European Journal of Inclusive Education (EJIE)
2024 Vol. 3, Issue 1, 43-59
https.//doi.org/ https://doi.org/10.7146/ejie.v3i1.142990

Productive Struggle as a Boundary Object between Co-Teachers in Grade 6 Mathematics Classrooms in the United States[1].

Angela R. Crawford[2]

Received: 8 January 2024, Accepted: 28 April 2024

Abstract

PURPOSE: Fostering coherence and sustainability of co-teaching partnerships can be challenging in inclusive mathematics classrooms. Establishing partnerships requires negotiating understandings and activities across disciplinary boundaries (mathematics education and special education) while implementing researcher-recommended practices in unique settings. This study investigates how a specific instructional practice, productive struggle, serves as a boundary object for co-teachers in negotiating those challenges.

APPROACH: I applied a pragmatic lens in a comparative case study to learn how two pairs of teachers engaged in co-teaching navigated disciplinary boundaries around a mathematics pedagogical practice, productive struggle.

RESULTS: The pairs of co-teachers authored meanings for productive struggle and developed distinctive approaches to foster this practice with all students in their inclusive classes. Their meanings and approaches were aligned in ways that were coherent with their goals and beliefs about teaching mathematics. Although productive struggle in their contexts contrasted in some ways with how the practice is portrayed in research literature, teachers described finding value for students' learning in their implementation. They also found inclusive settings and their co-teaching partnerships beneficial for student learning.

CONCLUSION: Teachers used authority and agency in boundary spaces to craft cohesive understandings and approaches to implement productive struggle, and they saw value in their partnerships for implementing the practice. As critical stakeholders in making inclusion coherent and sustainable, I advocate for teacher-educators and researchers to value teachers' authority and agency as they implement researcher-recommended teaching practices in inclusive settings.

Keywords: co-teaching, mathematics education, middle grades, productive struggle, boundary spaces

Points of Interest:

- Teaching mathematics in inclusive settings involves discontinuities between the disciplines of mathematics education and special education.
- Co-teaching is a form of boundary encounter between disciplines in which understandings, goals, and activities are negotiated.
- Research-based instructional practices can be seen as boundary objects that should capture multiple meanings in order to act as a bridge across disciplinary discontinuities.

[1] This material is based upon work supported by the US National Science Foundation under Grant No. [3414008].
[2] Corresponding author: angelacrawford1@boisestate.edu

ISSN: 2794-4417
https://tidsskrift.dk/ejie

- Teacher authority, agency, and value for productive struggle contributed to the coherence and sustainability in the boundary space of inclusive, co-taught mathematics settings.
- Efforts to incorporate inclusion in mathematics classrooms using co-teaching likely benefit from a focus on adaptability rather than fidelity to research-based and evidence-based practices.

Introduction

The United Nations Committee on the Rights of Persons with Disabilities identifies teacher partnerships characterized by "collaboration, interaction, and problem-solving" as a core feature of inclusive education (United Nations, 2016, p. 5). Despite their importance, co-teaching arrangements between general education teachers and special education teachers have been challenging to sustain. Common challenges are time for collaborative planning, maintaining equitable roles, and compatibility between teachers (Brendle et al., 2017; Friend et al., 2010; Mastropieri et al., 2005).

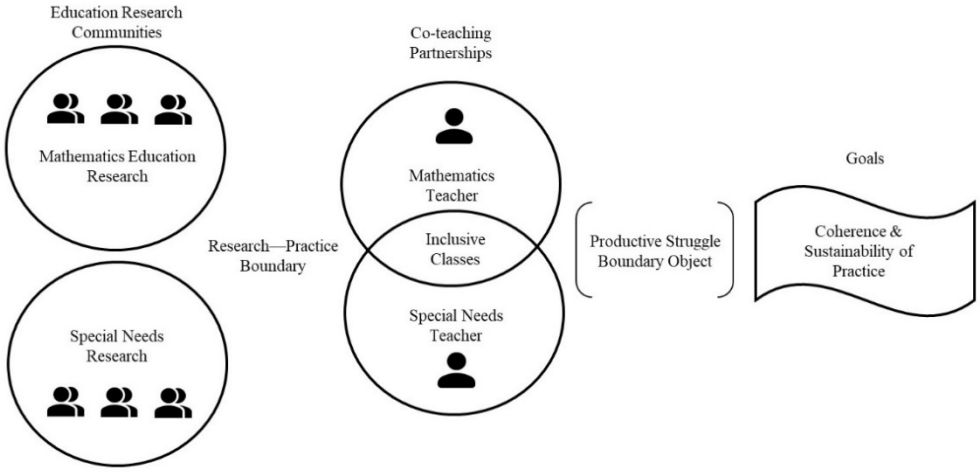
Implicit in the issue of compatibility is the need for coherence. Coherence among curriculum, goals, vision, and values is key to commitment to reform in education (Newmann et al., 2001) and thus to the sustainability of inclusive education (Naraian & Schlessinger, 2018). Mathematics teaching in inclusive classrooms adds a disciplinary challenge to this coherence, particularly because there are conflicting recommendations for instruction from researchers and professional organizations. This study explored how a specific instructional practice, productive struggle, acts as an object of negotiation across disciplinary boundaries in co-taught inclusive mathematics classrooms. The larger question is what impact implementation of productive struggle may have on coherence of teaching practices, given conflicting disciplinary perspectives, and sustainability, in terms of teachers' interest in continuing their partnership over time.

Theoretical Perspective

Boundaries, as an analytic concept, capture the idea of diverse perspectives and discontinuities in activity that occur when groups interact (Akkerman & Bakker, 2011). Boundaries form between communities of practice based on differences in beliefs and activities, and boundary spaces are characterized by ambiguity (Akkerman & Bakker, 2011; Sztajn et al., 2014). Objects or ideas may exist within boundary spaces as tools or common points of reference for interactions. These "boundary objects," whether physical or conceptual, are interpreted differently by each group, and meanings are negotiated (Star, 2010). To function successfully as a tool or point of reference, the boundary object needs to be inclusive of diverse perspectives (Akkerman & Bakker, 2011).

