A Simple Model for Current Speed in Tidal Channels

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A simple model for current speed in tidal channels is presented and tested in a tidal channel in the Danish Wadden Sea. The model is based on the continuity equation with water level as the only time dependant input parameter. The model can be used as a tool for extending and/or completing current speed time series in tidal channels. In four test periods the model proved to be able to explain over 95% of the current speed variation during periods with "normal" wind conditions, and 91% of the variation during a tested storm surge period.

Keywords: *Current speed, continuity equation, tidal channels, the Danish Wadden Sea.*

A precise description of the current speed variation in tidal areas is of great importance for a complete evaluation of a whole variety of environmental issues. Attempts to construct comprehensive models for current velocities in tidal areas are therefore numerous. Most of these are based on finite difference solutions of the basic hydraulic differential equations in a two-dimensional frame work. These type of models are often referred to as the Leendertse-type (Leendertse, 1970). When used in areas with a complicated topography as the Wadden Sea, these models require relatively large computers and are not seldom rather difficult to calibrate. The theory, on which these hydrodynamical models are based, is described in several textbooks, for instance McDowell & O'Connor (1977).

The purpose of this paper is to present a simple non-distributed, PC-based model describing current speed in a tidal channel based on the continuity equation with tide gauge water level as the only time dependent input parameter. This type of model can be used to extend and/or complete time series of current speed in a particular tidal channel and to make quick estimates of the current speed, with a minimum of information and CPU consumption.

Area of Study

The tidal channel for which the model is set up is situated in the southern part of the tidal area "Grådyb" in the Danish Wadden Sea, described by e.g. Bartholdy & Pheiffer Madsen (1985). The modeled cross section is situated immediately west of the harbour of Esbjerg (Fig. 1).

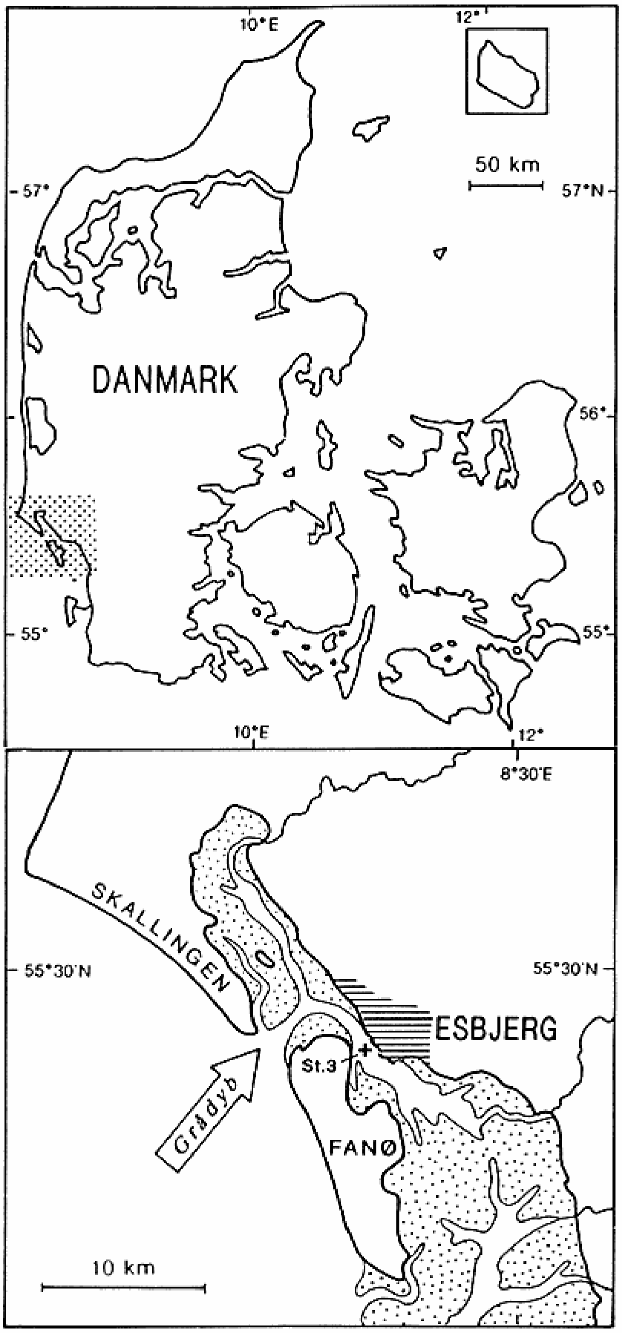

Figure 1: Map of the study area. The current speed was measured at St.3.

