Counting on the Census Numbers: Mathematical Approaches to the *elep* in Numbers

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Abstract: Censuses of the Israelites found in Numbers have been mulled over because of both their large values and the particulars of those values. Efforts to fit the large numbers of Israelites escaping Egypt into something historically plausible or suggest a lexical development focus on reinterpreting the standard counter for “thousands”, *elep* (אלף). The most recent attempts use mathematical models, but the results have not been rigorously tested. A statistical evaluation finds the efforts wanting, and the underlying numbers are most likely artificial. The origin of the census values is then proposed, indicating source relationships within the Pentateuch rather than some archaic, pre-CBH features and sources. Mathematical tests also limit what esoteric meanings the census numbers might have.

Keywords: Book Numbers, Census, Source Criticism

Measuring the Problem

In the traditional Exodus story, the number of Israelites fleeing Egypt is so large it gives pause to readers. Exodus 12:37b says the number of fighting-age men as over 600,000, while Numbers 1 counts the soldiers from each tribe, totaling 603,550 (Num 1:46). The conservative assumption of one woman and two children per soldier gives a total population at over 2.4 million, on par with the entire population of ancient Egypt. Similarly large numbers in the Deuteronomistic History are at odds with the archaeological record, as the territory only began to hold millions of residences in the 20th century.

While several approaches to this numerical issue have been published, the most common way to maintain the historical plausibility of these numbers is suggesting the term for ‘thousand’ (*elep*, אלף) does not mean a numerical value but a unit, at least when used in these census counts. If one hypothesizes an older source for the census numbers, perhaps אלף would have had a different meaning, thus confusing a later editor and implying a massive army. As a working hypothesis, אלף may have a pre-CBH meaning that was lost with developments within the Hebrew language and conflated it with a numerical value. The time of this other meaning must be prior to all CBH outside of the Pentateuch and the Deuteronomistic History, since Amos 5:3 and Isa 7:23; 30:17; 37:36 use אלף as the numeric value of 1,000.[1]

[1] Cf. Mic 5:2 as the only non-Pentateuchal or Deuteronomistic CBH example of אלף as different from ‘thousand’.

Though *elep* can mean a clan or tribe, and perhaps it could mean a military unit, the question is if this fits the context of the censuses in Numbers 1 and Numbers 26. The fullest mathematical treatment, following a suggestion going back to Flinders Petrie,[2] comes from Colin Humphreys,[3] wherein he argues that the *alapim* for each tribe in the Numbers censuses were small military units. When Reuben, for example, counted 46 *elep* and 500 men, this is not 46,500 soldiers, but 500 fighting men in 46 military units (see Table 1). This goes against the natural reading of the numbers as 46 *elep* plus 500 men (using a *waw* to indicate addition), so there must be textual issues to explain why we should read "46 אלף *of* 500 men" as if it were a construct state (cf. Num 10:36) instead of "46 אלף *and* (ו) 500 men" with the clear *waw* conjunction. The use of the singular *elep* instead of the plural *alapim* (אלפים) is also odd, if אלף were being used as a noun instead of as a number (cf. Num 31:14).

Currently, these grammar issues are not discussed, let alone explained, by advocates. While Petrie, Mendenhall, and Humphreys suppose there was another, similar word to *elep* (perhaps with a different vocalization) that meant "contingent," this would not avoid the need to explain the grammatical points. Instead, Denise Flanders provides a detailed analysis of the grammatical and lexical issues with the argument that *elep* could mean a military unit or contingent.[4] Her focus was primarily on the Deuteronomistic History, but the results apply to the Numbers censuses as well. However, she does not address the mathematical arguments of Humphreys, and his hypothesis might overcome the grammatical issues, though with *ad hoc* suppositions about the Priestly writer not understanding prior sources both lexically as well as grammatically. However, an additional problem for Humphreys is his requirement that the census values are very ancient (perhaps even from Moses), but not only are the census numbers from P, but a late stratum of P.[5] Humphreys does not engage in source criticism. Nonetheless, his hypothesis can and should be examined on mathematical grounds.

To date, several critiques of Humphreys exist,[6] but they do not carefully examine his modeling or perform tests with a statistical method,[7] and others seem to find no fault with the mathematics presented.[8] This may be in part because most published researchers in biblical studies do not have a strong mathematical background, while Humphreys is a physicist. To fill in this gap, the following

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[2] W.M. Flinders Petrie, *Researches in Sinai*, 207–217. Similarly, George E. Mendenhall, "The Census Lists of Numbers 1 and 26", 52–66.

[3] Colin J. Humphreys, "Number of People", 196–213; "Numbers in the Exodus", 323–328.

[4] Denise Flanders, "A Thousand Times No, אלף Does not Mean 'Contingent' in the Deuteronomistic History", 484–506.

[5] Suzanne Boorer, *The Vision of the Priestly Narrative*, 70–71.

[6] Jacob Milgrom, "On Decoding Very Large Numbers", 131–132; Mark M. McEntire, "A Response to Colin J. Humphreys's", 262–264; Rüdinger Heinzerling, "On the Interpretation of the Census Lists", 250–252.

[7] McEntire, "A Response to Colin J. Humphreys's", 262 already found Humphreys' math to be "torturous." I fear what might be said about the present article.

[8] McEntire, "A Response to Colin J. Humphreys's," 262 initially said the math was "convincing" and "internally consistent." Gary A. Rendsburg, "An Additional Note to Two Recent Articles on the Number of People in the Exodus from Egypt and the Large Numbers in Numbers I and XXVI", 394 says that Humphreys' mathematics is "most convincing."

analysis will critique the population modeling proposed by Humphreys, examining both its internal consistency as well as what statistical tests can say about the quality of the hypothesis. Moreover, an analytical approach to the numbers in the censuses will help indicate how they were generated, if they have a deeper purpose, and what might be the origins of those values given the sources of the Pentateuch.

Real or Artificial Numbers

Before addressing Humphreys' model, one should address how reliable are the numbers in the censuses in the first place. Humphreys, as well as Petrie before him, assumed the general historicity of the narratives that contain the counts of Israelites, and they made observations that they alleged supported accepting those numbers, though after reinterpreting the term *elep*.

