On variations of mean temperatures of the air.

Conclusive remarks.

By J. Egedal.

In connection with the discussion on the variations of the mean temperatures of the air, to which a paper by C. Benedicks[1]) gave rise, and to which Dr. A. Ångström[2]) and more recently Mr. G. H. Liljequist[3]) have made their contributions I would like to make the following conclusive remarks.

The material used.

Some part of the mentioned discussion is based on data taken from a table in C. Benedicks' paper which was erroneously supposed to contain the mean temperatures for Stockholm and Berlin for the winters 1767—1920. According to A. Ångström[4]), however, the mentioned values do not represent the means of the daily mean temperatures, but are sums of the negative daily means for Berlin only. I regret to have used these values in the discussion.

In G. H. Liljequist's paper are given the mean temperatures for Stockholm for the winters 1757—1942[5]). It must be admitted that the data are compiled with great care, but it should be borne in mind that, in the 18th century, measurements of air temperatures were taken with thermometers of a quality inferior to those used

1) C. Benedicks: Influence of lunar periodicity on climate according to O. Pettersson, Ark. f. Mat., Astron. och Fys., Bd. 27 A, No. 7, Stockholm 1940.

2) A. Ångström: Principiella synpunkter på undersökningar över klimatets förändringar. Med tillämpning på det svenska klimatet. Geogr. Annaler, Vol. XXIII, 1941, p. 276—292. Remarks on variations of mean temperatures of the air in north-western Europe. Geogr. Annaler, Vol. XXIV, 1942, p. 274—278.

3) G. H. Liljequist: The severity of the winters at Stockholm 1757—1942. Geogr. Annaler, Vol. XXV, 1943, p. 81—104.

4) Geogr. Annaler, Vol. XXIV, 1942, p. 277.

5) Winter 1757, f inst., means Dec. 1756—March 1757.

nowadays, and, also, that at that time less care was often exhibited in placing the thermometers in the most suitable way.

Discussion.

With regard to the question of the reality of *periodic variations* of the mean temperatures of the air with periods of several years, for instance for north-western Europe, it seems to be generally agreed that no periodic variation, perhaps with the exception of the sunspot period only, has so far been determined with certainty.

A more important question is that of the *non-periodic variation* of the mean temperatures. It has been established, in respect of north-western Europe, that the mean temperature has been rising during the period from the 18th century and up to now, and that this rise is most pronounced for the winter season, for Stockholm, f. inst., the rise for the winter season amounts to 2° C. In nature there are many different indications of this increase of the mean temperature.

The non-periodic variations have been the main subject of the present discussion and especially the question, whether the established change of the mean temperature was to be regarded as a real change of the climate[6]), or whether the change should simply be attributed to a particular distribution of cold and warm periods.

There is a marked difference between the two views mentioned. If the former view is correct, it should under certain circumstances be permissible to draw conclusions with regard to future temperatures. If the latter view is correct, no predictions with regard to future non-periodic variations of the temperature can be made. If, f. inst., the rise of the mean temperature in Stockholm for the winter season has been going on smoothly from 1757 and up to now, and, further, if it is possible to lay down that a change has taken place, it should be permissible from the smoothness of the change to draw the conclusion that for the ensuing years the mean temperature in Stockholm for the winter season would be high. If such a forecast proves correct, it may be of immense value, but on the other hand if the forecast has been made without sufficient base and proves to be wrong, severe losses might result.

For the purpose of examining which of the two views is the more probable one, the following method should be used. In order to lay down that a real change of the mean temperature has taken place,

[6]) This expression may — as far as the temperature is concerned — mean that a change in the mean temperatures for longer periods, say 30 years, can be established with certainty.

it must be proved that the change itself can with sufficient certainty be determined to be different from 0. In order to state, whether the change should be attributed to a particular distribution of cold and warm periods, it must be proved that a distribution like the one found is probable if cold and warm periods are distributed at random.

