of information and model construction in most business decision situations. Consequently, for the

[5]) L 3, p. 630.

[6]) This is Friedman's statement (L 3, p. 630) of what is, according to him, a necessary implication of Lange's premises.

purpose of locating optimal initial decisions, the plan of the entrepreneur can not be assumed to be that of liquidating his enterprise at the model horizon. On the contrary, he plans to have his firm remain a going concern for a much longer period of time. But, in choosing at the point of planning, he does not know what is, at the horizon, the optimal size and structure of his firm[7]).

Typical for many theoretical models of this type, however, is that the description of the states may be reduced to a form very much simpler than the detailed drawing up of each possible plan ending in one of these states. Even with a small number of initial decisions possible, and a small number of states at the horizon, each initial decision and each state may often be connected in a very large number of ways by the intermediate decisions. In treating the initial decision and the state as variables, the remaining part of the sequence of decisions may be "maximized away". The decision problem thus being reduced to one in two variables, the sensitivity of the optimal value of one of them to variations in the other may be analysed explicitly and, in the author's experience, often in quite simple terms.

It should be emphasized that the purpose of this sensitivity analysis is not that of finding out whether environmental parameters are worth estimating. Rather, it is that of judging whether arbitrary assumptions about the state at the horizon, as often found in the theory, are justifiable or not. Whenever absolute insensitivity or negligible sensitivity is found, it clearly does not make any difference which horizon state is assumed in the model. On the other hand, if an arbitrary assumption about the horizon state is included in the theory, (often a zero inventory level or similar assumptions corresponding to a liquidation of the enterprise), and the optimal initial decision shows considerable sensitivity to variations away from this state, the theoretical conclusions as to optimality of initial decisions are misleading. They may be directly wrong if the horizon state assumed should later prove not to be the

8. Thus, by means of the insensitivity criterion for irrelevance developed above, we are capable of testing the validity of the conclusions of

[7]) In Value and Capital, Hicks suggests that "the plant he (i. e. the entrepreneur) plans to have left over at the end of that time" (i. e. the horizon), may be regarded "as a particular kind of output – –, a kind which is only produced in the last week" (L 4, p. 194). In normative analysis, however, a determination of the model by putting the value of this product equal to its market value, does not solve our problem, even if the market value is known, since it begs the question of the optimality of selling the enterprise.

optimal one. If it should actually prove to be optimal, the conclusions are correct when applied to a special case. But as general conclusions they are still misleading, since optimality of the assumed state is not an inference drawn from analytical premises.
a large body of models of such dynamic theories as those of production, inventory, and investment. It is clearly seen, however, that the outcomes of these tests turn on the number and variety of the different states considered, and this fact leads naturally to an extension of the scope of the analysis through a closer examination of the problems involved in limiting the set of states to consider. Although a sufficient horizon may often be located at a relatively short distance on the basis of reasonable assumptions about the set of possible states, the criterion of insensitivity is admittedly quite weak. But a number of suggestions for the development of stronger criteria present themselves.

The choice of a set of possible states as a basis for the sensitivity analysis rests of course on an evaluation of the possibility that any given state may prove to be optimal. This evaluation introduces into the analysis a consideration of the decision process beyond the horizon. Furthermore, any criterion stronger than the one developed above must be based on assumptions about the future that come in addition to those needed by the sensitivity approach. Nevertheless, considerable strengthening of the criterion is possible through very simple and reasonable assumptions. For instance, each state may be assigned a maximum and a minimum value, in the sense of upper and lower extremes of the discounted expected streams of profits initiating in each of these states. Such parameters suffice for the construction of a much stronger criterion than that of insensitivity to variations in the state itself. However, a further examination of the logic and interpretations of an extended analysis along these lines lies beyond the scope of the present paper[8]).

The point I have wanted to make here, is that meaningful conclusions as to the sufficiency of a limited model may often be reached on the basis of very little information except that contained in the model itself. Even if all those states are considered that result from at least one sequence of the decisions descibed by the model and satisfying its constraints, insensitivity may often be attained through such advances of the horizon which are analytically manageable. Thus the insensitivity criterion, however weak, tends to supply at least some indication as to the length of time that has to be considered in different types of decision

[8]) Two additional criteria are indicated in L 8.

situations. It presents a mean of checking, in individual cases, the penetrativeness of a structural phenomon, an intuitive acknowledgement of which seems to constitute the motivation for early model cut-offs all throughout the dynamic theory of the firm and is stated as follows, in the words of Boulding[9]): "We may reasonably expect that the longer the period of time we take into consideration, the less important will be inventory changes relative to the total volume of output and sales".

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[9]) L 1, p. 115.

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