

Central Places and Functional Regions in Denmark

Factor Analysis of Telephone Traffic

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Abstract

Factor analysis is applied to a matrix of inter-district telephone calls in Denmark. The resulting factors are interpreted as indicating regional centres and their influence zones. Factor analysis is compared to other methods of measuring the centralities of central places.

1. The data

The purpose of this paper is to discern places with a high degree of centrality in terms of telephone traffic with their surroundings, and to delimit the zones of influence of these central places.

The data available for this purpose were composed of the number of calls between each of the 62 districts, into which the telephone companies have divided the country. A district usually consists of a town and its hinterland. The inter-district telephone calls were counted in 12 sample days in 1961. In some cases where telephone traffic had become automatic in 1961, the number of calls had to be estimated. Thus the number of calls between districts in Sjælland was estimated on the base of 1958-data, and the number of calls between districts in Lolland-Falster and in Nord Slesvig had to be estimated through analogies to districts of similar sizes and similar distances apart in other parts of the country.

On the base of these data, a matrix was established including the number of telephone calls from each of the 62 districts (columns) to each of the 62 districts (rows). Intra-district calls were excluded, the elements of the matrix diagonal thus being zeros.

2. The factor analysis

Linear factor analysis was then applied to the telephone call matrix. The inherent hypothesis in the factor analysis is that the 62 (n) fields of origination of telephone calls, which are represented in the 62 (n) rows of the matrix, can be reproduced as linear functions of a smaller number (m) of fictive origination fields represented by the (m) factors of the factor analysis.

In matrix notation this means that we can reproduce the original 62 x 62 matrix (X) by a product of two matrices:

$$\begin{array}{cccccc}
 \text{from} & & & & & \\
 \text{to} & X_{11} X_{12} \dots X_{1N} & a_{11} a_{12} \dots a_{1m} & F_{11} F_{12} \dots F_{1N} & a_1 0 \dots 0 & U_{11} U_{12} \dots U_{1N} \\
 & X_{21} X_{22} \dots X_{2N} & a_{21} a_{22} \dots a_{2m} & F_{21} F_{22} \dots F_{2N} & 0 a_2 \dots 0 & U_{21} U_{22} \dots U_{2N} \\
 & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & X_{n1} X_{n2} \dots X_{nN} & a_{n1} a_{n2} \dots a_{nm} & F_{m1} F_{m2} \dots F_{mN} & 0 0 \dots a_n & U_{n1} U_{n2} \dots U_{nN} \\
 \\
 & (62 \times 62) & (62 \times m) & (m \times 62) & (62 \times 62) & (62 \times 62) \\
 & (n \times N) & (n \times m) & (m \times N) & (n \times n) & (n \times N)
 \end{array}$$

The m rows of the F-matrix represent the factor scores of the m common factors of the factor analysis. In the present case, districts showing a high score on factor no. 1 can be said to form the factor 1 origination field. The 62 (n) rows of the a_{ji} -matrix are the factor weights, which show by what weights the factors enter into the description of the 62 origination fields. In districts with high weights on factor 1, this factor explains a high proportion of the total variance. The squared factor weights are equal to those proportions of the total variance of the origination fields which are explained by the factors.

The 62 (n) rows of the U-matrix represent the unique factors which describe the parts of the origination fields not described by the common factors of the F-matrix. Each of the unique factors only influences one of the origination fields, and the coefficient matrix (a_j) therefore is a diagonal matrix.

For a single origination field the above matrix writes:

$$X_{ji} = a_{j1} F_{1i} + a_{j2} F_{2i} + \dots + a_{jm} F_{mi} + a_j U_{ji}$$

3. The results: Ten factors and seven regional centres

In the present analysis 10 common factors were extracted of the original matrix. The 10 common factors were extracted by means

of the principal factor method. This means that the first factor extracted explains as much as possible of the total variance, the second factor explains as much as possible of the remaining variance not explained by factor 1, etc. In the present case the first ten factors together explained 79 % of the total variance in the matrix. The contribution from each of the ten common factors appears from table 1.

Table 1

Factor no.	Proportion of total variance explained by the factor:
1	0.335
2	0.125
3	0.079
4	0.068
5	0.056
6	0.044
7	0.025
8	0.021
9	0.020
10	0.017
Total	0.790

Factor no. 1 (see figures 1-2) showed a very high score in the Copenhagen district and low positive or negative scores in all other districts. The factor thus represents the calls from Copenhagen to the rest of the country. In accordance with this interpretation, the factor had high factor weights in all districts east of Storebælt (except Copenhagen itself) and in the districts of the main regional centres of western Denmark: Århus, Odense, and Ålborg.

Factors 2, 3, and 4 (see figures 3-8) had high factor scores in the districts of Århus, Odense, and Ålborg, respectively. The factor weights of factor 2 were high in the districts of central-eastern Jylland (except in Århus), and in Ålborg and Copenhagen. For factor 3 the factor weights were high on Fyn (except in Odense), in the Jylland districts located nearest to Fyn, and in Copenhagen. For factor 4 the factor weights were high in northern Jylland (except in Ålborg), in Århus, and in Copenhagen. These three factors represent the telephone calls from Århus, Odense, and Ålborg, respectively.

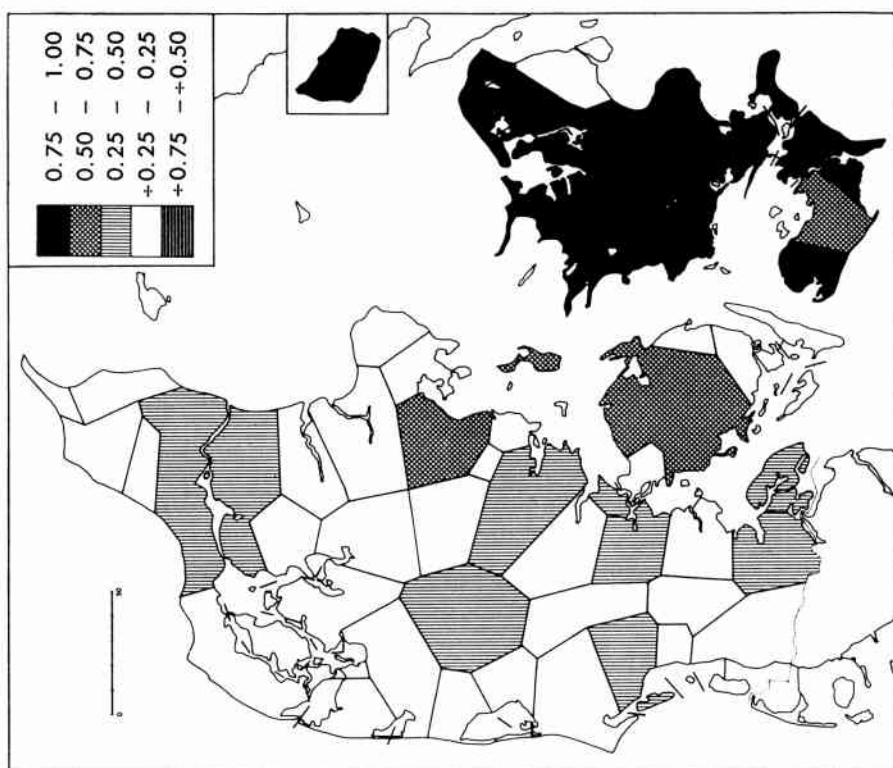


Fig. 2. Factor weights, factor 1.
Fig. 2. Faktorvægte, faktor 1.