It has an approximate width of 800 m with an additional 200 m of low-lying tidal flats on both sides. The mean depth is approximately 10 m in the channel. It controls the filling and emptying of an area of approximately $4.1*10^7$ m^2 at 0 m above Danish reference level (DNN), which, with a mean tidal range of approximately 1.5 m, produces a mean tidal prism of approximately $60*10^6$ m^3 . This location was chosen because of the access to large data series of current data from the eastern central part of the cross section, recorded for the harbour authorities in Esbjerg during an environmental study (Bartholdy, 1993a&b and Bartholdy & Anthony, 1994).

Methods

Measurements

The current speed was measured with a General Oceanics Niskin winged current meter, model 6011 MkII, adjusted to record the mean current speed every 5 minutes based on 40 instantaneous measurements at intervals of 7.5 seconds. The instrument was mounted on a moored wire and kept in a fixed position of approximately 4 m above the bottom by means of buoyancy balls. This level is close to the mean speed level at 0.4*depth above the bottom. The recorded speed is thus assumed to be equal to the mean speed in the water column. Because of the varying water depth, this assumption is not strictly correct, although acceptable in this context. An estimate based on the logarithmic velocity distribution with a hydraulic roughness coefficient, $k = 0.1$ m, shows a variation within ± 3 % in the relevant depth interval of 9-13 m.

Water level and wind speed (15.5 m above the ground) were recorded at 15 minutes intervals by the harbour authorities in Esbjerg at a distance approximately 1 km from the current meter.

The water-covered area "inland" of the cross section is based on a hypsographic curve constructed on the basis of digitised navigation charts (Danish Hydraulic Institute, pers.com.).

Theory

In the following the measured current speed is compared with calculated current speed, by means of a transformation based on the Manning formula (Bartholdy, 1984) and the mid-section method. This makes it possible to calculate the mean speed for any position with known depth in a cross section by means of one known mean current speed and the corresponding depth.

The mid-section method is frequently used for water discharge calculations in irregular cross sections, the cross section being split up in several sections with each section having a unique water depth. The mid-section method states that:

$$Q=\sum_{i=1}^{n} W_i D_i V_i \qquad (1)$$

in which n is the number of the individual sections, W_i is the width of the individual sections, D_i is the water depth of the individual sections and V_i is the mean current speed of the individual sections. Q is water discharge.

The transformation of the Manning formula states that:

$$V_x=V_i\left(\frac{D_x}{D_i}\right)^{\frac{2}{3}} \qquad (2)$$

in which V_x is the mean current speed over depth, at the depth D_x , at any sub-section in the cross-section.

The combination of (1) and (2) is able to express the water discharge as:

$$Q=\sum_{i=1}^{n} W_i D_i V_x \left(\frac{D_i}{D_x}\right)^{\frac{2}{3}} \qquad (3)$$

Isolating V_x gives:

$$V_x=\frac{Q}{F} \qquad (4)$$

where:

$$F=\sum_{i=1}^{n} W_i D_i \left(\frac{D_i}{D_x}\right)^{\frac{2}{3}} \qquad (5)$$

If the principle of continuity is used to calculate the discharge, and it is assumed that in the water-covered area beyond the cross-section, the surface of the water is horizontal, the discharge can be calculated as:

$$Q = A\frac{\Delta h}{\Delta t} \qquad (6)$$

in which A is the water covered area beyond the cross section, Δh is the change in water level and Δt the change in time.

The water-covered area A depends on the water level and the hypsographic curve. Knowing the size of A, the current speed at a given location in the cross section, can be expressed as a function of the changing water level under the above-stated conditions using Equation 6 and Equation 4. In this way, the recorded current speed can be compared with the output from the model, provided that the location of the calculated current speed corresponds to the place in the cross section where the current speed is measured.

As the water level reflects the constantly changing conditions of the tidal wave, the assumption of a horizontal level beyond the cross section is not valid. The water level will experience an inclination regulated by flood and ebb, and it will not follow a straight line, but bend according to the changing conditions. This part of the problem can only be solved theoretically by a complete solution of the hydrodynamic equations. In order to incorporate some kind of consideration for this effect in the model, it is described empirically and used to calibrate the model.

