Sales Forecasting in the Beer and Soft Drink Industry.

By GEORG J. KJÆR*)

Resumé.

Der beskrives en metode til forudsigelse af fremtidige salg, som disse må forventes at blive, dersom de fundne salgsbevægelser fortsætter i fremtiden, d. v. s. et forecastingprogram.

Den praktiske udnyttelse af metoden er muliggjort ved kodning af et regneprogram til cifferregnemaskinen DASK.

Efter regneprogrammet antages salget at være deleligt i en langtidskomponent (trend), en sæsonkomponent og en tilfældig variabel.

Sæsonkomponenten findes ved kvotientmetoden, der er beskrevet detailleret, med en tiårig salgsperiode som beregningsgrundlag. Ved multiplikation med salget i en basis-måned forvandles kvotienterne til en salgssæson i absolutte tal.

Selv om denne beregningsmetode forudsætter multiplikative sæsonudsving, tillader multiplikationen med den senest kendte basismåned, at langtidskomponenten findes ved subtraktion af sæsonen fra totalsalget i de seneste 12 måneder.

Differensen indeholder langtidskomponent plus tilfældig komponent. Langtidskomponenten tilnærmes ved antagelse af linearitet og beregning efter mindste kvadraters metode.

Prognosetallene fremkommer ved extrapolation af trendlinier og påfølgende addition af sæsonkomponenter.

I vurderingen af regneprogrammet og resultaterne tages hensyn til spredningen om trendlinierne, der angiver den tilfældige komponents spredning, og en beregning af denne spredning muliggør en fastsættelse af længden på perioden, der bør anvendes til be-

*) Chemical engineer, Carlsberg Breweries, Copenhagen.

Sales Forecasting in the Beer and Soft Drink Industry.

By Georg J. Kjær*)

Resumé.

Der beskrives en metode til forudsigelse af fremtidige salg, som disse må forventes at blive, dersom de fundne salgsbevægelser fortsætter i fremtiden, d. v. s. et forecastingprogram.

Den praktiske udnyttelse af metoden er muliggjort ved kodning af et regneprogram til cifferregnemaskinen DASK.

Efter regneprogrammet antages salget at være deleligt i en langtidskomponent (trend), en sæsonkomponent og en tilfældig variabel.

Sæsonkomponenten findes ved kvotientmetoden, der er beskrevet detailleret, med en tiårig salgsperiode som beregningsgrundlag. Ved multiplikation med salget i en basis-måned forvandles kvotienterne til en salgssæson i absolutte tal.

Selv om denne beregningsmetode forudsætter multiplikative sæsonudsving, tillader multiplikationen med den senest kendte basismåned, at langtidskomponenten findes ved subtraktion af sæsonen fra totalsalget i de seneste 12 måneder.

Differensen indeholder langtidskomponent plus tilfældig komponent. Langtidskomponenten tilnærmes ved antagelse af linearitet og beregning efter mindste kvadraters metode.

Prognosetallene fremkommer ved extrapolation af trendlinier og påfølgende addition af sæsonkomponenter.

I vurderingen af regneprogrammet og resultaterne tages hensyn til spredningen om trendlinierne, der angiver den tilfældige komponents spredning, og en beregning af denne spredning muliggør en fastsættelse af længden på perioden, der bør anvendes til be-

*) Chemical engineer, Carlsberg Breweries, Copenhagen.

regning af sæsonen. Desuden vises fremgangsmåden ved valg af antallet af måneder, der skal bruges til beregning af langtidskomponenten, og det anbefales at give salgsprognosen på basis af såvel en relativ kort salgshistorie som på basis af en noget længere.
Herved opnås dels en hurtig reaktion på ændringer i salgstendensen ved den kortperiodiske prognose, dels en prognose, der ikke reagerer på pludselige tilfældige salgsudsving.

Der beskrives en metode til beregning af kontrolgrænser for en kurve over den akkumulerede differens mellem salgs- og forecast værdier. Kontrolgrænserne bruges som indikator for skift mellem anvendelsen af flere eller færre måneders salgshistorie i trendberegningen.

Regneprogrammets reaktion på visse typiske ændringer i salget er beregnet.
Endelig indeholder artiklen en beskrivelse af regneprogrammets praktiske udnyttelse på DASK, samt en omtale af resultaterne ved anvendelse på ølsalget på Carlsberg Bryggerierne.

In modern industrial developments there is an increasing need for reliable methods of estimating future sales.

In the inventory planning it is necessary to have an approximate knowledge of the demand for raw materials in the production process, and if possible the oscillation in the demand too. For this purpose a sales prediction should be very useful.

As a basis for a sales prognosis it is possible to employ a statistical forecasting method.

Such forecasting includes periodical recalculations as knowledge is gained of new sales figures, whence the use of an electronic computer is imperative to make the latest corrections of the expected sales figures available for the planning departments without undue delay.

On the other hand it is not advisable to use the unscrutinized forecasting results in the inventory planning.

The word forecasting means only the projection of the past into the future, whereas the influence of future sales efforts, competition, expanding markets and other independent variables are not taken into consideration in a sales forecasting programme. Accordingly the statistical computations will not be ready for use before further adjustments have been carried out in the sales and marketing departments, where the effect of planned advertising and other sales efforts is estimated.

As any sales prediction may be subject to more or less grave errors it is the objective of the sales and advertising departments to keep the sales as close to the stipulated figures as possible to insure optimum