Such a boundary exists between mathematics education and special education, disciplines which often hold contrasting views of mathematics and mathematics learning (Munter et al., 2015). These differences stem from the historical roots of the field of mathematics education in constructivism and special education in behaviourism and information processing (Woodward, 2004). An example of the contrasting views is seen in the instructional practice of productive struggle, the boundary object in this study. To function successfully in terms of coherence of instructional practice and thus contribute to sustainability, teachers will need to negotiate compatible meanings and implementation approaches (see Figure 1).

Figure 1.
Communities and boundaries implicated in mathematics co-teaching partnerships.



In the mathematics education community, productive struggle has garnered considerable support in recent decades (Otten et al., 2017; Young et al., 2024). Productive struggle, and a similar practice known as productive failure, is understood as an opportunity for students to make sense of a problem for which there is no immediately obvious solution path or a concept which is not readily apparent to them (Hiebert & Grouws, 2007; Kapur, 2016). Features such as problem solving, sense-making, and perseverance center attention on cognitive processes rather than solutions to problems. Teachers can implement productive struggle through tasks which ask students to make connections between representations, look for and make use of structure, construct arguments, and critique the reasoning of others (National Council of Teachers of Mathematics, 2014). The field of mathematics education has adopted the idea of maintaining high cognitive demand as a critical feature of productive struggle (Otten et al., 2017; Stein & Lane, 1996), and there is a concern for not lowering the cognitive demand through too much guidance. When students have difficulty, researchers recommend eliciting students' thinking and pressing them to extend it, not simplifying the problem, suggesting an action, or breaking it into smaller parts (Stein & Lane, 1996; Warshauer, 2015).

In stark contrast, special education researchers argue that instruction for students with special educational needs should be explicit (Grigorenko et al., 2020; McLeskey et al., 2022). Teachers should provide clear demonstrations and explanations of the concepts and procedures (Powell et al., 2022; McLeskey et al., 2022). Given the potential for difficulty arising from cognitive, behavioural, or affective differences exhibited by students with special educational needs (Nelson et al., 2022), the aim is to minimize struggle and ensure learning of accurate concepts and procedures. Recently, a number of researchers have pressed this perspective and challenged the value of productive struggle through presentations at practitioner conferences, social media, and a webpage (The Science of Math, 2023). They argue only after teachers have demonstrated, explained, and verified students have learned the content would it be appropriate for opportunities to incorporate struggle. Doing otherwise leads to misconceptions, frustration, and lost instructional time. They re-frame productive struggle as working to generalize learning to a similar but novel, challenging problem or task. Thus, this meaning for productive struggle is in conflict with that of mathematics education researchers.

The disciplinary differences between mathematics education and special education create another boundary, that between research and practice. From this vantage, the boundary object, productive struggle, needs to be inclusive of the vision from research and the contexts in which it will be enacted. The tendency from the research community is to articulate visions of high-quality instruction or call for

fidelity of implementation with the aim of shaping teachers' practice to align with researchers' ideals (Cai et al., 2020; Cobb et al., 2017; DeFouw et al., 2019; Kretlow & Bartholomew, 2010; Sanetti & Luh, 2019). However, direct translation is a rarity. Teachers often do not find researcher-recommended practices useful or feasible; they commonly adapt and author practices to fit their needs and goals within their local contexts (Jaworski & Potari, 2021; Helenius, 2021; Tamborg, 2021).

Given this adaptation, some researchers have called for more attention to the nature of implementation in classrooms. These calls acknowledge the ambiguity of the boundary between researchers and teachers and recognize a need for better understanding of how instructional practices work in varying contexts (Koichu et al., 2021; Cai & Hwang, 2021). Considering a specific object of innovation, such as productive struggle, this research could address the extent to which meanings are negotiated and the forms in which it is implemented (Koichu et al., 2021).

Co-teaching arrangements between general educators and special educators in inclusive mathematics classrooms are examples of boundary encounters, and in this study, productive struggle is viewed as a boundary object for these teachers. The meaning of productive struggle, how to appropriately implement it, and its value are things that are negotiated when teachers work in partnership. Through a comparative case study, I explored how co-teachers interacted with productive struggle as a boundary object. I also aimed to understand if and how this boundary object might be related to coherence and sustainability of co-teaching partnerships in inclusive, grade 6 mathematics classrooms.

Method

Research Context

This comparative case study was part of a larger crossover study of sequences of mathematics instructional strategies that involved 98 teachers. The instructional strategies, explicit attention to concepts and students' opportunity to struggle, were first explicitly articulated in a synthesis of research by Hiebert and Grouws (2007). In the context of this study, the researchers described productive struggle as comprising three features: a) sustained mental effort, b) focus on sense-making, and c) engagement with important mathematics (Crawford et al., 2022). These features were explicitly shared with teachers during professional development modules and through a two-page guide (Champion et al., 2020). The guide suggests techniques for encouraging productive struggle which include assigning contextual problems with multiple solution strategies, asking students for extended explanations, asking students to look for patterns or make conjectures, and promoting discourse around emerging ideas. Teachers were encouraged to use their expertise about their contexts and their students to translate these techniques into opportunities for productive struggle for their students.

Throughout this comparative case study, I have tried to incorporate reflexivity. I worked as a mathematics teacher for 16 years before becoming a researcher. My research has been situated at the intersection of mathematics education and special education, though I find my beliefs more closely aligned with mathematics education. To challenge my biases, I have attempted to maintain an open-mind about productive struggle and throughout this project regularly recorded memos asking questions such as what might be an affordance or a constraint of any point-of-view or decision.

Participants & Data Sources

Two pairs of teachers began teaching partnerships in inclusive, grade 6 mathematics classrooms during our project. Each pair consisted of a general education teacher and special education teacher. All teachers held bachelor's degrees in education or special education; one also held a master's degree in special education. The teachers were white women with years of experience ranging from 2 to 23. All worked in schools in the western United States. All teachers had participated in the first year of the research project independently. In the second year, when this study took place, their schools changed to a full

inclusion model and the teachers began co-teaching partnerships. The teachers were not given any training on co-teaching prior to beginning their partnerships.