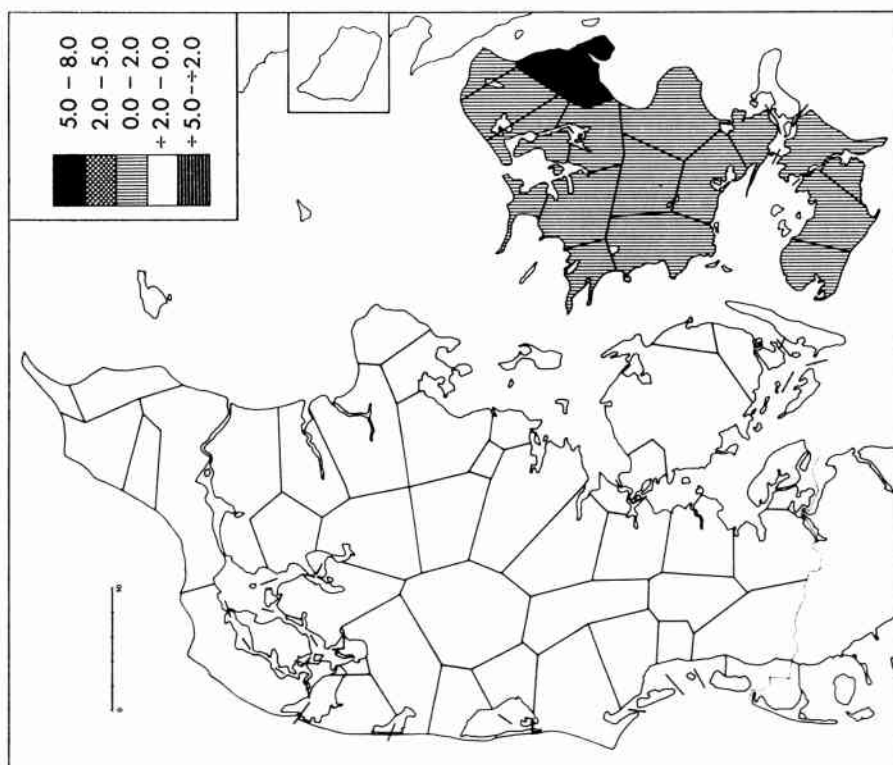


Fig. 1. Factor scores, factor 1.
Fig. 1. Faktorværdier, faktor 1.

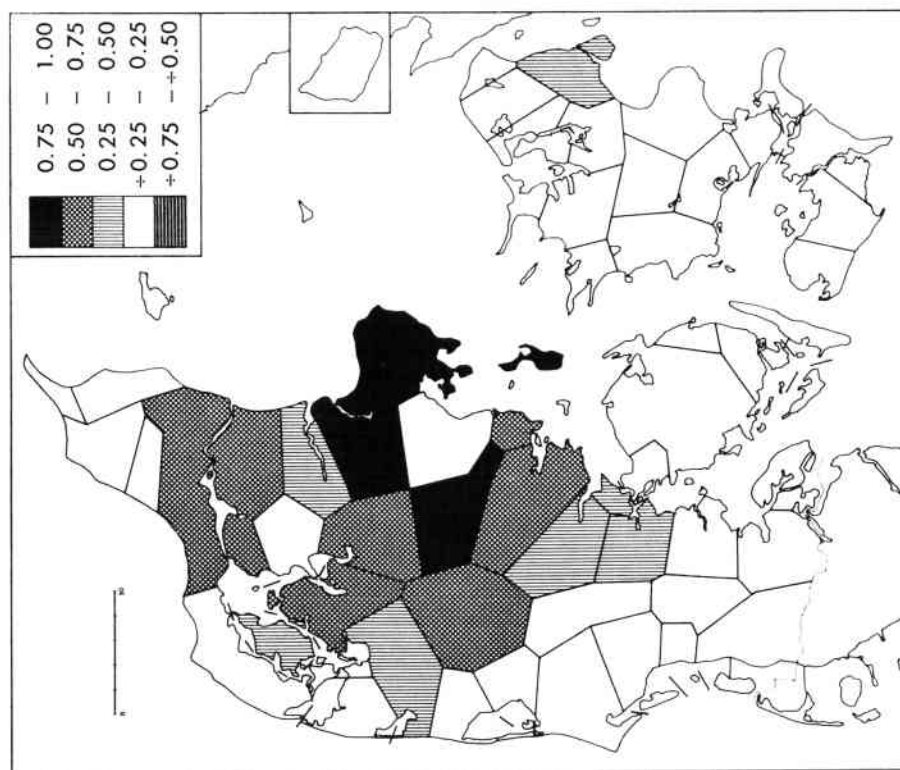


Fig. 4. Factor weights, factor 2.
Fig. 4. Faktornægte, faktor 2.

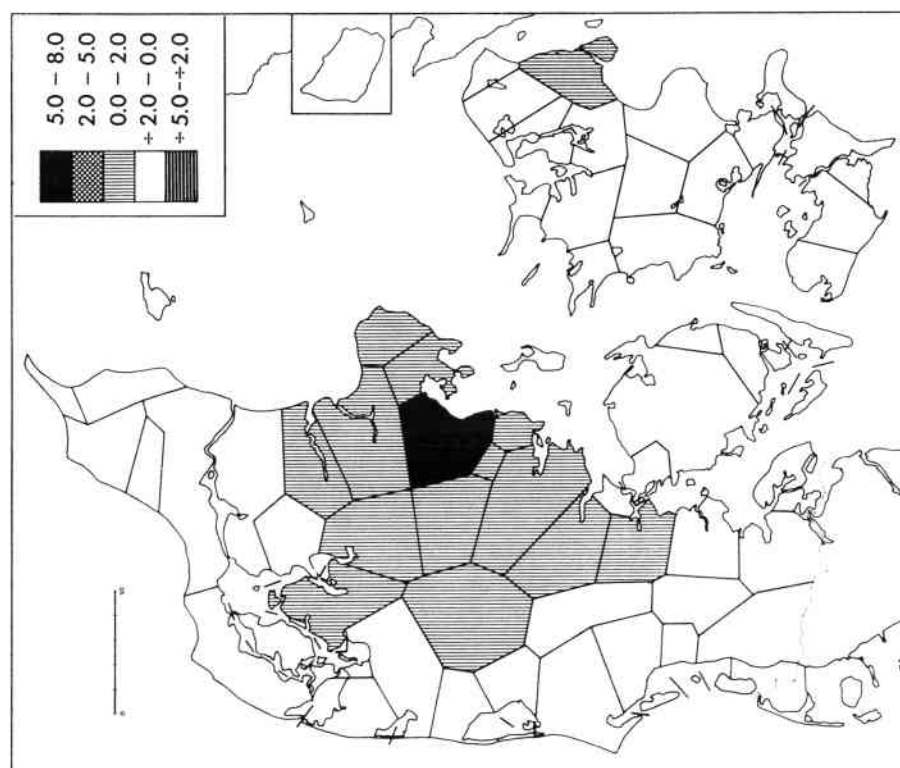


Fig. 3. Factor scores, factor 2.
Fig. 3. Faktornærdier, faktor 2.

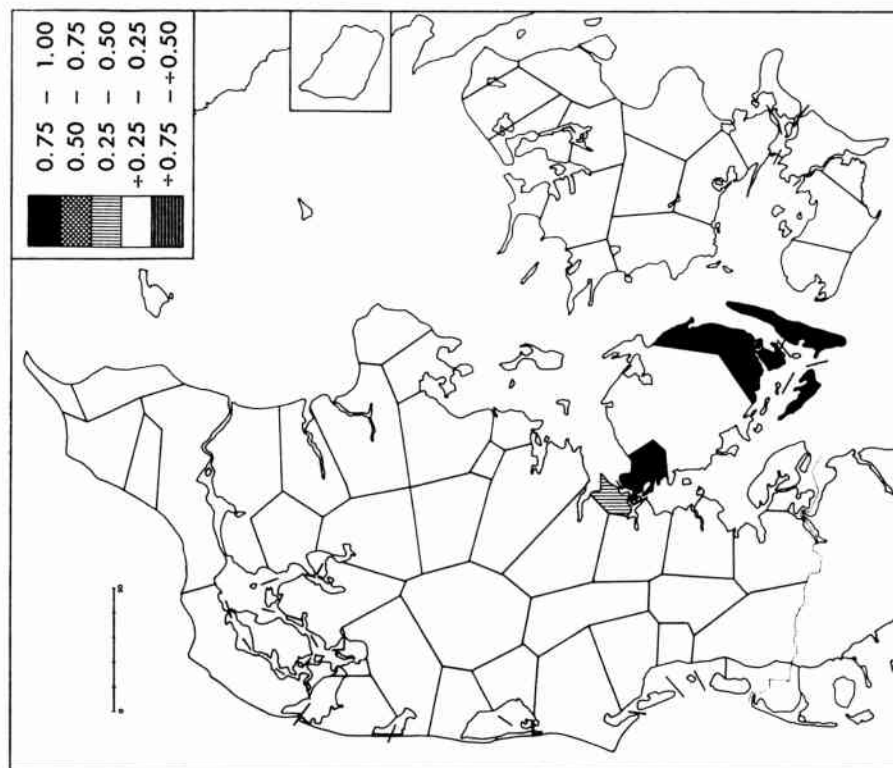


Fig. 6. Faktorvægte, faktor 3.
Fig. 6. Faktorevægte, faktor 3.