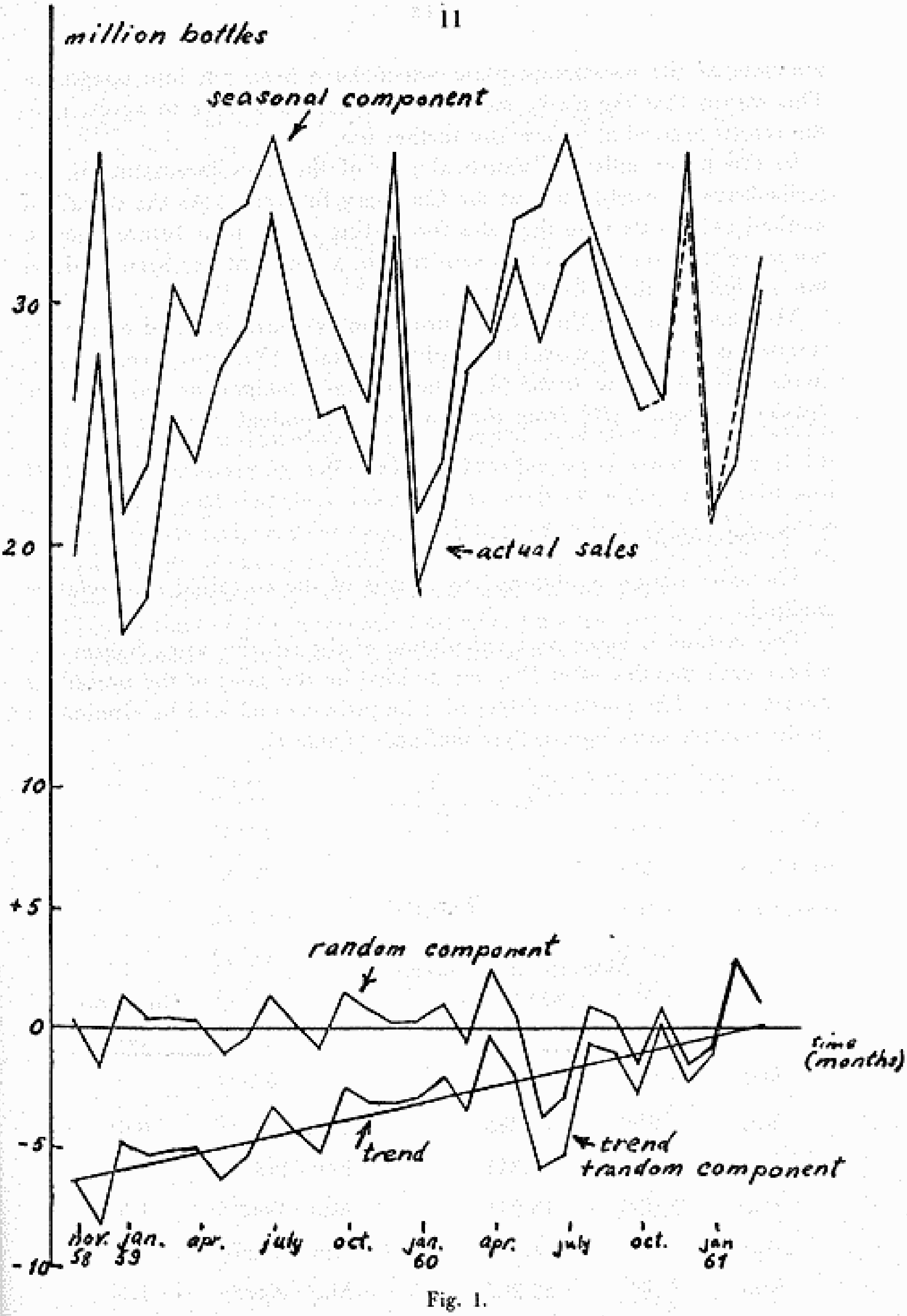
million bottles
seasonal component
30
20
actual sales
10
+5
random component
0
time (months)
-5
trend
trend +random component
-10
nov. 58
jan. 59
apr.
july
oct.
jan. 60
apr.
july
oct.
jan 61

Fig. 1.

economy of the investment plans which have been put into operation. This means that the above mentioned departments have to agree upon the results arrived at before any further use.

In this paper only the statistical part of the sales forecasting is described as it is used to-day at the Carlsberg Breweries. As the described method only deals with the sales forecasting in the near future, that is, not more than six months to a year ahead, a short cut has been made in the analysis of the sales history.

The forecasting of the sales figures months ahead is based on a very simple mathematical model that splits beersales (Y_t) into three components defined as the *trend* (T_t), the *seasonal component* (S_t), and the *random deviation* (R_t) from the seasonal component. (Figure 1).

$$Y_t = S_t + T_t + R_t \tag{1}$$

I. *Estimation of seasonal component,* (S_t).

The sales season is obtained by means of the so-called link relative method.

This method is based on a calculation of the relative sales figures (q_t) where each month's sales (Y_t) are divided by the sales of the preceding month Y_{t-1}. The possible effect of a long time trend will be eliminated in the relative sales figures thus obtained. (Table 1).

$$q_t = \frac{Y_t}{Y_{t-1}} \tag{2}$$

TABLE 1

		Monthly lager sales (1.000 bottles) (Y_t)		Quotients (q_t)
Dec.	58	21.784		
Jan.	59	13.288	Jan./Dec.	0,61
Febr.	59	14.241	Feb./Jan.	1,07
Mar.	59	18.771	Mar./Feb.	1,32
Apr.	59	18.958	Apr./Mar.	1,01
May	59	22.267	May/Apr.	1,17

June	59	23.757	June/May	1,07
July	59	27.704	July/June	1,17
Aug.	59	23.833	Aug./July	0,86
Sept.	59	20.808	Sept./Aug.	0.87
Oct.	59	20.962	Oct./Sept.	1,01
Nov.	59	17.799	Nov./Oct.	0,85
Dec.	59	24.685	Dec./Nov.	1,39
Jan.	60	14.482	Jan./Dec.	0,59

To arrive at a reasonably well founded expression for a seasonal factor it is necessary to use figures from a period of some length as for instance ten years monthly sales figures, each of which is transformed into a monthly link relative by a division as described above. The choice of the length of the preceding period used for the calculation of the seasonal component will be discussed later in the paper.

Using a total of 121 consecutive months, January will be estimated by ten link relatives, February also by ten and so on.

TABLE 2

Year	Quotients Jan./Dec. (q_t)	
51/52	0.674	(excluded)
52/53	0.661	(excluded)
53/54	0.559	(excluded)
54/55	0.612	
55/56	0.604	
56/57	0.616	
57/58	0.625	
58/59	0.610	
59/60	0.587	(excluded)
Mean: =	0.616	
Mean of 6 central values: $\bar{q}_{Jan./Dec.}$ =	0.611	

As the ten relatives are varying due to the random component of the season, a mean must be found.

Besides the random variation a certain number of sales anomalies may occur, causing abnormal deviations of the relatives during a ten year period. In order to obtain a mean relative which is not influenced by such anomalies, a so-called positional mean is calculated for each month.