Gayle and Susan taught in a rural school with a student population which comprised 15% students with disabilities, 62% students from low-income families, and 17% students learning English. Gayle and Susan taught the same students in sequential class periods; that is, Gayle taught a mathematics lesson in one classroom and the next period those students went to Susan's classroom for a follow-up mathematics lesson. Gayle and Susan regularly planned together and coordinated their sequential lessons. Gayle was the general education teacher, and Susan was the special education teacher.

Gemma and Sally worked in a suburban school with a student population that was 16% students with disabilities, 42% students from low-income families, and 2% students learning English. Gemma and Sally worked together with students in the classroom at the same time during one class period, adopting a one-teach, one-assist model. Gemma and Sally planned together a couple of times a week. As the special education teacher, Sally was also obligated to attend meetings related to provision of services for students which prevented her from planning with Gemma, the general education teacher, on a daily basis.

This research occurred toward the latter part of the COVID-19 pandemic when teachers and students had returned to in-person instruction. Due to the continuing pandemic, I worked with teachers to create a flexible data collection protocol and timeline. Video recorded data included interviews (two with Gayle and Susan; three with Gemma and Sally), think-alouds during planning meetings (four with Gayle and Susan; seven with Gemma and Sally), classroom lessons (four each from Gayle and Susan; three from Gemma and Sally), and post-lesson reflections (three from Gayle and Susan; four from Gemma and Sally). Also, each pair of teachers submitted a collection of artifacts which included lesson plans and annotated examples of students' work. Semi-structured interviews, which the teachers elected to do together, focused on their thoughts about productive struggle, its benefits and challenges for students, and how they were coordinating their approaches for implementing it. I shared the research questions with the teachers, and I offered prompts for things they might talk about during their planning and reflection (i.e., what would/did productive struggle look like, what would/did unproductive struggle look like, how would/did you support all the students in your class, what challenges might/did you experience).

Data Analysis

I used reflexive thematic analysis (Braun & Clarke, 2022) and narrative analysis (Polkinghorne, 1995) to analyse these data. Teachers' perspectives as provided through interviews, planning meetings, and reflections were the primary data sources. I used videos of classroom lessons and artifacts to look for confirming and disconfirming evidence. In the familiarization phase I watched the videos multiple times, corrected auto-generated transcripts, and wrote summaries. I used NVivo© during the second phase of analysis to define participants as cases and descriptively code segments of video applying the following codes: struggle, productive, scaffolding, and challenges. I used queries to sort coded data by teacher pair, and then I identified themes for each teacher pair and each code. The third phase was a narrative analysis to synthesize data across the school year. This phase comprised four steps: (1) arrange the data chronologically, (2) identify segments of video and associated transcripts that relate to productive or unproductive struggle and teachers' supports for students, (3) look for cause and influence, identifying or inferring contextual, personal, or interpersonal contributors to outcomes, and (4) writing a narrative description of productive struggle for each pair of teachers. The final phase of analysis involved a cross-case analysis, integrating the patterns identified in coding and narrative descriptions to answer the research questions, and specifically noting and describing evidence of authorship and agency.

Findings

I found the pairs of teachers did not wholly adopt a mathematics education or a special education perspective in regard to productive struggle but developed a system which was coherent within their partnerships. All teachers authored meanings for productive struggle as students making sense of mathematics but with sense-making focused on different goals. The pairs of teachers demonstrated agency in the ways they implemented productive struggle. Finally, the pairs of teachers found productive struggle and their partnerships to be valuable for students' learning, and they aimed to sustain the instructional practice, preferring to do so in partnership in the future. In this section, I will first describe productive struggle from Gayle's and Susan's perspectives before sharing Gemma's and Sally's perspectives.

Gayle and Susan – Sequential Co-teaching with a Focus on Models

Gayle and Susan taught coordinated lessons, with Gayle teaching first and Susan teaching second. They created a system of co-teaching that was coherent with their individual values and beliefs about quality instruction and in their view sustainable as long as they had two class periods. The meanings for productive struggle which they developed were not precisely the same but were compatible. They used the same supports, though they placed emphasis on different ones in discussions of their approaches. Finally, the time they had for teaching allowed them to include other instructional practices they believed were crucial.

Meanings for Productive Struggle

When Gayle and Susan discussed productive struggle, they consistently focused on struggle employing visual models or diagrams, such as area diagrams, plastic counters, and ratio tables. Their plans for productive struggle asked students to apply visual models or diagrams to mathematical problems which students had not yet been taught to solve or to develop understanding when students already knew an algorithm. Gayle said that her students "tend to jump right to [the equation]. And today I didn't feel like there was very much real understanding because of that. So we talked about models, and they were successful with the problems in the lesson." Thus, their understanding of productive struggle was it is what happens when students make sense of models as tools for successfully solving problems.

They described models as a way to support learning for all students in their classes. Regarding students that generally have difficulty in mathematics, Gayle said, "The modelling makes sense to them, it gives them a way to solve it, even though they don't know how to do it." Susan added, "And the ones who don't normally struggle [with mathematics], with the model, they might really struggle with it." Gayle described models as "scaffolds" for some students to access mathematics they might otherwise find challenging, while Susan added models offered a way to introduce productive struggle for other students.

There were some differences in the ways Gayle and Susan understood the models themselves. Gayle viewed the models as representations of mathematics. For example, in a reflection she talked about students needing to understand the meaning of an area grid to model fraction multiplication, "There was a really big misconception on how the model should be drawn. They were not being faithful to the model itself." This indicates Gayle views models as having an inherent mathematical meaning. In contrast, Susan viewed the models as expressions of students' thinking. In the same reflection, Susan described how she adjusted her lesson, "I did have kids *not* use the grid for their model. They just drew it out, which…I think that…it kind of got them thinking about 'How else could I see this?' maybe. And they stretched their thinking on that problem." Whereas Gayle asked students to struggle to determine how to use a particular model to solve a problem, Susan described asking students to generate models that represent their way of thinking about the problem.