If the change of the mean temperature is determined by computing the difference between the mean temperature for two separate periods, it should be remembered that also in cases where the mean temperature remains constant, it will be possible to obtain a considerable change, f. inst., when the greatest negative and the greatest positive deviations from a common mean are considered. In this case the real deviation from the common mean will be roughly half the total change. As a numerical example hereof, mean temperatures for the winter season in Stockholm may be used. According to Liljequist the mean temperature in Stockholm for the period 1761—90 is —3.°46 C, for 1911—40 —1.°53 C which makes a total change of 1.°93 C; the common mean is —2.°63 C so that the greatest deviation from this value is +1.°10 C. If the difference between the means considered is 6 times the probable deviation for the difference between two means or $6\sqrt{2}$ times the probable deviation for a single mean, the deviation of one of the considered means from the common mean would be about $3\sqrt{2}$ times the probable deviation or about 2.8 times the standard deviation.

It should also be remembered that the distribution of the deviations does not follow the Gaussian law so that conclusions drawn on the assumption of the existence of such distribution will to some extent be incorrect. This question will be considered once more below.

In the following the mean temperature in Stockholm for the winter season (Dec.—March) as given in Liljequist's paper will be considered in order to examine whether or not the non-periodic variation found may be due to a particular distribution of the deviations of the mean temperatures.

Mild winters.

From Liljequist's Table X the mean value of the mean temperatures for the three mildest winters in Stockholm in each of the 30-year periods from 1761 to 1940 has been computed, and the result is given in the table below.

Table 1. — Mean temperatures of the three mildest winters in Stockholm in 30-year periods from 1761 to 1940.

Period	Mean	
1761—1790	+0.°57 C	
1791—1820	+0.°23 C	1761—1850: Mean: +0.°57 C
1821—1850	+0.°90 C	
1851—1880	—0.°03 C	
1881—1910	+0.°37 C	1851—1940: Mean: +0.°35 C
1911—1940	+0.°70 C	

From Table 1 it will be seen that the means do not show any signs of an increase, the mean for the period 1851—1940 is even a little lower than that for the period 1761—1850. If the above mentioned change of the temperature of 1.°93 C should be the consequence of a change in the climate, it may be expected that also the upper limit of the mean temperatures would show a change similar to that of the found change of the mean temperature. Therefore, the found change (1.°93 C) must be due to a particular distribution of the deviations from a common mean.

Severe winters.

Compiling from Liljequist's Table X the temperatures of the three coldest winters for each of the 30-year periods and computing the means thereof, the result, given in Table 2, is obtained.

Table 2. — Mean temperatures of the three coldest winters in Stockholm in 30-year periods from 1761—1940.

Period	Mean	
1761—1790	—6.°73 C	
1791—1820	—6.°87 C	1761—1850: Mean: —6.°46 C
1821—1850	—5.°77 C	
1851—1880	—5.°67 C	
1881—1910	—5.°77 C	1851—1940: Mean: —5.°50 C
1911—1940	—5.°07 C	

From Table 2 it will be seen that there is an increase in the values, thus, the mean value for the period 1761—1850 is nearly 1° C lower than that for the period 1851—1940. This is in conformity with the opinion generally agreed upon that the non-periodic variation in the mean temperatures takes place during the winter season, however, so far there is no actual basis for drawing conclusions as to the problem in question.

As the non-periodic variation to some extent is due to the distribution of the coldest winters, it must be stated that it is probable that the found distribution of the coldest winters may occur even if their distribution is a chance distribution.

In the following a winter in Stockholm is indicated as severe, when its mean temperature is equal to or below —5.°0 C. In the table below are given the numbers of severe winters in Stockholm for the 30-year periods from 1761 to 1940 according to Liljequist's Table V.

Table 3. — Numbers of severe winters in Stockholm in 30-year periods from 1761 to 1940.

Periods	1761—1790,	1791—1820,	1821—1850,	1851—1880,	1881—1910,	1911—1940
Numbers	8	8	2	4	3	1

During 180 years 26 severe winters have occurred, that is $4^1/_3$ severe winters in a 30-year period.

On the assumption that $4^1/_3$ severe winters normally occur in a 30-year period and that the distribution of severe winters is a chance distribution, the probability of having different numbers of severe winters in a 30-year period is computed. The table below gives the result.

Table 4. — Probability of severe winters in Stockholm within periods of 30 years (chance distribution).