In the programme for the electronic computer the two highest and the two lowest link relatives are excluded from the estimated mean link relative, $\overline{q}_t$ (table 2).

Until now the calculations have resulted in one link relative ($\overline{q}_t$) for each of the twelve months of the year.

In the further calculations it is necessary to get a sales season where the individual month is represented by a real, not by a relative figure.

To reach such a season a so-called basic month is chosen. In the present instance the month of November has been used for this purpose.

The typical link relative for November, the basic month, is fixed at $Q'_0 = 1.00$.

The typical relative for December is thus 1.00 multiplied by the link relative for December (Table 3). $Q'_1 = Q'_0 \cdot \overline{q}_1$.

The typical relative for January is found by a multiplication of the link relative for January by the newly found typical relative for December. (Table 3).

In this way the typical relative season is found through a series of consecutive multiplications. (Table 3).

$$Q'_t = \overline{q}_t \cdot Q'_{t-1} \tag{3}$$

TABLE 3

Mean link relative ($\overline{q}_t$)		Typical mean link relative (Q'_t)		Typical mean link relative corrected (Q_t)
Dec./Nov.	1,34	Dec./Nov.	1,34	1,34
Jan./Dec.	0,62	Jan./Nov.	0,83	0,83
Feb./Jan.	1,07	Feb./Nov.	0,89*)	0,88
Mar./Feb.	1,22	Mar./Nov.	1,09	1,08
Apr./Mar.	1,02	Apr./Nov.	1,11	1,10
May/Apr.	1,16	May/Nov.	1,29	1,28

June/May	1,04	June/Nov.	1,35**)	1,34
July/June	1,08	July/Nov.	1,46	1,45
Aug./July	0,89	Aug./Nov.	1,31	1,29
Sep./Aug.	0,91	Sep./Nov.	1,20	1,18
Oct./Sep.	0,96	Oct./Nov.	1,14	1,12
Nov./Oct.	0,89	Nov./Nov.	1,02	1,00

*) ex: $0{,}89 = 1{,}07 \cdot 0{,}83$

**) –: $1{,}35 = 1{,}04 \cdot 1{,}29$

In the results thus obtained the fixed November relative ($Q'_0 = 1{,}00$) will rarely coincide with the November relative found as the last result in the series of successive multiplications, (1,02 in table 3). This lack of coincidence is due to the resulting trend component for the used ten years, mostly different from zero. Because of this difference from 1.00, $^1/_{12}$ of the difference is subtracted from the December typical relative, $^2/_{12}$ from the January typical relative and so forth, concluding in the subtraction of $^{12}/_{12}$ from the last November typical relative, transforming it to 1.00. $Q_t = Q'_t - (Q'_{12} - Q'_0)\ t/12$. (Table 3).

The sales season is now easily obtained by a multiplication of the newly found typical relative season by the figure from the latest known November sales. (Table 4).

$$S_t = Q_t \cdot Y_{Nov.} \qquad (4)$$

TABLE 4

	Quotient after correction (Q_t)	Sales season in absolute figures (S_t)
Nov.	1,00	17.799
Dec.	1,34	23.851
Jan.	0,83	14.773
Feb.	0,88	15.663
March	1,08	19.223
April	1,10	19.579

May	1,28	22.783
June	1,34	23.851*)
July	1,45	25.809
Aug.	1,29	22.961
Sept.	1,18	21.003
Oct.	1,12	19.935

*) ex: $1{,}34 \cdot 17.799 = 23.851$

Even if the link relative method fundamentally is based on the assumption that the seasonal variations are proportional to the sales, the multiplication with the November sales will make it possible to consider the seasonal component additive or multiplicative in the further use. That is, a quite satisfactory estimate for the seasonal component has been reached.

The practical adaption, however, will be much too time-consuming for routine calculation unless electronic computers are available.

II. *Estimation of trend*, (T_t).

Complying with the hypothesis of the possibility to split the sales into seasonal, trend and random components, it is now possible to find *the trend plus the random component* by means of a simple subtraction of the seasonal component from the total sales. (Fig. 1).

$$Y_t - S_t = T_t + R_t.$$

In the computer programme the subtraction is carried out for the last known twelve months only, as the hypothesis of an additive season will be less correct for a longer period of time. The twelve figures obtained will contain the trend plus the random deviations of the last twelve-months' season from the calculated typical season. (Fig. 1).

If there is a systematic deviation from the calculated season during the last twelve months, this will cause an abnormal distribution of the resulting figures, illustrated in fig. 1 as trend + random points.

If there is no reason to assume that the recent season has been abnormal, a trend curve may be fitted to the resulting points for the last twelve months.

The fitting of a polynomial expression to the points will be possible, but the values found by extrapolations will be less correct the more complicated the used polynomium is.

To avoid this complication *the trend is considered linear, and estimated as the regression line corresponding to the points using a trivial least squares method.* (Table 5).

TABLE 5

Least squares method:
Equation of the regression line.

$$T_t = bt + a \tag{5}$$

where

$$b = \frac{\Sigma t T_t - \frac{\Sigma t \Sigma T_t}{n}}{\Sigma t^2 - \frac{(\Sigma t)^2}{n}}$$

$$a = \frac{\Sigma T_t - b \Sigma t}{n}$$

$$n = 2, 3, 4, 5, \ldots\ldots 12.$$

Complying with

1) Jan.. – Feb . trend $n = 2$
2) Dec., – Jan.. – Feb., trend $n = 3$
3) Nov.. – Dec. – Jan.–Feb. trend $n = 4$
– – – – – – – – – – –
– – – – – – – – – – –
– – – – – – – – – – –
11) Mar.–Apr. ...... Jan.–Feb. trend $n = 12$

III. *Estimation of random component, (R_t).*

It should now be possible to make calculations of the random deviations from the season during the period treated.

The trend is considered linear in each of the twelve-months' periods used for the calculation of the sales season.