Implementing Productive Struggle

Gayle and Susan described general approaches to implementing productive struggle that could be used across any lesson. Gayle brought the idea of cooperative groups to the partnership. She relied on the structure of these groups which gave students consistent expectations. Every member of the group was jointly responsible for giving a short presentation on their solution which included restating the question, getting a correct answer, offering a good explanation and model, and using mathematical vocabulary. These groups were present in all of Gayle's lesson plans and videos. Gayle describes these groups as "strategic," with a balance of students with different levels of prior achievement in mathematics: "So that anyone that's low has someone that's higher right next to them to support and we know that that isn't total support, we understand that. But it does give them additional support." Gayle felt the cooperative groups made productive struggle accessible to students who have more difficulty in a fully inclusive classroom, "because we can't do it all by ourselves." Other means of implementing struggle were referenced secondarily to the groups. For example, when asked about support during an interview, Gayle replied, "Well, there's our groups. Oh, and we use manipulatives and then those questions that keep them on track."

Susan's primary means of support was asking "leading questions." She described her questioning as intuitive, "I think adjusting…I don't know. Some of it comes kind of natural. It's hard to think about why." This indicates her questioning approach was part of her typical practice, something that comes naturally to her. These questions emerged as breaking the problem into small tasks and asking students to consider the situation with simpler numbers. In the second interview, Susan's comment about solving missing value ratio tasks shows how she used leading questions:

> I start with 'What would one look like?'...and then just start scaffolding from there. 'How could you build on to that?' Just asking those questions. So that' s kind of where I went with at least one of the groups and some of them, once they got it and made [the model], you know really got that base, could go a little deeper. But yeah, the question is really important.

Susan described asking questions that focus the student on modelling the basic ratio with a unit rate ("what would one look like?") before scaling the ratio up ("build on that"). She felt asking questions that focus attention on the mathematical relationship (a unit rate) would help students to understand and apply the relationship to the more complex ("deeper") aspects of the task.

Value in Co-teaching & Productive Struggle

Gayle and Susan described seeing value in using productive struggle as an instructional practice. Gayle in particular had overcome initial scepticism to embrace productive struggle:

> What I love about what I'm doing is when we go into [the demonstration], I can say, 'Okay, this is what we are learning today, these are the objectives. We just did that, right? You guys just did that.' and see the connection. And I really like that better. In the past, I've kind of done a real quick check or a review. And I really like the struggle, because it's an introduction. And even if they don't always quite get it when they get it, I felt like it was worth going through. But I like that, even if they don't quite get it in the beginning with the struggle, when we do the lesson, they make the connection. And I think there's value in that, and I really love doing struggle first. And I didn't think I would.

Susan confirmed Gayle's initial resistance to using productive struggle by saying, "Yeah, she really had her heels dug in." Gayle specifically identified the value in productive struggle as offering a foundation for the teacher-led instruction that follows.

Susan realized she had already included some struggle in her teaching in the past but found new benefits,

> It's built into a lot of the stuff we do, we just didn't recognize it as struggle. Like we didn't call it that, we didn't have a name for it, but you already do that a lot. Since we've focused on it, it's intentional. We intentionally find something that is going to challenge them, you know, to make them have to think about it more. And, it's been fun. I think that kids have enjoyed math. Surprisingly, they do not complain about having two hours of math a day.

Susan noticed that by being more intentional about incorporating productive struggle, her lessons asked students to engage more thoughtfully with the mathematics. She specifically identified the value in productive struggle as bringing more enjoyment to the lessons.

They found their co-teaching structure, teaching sequential classes, was pivotal in making productive struggle work in their classrooms. They were able to coordinate their lessons daily, the structure gave them two full class periods for teaching, and this in turn provided time for them to incorporate other instructional practices they felt were crucial. For Gayle, she felt it was crucial to have a sufficient amount of time following the problem solving and collaborative group presentations to show students how the model should be used. By having Susan's class follow hers, she felt students were given adequate practice time after her demonstration. Despite Gayle's enthusiasm for productive struggle, she felt that she would have to give up the practice of productive struggle if there was only one class period.

The sequential classes provided time for Susan to incorporate something she viewed as crucial – increasing the background knowledge of students who struggle. Because they come to her class with "a little bit of knowledge that day from what [Gayle's] working on first", Susan had time to incorporate productive struggle alongside a "concept board." This concept board focused on review of important concepts and skills that are fundamental to grade 6 mathematics. She felt strongly about being able to continue this practice because she felt it contributed to growth in students' scores on achievement tests in the past, "It's proven to work, and I use that. I mean obviously there's my other instruction, but [for some students] seeing that and just reviewing and seeing and hearing it. I'm kind of hooked on it. I would have a hard time giving that little portion up." Despite Susan's appreciation of productive struggle, her comment suggests she would choose to implement the concept board if time were more limited.

Overall, Gayle's and Susan's partnership can be described as negotiating boundary spaces through coordination (Akkerman & Bakker, 2011). The shared focus on sense-making, with some differences in their views of models, illustrates how they negotiated compatible meaning for the boundary object. Gayle and Susan confirmed they both used the cooperative groups and leading questions. However, they emphasized these different ways of supporting productive struggle in their interviews and artifacts. This is indicative of coordinating some approaches while valuing them differently. Their implementation of productive struggle did not include extended opportunities for problem solving and perseverance described by mathematics education, nor was it application of a previously taught idea in a new context as described by special education. Instead, it was a scaffolded opportunity to use a model to find a solution to a novel problem. Productive struggle was viewed as a valuable addition to their existing routines.

Gemma and Sally – Concurrent Co-teaching with a Focus on Mathematical Relationships

Gemma and Sally co-taught together in two mathematics classes with a "Gemma teach, Sally assist" approach. Gemma and Sally described understandings and approaches for working with productive struggle which were similar to one another and became more nuanced over time. Their attention was on the mathematical goal of the particular lesson and the specific needs of individuals in their class.