Petrie noted that the hundreds place in the census numbers was oddly distributed, with most values a 4 or 5, and no 0s, 1s, 8s, or 9s.[9] He suggested that the distribution indicates the reality of the numbers, because getting such a distribution by chance is highly improbable. Petrie does not specify what a natural distribution would look like, or why an artificial selection of numbers would not have the features he noticed. Perhaps he assumed that fictitious numbers would follow a uniform distribution, where each digit is just as likely to appear; or if the numbers were poorly transmitted, they would randomize into a uniform distribution. Rather, the natural distribution of digits should follow Benford's law: a randomly generated number is most likely to start with a 1 (30.1% probability), then a 2 is the next-most likely (17.6% probability), and so on.[10] There are also expected probabilities for the next digits and combinations of digits. Especially after the first digit, so that there are orders of magnitude above and below to allow for variance for the numbers, Benford's law indicates there should be more 1s, not fewer, let alone none. This is most extreme when looking at the tens place; it is almost all 0s, except for a 5 and a 3, as if the writer generally rounded to the nearest hundreds place but included just a couple arbitrary examples of additional precision. Petrie's observations should make us doubt the values more than accept them, as the process that generated the numbers looks unnatural.[11]

But there is no reason to only look at the hundreds place. Observing both the ten-thousands place and the thousands place (the first and second *elep* digits), the distributions do not suggest a natural distribution. The ten-thousands place distribution is similar to that of the hundreds place—mostly 4s and 5s, and no 0s, 1s, 8s, or 9s. That the first and third digits should have such similar distributions is highly unexpected with naturally occurring numbers. However, the thousands place does not follow the same pattern, as there are now 0s, 1s, and 9s, the mode is 2, the 4s and 5s are in the minority instead of the majority, and the general shape is closer to Benford's law. Still, 1 is underrepresented, while 2, 4, and 5 are overrepresented (see Figure 1). Measures of the divergences between the ex-

[9] Petrie, *Researches in Sinai*, 210.

[10] Mark J. Nigrini, *Benford's Law*.

[11] Heinzerling, "On the Interpretation of the Census Lists", 251 also noted other anomalous findings in the digits of the census counts.

pected counts and the actual counts, such as either the Kullback-Leibler or the Jensen-Shannon Divergence,[12] indicate that the thousands place distribution is much better than the ten-thousands place when compared to Benford's law. The digit distribution suggests the leading digits for *elep* and for hundreds were artificial, while the next *elep* digit was closer to a natural distribution.

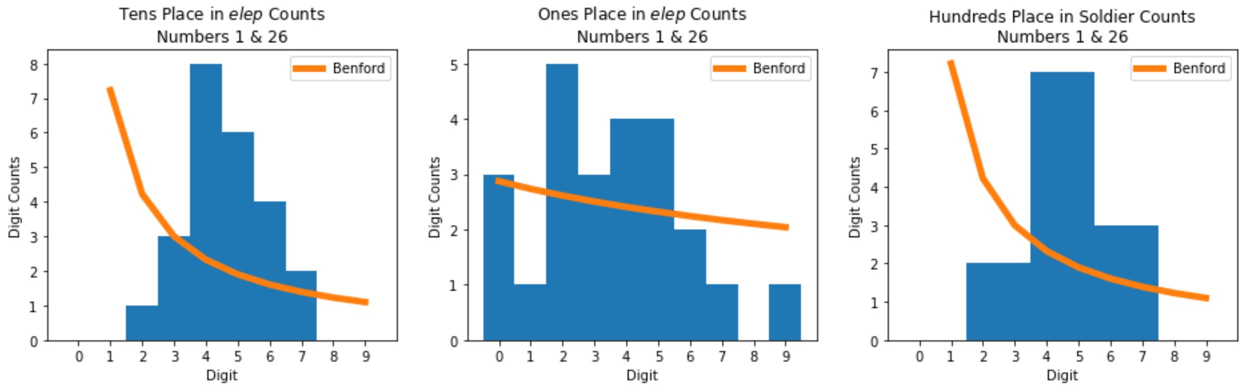

Figure 1. Counts of digits in censuses in Numbers, plotted against expected counts following Benford's law. Note that the hundreds place was plotted against the first digit predictions of Benford's law because Petrie treated this as the first digit in the actual soldier counts. If instead the third digit predictions of Benford's law were used, there should be approximately a uniform distribution.

However, given the small dataset, sweeping claims about the digits cannot be argued based on Benford's law alone. Instead, the problem is Petrie's vague logic going from the distribution of the hundreds place digits to historicity. Not only was Petrie unclear about why the distribution was an indicator for the reliability of the numbers, but the reality is the opposite. Similarly, when Humphreys says that it is "statistically extremely unlikely that a set of fictitious invented numbers would display such curious characteristics,"[13] this is without support, though he thinks he can explain the distribution of digits with his population model (see below). Petrie's logic is not sound, while Humphreys' discussion requires his modeling of the size of the tribes of Jacob.

Humphreys' analysis is based on several questionable assumptions. In particular, he assumes that the tribe of Levi is the about same size as any of the other tribes.[14] However, the straight-forward reading of Numbers 3 suggests that all one-month and older male Levites are 22 *elep*,[15] while other tribes have fighting forces several times that (see Table 1). This makes Levi's tribe far smaller. Their smaller size is also indicated by Deut 18:1, as the Levite were allotted no lands and were to live on sacrificial offerings. Special pleading is needed to suggest that the *elep* in Numbers 3 is different in size than the *elep* in Numbers 1 in such a way to make Levi the same size as Judah (more on this below). Additionally, not all these Levites can be in "priestly troops" as Humphreys sug-

[12] These measures are calculated using the implementations in the *scipy* library for the Python programming language.

[13] Humphreys, "Numbers in the Exodus", 327.

[14] Humphreys, "Number of People", 201–202.

[15] Petrie, *Researches in Sinai*, 216–217 suggests the Levite numbers were a later addition, coming from perhaps as late as the monarchic period.

gests,[16] because priests had to be between the ages of 25 and 50 (Num 8:24–26), but the census counted any male at least one month old. The smaller number of priestly men is confirmed in Numbers 4, where 8 *elep* and 580 were of age and worked in the tent of meeting. Therefore, a Levite *elep* in Numbers 3 cannot be a priest troop, and the straight-forward reading of the Book of Numbers indicates Levi is a comparatively small tribe.