When the sales season is subtracted from the actual sales figures throughout the said period, the difference will consist of the trend plus the random deviation from the season, as pointed out in the description of the estimation of the trend component.

$$Y_t - S_t = T_t + R_t.$$

For each twelve-months' period a calculation of the regression line will give the trend component, considered linear, and the further subtraction of the trend

$$Y_t - S_t - T_t = R_t$$

will give the values of the random component. Using separate twelve-months' periods, the use of an additive seasonal component in the computation can still be justified. The standard deviation about the regression line is thus the standard deviation of the random seasonal component.

$$\sigma^2_R = \Sigma (Y_t - S_t)^2 - \frac{[\Sigma (Y_t - S_t)]^2}{12} - \frac{\left[\Sigma t (Y_t - S_t) - \frac{\Sigma t \Sigma (Y_t - S_t)}{12}\right]^2}{\Sigma t^2 - \frac{(\Sigma t)^2}{12}} \quad (6)$$

IV. *Length of period for calculation of seasonal component, (S_t).*

As previously mentioned the length of the period used for the calculation need some thought. Given a value for the standard deviation of the random seasonal component for each year used for the calculation of the season, it is possible to detect systematic alterations of the sales season from year to year.

If the figures from the earlier part of the period are considerably higher than those from the later part, this earlier period may be excluded in the estimation of the sales season.

A statistical test on the standard deviation found to detect a "jump" in the figures will hardly be necessary.

V. *The sales forecasting.*

Roughly spoken the sales forecasting when looked upon as a geometric problem will result from an extrapolation of the trend curve followed by an addition of the seasonal components to the extrapolated future trend curve.

More detailed the arithmetic treatment by the computer programme will split the 121 known sales figures in a typical seasonal component, and a trend plus a random component for the last twelve months.

Furthermore the programme will make extrapolations on the straight trend lines resulting from the trend plus random components for the last 2, 3, 4, – 12 months, that is, a total of 11 trend lines. (Fig. 2). The extrapolations are made right up to twelve months after the last known, and the values of the typical season are added resulting in a total of $11 \cdot 12 = 132$ forecast figures. Furthermore the typical season and an estimate of the standard deviation in the figures used for computation of the link relatives are typed of the computer.

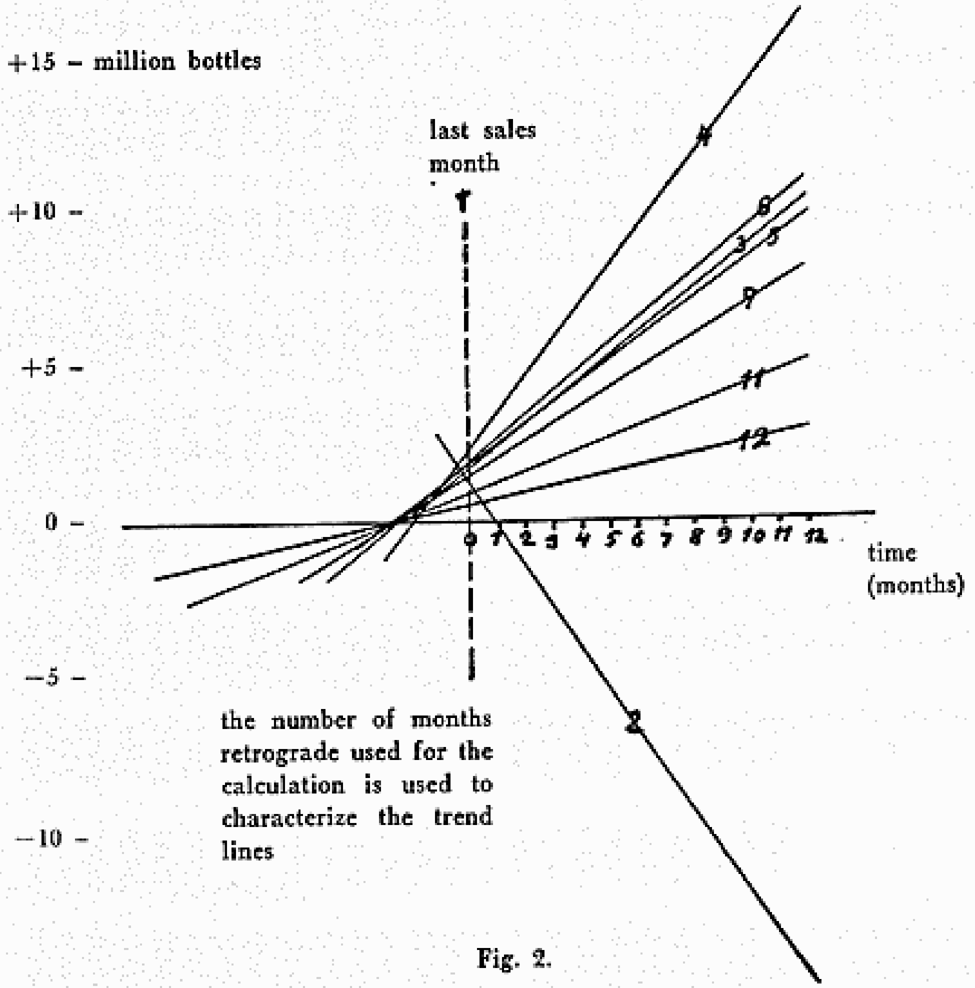

Fig. 2.

VI. *Practical use of the computer programme.*

Every month the newly realized month's sales are used as input to the computer programme together with the 120 sales figures from the preceding ten years. The computer result will be a tabular arrangement as shown in table 6.