This is done by replacing the area A from the hypsographic curve, with the area A', which, based on the recorded data, satisfies Equation 4 and 2. It was found that the difference between A and A' can be described by an empirical relation with water level velocity $\Delta h/\Delta t$ and water level acceleration $\Delta^2 h/\Delta t^2$ as independent parameters.

Results

Calibration

In order to calibrate the model with a minimum of disturbance from other factors than the tide, a reference period with low wind speeds was selected. It is a 8½-day period from October 27 to November 4, 1993 with an average wind speed of 2.9 ms^{-1} from NE, only exceeding 5 ms^{-1} in 1.5 % of the time.

The calculated values of A' for flood and ebb respectively are plotted against water level and compared with the hypsographic curve in Fig. 2. It is obvious that A and A' actually corresponds relatively well in the interval between -0.5 and 0, but also that A', apart from being dependant on

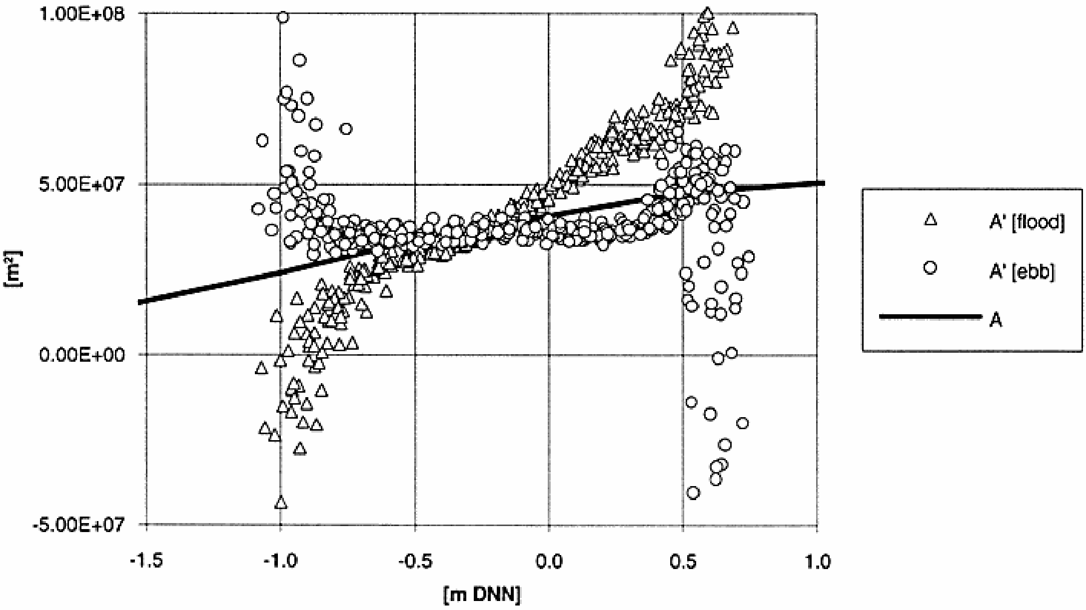

Figure 2: The tidal area "inland" of the selected cross section, based on the hypsographic curve, A, compared with the calculated values A', satisfying the model in the reference period. The deviation between A and A' is used to calibrate the current speed model.