Number af severe winters	Probability	Number af severe winters	Probability
0	0.0093	7	0.0739
1	0.0470	8	0.0358
2	0.1150	9	0.0148
3	0.1813	10	0.0052
4	0.2066	11	0.0016
5	0.1814	12	0.0004
6	0.1276	13	0.0001

The probability of having the numbers of severe winters in 30-year periods as given in Table 3 are found by means of Table 4 and compiled in Table 5.

Table 5. — Probability of having the numbers of severe winters in 30-year periods as given in Table 3.

Period	Number of severe winters	Probability
1761—1790	8	0.0358
1791—1820	8	0.0358
1821 – 1850	2	0.1150
1851—1880	4	0.2066
1881—1910	3	0.1813
1911—1940	1	0.0470

The probability-figures given in Table 5 are so high that it is permissible to conclude that the found numbers of severe winters may occur also, when the distribution of severe winters is a chance distribution, so that it will not be necessary to assume any change in the climate or to make any other assumptions.

The distribution of mean temperatures for the different winters.

G. H. Liljequist has examined the frequency of winters of different severity and for this purpose used the Gaussian distribution curve, though, as he says, "the distribution is not in general symmetrical". Another difficulty would arise if the values used — as Liljequist seems to suppose — are steadily increasing with time.

It is however possible by means of values from Liljequist's Table V to examine the question concerning the distribution of the values by another method.

From the mentioned Table V the numbers of winters with mean temperatures $\leqq -4.^\circ0$ C; $> -4.^\circ0$ C $\leqq -1.^\circ0$ C and $> -1.^\circ0$ C are compiled and given in Table 6 below. From the sums in the last column it will be seen that the number of values below —4.°0 C and above —1.°0 C are nearly equal and added together nearly equal to half the total number. Thus, the probability of having a value below —4.°0 C or above —1°0 C is nearly ½. It should be added that the number of values lying between —4.°0 C and —1.°0 C is fairly constant for all periods. Using the above probability, ½, it will, assuming the distribution of the values to be a chance distribution, be possible to determine the probability of distributions of severe and mild winters like those found. The found probability are given in Table 6.

Table 6. — Numbers of winters with different mean temperatures (Stockholm) and the probability of their distribution (chance distribution).

	1761-1790	1791-1820	1821-1850	1851-1880	1881-1910	1911-1940	Sums
$\leq -4.^\circ 0$ C	13	10	9	9	3	3	47
$> -4.^\circ 0$ C $\leq -1.^\circ 0$ C	13	16	15	15	17	13	89
$> -1.^\circ 0$ C	4	4	6	6	10	14	44
Probability	0.0182	0.0611	0.153	0.153	0.0349	0.00519	

From Table 6 it will be seen that only the probability for the period 1911—1940 is rather low, but the highest value is not more than 30 times as high as the one considered. Thus, the found distribution of mild and severe winters may occur, when it is the question of a chance distribution of the values, and there is no cogent reason to assume a change in the climate or to make any other assumptions.

In the years before 1940 the mean temperature in Stockholm for the winter season was rather high, but the three severe winters 1940, 1941, and 1942 changed the state considerably and underlined the unstable character of the non-periodic variation.

Postscript.

H. Lundgren, M. A., has examined the degree of certainty with which it has been possible to determine the yearly increase of the winter temperature in Stockholm for the period 1757—1942, supposing this rise to be constant during the whole period, and he has also examined the deviations of the different values from values computed in accordance with the above assumption in order to test the distribution of the deviations. The results, given with his permission, are as follows:

The yearly increase was found to be:

$$+0.^{\circ}0098 \pm 0.^{\circ}0027 \text{ C}$$

where the latter quantity is the standard deviation. The standard deviation is so small in relation to the value of the rise that only once out of 6000 cases one may expect on the base of the treated values to have a fall of the temperature. The examination of the distribution of the deviations from the computed constantly rising temperature was carried out by means of the so-called χ^2 test which showed that there was no reason to suppose that the diviations did not represent a Gaussian distribution. Accordingly the rise found must be considered to be real and not the result of a chance distribution.