Table 6

Months retrograde / Months ahead	2	3	4	5	6	7	8	9	10	11	12	Typical season	
	4832	−1572	−1867	−2230	−2457	−2814	−2095	−1187	−1088	−541	−482	30040	September
1	34872	28469	28173	27810	27583	27226	27945	28853	28952	29499	29559		
	9544	−1662	−2135	−2679	−3004	−3495	−2536	−1356	−1229	−545	−472	27921	October
2	37464	26259	25786	25242	24917	24426	25385	26565	26692	27375	27448		
	14256	−1753	−2403	−3128	−3550	−4175	−2977	−1524	−1370	−550	−463	25822	November
3	40078	24069	23419	22694	22272	21647	22845	24298	24452	25272	25359		
	18967	−1843	−2670	−3577	−4097	−4856	−3417	−1693	−1512	−555	−454	35734	December
4	54702	33891	33064	32157	31638	30879	32317	34042	34222	35180	35280		
	23679	−1934	−2938	−4027	−4643	−5536	−3858	−1861	−1653	−559	−445	21187	January
5	44866	19253	18249	17160	16544	15651	17329	19326	19534	20628	20742		
	28391	−2024	−3206	−4476	−5190	−6216	−4299	−2029	−1794	−564	−436	23368	February
6	51760	21344	20162	18892	18179	17152	19070	21339	21574	22805	22932		
	33103	−2115	−3474	−4925	−5736	−6897	−4740	−2198	−1936	−568	−427	30517	March
7	63621	28403	27043	25592	24781	23620	25778	28320	28582	29949	30091		
	37815	−2205	−3742	−5374	−6283	−7577	−5180	−2366	−2077	−573	−418	28153	April
8	65968	25948	24412	22779	21870	20576	22973	25787	26076	27580	27735		
	42527	−2296	−4009	−5823	−6829	−8258	−5621	−2535	−2218	−577	−409	33065	May
9	75592	30769	29055	27241	26235	24807	27444	30530	30846	32487	32656		
	47239	−2386	−4277	−6273	−7376	−8938	−6062	−2703	−2360	−582	−399	33754	June
10	80993	31368	29477	27482	26378	24816	27693	31051	31395	33172	33355		
	51951	−2477	−4545	−6722	−7922	−9619	−6502	−2871	−2501	−587	−390	36407	July
11	88358	33931	31862	29685	28485	26788	29905	33536	33906	35821	36017		
	56663	−2567	−4813	−7171	−8469	−10299	−6943	−3040	−2642	−591	−381	32938	August
12	89601	30371	28126	25767	24469	22639	25995	29898	30296	32847	32557		

A. *Choice of number of months retrograde.* (Table 6).

In the computer output it is possible to choose between eleven different results for every month to come, depending on how many months retrograde are taken into consideration.

A smooth nearly linear development of the trend curve will make the use of many trend points reasonable, while an oscillating trend curve only allows a forecasting based on a few trend points retrograde.

In order to make the best choice of the number of months retrograde it is necessary to treat the forecasting results as "a dry running" of the results from the last one or two years known.

In the dry running the eleven possible forecast results for each month are compared with the actual sales in the same month, and thus it is found which result is closer to the actual sales.

In many cases the comparison will give two numbers of months retrograde giving a partial minimum difference between sales and forecast values, one corresponding to only a few months retrograde, another corresponding to some 9–12 months retrograde.

It is recommended to use both possibilities, as a small number of months retrograde will give a "nervous" curve rapidly tracking new developments in the sales trend, and therefore also reacting on random variations in a marked way.

A greater number will give a more stable curve where random variations will have only smaller effects, and on the other hand new sales developments will be tracked more slowly.

The two most frequent numbers occurring will consequently serve as the starting points of the forecasting.

B. *Control diagram.*

The control diagram for a forecasting sequence is obtained as follows:

The forecast is compared with the new sales result when known, and we start the drawing of a graph of the cumulated difference between the forecast and the actual sales plotted as a time series.

$$\Sigma\,(Y_t - F_t) = \Sigma D_t \qquad (7)$$

The cumulated diagram will serve as a control chart for the choice of the number of months retrograde to be used in the computation of the trend.

The control limits are an angle placed with the top point on a hori-

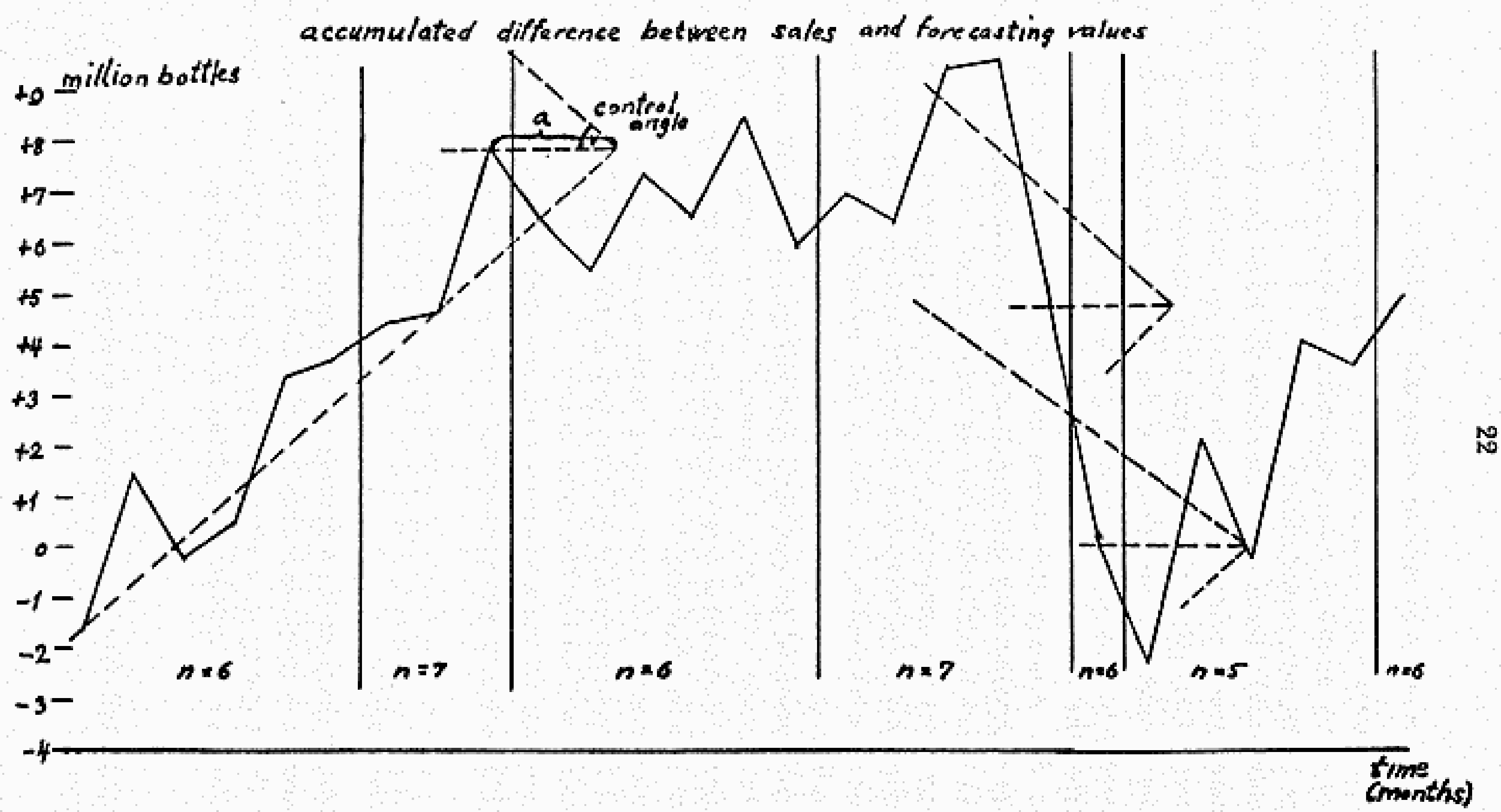
accumulated difference between sales and forecasting values
million bottles
+9
+8
+7
+6
+5
+4
+3
+2
+1
0
-1
-2
-3
-4
a
control angle
n=6
n=7
n=6
n=7
n=6
n=5
n=6
time (months)