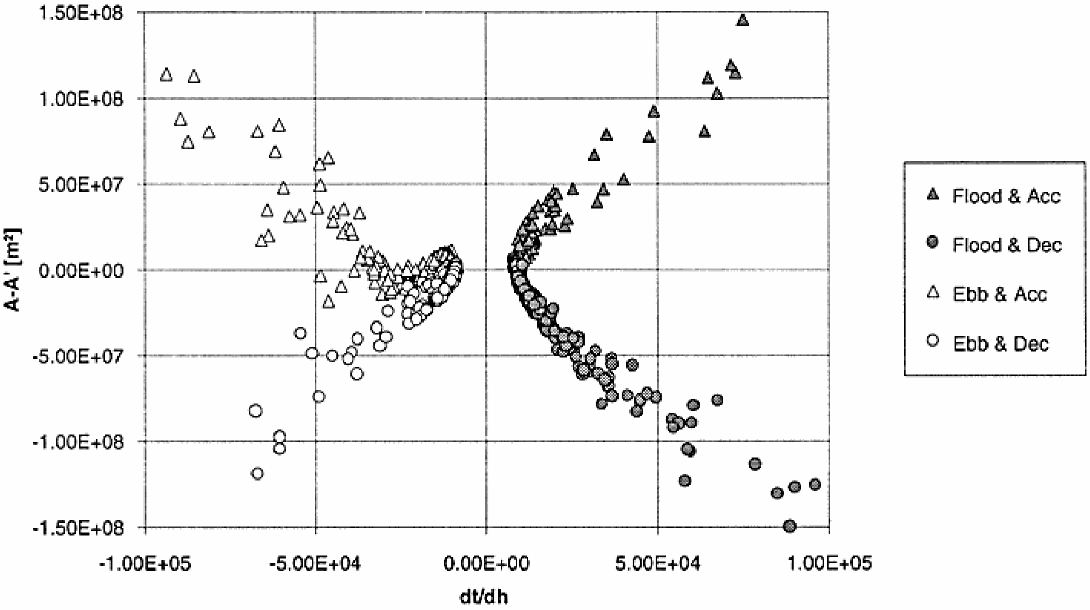

Figure 3: *The deviation between the size of the water-covered tidal area, A, based on the hypsographic curve, and calculated values of A', as a function of the reciprocal water level velocity* Δt/Δh *in the reference period.*

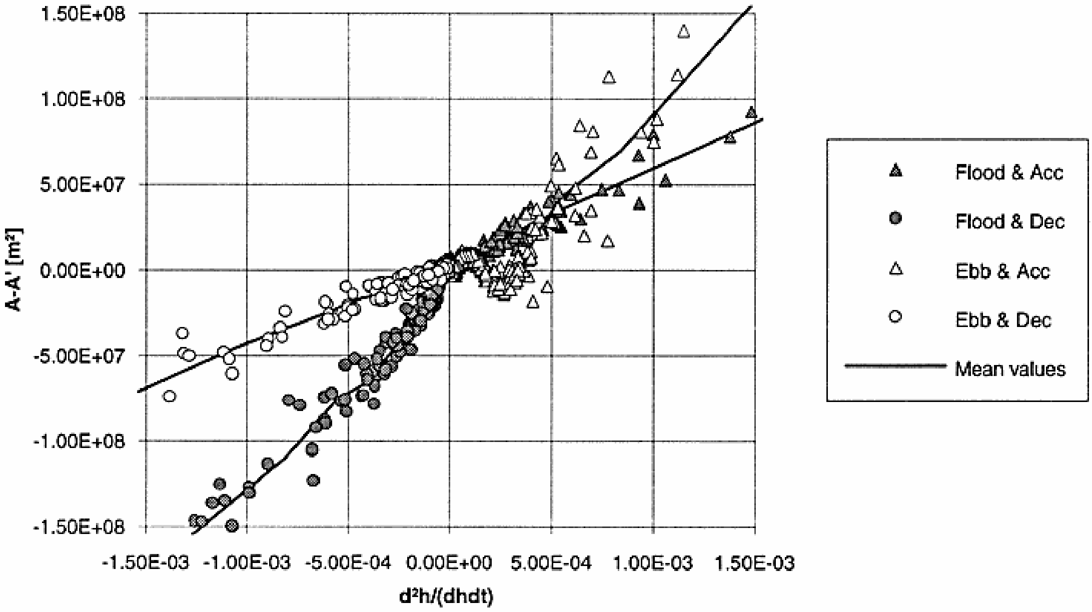

Figure 4: *The deviation between the size of the water-covered tidal area, A, based on the hypsographic curve, and the calculated values of A', as a function of the empirical term* $\Delta^2h/(\Delta h\Delta t)$ *in the reference period.*

the water level, depends on the current direction. As the divergence from the hypsographic curve is most prominent in the extreme parts of the plot, in which the acceleration values are greatest, this parameter most probably plays an important role as well.

