

Storage factor model.

An Application of Operations Research in Danish Industry.

By ROBERT F. WILLIAMS*)

In order to make a long range projection of space requirements for a Danish jerngrossist it was necessary to analyze the relationships between forecast sales and the warehouse space required to support those sales. Sales projections were first reduced from a Kroner basis to a physical dimension basis. Then the existing space requirements were determined by actual physical measurement. Adjustments in existing requirements were made to reflect planned rationalization of the method of storage on a projected ten year basis.

Having determined space requirements for the 15,000 items in inventory and having determined sales projections to the new period being considered, it was then necessary to determine the interaction between changes in sales levels and space requirements. It was most desirable to determine on an objective basis the space requirements for the new period.

A mathematical analog was developed which acts, very nearly, in the same manner in which inventories have been observed to act. The manipulation of this analog will yield answers concerning space requirements for sales levels projected to future dates, and under various conditions.

In general, this inventory consists of a picking stock from which orders are filled and a reserve stock from which the picking stock is occasionally replenished. The size of the storage unit in picking stock determines both the size of the replenishment and the cost per re-

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plenishment. The size of the picking storage unit also determines the average amount of walking and thereby the cost of picking. A study for the purpose of optimizing these costs is an important part of the whole problem, but it is outside the scope of this project.

The areas and volumes occupied by picking stock and by reserve stock were measured and recorded by subgroups for all of the 15,000 items carried in stock. There were 27 subgroups, some of which are very homogeneous and some of which are entirely heterogeneous. Recognizing that the cost of a detailed analysis of all 15,000 items would exceed the economic value of the improved accuracy which might be obtained, the mathematical model was developed to use the data available.

Assumptions. The development of the model required that certain assumptions be made. In working with the answers produced by this analog, it must be remembered that if management policies or other outside factors are changed, they (the answers) may require modification. The following are the principle assumptions made:

Turnover:

By definition, turnover is the ratio between the rate of sales (at cost) and the average inventory (also at cost) and may be expressed in a number of ways, for example, times per year, number of months per turn, etc.

It has been assumed that the turnover rates obtained in recent years reflect company policy and that future inventory practice will continue to achieve these rates. Sales to inventory ratios are assumed to be constant through the period considered.

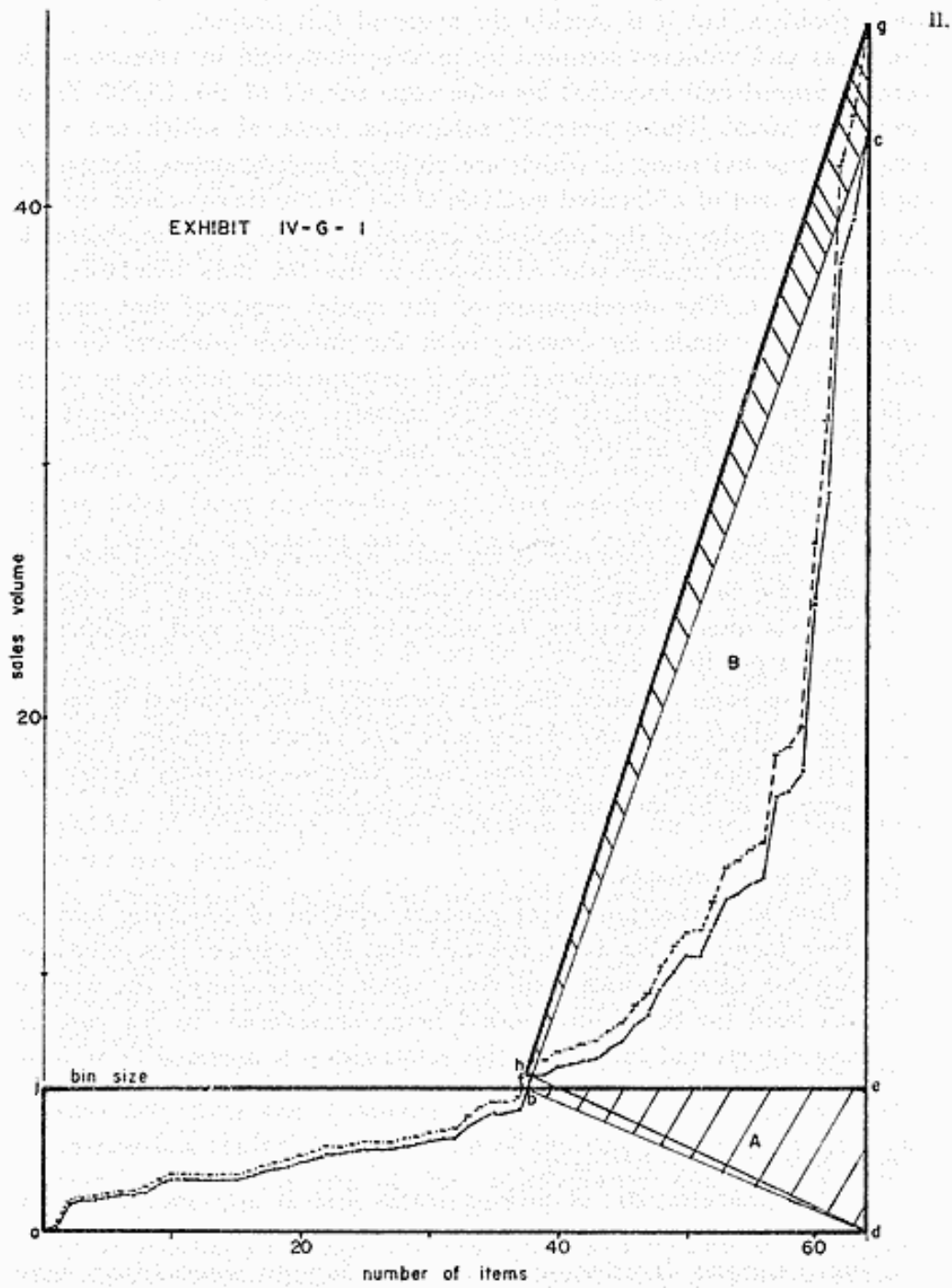
Volume-Area Relationship:

Due to characteristics of product, method of handling and/or other factors it has been assumed that the volume/area relationship for reserve stock and floor stock found during the survey will hold for the future period being considered. In the case of long objects and Finplader, this relationship is assumed to be constant at $2 \text{ m}^3/\text{m}^2$ of floor space.

Picking Stock Storage Efficiency:

It is characteristic of picking stocks that at any one time some of the storage units will be full, a few empty and the rest following a normal distribution in between. Sample observations were made of picking stock and the results proved to be characteristic.

It is, therefore, assumed that all Picking Stock storage units (excepting Floor Stock) will be on the average one-half full.



Floor Stock and Reserve Stock:

In most cases where goods are stored on the floor, maximum utilization of area is achieved within the volume-area relationship found to exist. It has been assumed that this condition will continue to obtain, so that floor stock is treated as reserve stock.

Sales Item Distribution:

The relationship of sales by item within samples subgroups was closely examined. The form of the curve best representing the distribution of sales among the items was thereby determined.

Several different methods of treating these data were explored. Among these methods were Poisson and Monte Carlo.

The method finally selected was to array the items along the horizontal axis in ascending order of sales and with annual sales volume the vertical axis. In so doing, a distribution pattern develops which is depicted in Exhibit IV-G. 1, which follows. Above line $j-e$ is that part requiring reserve storage. This method of treating these data appeared to be best.

Data and time were not available to determine the actual distri-

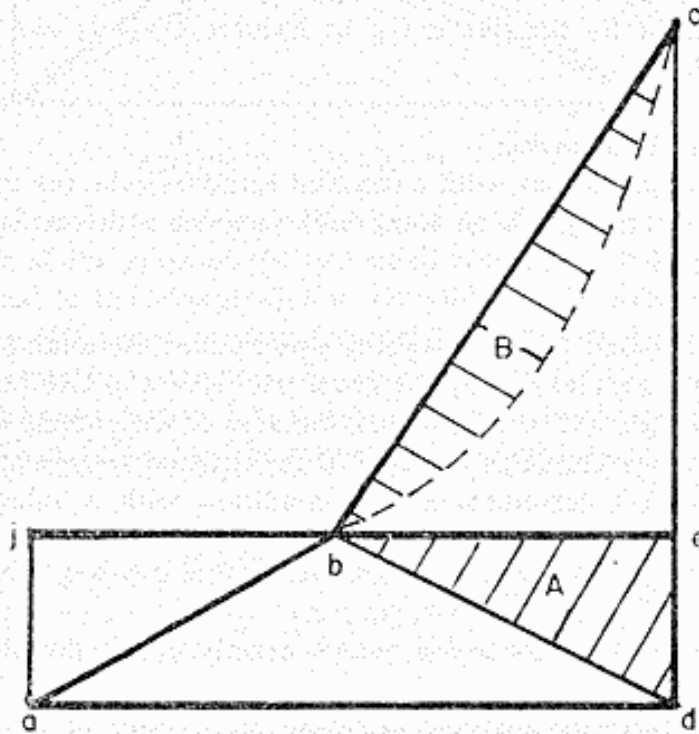


EXHIBIT IV - G - 2

bution for each product subgroup. By representing the distributions in triangular form, an easy to manipulate analog results. The real distribution pattern in Exhibit IV-G-1, resembles such triangular form, but differs by an amount equivalent to the region bounded by the line b-c and the distribution curve.