Meanings for Productive Struggle

Gemma and Sally initially described productive struggle as sense-making about mathematical patterns or relationships, but over time their descriptions added other dimensions. Early in the year, these descriptions were about "stumbling into" seeing mathematical relationships. When describing the goal of a lesson early in the year, Gemma said, "I wanted them to stumble into finding what a unit rate was on their own because we had not yet discussed the vocabulary term itself." In a reflection, Sally identified the struggle as making sense of ratio relationships and found it was productive even if students did not reach the level of understanding the teachers hoped for, "There was productive struggle because they could recognize additive [relationships] in the table [of values], but I want them to do more multiplicative [thinking]." The idea of students discovering patterns or relationships remained important later in the year when Gemma said, "I just want to keep using the word stumble," as she described the goal of sense-making about rational number magnitude with number lines. Each of these quotes indicates productive struggle involved students recognizing mathematical patterns or relationships in content that was new to them.

These teachers described the models as flexible tools for seeing relationships. During a planning session, Gemma wanted to give students freedom to choose what tools they would use because she did not want to "stifle any kind of potential." In an interview, Sally said, "We want them to use tools to figure how to represent the ideas we're focusing on." They focused their conversations with individual students and with the whole class on models which students had employed. They asked students to explain their models to them and asked questions that directed attention to the mathematical relationship that was the goal of the lesson. For example, they asked students to explain what one portion of a diagram represented and then asked how it would change if quantities in the task changed or were added (e.g., "what if we cut our starting amount in half," or "how would this look if you had to include a fraction like ¼?")

Over time, Gemma and Sally added descriptions of mental effort and students' sharing their emerging ideas to their descriptions of productive struggle. In a reflection Gemma noted that "just trying was a big part of my vision for 'productive' today." In another reflection, Sally described being happy to see a particular student with more intensive support needs engage in productive struggle by working with peers instead of getting upset and giving up. In the third interview Gemma described how their idea of productive struggle had evolved during the year, "I think our idea of struggle was not helping, just letting them do everything on their own rather than engaging. But you know, as we did that, I realized that's not what productive struggle is. It evolved." Sally added, "Yeah, so I think it's a lot more like...the discussion and having to share thoughts…It really was the struggle part because it was a lot of them guiding conversation rather than us." Gemma continued:

> Yeah, and then, struggling through the math themselves was the struggle, more than just trying it…struggling in persevering. That was the bigger part that I've seen that is like, okay, so if I give them something that's tough, it still needs to be attainable. Regardless of, I kind of want to say regardless of what their abilities were, to just be reachable in some way. So [we're] just thinking about it a different way than we had before.

Gemma and Sally incorporated new ideas into their understanding of productive struggle that involved more than just the goal of discovering mathematical patterns and relationships. Their use of phrases such as "just trying," "discussion and sharing thoughts," and "struggling in persevering" are indicative of incorporating more attention to the process of learning along with the goal of stumbling on patterns or relationships.

Implementing Productive Struggle

Though they began with generic structures to support students in productive struggle, Gemma and Sally used more lesson specific supports over time. At the beginning of the year, an artifact shows Sally's

planned supports for students with special needs could be applied no matter the lesson content: to scribe for students who had difficulty with writing or working memory, to read instructions and problems to students, and shorten assignments. She added a note to this artifact stating she felt it was important to plan for specific needs of individuals. In the same artifact Gemma described making sure that the numbers in the problems would be "friendly" enough for students to use. These kinds of generic supports for access were used throughout the year.

In keeping with their evolving views of productive struggle, Sally told me she "intentionally added a student to each small group who is confident and friendly to promote discourse." She modified an assignment for non-verbal student by giving him conversation cards to try to include him in the group discussion. By the third and fourth planning recording, they were explicitly planning for questions or tasks that would extend the activity for students who identified the relationship or solved the problem more quickly than others.

Over time, their discussions attended more to lesson-specific support. In the midst of a planning meeting, Sally said she was having trouble figuring out how to support students because she was not sure what the primary goal of the lesson was. Gemma shared her perspective on the conceptual focus and how it fit into prior and future lessons. Once they agreed upon the goal of the current lesson, they began brainstorming how to offer specific guidance. They discussed which visual models would better support students in comparing ratios based on how students had solved problems in the past. Gemma said the ratio table is good because it stores information, but "I really think the tape diagram helps kids visualize what's going on." Sally added, "It helps them visualize the total part." Together they decided what types of responses from students, rather than which students, would lead them to guide students to use a tape diagram.

Value in Co-teaching & Productive Struggle

Gemma and Sally described seeing a lot of student learning through productive struggle, and they strongly valued their co-teaching partnership. During an interview, I specifically focused discussion on the question of co-teaching in inclusive classrooms when aiming to incorporate productive struggle. Gemma commented on the value of inclusion in lessons that ask students to generate ideas,

> [Student A] does math so differently than how anyone else does. So having him be able to work with someone else and others be like, 'Oh! Okay! I can kind of see how you're doing that.' So it's been beneficial on both sides. And having [student A] work with [student B], that would have never happened before, right? That happened yesterday, and that would never, there was no way that would ever have happened...

Sally finished the sentence, "Organically." Gemma said, "Yes, organically." They went on to describe how together they were able to plan for students' engagement with one another. In this interview Sally also added, "Definitely these kids are achieving higher than I've seen kids before. Like what they're able to do is a drastically higher level than in past years." These teachers felt productive struggle supported by their co-teaching partnership was benefiting all students.

Albeit limited to two meetings per week, Gemma felt their planning time together helped her to address struggle for a more diverse group of learners:

> And I know that was a really big help for me to have going into the day…I knew what they were going to struggle with, and I had a plan as to how I was going to address that struggle with everyone and kind of keep that more consistent than I have in the past, ever.

Gemma also specifically described a reason she valued working with Sally:

> If I'm not feeling super confident in what accommodation to give or what an accommodation could be for something, [working with Sally] takes the pressure off me to have whatever it is right. I don't know how much to take off, or I don't know if changing this word or changing this number is, you know, doing more than just accommodating. So that's, you know, that is helpful.