Additionally, the argument that all the tribes were about the same size at their inception and at the time of the censuses are problematized by modern studies and refuted by Humphreys' own counting. There is already the assumption that the tribal system existed in the mid-2nd millennium BCE and following the basic historicity of its origins as formed from the sons of Jacob. This is not supported by Humphreys but implicitly assumed while not addressing the scholarship questioning that assumption.[17] Nonetheless, looking at the biblical numbers from the various tribes through time show large disparities. For example, First Chronicles 12 had the number of soldiers coming to David's call differ by an order of magnitude; Judah and Simeon could only muster several thousand apiece, Levi less than 5,000, while Zebulun provided 50,000. As for the time of the Exodus, the tribal sizes are found to vary significantly.[18] The Numbers 1 census has Dan more than double the size of Simeon (assuming *elep* does not mean 1,000); the traditional reading of Numbers 26 has Judah more than triple the size of Simeon. Moreover, while Humphreys suggests that a few centuries would not make one tribe significantly larger than another, the change in counts between Numbers 1 and Numbers 26 shows Manasseh more than tripled in size (assuming the *elep* was not a count of people), while Simeon lost a third (or more than half, when reading *elep* traditionally), all in a matter of decades. If such fluctuations were real, then there are no grounds to say the tribes would be of the same order of magnitude in size centuries after their formation.

Instead of Humphreys' explanation, another hypothesis can explain the oddities in the digit distributions. As noted by Eryl Davies,[19] the earlier Yahwist (or simply non-P) source had a population of about 600,000 men leaving Egypt (Exod 12:37b;[20] Num 11:21). An even division of 600,000 gives 50,000 per tribe. Making half of the tribes larger than 50,000 and half smaller gives the abundance of 4s and 5s in the first digit (the ten-thousands place). As for the hundreds, when all added together this total is close to 6,000; again, taking 6,000 and dividing by the number of tribes, and there would be an additional 500 men per tribe. Adjusting the number up or down would similarly give an over-abundance of 4s and 5s. Also, Numbers 1 has half of the hundreds counts below 500, just like how half of the total counts were above and below 50,000. In other words, the same process of taking a number, evenly dividing it by 12, and then semi-randomly adjusting the numbers up and

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[16] Humphreys, "Number of People", 205.

[17] See Andrew Tobolowsky, *The Sons of Jacob and the Sons of Herakles*, for summary of modern research and further analysis.

[18] McEntire, "A Response to Colin J. Humphreys's", 262–263 for a similar critique.

[19] Eryl W. Davies, "A Mathematical Conundrum", 466.

[20] R. S. Driver, *The Book of Exodus*, 100–101; John Van Seters, *The Life of Moses*, 229. Joel S. Baden, *The Composition of the Pentateuch*, 199 argues Exod 12:37 was from P, but his argument only applies to 12:37a, in agreement with Driver.

down produces a very artificial result. That semi-randomness in the adjustments especially effected the thousands place digits, which was noted earlier to more closely follow Benford's law. Overall, the evidence better fits the hypothesis that the census numbers are artificial creations.

However, these confounds only undermine Humphreys' explanation for the digit distributions, his population estimate (c. 20,000 Israelites), and the reliability of the numbers themselves. His major premise is not yet addressed: the number of men per tribe were divided up into the specified number of *alapim*. This can be tested in three ways: correlations between counts of *elep* and soldiers, mathematical consistency, and narrative consistency.

Correlations and Testing the Null Hypothesis

Given the hypothesis that the numbers are soldiers in *elep*-units, there ought to be a correlation between the number of *alapim* from a given tribe and their number of soldiers. If there are more soldiers, there ought to be more military units/*elep*-groups, following a simple linear model. A very simple linear model would say as the number of soldiers doubled, the number of units would double. The basic linear model (Figure 2) for this census hypothesis is expressed by equation (1), where y is the count of soldiers, x is the *elep* count, m is the number of men per *elep*, and y_0 is the y-intercept (discussed below).

$$y = mx + y_0 \ (Eq. 1)$$

To see if this model fits the data well,[21] indicating a relationship between soldier and *elep* counts, I calculate the (Pearson) correlation coefficient,[22] R, in four places: the counts in Numbers 1, in Numbers 26, the combination of Numbers 1 and Numbers 26, and the changes between Numbers 1 and Numbers 26. If there is a strong correlation between the count of soldiers and the count of *elep*-groups in each tribe, such that more soldiers means more *elep*-groups, then R should be close to 1. Because the counts of soldiers are round numbers and no more precise than at the tens place, then the correlation will not be perfect. Nonetheless, a value greater than 0.7 is expected for the model to explain most of the variance,[23] while a value close to 0 indicates no relationship at all.

[21] The standard fitting method is known as ordinary least squares (OLS), which draws a straight line through the data points that minimizes the vertical distance between the data points and the line. Calculations were done with the *statsmodels* library for the Python programming language.

[22] The Pearson correlation is the ratio of the covariance between two variables to the product of each variable's standard deviation. The covariance is the average of how much the product of each value's difference from the variable's mean, while the standard deviation is basically the average difference between a value and the mean of all values. Basically, it is a ratio of the correlated variance to the amount of variance in the data. A correlation of 0 implies no relationship between two variables (a change in x has no predicted change in y), while a correlation of 1 implies a direct, positive relationship (a positive change in x will have an exact, positive change in y), and a correlation of -1 implies a direct, negative relationship. Normally, the correlation is between -1 and 1, as most relationships are not direct or sufficient to explain all the variance in one variable in terms of another.

[23] Variance explained is given by R^2. If $R = 0.7$, then $R^2 \approx 0.5 = 50\%$.

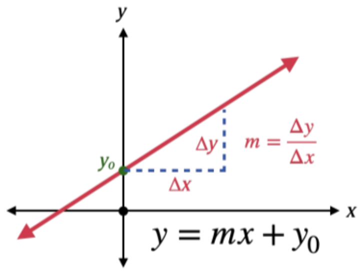

Figure 2. Equation of a line (red). The coordinates are on an x-y plane, the slope (m) is the change in y (Δy) given a change in x (Δx), the green dot is the y-intercept (y_0) where the red line intersects the y-axis (where x = 0), and the black dot is the origin of the coordinate system (x = 0, y = 0).

The results should also be statically significant, meaning they should not be plausibly achieved by randomness. The common rule-of-thumb in statistics is that the probability, p, of having such a correlation R or better when there is no actual correlation (the null hypothesis) should be less than 1 in 20 ($p < 0.05$); only then is a result considered significant. With the small amount of data involved in this analysis, the less strict $p < 0.10$ threshold may be acceptable.[24] Having the criteria of a minimum R as well as a minimum p will balance weak statistical power (that is, few data points) with minimizing Type II errors (inadvertently excluding true positives). Now, we can test Humphreys' hypothesis and see if $R > 0.7$ while also being statistically significant in any of the four cases. The results are in Table 2. Additionally, scatter plots of these counts are displayed to see if there are any patterns.