Fig. 3.

zontal line through the last point plotted on the cumulated curve, and at a predestinated distance (a) after this point.

The characteristics of the angle can be set up as empirical values found suitable in the dry running or estimated through a simple bit of reasoning.

In fig. 3 and 4 the control angle is shown.

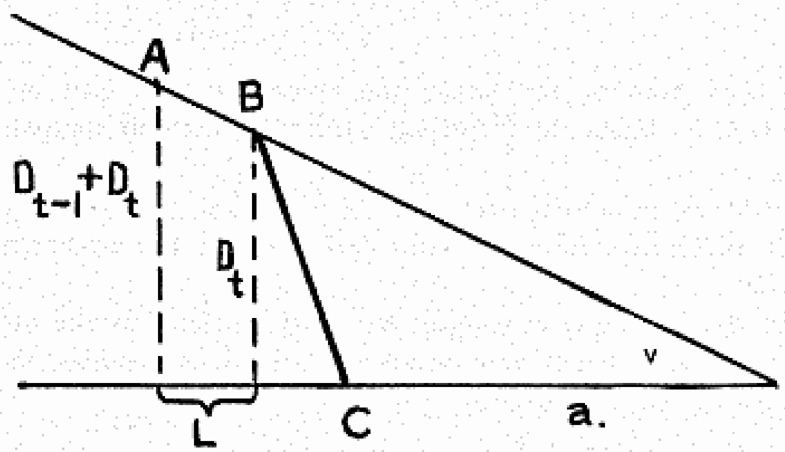

Fig. 4.

The letters A, B, C illustrate three accumulated differences between the actual sales and the forecast values when the values A and B fall just on the borderline of the control limits. (Fig. 4). L is the distance chosen corresponding to the time interval between the successive forecast values.

Given the values of A, B, C and L – the values of a and v can be found.

The maximum allowable difference that should not give rise to reactions is set to $D_{tmax} = 2 \cdot \sigma$, where σ is the standard deviation corresponding to the random seasonal component.

The solution of the following two equations will give the values for $\tan v$ and a

$$\tan v = \frac{D_{tmax}}{L + a} \qquad (8) \qquad\qquad \tan v = \frac{(D_t + D_{t-1})_{max}}{2L + a} \qquad (9)$$

The value of $D_t + D_{t-1} = (Y_t - F_t) + (Y_{t-1} - F_{t-1})$, the sum of the differences between actual sales and forecast figures for the last two sales periods (months) should not exceed $2\sigma\sqrt{2}$ if we consider the random component normally distributed with standard deviation σ and mean equal to zero.

As the forecasting programme, however, reacts on every deviation from the forecast value, a certain amount of quantitative knowledge of the reaction of the programme is required.

Let us assume that the trend line used for the extrapolation in the forecasting is based on n months retrograde.

The equation

$$T_r = T_{r-\frac{n+1}{2}} + b\left(\frac{n+1}{2}\right) \tag{10}$$

will then describe the trend line when the time unit on the abscissa equals 1, T_r being the (forecast) value of the trend component.

$$T_{r+1} = T_r + b \tag{11}$$

if no difference between sales and forecast has occurred in the r'th month.

(In the following reasoning the disappearance of the oldest point on the trend line, when new points are added, is assumed to have no influence on the calculations).

If the new sales have exceeded the forecast figure with the amount $a\sigma$ in the r'th month,

$$Y_r = T_r + S_r + a\sigma$$

then $$T_{r+1} = T_r + b + \frac{a\sigma}{n} + \frac{(n+1)}{2} \cdot \frac{a\sigma\,(n-1)}{2} \cdot \frac{12}{(n-1)(n+1)\,n}$$

$$T_{r+1} = \frac{4\,a\sigma}{n} + T_r + b \tag{12}$$

This reaction of the programme on deviations of sales from forecast values means that a theoretical maximum value of $D_t + D_{t+1} = 2\,\sigma\sqrt{2}$ will be reduced with $\frac{4\,a\sigma}{n}$ when the deviation of the first of the two months considered has been $a\sigma = D_{t-1}$.

The maximum allowed $D_t + D_{t-1}$ value is thus reduced from the "ideal" value $2\,\sigma\sqrt{2}$ to

$$(D_{t-1} + D_t)_{max} = 2\,\sigma\sqrt{2} - \frac{4\,a\sigma}{n}\,. \tag{13}$$

As $D_{t-1} = a\sigma$ and $D_{t\,max} = 2\,\sigma$

we have also

$$(D_t + D_{t-1})_{max} = 2\,\sigma + a\sigma. \tag{14}$$

From equations (13) and (14) a is found to

$$a = \frac{2\,(\sqrt{2}-1)\,n}{4+n}\,, \tag{15}$$

and by substituting (15) in (13) or (14)

$$(D_t + D_{+1})_{max} = \frac{8 + 2\,n\,\sqrt{2}}{4+n}\,\sigma \tag{16}$$

Entering the expressions for $(D_t + D_{t-1})_{max}$ and $(D_t)_{max}$ in (8) and (9) we have:

$$\tan v = \frac{(D_t)_{max}}{L+a} = \frac{2\,\sigma}{L+a}$$

$$\tan v = \frac{(D_t + D_{t-1})_{max}}{2\,L+a} = \frac{8 + 2\,n\,\sqrt{2}}{(4+n)\,(2\,L+a)}\,\sigma$$

from which

$$\frac{2\,\sigma}{L+a} = \frac{8 + 2\,n\,\sqrt{2}}{(4+n)\,(2\,L+a)}\,\sigma$$

$$a = L\,\frac{4\,\sqrt{2} + 4 + n\,\sqrt{2}}{n} \tag{17}$$

and

$$\tan v = \frac{2\,\sigma n}{L\,(4+n)\,(\sqrt{2}+1)} \tag{18}$$

The control angle is now fully described when σ, L and n are known.