Gemma also valued having Sally in the room because it allowed for "feeling things out in the moment, and learning from in-the-moment input for future planning." Despite taking on the role of leading the instruction while Sally assisted, it was clear that Gemma felt she was becoming a more consistent, confident teacher and learning about supporting diverse learners as a result of working with Sally.

Sally did not voice discontent with her role as "assistant" during lessons, perhaps because of their more equitable contributions during planning sessions. In fact, Sally voiced how important she felt the partnership was: "I wish all classes had two teachers. Let's get rid of all of SPED [special education services outside the general education classroom] and just put more teachers in classrooms." She continued, "Everybody needs individualized education, you know? Right. Let's just have enough people that can do that for everybody." Sally attributed their partnership to their ability to individualize instruction to meet student needs and hoped to sustain the partnership in the future.

Overall, Gemma's and Sally's partnership can be described as negotiating boundary spaces through reflection (Akkerman & Bakker, 2011). Their meanings for productive struggle initially focused on modelling to understand relationships but gradually added processes such as perseverance and communicating emerging ideas. They learned about practice from one another—Gemma learned about adapting for individual needs and Sally learned about conceptual trajectories for mathematics lessons. They aimed continue their partnership, finding that it would benefit their future practice to jointly use their expertise to help meet the needs of individual students.

Discussion

Productive struggle, as a boundary object between the fields of mathematics education and special education, is a potential source of disharmony for co-teachers working in inclusive mathematics classrooms. However, the teachers who participated in this comparative case study authored understandings of this boundary object that were harmonious. Their agency is evident in implementation of productive struggle in ways which fit their pedagogical beliefs and commitments. Also, they described the practice as valuable and attributed their partnerships as helping to make it successful. Thus, the authority and agency they used around this instructional practice may contribute to its coherence within their practice more broadly and to the sustainability of the partnership.

Authoring Meanings

Meanings for the boundary object, productive struggle, emerged in ways that were coherent for each pair of teachers. The pairs of teachers described productive struggle in different ways, but the meanings held by the individual teachers were compatible with those held by their partner. Gayle's and Susan's descriptions of productive struggle remained consistent. They described productive struggle as sense-making, specifically making sense of a mathematics problem using a visual model, though they discussed models differently. Gayle viewed models as representations of a mathematical idea, while Susan viewed them as representations of students' thinking. Gemma and Sally viewed productive struggle as sense-making about mathematical patterns and relationships, but their perceptions co-evolved over the school year to include students' persistence in the face of challenge and communicating emerging ideas.

Teachers' authority is evident in these characterizations of productive struggle; they authored meanings that work in their contexts rather than align to an external disciplinary definition. Unlike a position

common in the field of special education, they did not position productive struggle as the application of a previously taught procedure (The Science of Math, 2023). Their focus on sense-making is aligned in part with the descriptions of productive struggle found in mathematics education literature (Hiebert & Grouws, 2007; Henningsen & Stein, 1997; National Council of Teachers of Mathematics, 2014). The purpose and measure of success for sense-making in Gayle's and Susan's instruction was to find the right answer, rather than develop cognitive processes related to problem solving (Hiebert & Grouws, 2007; Kapur, 2016). Gemma's and Sally's gradual incorporation of perseverance and communication demonstrates more alignment with mathematics education descriptions over time. Their perspective also became more aligned with the features provided by the research project–sustained mental effort, focus on sense-making, and engagement with important mathematics (Crawford et al., 2020). However, the evidence suggests this was authored, not given to them, as this alignment emerged gradually and was elicited by their observations of their students.

Agency in Implementation

These pairs of teachers planned to implement productive struggle in contrasting ways, yet both pairs developed an approach that was coherent with the practices they valued. Gayle and Susan employed consistent structure and supports for struggle. This consistency indicates they were able to add productive struggle to their practice in a way that was coherent with their priorities for teaching – teacher demonstration (in Gayle's case) and development of background knowledge (in Susan's case). Both Gayle and Susan offered support by suggesting students use simpler number sets or breaking down the process into smaller steps through intentional questioning. These supports might be considered as lowering the cognitive demand by some mathematics education researchers (Stein & Lane, 1996; Warshauer, 2015), but Gayle and Susan found they helped all students achieve the goals they set for them. Gemma and Sally had a dynamic approach for encouraging productive struggle. They identified goals for particular lessons and ways to support productive struggle that aligned with those goals. This approach was coherent with Gemma's belief in the importance of students constructing their own understandings and Sally's belief in the importance of meeting needs of individuals to assure accessibility.

Adaptations can be seen as a form of agency, equalizing teachers in the teacher-researcher boundary encounter. The co-teachers in these cases implemented productive struggle by providing support that guided or focused students' thinking. They fit productive struggle into their practice in ways that were consistent with their characterizations of the practice and which they felt encouraged all their students to participate. Agency rather than fidelity to researchers' visions allowed for cohesion between beliefs about mathematics pedagogy, meanings for struggle, and implementation for struggle. The findings in this comparative case study reaffirm the importance of viewing teachers as experts in their local contexts and learning from them as they adapt practices for their contexts (Cai & Hwang, 2021; Krainer, 2021).

Co-teaching & Productive Struggle Add Value to Their Practice

Productive struggle and their co-teaching partnerships were viewed as valuable by all four of these teachers. Contrary to claims by some in special education (The Science of Math, 2023), these teachers did not find struggle led to misconceptions, frustration, or wasted instructional time. The teachers expressed a sense that their students were learning more than students had in the past. The partnerships were described as central in making productive struggle happen. Further, each teacher contributed something to the implementation of productive struggle that was rooted in their professional backgrounds and valued practices: Gayle established cooperative groups and following struggle with clear demonstrations; Susan incorporated systematic review through concept boards; Gemma centered students' reasoning about key mathematical concepts; and Sally contributed carefully crafted supports individualized to lesson goals. As a result, all of these teachers believed the practice was contributing to student learning in noticeable ways.