If it is possible to express the divergence from the hypsographic curve (A-A') explicitly, A' can be found accordingly, independent of h. Plotting A-A' versus the reciprocal water level velocity term, $\Delta t/\Delta h$ (Fig. 3), an explicit relationship appears with a distinct discrimination between acceleration and deceleration. It is remarkable that small values of $\Delta t/\Delta h$ (rapid change in water level) actually produce the smallest deviation between A and A'. This demonstrates the influence of the acceleration of the water level, being most prominent when the change in water level is moderate. Thus, the size of the acceleration should be incorporated in the description. A multiplication of the acceleration term, $\Delta^2h/\Delta t^2$ with the reciprocal water level velocity term, $\Delta t/\Delta h$ giving the term, $\Delta^2h/(\Delta h\Delta t)$, therefore turned out to be relatively well correlated with $A\text{-}A'$ when distinguished between flood and ebb, as shown in Fig. 4.

Although this relation can be related to various interpretations, it is beyond the scope of this paper, and the term should be regarded solely as an empirical term. It should be noticed, however, that the acceleration term, being directly related as a multiplicator, has a very strong influence, which is clearly illustrated in the special case in which the acceleration term is 0 and A consequently is equal to A'. As the acceleration term is very sensitive to small fluctuations in the water level, this parameter is averaged (running mean) over 1½ hours for the reference period as well as for the test periods.

Instead of producing a mathematical function with a "best fit" through the data points in Figure 4, a reference table of the mean values of $A\text{-}A'$ as a function of $\Delta^2h/(\Delta h\Delta t)$, has been constructed for each of the four combinations of flood/ebb and acceleration/deceleration. The lines shown in Fig. 4 are based on these tabulated values.

Test of the Model

Four test periods (A, B, C & D), excluding the reference period, with different wind climates have been selected in order to test the model. Selecting the test periods, wind speed is used as a determining factor, because increasing wind speed disturbs the sea-level inclination, and thereby the models input parameter.

Period A (November 4 to November 11, 1993) is similar to the reference period, with a slightly higher mean wind speed. **Period B** (April 2 to April 12, 1994) is governed by variable wind conditions ranging from 0 to 16 ms^{-1} from almost all directions. The mean wind speed is 6.2 ms^{-1} and the mean wind direction is from NNW. **Period C** (March 28 to April 1, 1994) has a relatively high mean speed, with