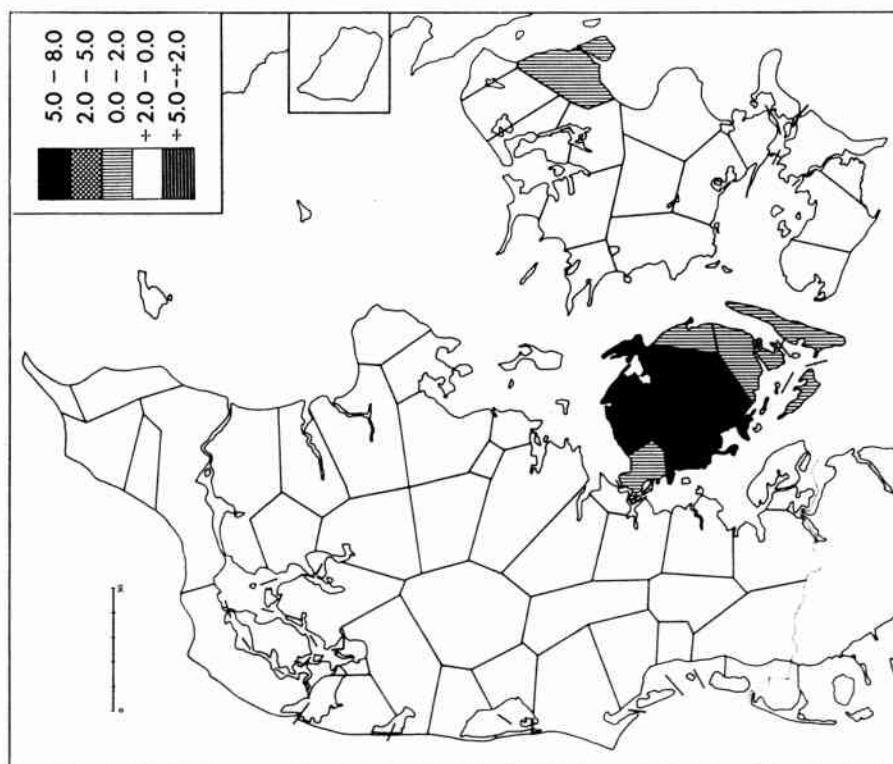


Fig. 5. Faktorscore, faktor 3.
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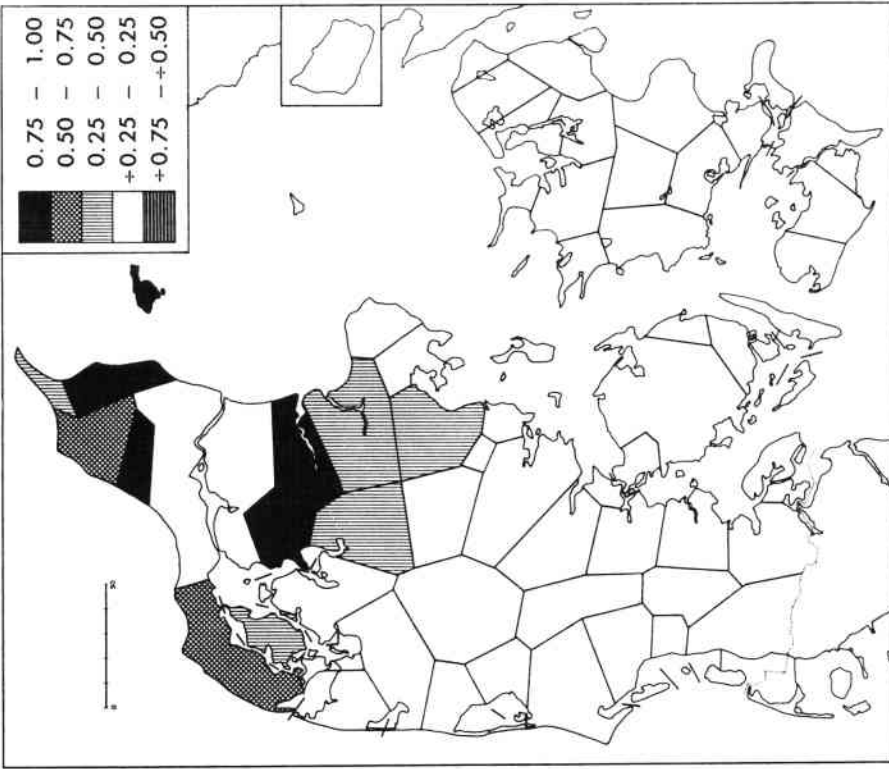


Fig. 8. Factor weights, factor 4.
Fig. 8. Faktørvegte, faktor 4.

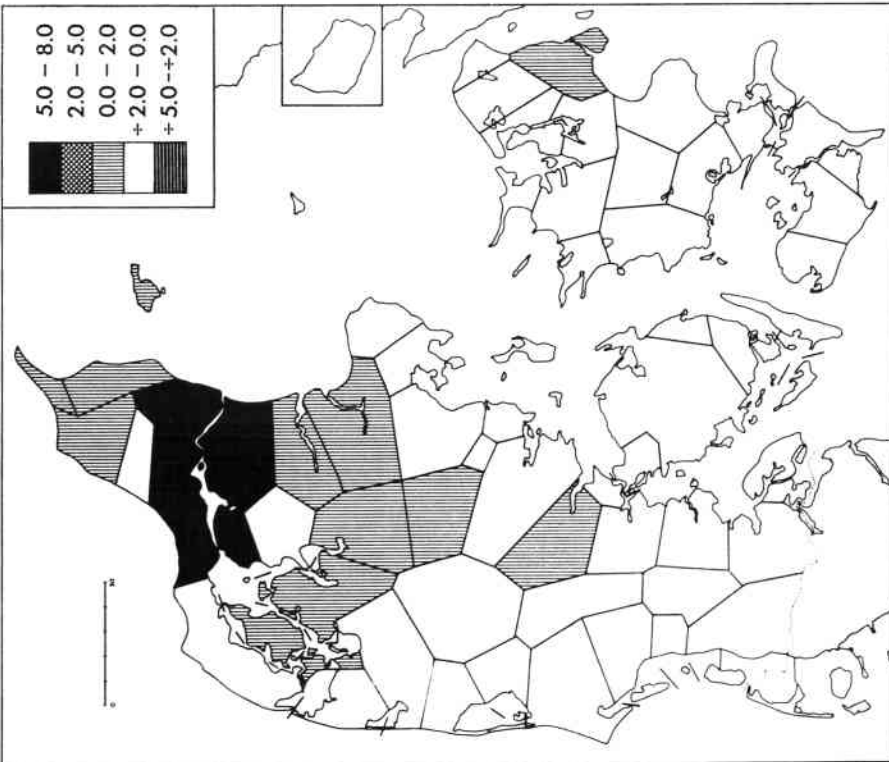


Fig. 7. Factor scores, factor 4.
Fig. 7. Faktørverdier, faktor 4.