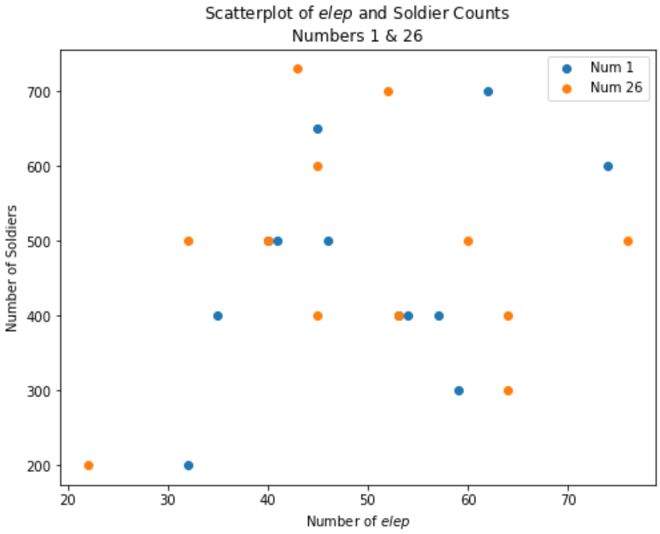

Figure 3. Scatterplot of the *elep* counts versus the hundreds counts. The source of the numbers, either Numbers 1 or Numbers 26, are color-coded.

Looking at the counts of men and *elep*-groups in Numbers 1, there is only a weak correlation without statistical significance. As for the counts from Numbers 26, the correlation is almost non-existent and not statistically significant. Combining these counts together provides more reliable

[24] For a correlation of $R = 0.7$, 12 samples, and an 80% chance of correctly rejecting the null hypothesis, then $p \approx 0.10$. See Stephen B. Hulley et al., *Designing Clinical Research*, 79.

statistics, as the statistical test is evaluating more data and will be more powerful. Unfortunately, the correlation is weak and not significant. A scatterplot of these counts indicates no pattern (Figure 3).

And yet, even these lackluster correlations are overestimates, because the y-intercept (y_0) was not constrained. The y-intercept indicates the value of y when x is zero. For the current hypothesis, the y-intercept is how many soldiers there are when there are no *elep*-groups. In some cases, there were significant results where y_0 was well above zero, indicating that even without a single *elep*, there would be hundreds of soldiers. This is not possible if soldiers were all assigned to an *elep*. Worse, these statistically significant intercept values are much larger than any *elep*, which ranged in size between 5 and 17 soldiers; y_0, on the other hand, was over 340 for the combined dataset, when there were only about 460 soldiers per tribe. It is as if most (~75%) of the troops cannot be accounted for. The fact that the y_0 results were ever statistically significantly different than zero is unexpected given the hypothesis.

Instead, the hypothesis of *alapim* related to the soldier counts requires this constraint: zero soldiers implies zero *elep*-groups and vice versa, so the y-intercept ought to be set to zero. Therefore, equation (1) should have $y_0 = 0$, and equation (2) ought to be fitted to the data.

$$y = mx \ (Eq. 2)$$

This model, which more accurately conforms to Humphreys' hypothesis, provides uninspiring coefficients (***R* (y_0 = 0) Pearson** in Table 2).[25] Some are not real numbers (*NaN*), suggesting that a horizontal line with no trend is better than the fitted model. In other words, in several cases there was no relationship between *elep* counts and soldier counts at all. With such small or non-existent correlation values and large p-values, almost all the variance is either explained by other variables or just random noise.

As another approach, instead of the Pearson correlation, the Spearman correlation (ρ) looks at the rank of the values.[26] This tests the hypothesis: the largest *elep* counts go with the largest soldier counts, and the converse. This does away with the restrictions of the linear model, wherein the values must always proportionally increases together rather than just generally increase together. Spearman also does away with the y-intercept, removing an additional modeling constraint. Thus, the Spearman correlation may work better for the sake of Humphreys' hypothesis. However, this results in similarly bad correlations (**ρ Spearman** in Table 2). The negative correlation for the Numbers 26 counts suggests more *alapim* means fewer troops, the opposite of the expected trend. No results were statistically significant.

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[25] R is calculated differently when there is a set y-intercept as in equation (2), so these coefficients are not directly comparable with that calculated using equation (1). The p-values also cannot be calculated. Nonetheless, small and non-existent R-values are informative.

[26] Spearman correlation uses the rank of each value (smallest to largest) from 1 to n in a set with n distinct values. Then the rank values are used to perform the same calculations as in the Pearson correlation, using covariance and standard deviation. If all the values are different, then an easier calculation of the coefficient uses the differences in rank (d_i) for each set of points (x_i, y_i) and the number of data points (n). If all x_i and y_i values have the same rank, then the correlation will be 1. If the ranks are random and the variables have no relationship, we expect the correlation to be close to 0.

The failure to reject the null hypothesis ought to be sufficient to no longer pursue the *elep*-group hypothesis. However, for completeness, suppose that another variable is each tribe had its own rules for organizing a military unit. While the hypothesis is *ad hoc*, it is already required to explain why one tribe had as few as 5 men in an average *elep*, while another had as many as 17. Perhaps then these divergent-sized *alapim* could explain the weak correlations. One way to account for this is by looking at proportional changes in the counts of *elep*-groups and men between Numbers 1 and Numbers 26. In that case, looking at the correlation between proportional changes in *elep* counts and in soldier counts within a tribe will avoid the possible confound of inter-tribal differences. At first, there appears to be a better correlation (**26 - 1**$_{\textbf{prop.}}$ in Table 2), but it is almost completely driven by outlier values, particularly Manasseh skewing the results with high statistical leverage—that is, the value is so different from the rest of the data points that it has an undue influence on the fitted line. If removing one data point notably changes the overall trend, then it is likely an outlier. Without Manasseh, the result is no better than before (**26 - 1**$_{\textbf{prop.+clean}}$ in Table 2). The scatterplots have no pattern, and Manasseh was clearly an outlier (Figure 4), driven by the near tripling in size of Manasseh in 40 years—a sure sign that there is something amiss with the value, given Humphreys' model. There is simply no statistical support for the hypothesis.