C. *Use of control diagram.*

In the application of the sales forecasting programme the following practice may be suggested.

n, the optimum number of months retrograde to be used, is found from a two year period.

The following reasoning covers the "nervous" curve ($n = 3, 4, 5, 6$) as well as the "stable" curve ($n = 10, 11, 12$).

σ is found as the mean standard deviation about the regression lines calculated for each twelve-month's period in the years used for calculation of the sales season.

L is fixed as the time interval on the abscissa of the diagram illustrating the accumulated difference between sales and forecast figures ΣD_t.

The characteristics of the control angle, a and $\tan v$, are found in accordance with the values of n, σ and L, from equations (17) and (18).

The control chart (showing the aforementioned accumulated difference ΣD_t between sales and forecast values) is started with n months retrograde as the basis for the forecasting.

The points on the graph are tested in respect to the control limits. If a newly plotted point results in part of the accumulated curve falling outside the control limits this may be due to alterations in the trend, and the consequence will be a shift to $n - 1$ months retrograde as the basis for the forecasting.

Only the part of the curve corresponding to the last n months will have to be taken into consideration in the use of the control angle.

The shift, however, should be made only when the diagram goes out of control because of *increasing* or *decreasing* ΣD_t-values during two-three months. If the lack of control follows a shift from positive to negative elevation of the accumulated curve, i. e. abnormal high sales followed by abnormal low sales et vice versa, the matter must be given further consideration.

It should be noted that the programme reacts on an increase $a\sigma$ in the value of ΣD_t in the way that the next forecast figure will be increased with $\frac{4\, a\sigma}{n}$ as calculated in equation (5).

A decrease ΔD in value of ΣD_t in this situation therefore should be corrected to $\Delta D - \frac{4\, a\sigma}{n}$, which value should be considered in relation to the control diagram.

As the reaction of the programme on alterations in the elevation of the trend curve is very quick, there should be no need to stay on $n - 1$ months retrograde as the forecasting basis for more than a few months.

If in the forecasting a constant number of months retrograde has been used for some time it should be possible to consider the trend so stable that a shift to $n + 1$ months could give just as good a basis for the forecasting.

Besides the control diagram a check on the "best" choice of the number of months retrograde to be used is made in the form of a calculation repeated after every twelve months for those in each case foregoing two years to insure that the most frequent numbers of months used are still unaltered.

The numbers of months are then corrected corresponding to the result of the computation.

In the application of the programme to beersales it has been found that a correction should be made only to the extent that the number of months retrograde used is not allowed to differ more than one from the result found in the computation.

The shift to $n + 1$ months because of an apparently stable trend has been made when n months retrograde has been used for n months.

If a month shows a marked low or high value compared with the forecast value it must be noted that this abnormal value will influence the future forecasting especially when the said month appears as earliest of the months retrograde used in the determination of the trend line.

When such a situation occurs it is advisable either to use the mean of the two surrounding forecast values corresponding to $n - 1$ and $n + 1$ months retrograde, or to exclude the value in future computations. However, it should be noted that this practice should not be carried too far. Only when it is evident that something really abnormal has taken place, such correction may be allowed.

To test the programme the influence of theoretical alterations in the input figures i. e. sales figures have been examined.

1) a "jump" J occurring in one month only (month 0) will cause the following oscillations.

Increase of forecast value:

first month afterwards	$\frac{4J}{n}$
second – –	$\frac{4J}{n} - \frac{6J}{n(n-1)}$
third – –	$\frac{4J}{n} - \frac{12J}{n(n-1)}$

etc. the disturbing influence of the jump disappearing after n months.

For $n = 6$ the values are $\frac{10}{15}J$, $\frac{7}{15}J$, $\frac{4}{15}J$, $\frac{1}{15}J$, $-\frac{2}{15}J$, $-\frac{5}{15}J$, 0 (fig. 5).

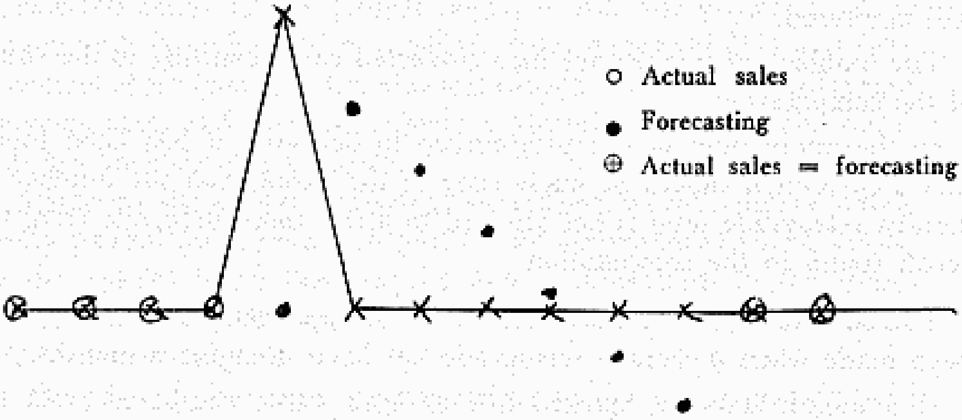

Fig. 5.

2) a "step" S shifting the niveau of the trend will be tracked as follows.

Increase of forecast value:

first month afterwards	$\frac{4S}{n}$
second – –	$\frac{8S}{n} - \frac{6S}{n(n-1)}$
third – –	$\frac{12S}{n} - \frac{18S}{n(n-1)}$
fourth – –	$\frac{16S}{n} - \frac{36S}{n(n-1)}$
fifth – –	$\frac{20S}{n} - \frac{60S}{n(n-1)}$

etc.