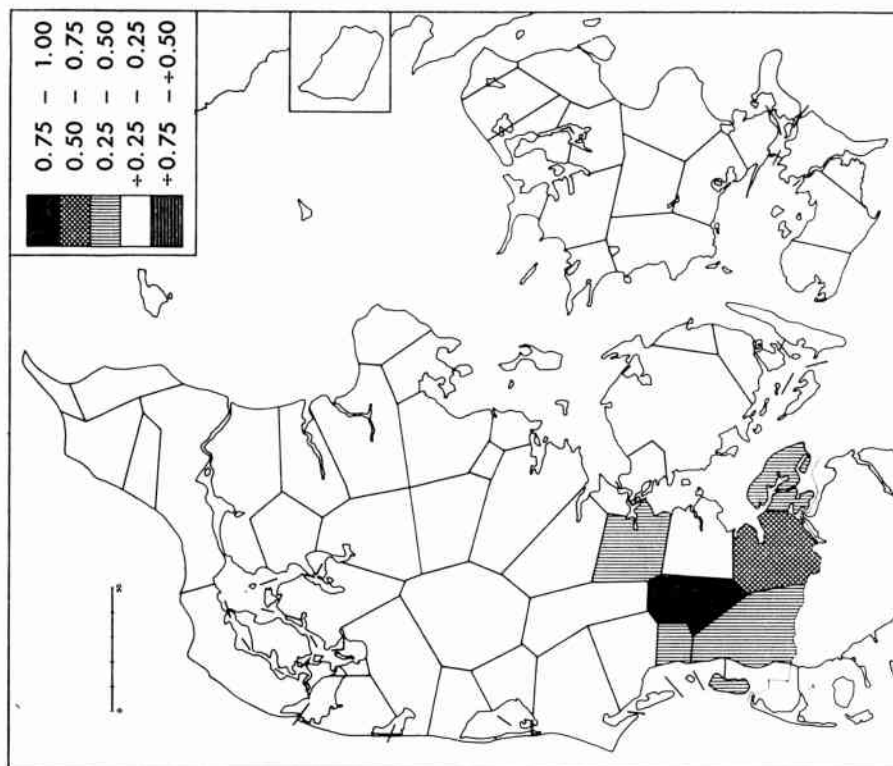


Fig. 10. Faktorvægte, faktor 5.
Fig. 10. Faktorvægte, faktor 5.

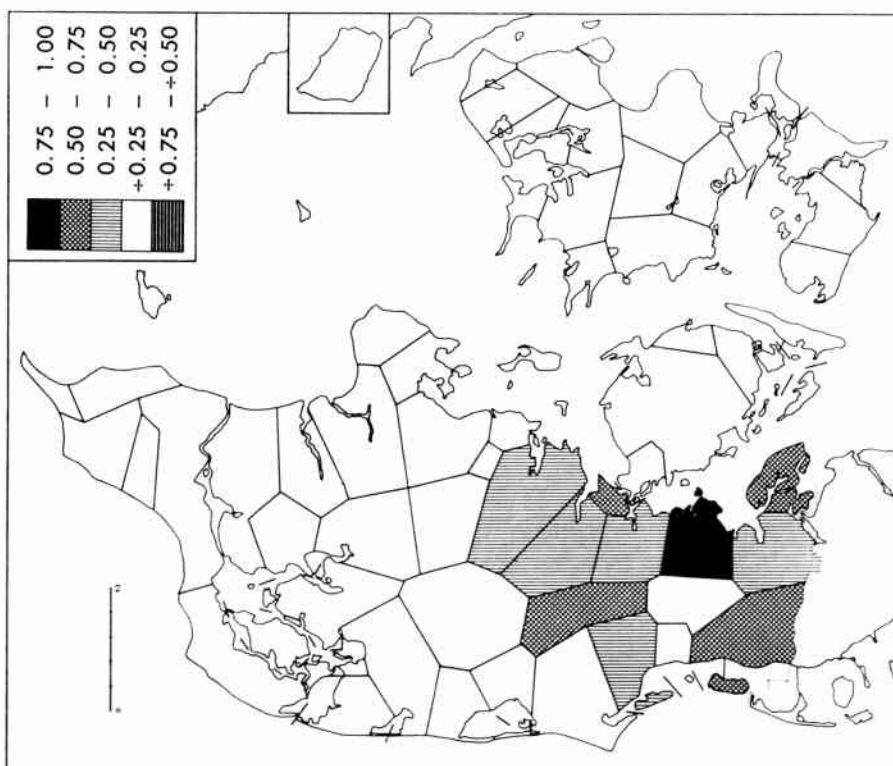


Fig. 9. Faktorværdier, faktor 5.
Fig. 9. Faktorværdier, faktor 5.

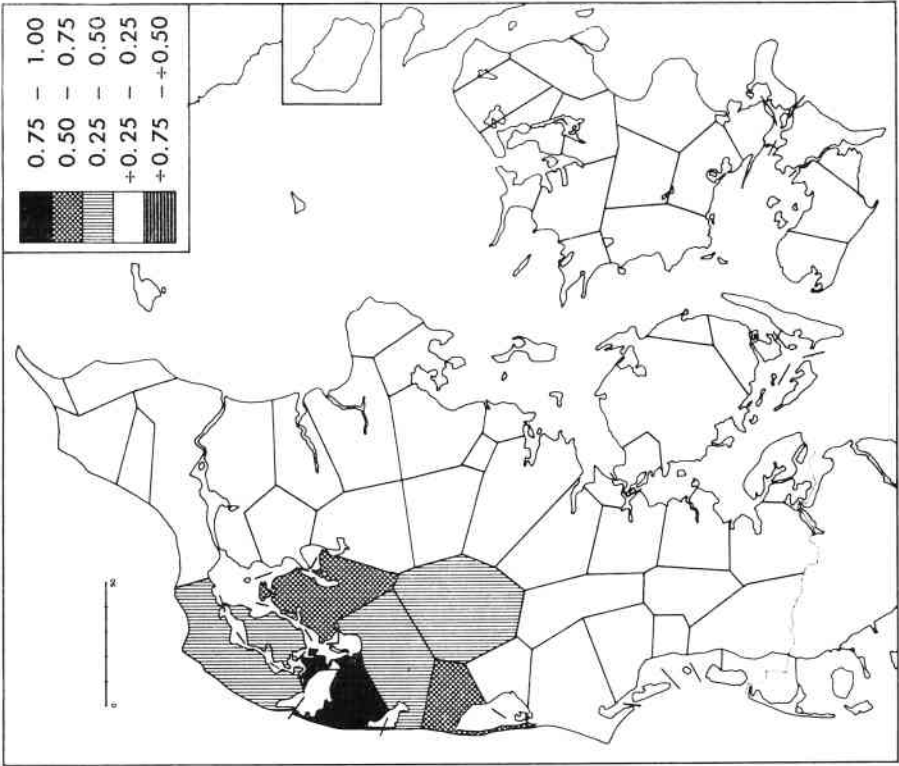


Fig. 12. Faktorvægte, faktor 6.
Fig. 12. Faktorvægte, faktor 6.