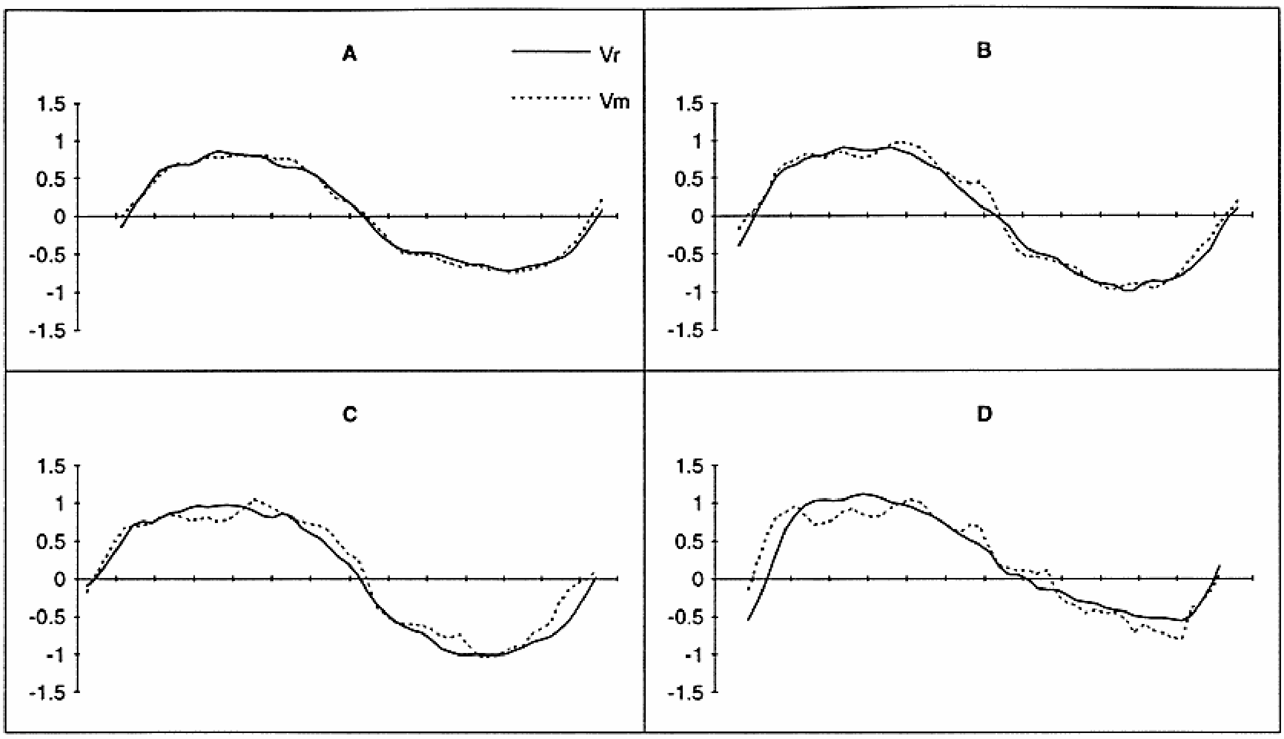

Figure 5: Modelled and recorded current speed (V_m and V_r) for a selected tidal period in each of the four test periods (A-D). The speed is in ms^{-1}, and the x-axis covers 14 hours. Positive speed is in the flood direction and negative speed is in the ebb direction.

Table 1: Wind data and determination coefficients for the four test periods. The periods are ranked according to mean wind speed for the whole period. The wind speed in brackets is the mean wind speed for the selected tidal periods shown in Figure 5. Notice that uniform wind conditions only occur in period A and B.

Period		Number of days (approx.)	Mean wind speed [ms^{-1}]	Wind direction [deg. North]	Determination coefficient, r^2, between V_r and V_m	Notes
A	Nov.93	6	4.4 [3.9]	ENE	0.98	Uniform wind
B	Apr. 94	11	6.2 [7.2]	NNW	0.95	Mixed wind
C	Mar. 94	4	8.9 [11.2]	SSV	0.95	Uniform wind
D	Jan. 94	1.5	13.9 [11.9]	W	0.91	Storm surge

the major part centred around the mean value of 8.9 ms^{-1}. The mean direction is also without major fluctuations primarily from SSW. **Period D** (January 30 to January 31, 1994) includes a minor storm surge reaching 2.36 m above DNN with wind speeds reaching 19.5 ms^{-1}. The mean wind speed was 13.9 ms^{-1} and the mean wind direction from W.

In Figure 5 (A-D) modeled and recorded current velocities (V_m and V_r) are compared during four selected time series, one from each of the test periods. As it appears there is a good agreement between the modeled and the recorded velocities. The model is able to describe the current speed variation quite well even during the storm surge, where water levels and wind conditions vary significantly from those of the calibration period. In order to evaluate the validity of the model in an objective way, Table 1 shows the determination coefficient (r^2) of the model for all the recorded data in the four test periods. Based on this material it is concluded that the model is able to explain over 95 % of the variation in the current speed under "normal" wind conditions, and 91 % of the variation during the tested storm surge period.

Summary and conclusion

This paper presents a simple non-distributed, PC-based model describing current speed in a tidal channel based on the continuity equation, with water level as the only time dependent input parameter. The model can be used as a tool for extending and/or completing time series of current speed in a particular tidal channel and to make quick estimates of current speed, with a minimum of information and CPU consumption. By means of a theoretical distribution of the mean current speed over depth, the continuity equation and the hypsographic curve for the tidal area beyond the cross section, the model transforms tide gauge information into current speed in the selected cross section.

The calibration of the model is accomplished by means of an empirical relation between the term, $\Delta^2 h/(\Delta h \Delta t)$ (in which h is the water level and t the time), and the deviation between the size of the tidal area beyond the cross section, found on the basis of the hypsographic curve, and the calculated ditto, based on the continuity equation and of the recorded current velocities in the cross section.

Used in the tidal channel forming the southern part of the Grådyb tidal area in the Danish Wadden Sea, the model proved to be able to explain over 95 % of the current speed variation during "normal" conditions and 91 % of the variation during a tested storm surge period.

Acknowledgement

This paper is based on data material from an environmental investigation carried out for the Harbour Authorities in Esbjerg in a cooperation between the Danish Hydraulic Institute (DHI), the Institute of Water Quality (VKI) and the Institute of Geography, University of Copenhagen. I thank Esbjerg Harbour and especially the crew on board the working vessel "Grådyb" for god seamanship and coorporation, and J. Jønsson for the drawing of the study area. Finally I am grateful to Jesper Bartholdy for valuable comments and advise.

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