It was previously assumed that picking area storage efficiency is 50 %. Then for those items where picking storage is backed up by reserve storage, an average of 50 % of the inventory which could be in picking, is in reserve. Area A in IV-G-2, which follows, represents the 50 % area of the picking stock which could store items but which does not, Area B, in the reserve area is assumed to store those items not stored in Area A. In the subgroups examined, area B and A were very nearly equal. Thus, adding area B to the form representing the distribution depicted in IV-G-1, results in a triangular distribution, as was assumed.

Sales Increase:

It has been assumed that a sales increase of a certain percentage for an entire subgroup will be distributed over all the items in the subgroup by a like percentage applying to each item. This is represented by the line a-f-g in Exhibit IV-G-1, and a-h-g in IV-G-3.

Sales-Volume Relationship:

It is assumed that, with a constant turnover rate, the volume occupied by the goods in stock will vary as a direct function of sales level.

The Model. Based on the foregoing assumptions the analog has been developed. It is represented by the geometric figures in Exhibit IV-G-2. Here the rectangle a-b-e-j represents the area of the picking stock, and the triangle b-e-c represents the area of the original reserve stock.

Exhibit IV-G-3 represents a new condition with a sales increase equivalent to $\frac{gc}{dc}$ when the new distribution line is a-h-g. The amount of stock in picking is now a-f-d, which exactly equals the old amount a-b-d.

The area of picking has not increased as a result of the sales increase but remains at a-j-e-d. The increase in total stock represented by

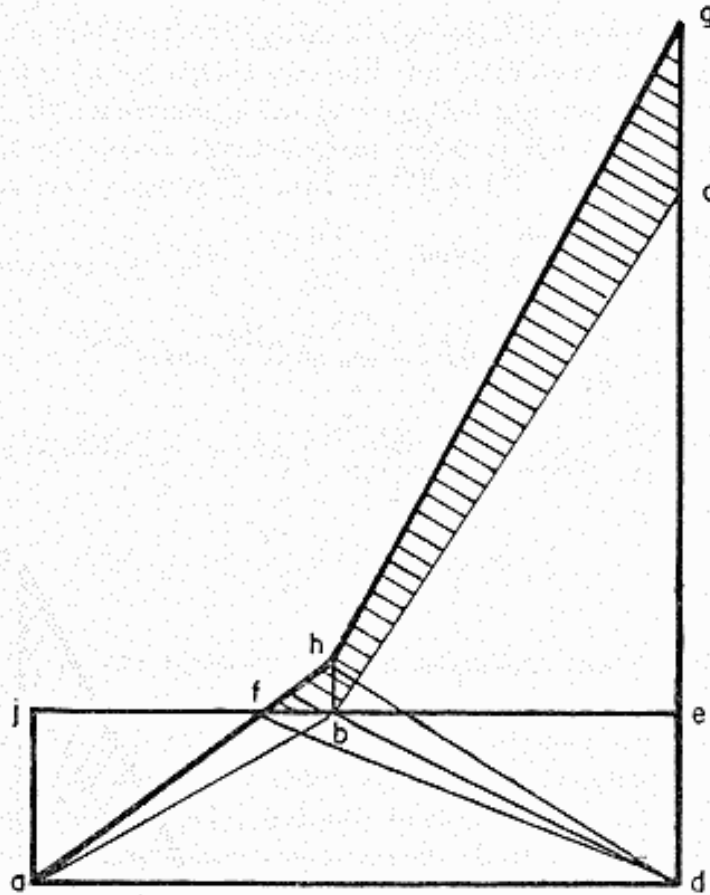


EXHIBIT IV-G-3

the component from picking which must now go into reserve is equal to the sum of the areas of the two small triangles $a-b-h$ and $d-b-h$ or is also equal to the triangle $d-f-h$. Its value can be determined by obtaining the difference in areas between triangle $a-h-d$ and $a-b-d$.