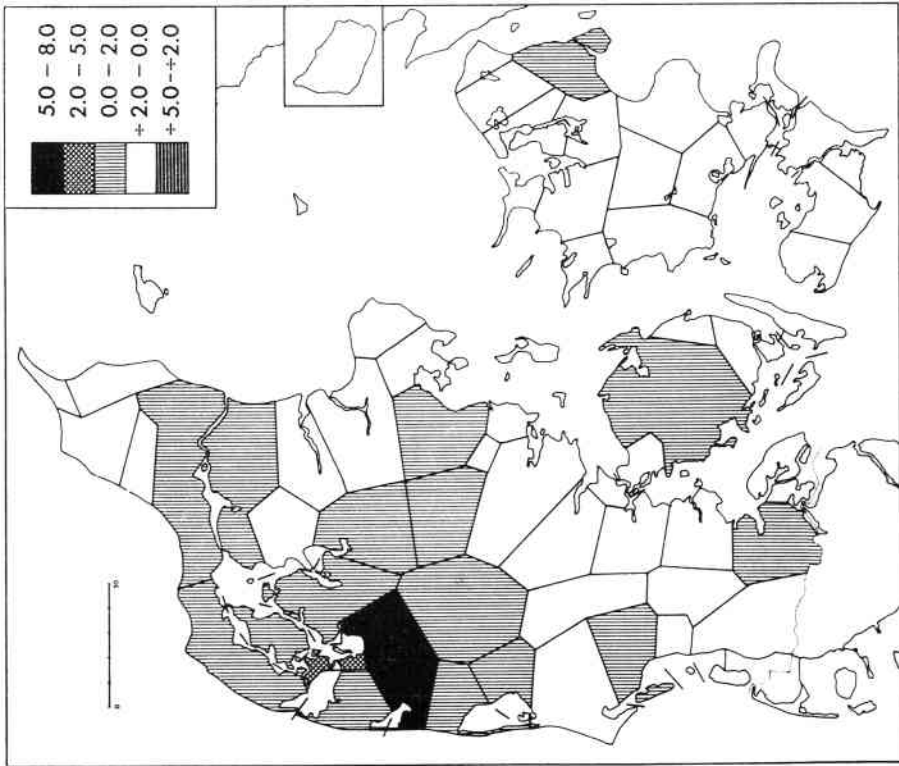


Fig. 11. Faktorværdier, faktor 6.
Fig. 11. Faktorværdier, faktor 6.

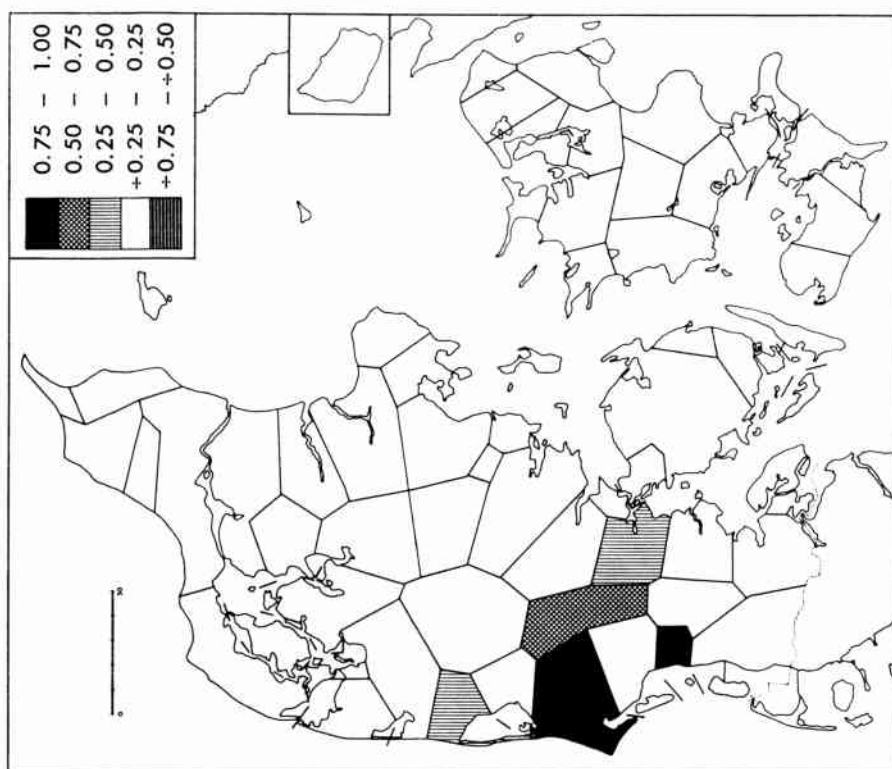


Fig. 14. Factor weights, factor 7.
Fig. 14. Faktorrægte, faktor 7.

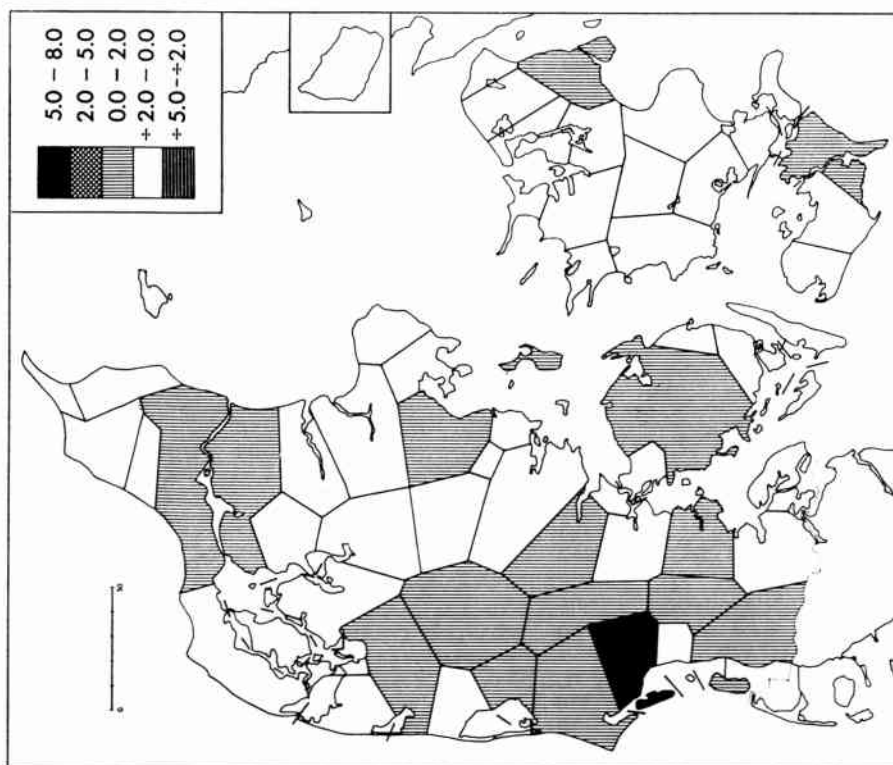


Fig. 13. Factor scores, factor 7.
Fig. 13. Faktornedier, faktor 7.

Factors 5, 6, and 7 (see figures 9-14) had the highest factor scores in the districts of Haderslev, Holstebro, and Esbjerg respectively, but also other districts around Haderslev and Holstebro had rather high factor scores on the factors 5 and 6. The factor weights for the three factors were high in Nord Slesvig, in northwest Jylland, and in southwest Jylland, respectively. The three factors indicate Haderslev, Holstebro, and Esbjerg as being regional centres for the telephone traffic. The picture, however, is not as clearly accentuated as was the case with the first four factors. Factor 5, especially, must be taken with some reservation, since the data for the Nord Slesvig districts are estimated on a rather weak base.

Factor no. 8 showed a pattern very much different from the first 7 factors (see figures 15-16). For factor 8, the 7 centre-districts pointed out by the first 7 factors distinguish themselves by scores having the opposite signs of those of their hinterlands. On Sjælland, the Copenhagen district had positive factor scores, while its hinterland districts had negative scores; in the rest of the country, the centre-districts had negative factor scores and their hinterlands positive scores. Around each of the centre-districts, there was a belt of districts with high factor scores. These characteristics were clear for the 4 most important centres, while for the next 3 centres, they only appear as weak tendencies.

The factor weights for factor no. 8 were high in the seven centre-districts, especially in the four biggest ones.

We can interpret factor 8 as representing the telephone calls to the main centres from their hinterlands. To Copenhagen, the greatest number of calls come from the districts in its immediate hinterland (i.e. Sjælland) and from the main west Danish centres; therefore these two groups of districts have the same signs. Contrary to this, the telephone calls to the west Danish regional centres come from their own immediate hinterlands and from Copenhagen; therefore the latter districts have got the same signs.