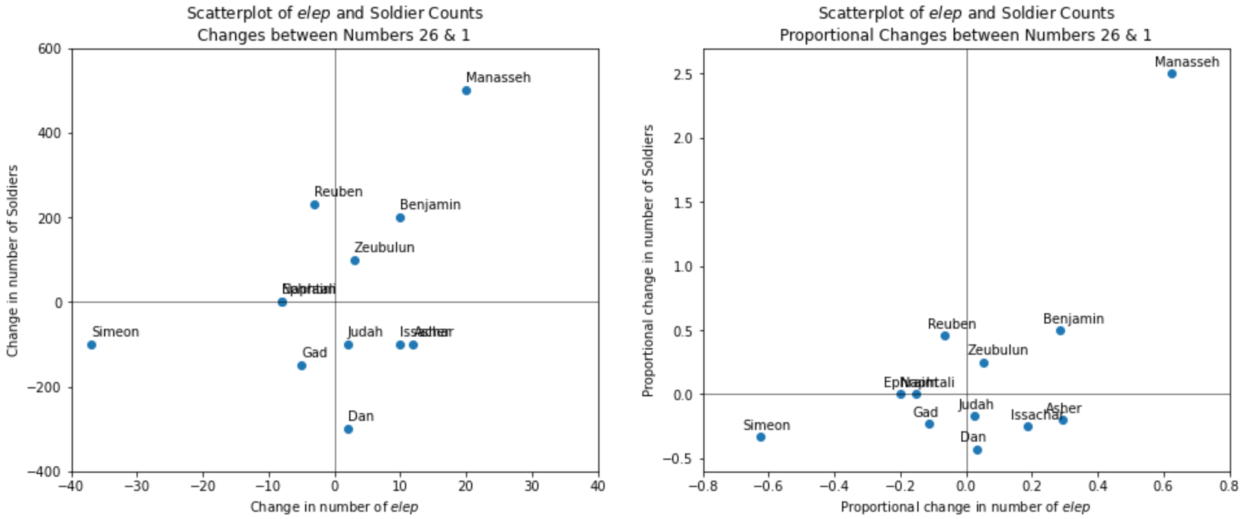

Figure 4. Absolute and proportional changes to *elep* and hundreds of soldiers counts between Numbers 26 and Numbers 1.

In fact, several tribal values show the folly in the approach advocated by Humphreys. Both Ephraim and Naphtali had 8 fewer *elep* in Numbers 26 than in Numbers 1, but their soldier counts were unchanged. More starkly, Asher gained 12 *elep*-groups, a 29% increase, but their soldier count went down by 100, a 20% decrease. Absurdly, Dan lost nearly half its population, but it gained 2 more *elep*. In about half of the tribes, the change in *elep* count went in the opposite direction as the soldier count. There is no discernable trend, indicating there is no underlying relationship between *elep*-counts and additional soldier counts.

Additionally, the hypothesis is directly refuted when looking at the Levite numbers. If the interpretation that only the hundreds count is the personnel count, while the *elep* is a group of arbitrary size, then there is the absurd situation wherein Kohath had only 300 males[27] (Num 3:27–28) yet sent 750 men into priestly services (Num 4:34–37). This impossibility falsifies Humphreys' reading, while the traditional reading of 8,300 males with 2,750 in service is perfectly consistent. The same problems exist for Gershon (500 total males, 630 in priestly service) and Merari (200 total males, 200 in priestly service).

This last impossibility was noted by Jerry Waite,[28] but he speculated that the next number after *elep* was originally a tens instead of a hundreds. The Priestly writer then assumed that *elep* was 1,000, and thus the next number was a hundreds. Not only is there no evidence of this, but it requires either the editor to have read a value like 50 (חמשים) as חמש מאות, or an earlier writer wrote חמש for חמשים, and then the editor imputed מאות. The first scenario is not plausible, while the latter requires mistakes on both the part of the original writer and the later editor, which is improbable. The fix Waite supposes also contradicts Humphreys' model that has the Levite tribe of similar size as other tribes; now, there were only 100 male Levites, confounding the population model in the first place. But these are all the least of the problems with Waite's *ad hoc* hypothesis. The Levite census counts in Numbers 4 makes the reading of tens values as hundreds impossible, because Numbers 4 includes the tens along with the hundreds, making the alleged misreading unviable. But even forcing the factor of ten change on Numbers 4, this would mean there are 158 men in the priesthood (75 from Kohath, 63 from Gershon, 20 from Merari)—this is still more men in the priesthood than total Levite males.

Finally, the very first equation used by Humphreys builds on a false premise, given his hypothesis, that leads to nonsensical results. Noting Num 3:46 that there were 273 more first-born males among the non-Levite tribes than there were male Levites, Humphreys builds his model on this difference.[29] He creates equation (3) (his equation (1)) where I_f is the number of first-born non-Levitical Israelites and L is the number of male Levites.

$$I_f - L = 273 \ (Eq. 3)$$

However, the count of first-born Israelites was given as 22 *elep* and 273 (Num 3:43). If *elep* is treated as 1,000, then there were more first-born males than all the Israelites as estimated by Humphreys. This also forces the number of Levite males to also exceed 20,000, and there are no grounds for following the *elep*-group hypothesis for any of the census data. If instead *elep* is treated as a group counter, as was done with in the *elep*-military unit hypothesis, then there were only 273 first-born Israelites (I_f) in 22 *elep*-groups (about 12 first-borns per *elep*). Following equation (3), then the number of Levite males (L) must be zero! This is not only an absurd result, but Humphreys also notes that his reading of Numbers 3 and his model indicate there were 1,000 Levite males. Using that as L, then the number of first-born Israelites would be 1,273, which corresponds to no other values in the censuses, particularly Num 3:43.

[27] MT says 600, but the total (Num 3:39) makes this impossible, suggesting Greek manuscripts more accurately preserved the number.

[28] Jerry Waite, "The Census of Israelite Men after their Exodus from Egypt", 489–490. Also, Heinzerling, "On the Interpretation of the Census Lists", 250–251.

[29] Humphreys, "Number of People", 201.