The increase in reserve area is represented in Exhibit IV-G-3 by the irregular shaped figure $f-h-g-c-b$. It is also represented in Exhibit IV-G-4, which follows by the sum of the two areas $h-g-m$ and $e-f-h-n$, therefore:

$$(1) \quad \Delta A_{R_{1-2}} = hgm + efn$$

$$= \frac{eb}{2} \cdot gm + en \cdot eb + \frac{bf}{2} \cdot en$$

$$(2) \quad = \frac{eb}{2} \cdot ec \left(\frac{gm}{ec} \right) + en \cdot eb + \frac{bf}{2} \cdot en$$

$$(3) \quad A_{R_1} = \frac{eb}{2} \cdot ec \quad A_{F_1} = ed \cdot ad$$

$$(4) \quad \frac{\Delta S_{1-2}}{S_1} = \frac{gm}{ec} = \frac{gm}{nm} = \frac{gc}{dc} = \frac{en}{ed}$$

$$(5) \quad \frac{en}{ed} \left(\frac{ed}{2} \cdot ad \right) = \frac{en}{2ed} (edad) = \frac{\Delta S_{1-2}}{2S_1} \quad A_{F_1}$$

From (5) and (2), $\frac{en}{ed} \left(\frac{ed}{2} \cdot ad \right) = en \cdot eb + \frac{bf}{2} \cdot en$

$$eb = \frac{ad}{2} - \frac{bf}{2}$$

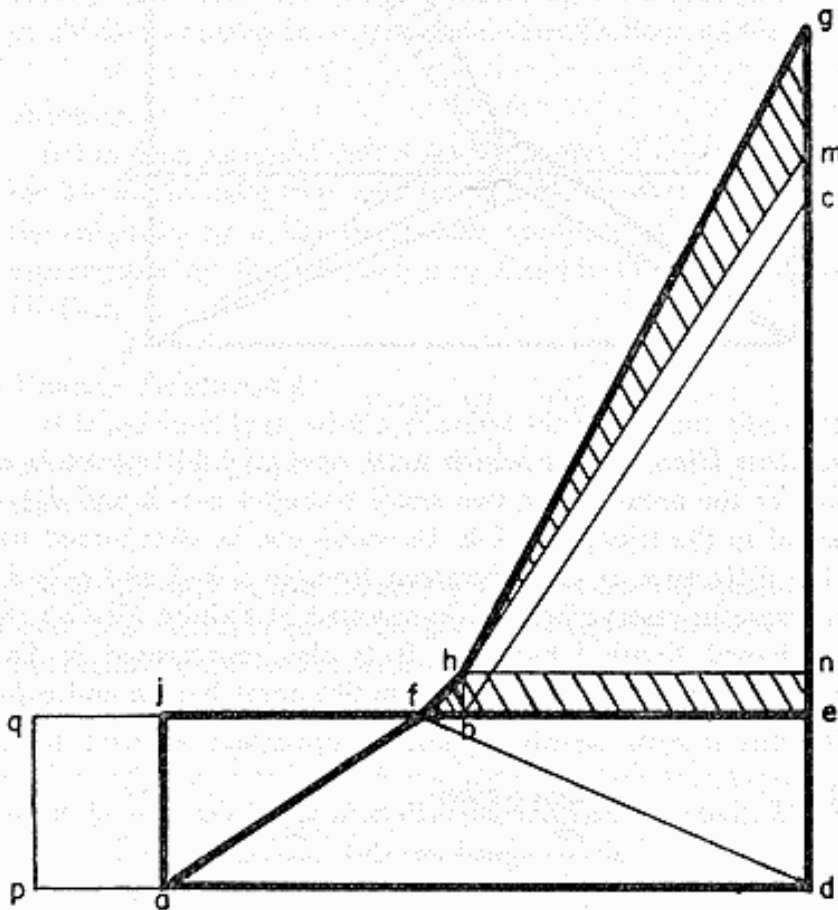


EXHIBIT IV-G-4

so that (5) holds exactly when the number of items in reserve at the time of the survey equals one-half the total items less $\frac{1}{2}$ the expected increase. By inspection it can be seen that the error would be small for any ratio of reserve items to total items as the area of triangle d-f-h is always very close to the area of e-n-h-f.

Substituting in (2)

$$\Delta A_{R_{1-2}} = A_{R_1} \left(\frac{gm}{ec} \right) + A_{P_1} \left(\frac{en}{2ed} \right)$$

$$(6) \text{ from (4) } \Delta A_{R_{1-2}} = A_{R_1} \left(\frac{\Delta S_{1-2}}{S_1} \right) + A_{P_1} \cdot \frac{1}{2} \cdot \left(\frac{\Delta S_{1-2}}{S_1} \right)$$

$$(7) \quad \Delta A_{R_{1-2}} = \frac{\Delta S_{1-2}}{S_1} \left(A_{R_1} + \frac{1}{2} A_{P_1} \right)$$

This (7) is the formula used in adjusting space requirements for changes in sales activity.

Region a-j-g-p represents changes in picking area resulting from changes in the number of items and treated elsewhere.

Expressed narratively, Storage Factor formula (7) states that the change in Reserve space requirements from time (1) to time (2) is equal to the change in Sales from time 1 to time 2 divided by Sales at time 1 multiplied by Reserve area at time one plus $\frac{1}{2}$ the Picking area at time 1.