Factors 9 and 10 showed a type of pattern similar to factor 8 (see figures 17-20). Factor 9 had high factor scores in the districts of Varde, Ringkøbing, Herning, and Struer, and it received high factor weights in the districts of Skjern, Holstebro, Esbjerg, and Grindsted. It may be interpreted as representing the telephone calls from the hinterland of Holstebro and from the northern hinterland of Esbjerg. In accordance with this interpretation, not only Esbjerg and Holstebro had high factor weights on this factor, but also the districts adjacent to the back-side of the hinterlands.

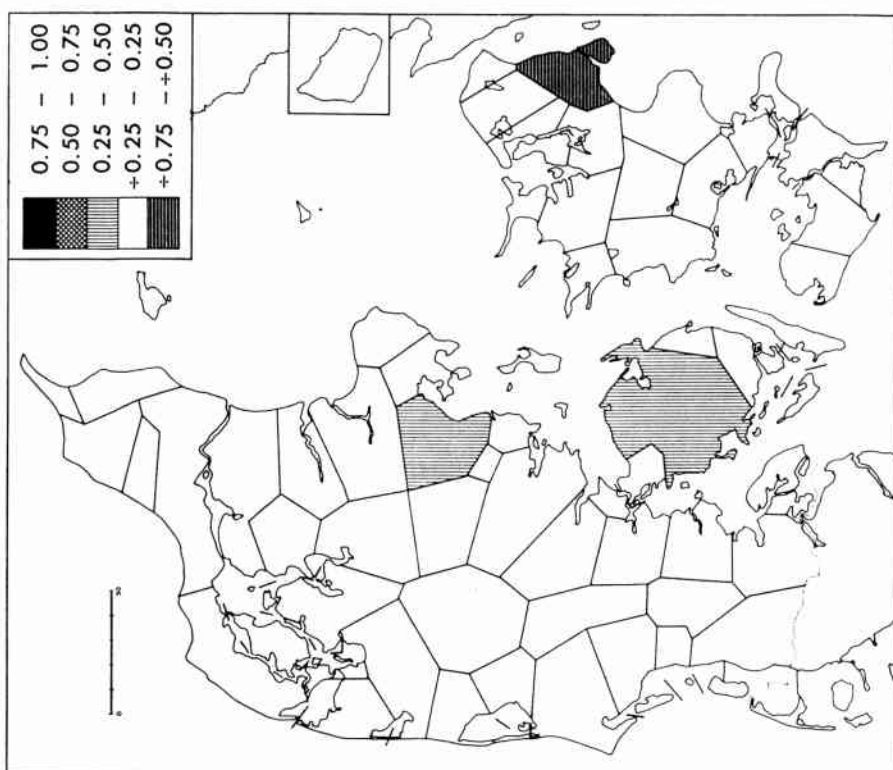


Fig. 16. Faktorvægte, faktor 8.
Fig. 16. Faktorvægte, faktor 8.

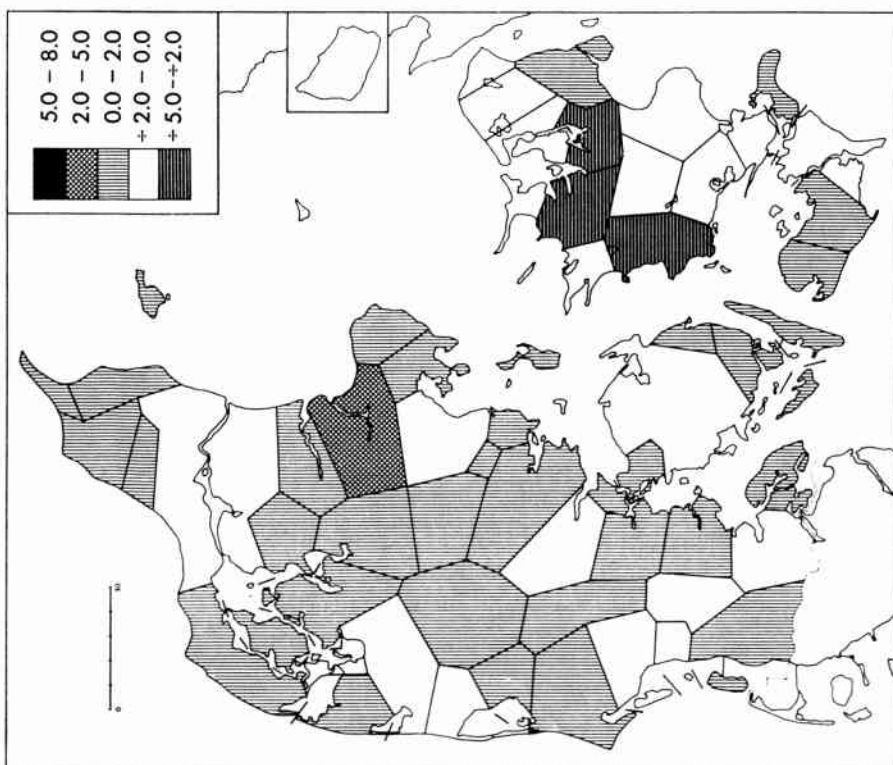


Fig. 15. Faktorværdier, faktor 8.
Fig. 15. Faktorværdier, faktor 8.

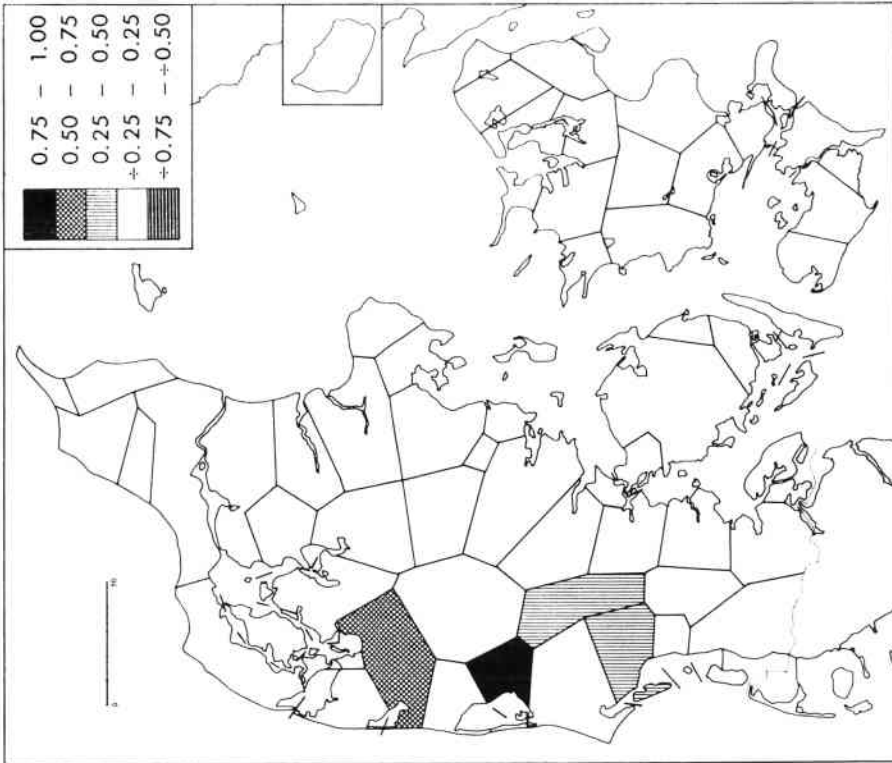


Fig. 18. Faktorvægte, faktor 9.
Fig. 18. Faktorvægte, faktor 9.