Both pairs of teachers felt that their co-teaching arrangement was pivotal in making productive struggle work. These teachers did not receive any training or support for co-teaching. These arrangements were negotiated in their boundary encounter as teachers with differing disciplinary backgrounds. For each pair, the negotiated arrangements were coherent with other practices and beliefs, and the affordances that emerged were viewed as valuable. The consistency seen in Gayle's and Susan's partnership offered affordances for the feasibility, and thus sustainability, of inclusion with co-teaching arrangements in mathematics classes: predictability for students, sufficient review, and an efficient framework for planning. The dynamic nature of Gemma's and Sally's approach offered different affordances for sustainability, affordances which offer equitable access to mathematics curricula: alignment to mathematical ideas and intentional tailoring of lessons for individual differences. The teachers clearly voiced a desire to sustain the partnerships into the future.

Co-teaching as a Boundary Space

Because of their ambiguity, boundary spaces afford different forms of interaction and exchange: co-existence of distinct communities; coordinated activity; creative reflection and reconstruction of practice; and transformative change (Akkerman & Bakker, 2011). Gayle's and Susan's partnership could be characterized as coordination (Akkerman & Bakker, 2011). They maintained a continual flow of information which coordinated their meanings for productive struggle and ensured their lessons were complementary. Thus, the boundary object and their shared learning around it permeated but did not reconstruct their practice. Gemma's and Sally's interactions in the boundary space were characteristic of reflective interaction (Akkerman & Bakker, 2011). They evidenced reflection on their understandings of productive struggle and how their pedagogy around it changed. Their planning sessions were dialogic and creative in the sense that they progressed toward designing lessons with consideration of specific students and specific learning goals. Gemma in particular described the partnership as informing her future practice. The contexts of consecutive and concurrent teaching may have influenced the forms of interaction that emerged, coordination and reflection respectively, revealing how context can be a factor in shaping these processes, and individuals can construct relationships that work for them (Akkerman & Bakker, 2011; Naraian & Schlessinger, 2018).

The boundary spaces these teachers constructed offer a few important considerations for researchers. The forms of interaction which occur in boundary spaces, from co-existence to transformation, can connote a continuum (Akkerman & Bakker, 2011). Yet, the success of these partnerships indicates this continuum should not be assumed to have a negative-to-positive valence. Also, despite some differences in meanings and unequal status in instructional delivery in a complex boundary space, these teachers established coherent practices and sustainable partnerships. They did this despite receiving no training in co-teaching models. This speaks to the professionalism of these teachers, and the value of having authority and agency in teaching practice. Further the teachers' participation in this study enables learning on the part of the fields of mathematics education and special education about the feasibility, utility, and value of productive struggle in real classroom contexts. Insistence on a researcher-defined vision may hinder the coherence and sustainability of its implementation in inclusive classrooms.

Conclusion

Teacher partnerships are a core feature of inclusive education. Many studies have documented challenges in sustaining these partnerships, and this one investigated productive struggle as potentially adding another source of challenge as a boundary object between two disciplines. However, the co-teachers in this comparative case study authored meanings for productive struggle that did not align perfectly with the disciplinary depictions, and they implemented these practices in ways that were coherent with their perspectives and experiences in teaching. Together authority, agency, and perceived value contributed to making what could be a problematic boundary object a valued component of these co-teachers' instruction. This study adds to emerging mathematics education literature which positions

teachers as key stakeholders in translating research-to-practice (see, for example, Cai et al., 2020; Helenius, 2021). This study also offers an illustration of how a boundary object can be successfully negotiated in teaching contexts when teachers have authority and agency in implementation. This in turn highlights the value of further investigations of authority and agency in developing coherent and sustainable co-teaching partnerships.

References

Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*(2), 132–169. https://doi.org/10.3102/0034654311404435

Brendle, J., Lock, R., & Piazza, K. (2017). A study of co-teaching identifying effective implementation strategies. *International Journal of Special Education*, *32*(3), 538–550. Retrieved from https://eric.ed.gov/?id=EJ1184155

Cai, J., & Hwang, S. (2021). What does it mean to make implementation integral to research? *ZDM — Mathematics Education, 53*(5). https://doi.org/10.1007/s11858-021-01301-x

Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V., Cirrillo, M., Kramer, S. L., & Hiebert, J. (2020). Working across contexts: Scaling up or replicating with variations? *Journal for Research in Mathematics Education, 51*(3), 258-267. https://doi.org/10.5951/jresemtheduc-2020-0007

Cobb, P., Jackson, K., Smith, T., Henrick, E. (2017). Supporting Improvements in the Quality of Mathematics Teaching on a Large Scale. In S. Doff & R. Komoss, (Eds.), *Making change happen*. Springer. https://doi.org//10.1007/978-3-658-14979-6_17

Crawford, A. R., Carney, M., Champion, J., & Schmidt, M. (2022). The role of a boundary object in a study of middle grades math instruction. In A. E. Lischka, E. B. Dyer, R. S. Jones, J. Lovett, J. Strayer, & S. Drown (Eds.), Proceedings of the forty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 1725—1732). Middle Tennessee State University. https://doi.org/10.51272/pmena.44.2022

DeFouw, E. R., Codding, R. S., Collier-Meek, M. A., & Gould, K. M. (2019). Examining dimensions of treatment intensity and treatment fidelity in mathematics intervention research for students at risk. *Remedial and Special Education*, *40*(5), 298–312. https://doi.org/10.1177/0741932518774801

Friend, M., Cook, L., Hurley-Chamberlain, D, & Shamberger, C. (2010). Co-teaching: An illustration of the complexity of collaboration in Special Education. *Journal of Educational and Psychological Consultation, 20*(1), 9–27. https://doi.org/10.1080/10474410903535380

Grigorenko, E. L., Compton, D.. L., Fuchs, L. S., Wagner, R. K., Willcutt, E. G., & Fletcher, J. M. (2020). Understanding, educating, and supporting children with specific learning disabilities: 50 years of science and practice. *American Psychologist, 75*(1), 37–51. https://doi.org/10.1037/amp0000452

Helenius, O. (2021). A stakeholder analysis of the development of a large-scale professional development program. *Implementation and Replication Studies in Mathematics Education, 1*(1), 227–256. https://doi.org/10.1163/26670127-01010009

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for research in Mathematics Education, 28*(5), 524-549. https://doi.org/10.5951/jresematheduc.28.5.0524

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K., Lester (Ed.). *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Information Age Publishing.