There is only one last way to save this estimate, which is to assume that originally Num 3:43 referred not to 22 *elep* of 273 first-born males, but 21 *elep*-groups, and then the number of first-borns is 1 *elep* + 273 = 1,243. In other words, the hypothetical source had to use *elep* in two different ways next to each other (i.e., עֶשְׂרִים וְאַחַת אֶלֶף אֶלֶף) in an unhelpful case of antanaclasis. Why the Priestly writer looked at this and neither understood two different meanings of *elep* nor treated it as a million (i.e., 1 Chron 21:5) is yet another grammatical issue Humphreys' hypothesis requires. Nonetheless, this opens the possibility that this same sort of issue would exist in all the other census counts for each tribe. For example, Reuben may have not had 46 *elep*-groups, but perhaps one or more of those *alapim* is actually 1,000. This means we would not know if the number of soldiers from Reuben was 500, 1,500, 2,500, and so on up to 46,500. Equations (4) and (5) expresses all the possibilities, where *n* and *m* are whole numbers ($\mathbb{N}_0$) that have to add up to the total number of *elep* of either kind.

$$n \cdot elep + 1000m + 500 = 46500\ (Eq. 4)$$

$$n + m = 46; n, m \in \mathbb{N}_0\ (Eq. 5)$$

Similarly, this uncertainty exists for all other tribal counts, so each tribal census requires its own version of equation (5) with different values of *n* and *m*. The mixture of *elep*-unit and *elep*-thousand is also needed to fix the problem with the Levite counts between Numbers 3 and 4, as noted earlier.

To solve for such counts analytically is impossible; each tribal count has two unknowns (how many *elep*-groups and how many 1,000s; *n* and *m*, respectively), and each tribal count is independent of the other. This requires solving for 60 unknown variables with 30 sets of equations and constraints (15 for each tribe or sub-tribe, repeated twice for two censuses, and each having two unknowns). At best, one can look at every possible combination of *elep*-counts in these 30 equation-sets and see which combinations seem to work. With literally thousands of possible combinations (on average, about 50 possibilities per equation), this is a daunting prospect, perhaps requiring numerical simulations and some objective function to see what results are best.

However, we can already find a solution that is far simpler: set *n* = 0 in all cases, and then *elep* = 1,000 in all cases.

Simply 1,000

Simple statistical tests falsify a relationship between the less-than-one-thousand-troop counts and the counted *elep* from a tribe, basic math disproves the hypothesis, and the underlying model is built from a demonstrably false premise. All ways to save the model require making it uninformative. Conversely, basic algebra shows that the writer of Numbers 1 interpreted *elep* as 1,000. Note that the totals provided in Num 1:46 are 603 *elep* and 550 men; summing the individual tribal counts, and there are 598 *elep* and 5550 men. Since these should be equal, as described by equation (6), then solving for the value of the *elep* is as follows:

$$603elep\ +\ 550\ =\ 598elep\ +\ 5550\ (Eq. 6)$$

$$5elep\ =\ 5000$$

$$elep = 1000$$

The same sort of algebra applied to Numbers 3, 4, and 26 give the same results for *elep*.

When looking at the excess number of first-born non-Levitical Israelites in Num 3:43 (22 *elep* + 273), the count of Levite males in Num 3:39 (22 *elep*), and equation (3) that was based on Num 3:46, we can prove the Israelite *elep* and the Levite *elep* are identical. If the Israelite *elep* is designated as $elep_I$ and the Levite *elep* unit is $elep_L$, without assuming these *alapim* are the same, then equation (7) again proves *elep* must have the same size in both cases.

$$I_f - L = 273\ (Eq.3)$$

$$(22elep_I + 273) - (22elep_L) = 273\ (Eq.7)$$

$$22(elep_I - elep_L) = 0$$

$$elep_I = elep_L$$

As the *elep* of the Israelite soldiers was shown to be 1,000, then the Levite *elep* must be 1,000. This also refutes Humphreys' estimation that the average military *elep* was about 9.3 soldiers, while the Levitical *elep* was about 48 priests.[30]

Independently, the same outcome is derived when examining the sum of half-shekels provided by each soldier (Exod 38:25–26): 100 talents and 1,775 shekels. As 1 talent was 3,000 shekels,[31] this gives 301,775 shekels. Doubling this to account for each soldier giving a half-shekel, and this gives 603,550 soldiers, exactly as reported by Exod 38:26 and Num 1:46, but only if *elep* is 1,000. If this shekel-count is not well-sourced, then it indicates the writer of Exodus 38 (usually believed to be P) is inventing numbers based on their own calculations. If some numbers are invented, then the probability that other numbers are invented increases necessarily. Conversely, if the shekel-count is historically reliable, then *elep* must be 1,000 and the soldier count was huge.

The Deuteronomistic History also contradicts the smaller *elep* model. In Gideon's fight against the Midianites (Judges 6–7), he first claimed his *elep* [family/clan] is the smallest in Manasseh. Nonetheless, his army started with 32 *elep*, but under instructions from God to reduce his army's size, it became 10 *elep*, and then finally 300 soldiers. If an *elep* were 9.3 soldiers as estimated, then Gideon started with 298 soldiers then reduced his army to 93; and yet it was reduced to 300—an impossibility. This example also undermines the argument that an editor would confuse the different possible meanings of *elep*, since the writer of Gideon's story fluently went between its possible valences.

Logistically, the Numbers narrative assumes much larger numbers of soldiers than estimated otherwise. For example, the war against the Midianites (Numbers 31) involved 12 Israelite *alapim* (one *elep* per tribe), and Israel won, killed five kings, and brought back hundreds of thousands of heads of livestock. Those 'thousands' of animals are confirmed by arithmetic results given in the narrative, wherein one five hundredth of the spoils are given to God, and the count of items sacrificed are given and perfectly match those values multiplied by 500 (i.e., 675 sheep for God, given 337,500

[30] Humphreys, "Number of People", 205.

[31] Carlo Zaccagnini, "Note on the Weight System at Alalah VII", 472–475.

(675×500) as spoils for the solders). As for the victorious Israelite army, given an *elep*-group of 9.3 soldiers, then all of this was accomplished by about 110 men. Each soldier had to shepherd over 6000 sheep, as well as 650 cattle and 550 donkeys. Also, these soldiers captured 32,000 virgin women, half of which would be spoils to the soldiers who participated (Num 31:27). If there were only 110 soldiers, then each of them received more than 140 sex slaves, a Solomonic proportion. Conversely, if there were 12,000 troops, then each soldier took one or two virgin women, and their shepherding duties would have been within the realm of plausibility. The tiny-*elep* model makes the story strange in a way the author likely did not intend.