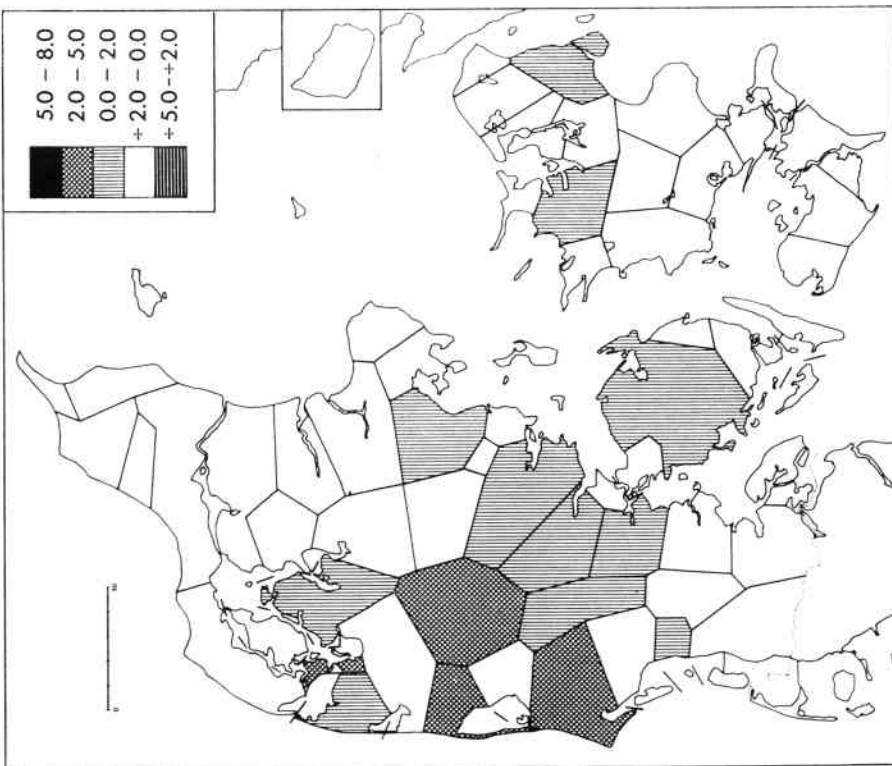


Fig. 17. Faktorværdier, faktor 9.
Fig. 17. Faktorværdier, faktor 9.

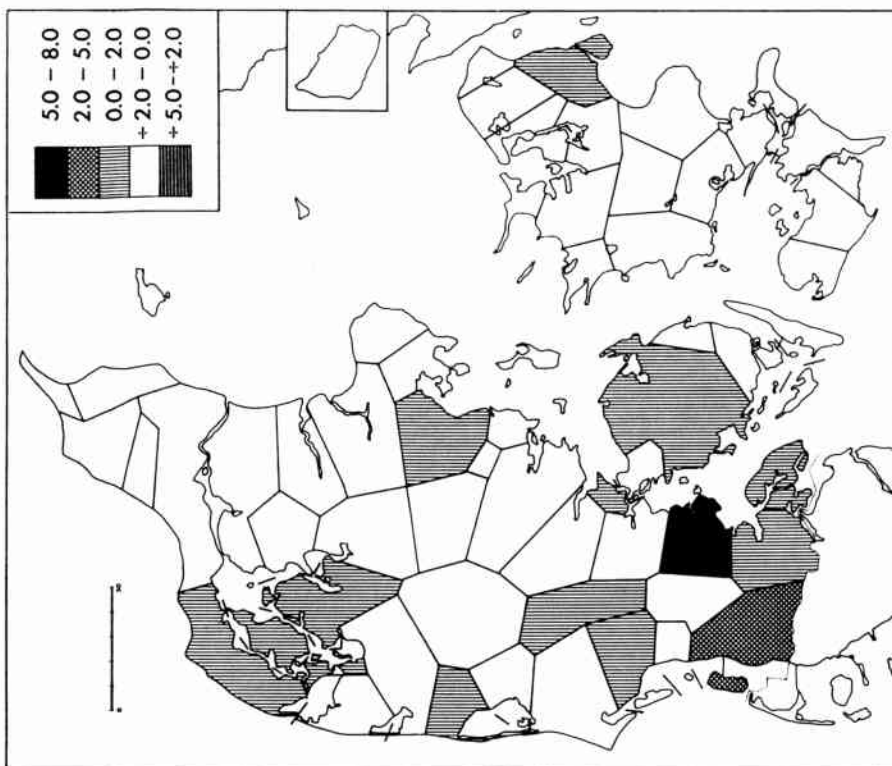


Fig. 20. Factor weights, factor 10.
Fig. 20. Faktørwægte, faktor 10.

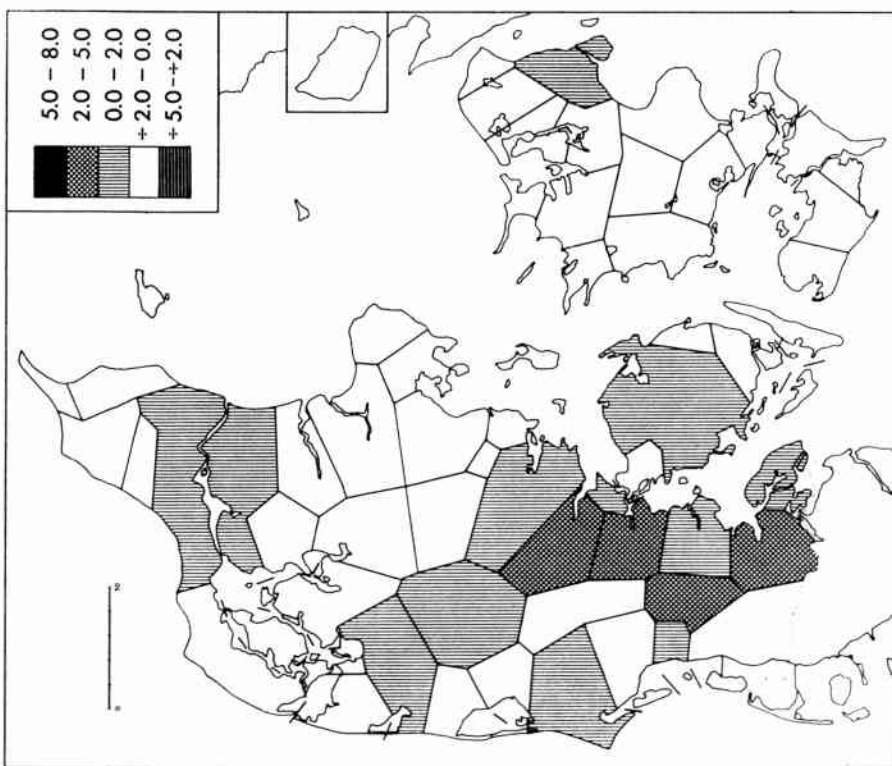


Fig. 19. Factor scores, factor 10.
Fig. 19. Faktørværdier, faktor 10.