Jaworski, B., & Potari, D. (2021). Implementation of a developmental model of teachers' and didacticians' learning through inquiry: Design, operationalisation and outcomes. *ZDM — Mathematics Education ,53*, 1073–1084. https://doi.org/10.1007/s11858-021-01290-x

Kapur, M. (2016). Examining productive failure, productive success, unproductive failure, and unproductive success in learning. *Educational Psychologist*, *51*(2), 289–299. https://doi.org/10.1080/00461520.2016.1155457

Koichu, B., Aguilar, M.S. & Misfeldt, M. (2021). Implementation-related research in mathematics education: the search for identity. *ZDM — Mathematics Education* 53, 975–989. https://doi.org/10.1007/s11858-021-01302-w

Krainer, K. (2021). Implementation as interaction of research, practice, and policy. Considerations from the Austrian initiative IMST. *ZDM — Mathematics Education, 53*(5), 1175–1187. https://doi.org/10.1007/s11858-021-01300-y

Kretlow, A. G., & Bartholomew, C. C. (2010). Using coaching to improve the fidelity of evidence-based practices: A review of studies. *Teacher Education and Special Education, 33*(4), 279–299. https://doi.org//10.1177/0888406410371643

Mastropieri, M. A., Scruggs, T. E., Graetz, J., Norland, J., Gardizi, W., & McDuffie, K. (2005). Case studies in co-teaching in the content areas: Successes, failures, and challenges. *Intervention in School and Clinic, 40(*5), 260–270. https://doi.org/10.1177/10534512050400050201

McLeskey, J., Maheady, L., Billingsley, B., Brownell, M. T., & Lewis, T. J. (Eds.). (2022). *High leverage practices for inclusive classrooms*. Routledge.

Munter, C., Stein, M. K., & Smith, M. S. (2015). Dialogic and direct instruction: Two distinct models of mathematics instruction and the debate(s) surrounding them. *Teachers College Record, 117*(11), 1–32. https://doi.org/10.1177/016146811511701102

National Council of Teachers of Mathematics (2014). *Principles to actions: ensuring mathematical success for all.* NCTM.

Naraian, S., & Schlessinger, S. (2018). Becoming an inclusive educator: Agentive maneuverings in collaboratively taught classrooms. *Teaching and Teacher Education, 71,* 179—189. https://doi.org/10.1016/j.tate.2017.12.012

Nelson, G., Kiss, A., & Crawford, A. (2022). Developmental distinctions in mathematics for students with disabilities. In E. Talbott & T. Farmer (Eds.) *Handbook of research on special education.* Routledge Press.

Newmann, F. M., Smith, B., Allensworth, E., & Bryk, A. S. (2001). Instructional program coherence: What it is and why it should guide school improvement policy. *Educational Evaluation and Policy Analysis, 23*(4), 297–321. https://doi.org/10.3102/01623737023004297

Otten, S., Webel, C., & de Araujo, Z. (2017). Inspecting the foundations of claims about cognitive demand and student learning: A citation analysis of Stein and Lane (1996). *The Journal of Mathematical Behavior*, *45*, 111–120. https://doi.org/10.1016/j.jmathb.2016.12.008

Polkinghorne, D. E. (1995). Narrative configuration in qualitative analysis. *Qualitative Studies in Education, 8*(1), 5–23. https://doi.org/10.1080/0951839950080103

Powell, S. R., Bouck, E. C., Sutherland, M., Clarke, B., Arsenault, T. L., & Freeman-Green, S. (2022). Essential components of math instruction. *Teaching Exceptional Children.* https://doi.org/10.1177/00400599221125892

Sanetti, L. M. H., & Luh, H. J. (2019). Fidelity of implementation in the field of learning disabilities. *Learning Disability Quarterly, 42*(4), 204–216. https://doi.org/10.1177/0731948719851514

Star, S. L. (2010). This is not a boundary object: Reflections on the origin of a concept. *Science, Technology, and Human Values, 35*(5), 601–617. https://doi.org//10.1177/0162243910377624

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2,* 50–80. https://doi.org/10.1080/1380361960020103

Sztajn, P., Wilson, P. H., Edgington, C., & Myers, M. (2014). Mathematics professional development as design for boundary encounters. *ZDM — Mathematics Education, 46*(2), 201–212. https://doi.org/10.1007/s11858-013-0560-0

Tamborg, A.L. (2021). Improving mathematics teaching via digital platforms? Implementation processes seen through the lens of instrumental genesis. *ZDM—Mathematics Education, 53,* 1059–1071. https://doi.org/10.1007/s11858-021-01282-x

The Science of Math (2023). *Common Misconceptions: Productive Struggle Causes More Robust Understanding and Learning.* https://www.thescienceofmath.com/misconceptions-productive-struggle-causes-more-robust-understanding-and-learning

United Nations. (2016). *United Nations Convention on the Rights of Persons with Disabilities Committee's General Comment No. 4 on the Right to Inclusive Education.* Retrieved from http://www.right-to-education.org/resource/general-comment-4-article-24-right-inclusive-education.

Warshauer, H. K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education, 18*(4), 375-400. https://doi.org/10.1007/s10857-014-9286-3

Woodward, J. (2004). Mathematics education in the United States: Past to present. *Journal of Learning Disabilities, (37)*1, 16-31. https://doi.org/10.1177/00222194040370010301

Young, J. R., Bevan, D., & Sanders, M. (2024). How productive is the productive struggle? Lessons learned from a scoping review. *International Journal of Education in Mathematics, Science, and Technology, 12*(2), 470-495. https://doi.org/10.46328/ijemst.3364