The plagues on Israel also demonstrate the numbers must be larger than Humphreys' estimated population of 20,000. For example, Num 25:9 says that 24 *elep* were killed by the plague. Here an *elep* cannot be either a military or priestly unit, since this is about how many Israelites in general were killed by the plague. There is also no plausibility that *elep* means clan or family, as this would suggest the plague killed 24 families entirely and all the others lost not a single member. In addition, an earlier plague killed 14,700 Israelites (Num 16:49), and similarly none of the arguments about other meanings of *elep* can consistently apply here. If it were 14 families of 700 people, then, applying that reading to Num 25:9, the 24 families killed had zero people; if this were 14 families and an additional 700 people, then this reading is contrary to how the same was done for the military and priestly units. When every hypothetical reading of *elep* must be exceptional, then the hypothesis is not informative. Instead, when reading the text straight-forwardly, the plagues are killing more than Humphrey's estimated population.

Clearly the writer(s) or redactor of Numbers interpreted the *elep* in the census as 1,000. Suggesting a misreading of one *elep* for 'military units' and another *elep* for 'thousands' early on would not explain the shekel count, nor the counts from battles against the Midianites, nor the plague deaths, nor does it help with the mathematical problems identified above. Whatever one thinks of the numbers, they cannot be minimized in a mathematically sound way, nor can it be done without doing violence to the narrative. As the hypothetical reading of *elep* as "military unit" or "contingent" also introduces grammatical anomalies, while reading *elep* as 1,000 is grammatically sound and mathematically consistent, all the evidence points away from the hypothesis.

Origins of the 600,000

As noted earlier, the P-sourced size of the tribes of Israel as all around 50,000 fighting men was plausibly derivate of the approximate 600,000 men that left Egypt according to the non-P source. This leaves the question as to how J/non-P derived a number like 600,000. One plausible reason is that J is writing after the creation of the Deuteronomistic History. While Deuteronomy has no census of Exodus-era soldiers, the Book of Joshua begins to indicate the size of Israel's fighting force during the conquest, as Joshua had around 40,000 soldiers laying siege to Jericho (Josh 4:13).[32]

[32] Liane M. Feldman, *The Consuming Fire*, 22–23, 241 suggests this verse is from P, but the only evidence is the mention of Jericho and the claim that only P mentions Jericho (which is contrary to most proponents of the Documentary Hypothesis; i.e., Deut 34:3 is non-P). If the Deuteronomistic history is older, mentions Jericho, and then influences P, then the same evidence is explained just as well, so Feldman's argument is

Even larger numbers are found in Judges 20 as 400,000 soldiers were arrayed for battle against the Benjaminites. Later, Saul gathered 330,000 men to his cause (1 Sam 11:8), and then David's own census found 800,000 fighting men from Israel and another 500,000 from Judah (2 Sam 24:9). Later still, when Rehoboam was planning to attack Israel, the tribes of Judah and Benjamin alone had 180,000 capable fighting men (1 Kgs 12:21).

A reader of these books would assume that the population of ancient Israel was huge, much like the Chronicler who put the size of David's army at over one million (a thousand thousand; 1 Chron 21:5). Perhaps then it was almost a requirement that a similarly large number left Egypt in the first place so that there could be these large armies to fight for Joshua, Saul, and David. A magnitude of tens of thousands would be too small to explain how there were enough Israelites to become the hundreds of thousands of soldiers that fought in Judges 20.

An additional indication of the size of the fleeing Israelite population could be gleaned from the size of the army Joshua used to seize Jericho. Only three of the twelve tribes (Reuben, Gad, and Manasseh) were involved when the siege was first described (Josh 4:12), which may suggest a quarter of Israel's fighting force was prepared for war. If a reader looked at this situation as similar to the story of Gideon (Judg 7:3–8), who started with 32,000 men, then cutting down by about a third to 10,000, and then cutting down by about a third again to 300, and he further divided his armies into thirds (Judg 7:16), then the same logic of a third of a third could be applied to Israel's fighting forces in Joshua. If only a quarter of the available tribes went to Jericho, and if only a quarter of each of those tribes participated, then the approximate 40,000 were a quarter of a quarter of all the fighting forces, which would be about 4×4×40,000 = 640,000. The approximate 160,000 that Reuben, Gad, and Manasseh had, given this hypothesis (4×40,000), is close to the total of soldiers given in Numbers 26 (136,930), so at least P may have made this deduction.

A value of about 600,000 may have been desirable to J/non-P if he already deduced a value of close to that from reading Joshua 4, but this rounded value would also have had a nice mathematical feature. As a number easily divisible by 12, this would give a round number of soldiers to each of the tribes. This would not have been true for 500,000 or 700,000, and the easy division of 600,000 by 12 seems to have been utilized by P.

While 600,000 can be represented in a sexagesimal number system (2:46:40:0), such as that of the Babylonians, there are no strong indications that the numerical values in the censuses were based on such a system, let alone complex numerical values for planetary cycles has had been suggested previously.[33] Furthermore, the distribution of census values is suggestive of there being no meaning to the particular numbers given to the tribes. As seen in Figure 5, the distribution of values closely follows a normal distribution (also known as a bell curve). A normal distribution is expected if there is one source for a measurement, along with random variation around the mean value of that distribution. Tests suggest that the distribution is surprisingly close to a well-shaped normal distribution;

not persuasive. Additionally, the approximate number of soldiers mentioned in Josh 4 clashes with the exacting numbers from P.

[33] For discussion, see Davies, "A Mathematical Conundrum", 457–458.

the main test for normality (the Shapiro-Wilk test) confirms this (the null hypothesis is consistent with the data, $p > 0.95$).

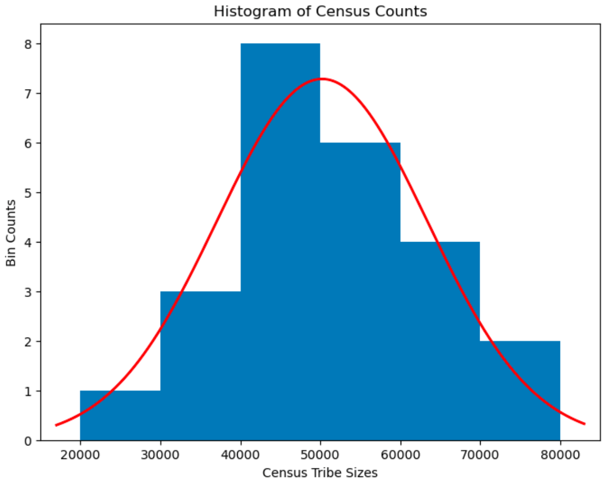

Figure 5. Histogram of Numbers 1 and 26 Censuses. A fitted normal distribution curve (red) is added to the bin counts.