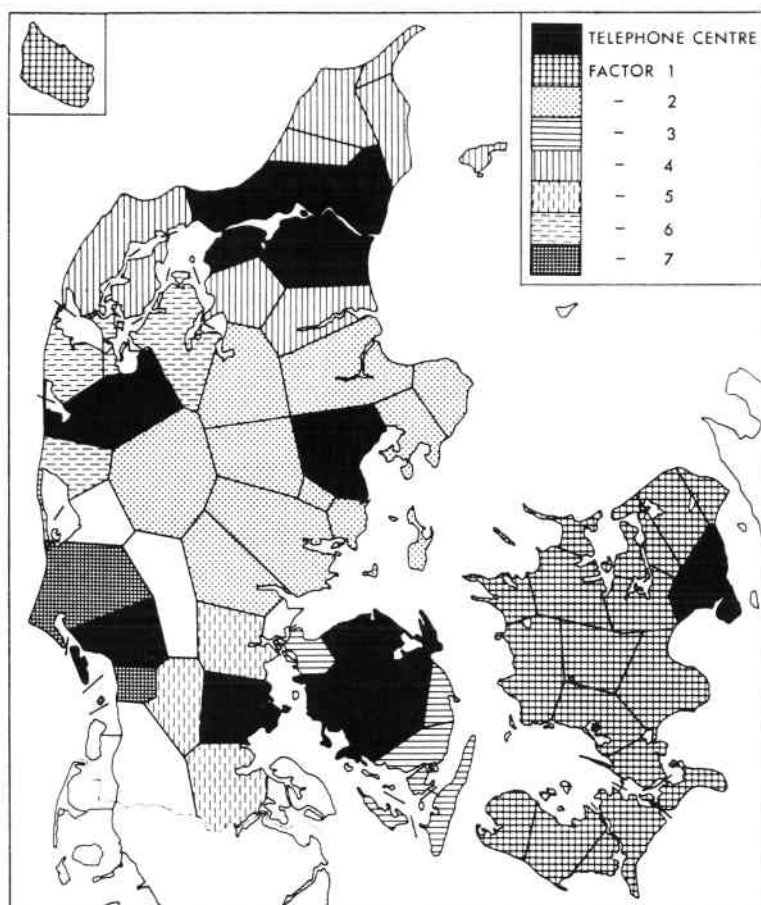


Fig. 21. Influence Zones. The influence zone of the telephone centre of Copenhagen is characterized by factor 1, of Århus by factor 2, of Odense by factor 3, of Ålborg by factor 4, of Haderslev by factor 5, of Holstebro by factor 6, of Esbjerg by factor 7.

Fig. 21. Telefoncentrenes oplande, afgrænset på grundlag af faktorerne 1-7.

Factor 10 had high scores in the districts of Åbenrå, Gram, Kolding, and Vejle, and it received high weights in the districts of Haderslev, Tønder, Sønderborg, Grindsted, Fredericia, and Esbjerg. The factor may be interpreted as representing telephone calls from the hinterland of Haderslev and from the eastern and southern hinterland of Esbjerg. In accordance with this interpretation, not only Esbjerg and Haderslev had high factor weights for this factor; also the districts adjacent to the back-side of the hinterlands (Tønder, Sønderborg, Fredericia, and Grindsted) had high factor weights.

The next factors, each of which represents only a very small part of the total variance, are not commented on here; they seem to represent telephone calls to and from lower order centres.

To sum up, the factor analysis of the telephone traffic between 62 districts in Denmark indicates 4 strong regional centres, namely Copenhagen, Århus, Odense, and Ålborg, and 3 weaker regional centres, namely Haderslev, Holstebro, and Esbjerg.

In figure 21, the influence zones of these 7 centres have been delimited, the maximum factor weight determining for each district to what hinterland it has been assigned. Thus districts with the highest weight on factor 1 have been assigned to the influence zone of Copenhagen, districts with the highest weight on factor 2 to Århus, and so forth up to factor 7. On the map, districts with higher weights on factor 8, 9, and 10 than on any of the first 7 factors have been shown as not belonging to any of the 7 influence zones.

The map shows the whole of Denmark east of Storebælt as belonging to the influence zone of Copenhagen, and central-eastern Jylland, Fyn, and northern Jylland as forming the regions of Århus, Odense, and Ålborg, respectively. Western and southern Jylland are only partly covered by the smaller hinterlands of Haderslev, Holstebro, and Esbjerg.

4. Comparison with other measures of centrality

The central place centrality measures generally used are the sizes of central activities or the occurrence of certain central functions. In table 2, the results of the present factor analysis are compared with the 7 most important central places assigned by other methods. As a quantitative measure, the employment in

Table 2. The 7 most important central places in Denmark, according to various measures of centrality.

	Factor analysis of telephone traffic, 1961	Employment in wholesale trade, 1958	Occurrence of 16 central functions, 1960
1	Copenhagen	Copenhagen	Copenhagen
2	Århus	Århus	Århus
3	Odense	Odense	Odense
4	Ålborg	Ålborg	Ålborg
5	Haderslev	Randers	Esbjerg
6	Holstebro	Esbjerg	Randers
7	Esbjerg	Kolding	Horsens

wholesale trade at the 1958 census of industries was used, while the occurrence of 16 selected central functions in 1960 as studied by *Biilmann* (1964) was used as a second measure.

While the positions of the four major centres are firmly established in the Danish central place hierarchy, the positions of the following medium-level towns depend on the measures applied. Quantitative measures of central activities and, to a smaller degree, qualitative measures of occurrence tend to assign high positions to towns situated in densely populated regions near major metropolitan centres (e.g. Randers). In contrast, the factor analysis points out centres of a more independent character, remote from other major urban areas.

These results are due to qualities inherent in the principal factor solution, by which – as already mentioned – each factor is extracted so that it explains as much as possible of the remaining variance in the observation matrix. Because of the distance dependency in most communication matrices, neighbouring districts are likely to have origination fields which are much alike, and so the first factor is likely to explain a large part of the origination fields of all the districts around the biggest city; in the present case: Copenhagen. Since the second factor explains as much as possible of the variance not explained by the first factor, it is likely to represent a big town located at some distance from the biggest one, because the origination fields of the towns close to the biggest one have already been explained to a large extent by the first factor. And so on for the following factors.

The centres resulting from the factor analysis thus will be spaced far apart, each dominating its own hinterland, but located outside the influence fields of other centres.

Regional centres pointed out according to these principles will often be the most appropriate ones for the location of regional activities, e.g. local government.

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RESUMÉ:

Telefonselskaberne har inddelt Danmark i 62 såkaldte netgrupper, hver omfattende en by og det omliggende område. For en stikprøveperiode på 12 dage i 1961 er antallet af samtaler mellem hver af de 62 netgrupper indbyrdes optalt. Mellem de sjællandske netgrupper indbyrdes måtte tallene dog beregnes ud fra et 1958-materiale, og mellem de lolland-falsterske og sønderjyske skønnes på et endnu løsere grundlag. Den således fremkomne 62 x 62 matrix er underkastet faktoranalyse.

De 10 vigtigste faktorer forklarer 79 % af matrixens samlede varians. Disse 10 faktorer danner grundlag for kortene 1-20, idet faktorværdierne er vist på kort med ulige numre og de tilsvarende faktorvægte på kort med lige numre. Faktorerne nr. 1-7 udviser hver høj faktorværdi i en enkelt netgruppe, nemlig henholdsvis København, Århus, Odense, Ålborg, Haderslev, Holstebro og Esbjerg. De tilsvarende faktorvægte er høje i de omliggende netgrupper. Faktorerne tolkes som hver repræsenterende telefonsamtalerne fra et center til dets opland. Det bemærkes dog, at billedet er klarere for de første 4 faktors vedkommende end for de følgende 3.

Faktor nr. 8 har i de 4 vigtigste centre (i svag grad også i de følgende 3) faktorværdier med modsat fortegn end i de omliggende netgrupper, og har høj faktorvægt i centrene. Faktoren må repræsentere samtalerne fra oplandene til centrene. Tilsvarende tolkes faktor nr. 9 og 10 som repræsenterende samtalerne fra Holstebros opland og den nordlige del af Esbjergs opland, henholdsvis fra Haderslevs opland og den sydøstlige del af Esbjergs opland.

På kort 21 er de syv nævnte centres oplande afgrænset på grundlag af faktorerne nr. 1-7, idet den højeste i hver netgruppe forekommende faktorvægt har været afgørende for, hvilket opland den er henregnet til. Netgrupper med højest faktorvægt for en af faktorerne nr. 8-10 er ladet stå hvide.

I sammenligning med centralitetsundersøgelser foretaget ved andre metoder er der ved faktoranalysen en tendens til at undgå udpegning af centre beliggende i nærheden af andre, større centre, og i stedet at fremhæve langt fra hinanden beliggende, mere uafhængige centre (jfr. center nr. 5, 6 og 7 i Danmark). Dette resultat, der beror på selve faktoranalysens egenskaber, giver denne fremgangsmåde særlig interesse med hensyn til praktisk anvendelse, f. eks. ved udvælgelse af lokale administrative centre.