For $n = 6$ the values are

$\frac{10}{15}S$, $\frac{17}{15}S$, $\frac{21}{15}S$, $\frac{22}{15}S$, $\frac{20}{15}S$, $\frac{15}{15}S$, $\frac{15}{15}S$ . . . . (fig. 6)

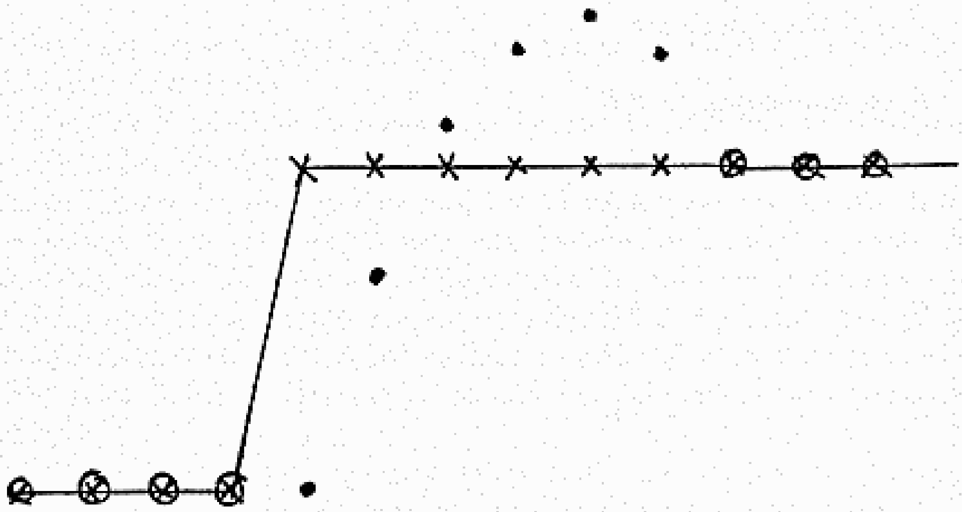

Fig. 6.

3) a "ramp" R shifting the elevation of the trend will be tracked as follows.

Increase of forecast value:

		Theoretical value
first month afterwards	$\frac{4R}{n}$	$2R$
second – –	$\frac{12R}{n} - \frac{6R}{n(n-1)}$	$3R$
third – –	$\frac{24R}{n} - \frac{24R}{n(n-1)}$	$4R$
fourth – –	$\frac{40R}{n} - \frac{60R}{n(n-1)}$	$5R$
fifth – –	$\frac{60R}{n} - \frac{120R}{n(n-1)}$	$6R$

etc.

For $n = 6$ the values are

$\frac{10}{15} R$ corresponding to $2 R$

$\frac{27}{15} R$ – – $3 R$

$\frac{48}{15} R$ – – $4 R$

$\frac{70}{15} R$ – – $5 R$

$\frac{90}{15} R$ – – $6 R$

etc. (fig. 7)

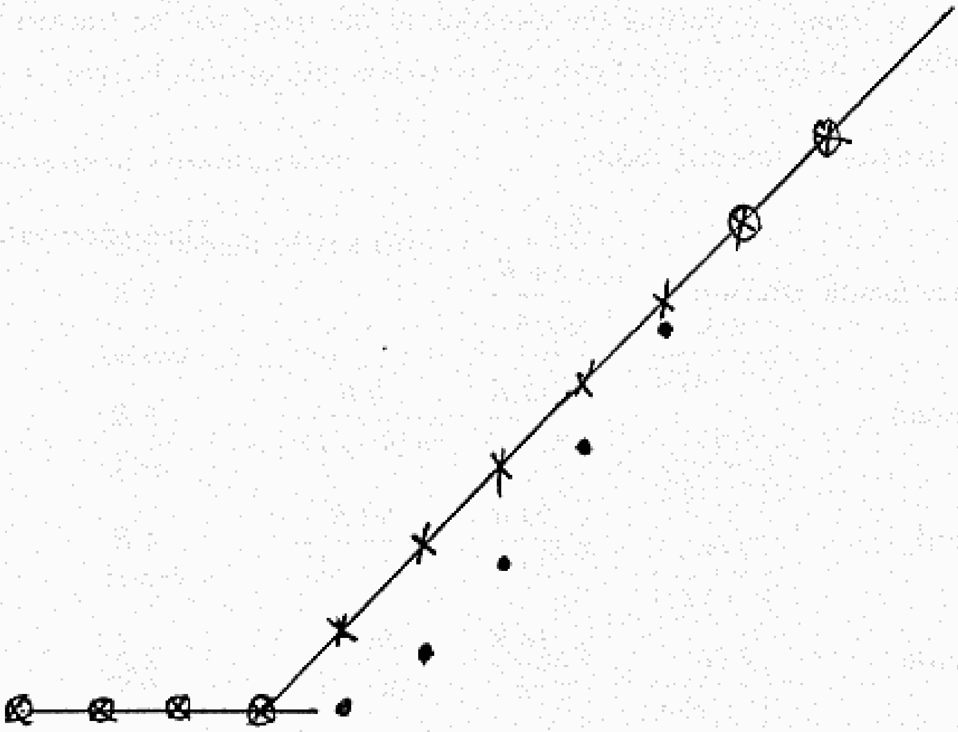

Fig. 7.

VIII. In some instances it has been possible to predict abnormal sales developments due to strike-situations, higher beer taxes, etc.

This may cause a jump in the sales figures followed by a retraction the month afterwards.

Under such peculiar circumstances it may be justified to straighten out the sales development in accordance with for instance stipulated storage figures for the retailers and the like, to avoid oscillations in the forecasting. Such precautions are only justified when first a clear relationship exists between sales figures and the said circumstances, and secondly such causal factors are not periodical, i. e. represents part of the seasonal component. If it is not possible to make a reasonable correction of the sales figure in this way, it should be excluded in the future calculations.

The programme may be used universally, not limited to beersales only, but minor corrections may be necessary in more specialized applications.

IX. In conclusion of this paper it should be noted that the programme has been used on the Carlsberg Breweries for some 18 months giving a relative standard deviation of the sales figures about the forecast values of 6 % taken over 2½ years, and a cumulated difference between the sales and forecast values not exceeding 9 million bottles on a sales total of about 400 million bottles per year.