The fact that the distribution is not multimodal (it has one peak) is suggestive that there is only one underlying cause for the values of the tribal populations. With a mean value of about 50,000 and more than half of the counts between 40,000 and 60,000, this distribution implies that the underlying cause is having tribes with about 50,000 soldiers with a bit of adjustment up or down (the standard deviation is about 13,000). If there were other causes for the particular numbers in each of the tribes, then such a clean distribution would have been unlikely. Values based on gematria,[34] for example, would not so nicely conform to a normal distribution, as each value would have a different cause, and this would have introduced additional modes (or "bumps") in the overall distribution. Instead, the deviations around the mean value are consistent with randomness around a central value.

The existence of a central value in a normal distribution is also more expected if P is using J/non-P rather than non-P using P and rounding P's numbers. If each tribe's size were randomly guessed by P, there is no reason to expect such a distribution (a uniform distribution seems most likely), let alone most tribes having sizes of nearly 50,000. The abundance of 4s and 5s in the first digit is also expected on the division of 600,000 by 12, while random generating numbers would be closer to Benford's law (discussed above). Instead, this is more expected if P had some total from which it started and then divided that value up nearly evenly between all the tribes. P is also not likely deriving his numbers directly from D, since the logic of getting a total of 600,000 from 40,000 at Jericho required rounding 40,000×4×4 down. If P directly used 40,000×4×4, then the census would have had a total closer to 640,000, and the average tribe would have had 53,000 fighting men. Instead, P is using the rounded-down value found in non-P, and P would not likely have rounded down, given the propensity for more exact numbers compared to D and non-P. This provides some evidence for P's knowledge of non-P.

[34] Davies, "A Mathematical Conundrum", 452–453.

Additionally, there is not a consistent size of the tribes between Numbers 1 and 26. Of the six tribes that had less than 50,000 soldiers in the first census, barely more than half (four of the six) continued to be so small; similar results exist for the tribes originally larger than 50,000. Also, barely more than half of the tribes (7 of 12) had their numbers increase, yet the total number of Israelites decreased, as if there were no trend but random occurrences of tribes getting larger or smaller. The changes in tribe sizes appear to be random and symmetric with a mean change close to zero. There is also only a weak and non-statistically significant correlation between the early and late census counts ($R = 0.45$, $p = 0.14$). As both sets of census numbers have an average of 50,000 soldiers per tribe, the counts in either census were largely independently generated from dividing up the 600,000 into twelve tribes. At most, a couple of values were made particularly large or small, but a larger project of making some tribes larger or smaller based on a criterion cannot be supported. Looking for meaning in exact values is unlikely to be fruitful.[35]

Of the sizes that may be meaningful, there are at most three examples that have some evidence. One is that Judah was always recorded as the largest tribe, and Judah was likely the tribe of special importance to the Priestly writer.[36] Second is the massive drop in Simeon, from 59,300 to 22,200. This may relate to the P-plague story in Numbers 25; Zimri of Simeon was the only person mentioned from a particular tribe who was killed because he was with a Midianite woman (Num 25:14).[37] But the motivation for the small size of Simeon, as well as Levi, may go back to Gen 49:5–7 wherein Jacob curses these two tribes who would be scattered. This could be another example of J/non-P influencing the numbers in P, and this would explain why Simeon and Levi had approximately the same counts in the censuses (roughly 22,000). Third, Benjamin is the smallest tribe in Num 1 (accounting for Ephraim and Manasseh as half-tribes from Joseph), and this may reflect 1 Sam 9:21 where Saul says that Benjamin was the smallest of all the tribes. However, Benjamin's numbers grew significantly between Numbers 1 and 26, so the random adjustments to the tribal sizes in Numbers 26 created an inconsistency in the final results. The case for Benjamin's initial size being made small is therefore only tentative.

Conclusions

The analysis performed above suggests both that the data in the censuses in Numbers are not likely to be historical or based on a pre-CBH relic, and instead they appear to be artificially generated, especially by P. The efforts to avoid the very large numbers in the Numbers censuses by finding another definition for *elep* were found to be wanting, either rejected on statistical grounds or refuted by simple arithmetic considerations.

Some evidence was found for the census numbers as a development between Pentateuchal sources. The P source was most likely working from a starting number from J/non-P, who was influenced by

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[35] Statistical inferences from the Levite numbers are even less likely to be fruitful, because there are so few data points.

[36] Davies, "A Mathematical Conundrum", 466 n. 44.

[37] Humphreys, "Number of People", 208–209.

the numbers in the Deuteronomistic History. The trend of D influencing J, then influencing P, follows trends others have found with the relationship between the Pentateuchal sources, such as the anti-Egyptian plague traditions.[38] This assessment adds to the growing consensus that P knew and used non-P.[39] However, P was not likely giving particular values for the size of tribal armies for some deeper meaning. Instead, P was producing counts around a set mean value of 50,000 and generally had random changes to that mean value.

[38] J. Van Seters, “A Contest of Magicians?”, 569–580.

[39] Boorer, *The Vision of the Priestly Narrative*, 90–110.

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Table 1. Counts of *elep* and Additional Soldiers per Tribe.

	Numbers 1		Numbers 26	
Tribe	***elep***	**+Soldiers**	***elep***	**+Soldiers**
Reuben	46	500	43	730
Simeon	59	300	22	200
Gad	45	650	40	500
Judah	74	600	76	500
Issachar	54	400	64	300
Zebulun	57	400	60	500
Ephraim	40	500	32	500
Manasseh	32	200	52	700
Benjamin	35	400	45	600
Dan	62	700	64	400
Asher	41	500	53	400
Naphtali	53	400	45	400
TOTAL	598	5550	596	5730
Stated Total	603	550	601	730

Table 2. Correlations and significance tests for *elep* and soldier counts. Numbers 1 and 26 censuses are treated individually, combined (1 & 26), and changes between the censuses (26 - 1). Statistically significant results ($p < 0.10$) have an asterisk (*).

Numbers Chapter	***R*** **Pearson**	p_R	y_0	**R ($y_0 = 0$) Pearson**	**ρ Spearman**	p_ρ
1	0.40	0.20	230	0.11	0.30	0.34
26	0.10	0.75	426*	*NaN*	-0.10	0.76
1 & 26	0.23	0.28	346*	*NaN*	0.08	0.70
$26 - 1_{\text{counts}}$	0.39	0.22	16	0.38	0.27	0.40
$26 - 1_{\text{prop.}}$	0.64*	0.02	0.13	0.62	0.34	0.27
$26 - 1_{\text{prop.+clean}}$	0.28	0.41	-0.03	0.29	0.15	0.66