It should be added that leaving the values for the years 1940, 1941, and 1942 out of consideration, the following yearly increase of the winter temperature in Stockholm was found:

$$+0.^{\circ}0123 \pm 0.^{\circ}00265 \text{ C}$$

which, in relation to the above result, gives a greater value for the yearly increase but nearly the same standard deviation.

In order to examine the rise of the winter temperature in another place the mean values of the temperature for Spitsbergen[1]) for the months November till March from 1912[2]) to 1938 have been treated

[1]) A. Wagner: Klimaänderungen und Klimaschwankungen. Die Wissenschaft, Bd. 92, p. 15—16, Braunschweig 1940.

[2]) 1912, f. inst., means Nov. 1911 till March 1912.

in the usual way. The deviations of the mean temperature from their common mean are given in Table 7.

Table 7. — Deviations of the mean temperatures Nov.—March from their common mean. Spitsbergen 1912—1938. Centigrade.

	0	1	2	3	4	5	6	7	8	9
1910			(—5.9)	—2.1	—3.5	—3.5	—7.3	—10.2	—5.8	—1.0
1920	+0.1	—0.2	+2.2	+0.6	+2.0	+3.2	—1.5	—0.1	—0.5	—2.3
1930	+0.6	+3.4	+3.5	+3.7	+4.4	+4,4	+3.9	+5.4	+5.9	

From the values of Table 7 the yearly increase of the temperature has been determined, and the following result was obtained:

$$+0.^{\circ}42 \pm 0.^{\circ}06 \text{ C}$$

where the latter quantity is the standard deviation.

From the magnitude of the standard deviation in relation to the determined yearly increase it is found that only once out of 1.3 billion cases one may expect on the base of the treated values to have a fall of the temperature. The total rise of the winter temperature during the considered period of 26 years from 1912 to 1938 amounts to 11° C, and the rise is taking place in the same period during which the winter temperature in Stockholm remains at a rather high value, about 1° C above the mean value for the period 1757—1942. In addition the values of Table 7 show variations from year to year which are remarkably small.

By computing means of the deviations of winter temperature for Spitsbergen for 8 years we get for the period 1912—1919:

$$-4.^{\circ}9 \pm 1.^{\circ}1 \text{ C}$$

and for the period 1931—1938:

$$+4.^{\circ}3 \pm 0.^{\circ}3 \text{ C}$$

The mean values for the considered two periods are deviations from the mean winter temperature. In accordance with the great rise of the temperature the means are rather different, and especially the latter value shows with almost complete certainty that a change has taken place.

Summing up how many times two consecutive deviations have the same sign (permanence) and how many times two consecutive values have opposite signs (variation), it is found that there are 19 permanences and 7 variations in the data from Spitsbergen. The facts that the permanences exceed the variations considerably and that no periodic variations of the mean temperatures in the considered regions have been discovered, make it obvious that each value depends on preceding values, and in this case the theoretical base for the application of the Gaussian method by the treatment of the values *does not exist*.

An examination of the data from Stockholm shows that they contain 97 permanences and 88 variations, which is only a very slight indication of dependence between the values.

However, the Gaussian hypothesis saying that the many small errors (elementary errors) to which the total error is due, must show a chance distribution, does not hold good in the case of the temperature of the air, f. inst. during stable weather conditions the deviations from the mean value of daily means of the temperature may have the same sign during a long period. The explanation of the found great yearly increase and of the great well-determined deviation of the mean winter temperature in Spitsbergen must be that it is *not permissible* to use the ordinary methods for the treatment of the values. Changes in the position of the edge of the arctic ice may possibly explain the observed variation of the temperature, Spitsbergen having a more continental climate in certain periods than in others.

Certainly, the whole question needs a much closer examination, and it is obvious that the old methods must be used with caution for stations in higher geographical latitudes in cases where small variations of the temperature of the air are examined. When means of periods containing more "swing"s are considered, it would be possible to use the old methods, but in this case the periods used, f. inst. for Stockholm, should be at least 200 years long.

In conclusion I wish to thank H. Lundgren for his contribution to the present "Postscript" and for his valuable suggestions which have made the addition of a postscript desirable.