

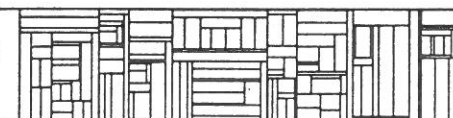
An Introduction to High-Level Petri Nets

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AN INTRODUCTION TO HIGH-LEVEL PETRI NETS

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Abstract

This paper describes a Petri net model, where information is attached to each token and when a transition fires, it can inspect and modify the information. The model is based on predicate/transition-nets (Genrich & Lautenbach) and on coloured Petri nets (Jensen).

This generalization of ordinary Petri nets allows, for many applications, more manageable descriptions, due to the fact that equal subnets can be folded into each other yielding a much smaller net. The paper investigates how to analyse high-level Petri nets, and it turns out that invariants and reachability trees, two of the most important methods for ordinary Petri nets, can be generalized to apply for high-level Petri nets.

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1. INTRODUCTION

In this paper we will assume the reader to be familiar with ordinary Petri nets, also known as place/transition-nets (PT-nets). On this basis we will introduce high-level Petri nets (HL-nets) by means of an example, which describes how the public telephone system works. Using the example we shall also indicate how HL-nets can be analyzed by means of invariants and by means of reachability trees.

Our aim is to introduce HL-nets, and to give a first impression of their modelling power and the suitability of their analysis methods. The present paper will be very informal and rely heavily on the chosen example. For the formal definitions of HL-nets and their analysis methods we refer to the literature, in particular to [1-5]. In practice, HL-nets have been applied to describe or specify a large variety of items, such as communication protocols, distributed databases, production flow in car factories, work organization in libraries, and the formal semantics of programming languages.

2. HL-NETS AS A DESCRIPTIVE TOOL

In HL-nets information can be attached to each token as a token-colour and each transition can fire in several ways represented by different firing-colours. The relation between a firing-colour and the involved token-colours is defined by expressions attached to the arcs. By the colours, it is now possible to distinguish between different processes, even though their subnets have been folded into a single subnet. It should be emphasized that the "colour" attached to a token or a firing

can be a complex information unit, such as the entire state of a process or the contents of a buffer area. New colours can be created by transition firings and there may be an infinite number of them.

In Figure 1 we use an HL-net to describe the public telephone system, as it may be viewed by users. The places are drawn in three different ways. Double circles represent states of calling phones, thick circles represent states of called phones, and finally thin circles represent the status of the telephone exchange. The first two kinds of places have the set of all phone numbers U as their set of possible token-colours, while the exchange places have $U \times U$ as possible token-colours. Each token on the first two types of places indicates by its colour a certain phone and by its position a certain state of that phone. Each token on the exchange places indicates by its colour a possible connexion (consisting of a calling phone and a called phone), and it indicates by its position the state of progress for establishing that connexion.

Analogously to the places, transitions are drawn in a way which makes it easy to distinguish between the behaviour of calling and called phones. All transitions have U or $U \times U$ as their set of possible firing-colours, depending on whether one or two phone numbers are involved in the corresponding action. As an example, LIFT1 represents the removal of the receiver at some phone $x \in U$ without involving other phones, and thus LIFT1 has U as the set of possible firing-colours. In contrast to this, LIFT2 represents the removal of the receiver at some phone $y \in U$ being called from another phone $x \in U$, and thus LIFT2 has $U \times U$ as possible firing-colours.

The arcs connecting places and transitions are also drawn in three different ways. It should, however, be noted that this has a formal meaning, in contrast to the drawing of places and transitions, which have only the purpose of enhancing readability. Each arc in an HL-net carries an expression describing how to calculate the token-colour (which is removed or added at the place) from the firing-colour of the transition. In Figure 1, these expressions are indicated by the shape of the arcs. A double arc indicates that a token with colour $x \in U$ is moved along that arc, a thick arc indicates the move of a token $y \in U$, and a thin (dashed) arc indicates the move of a token $(x, y) \in U \times U$. By using the variables x and y in a systematic way, we again obtain that double arcs relate to calling phones, thick arcs to called phones, and thin arcs to the manipulation at the telephone exchange.

When a transition fires all variables x and y on the surrounding arcs must be bound to values in U . If several x 's (or y 's) appear around a transition, they must all be bound to the same U -value when the transition fires. But at the next firing they may all be bound to another U -value. As an example, the transition FREE moves a calling phone $u_1 \in U$ from a state with NO TONE to a state with LONG intervals, and a called phone $u_2 \in U$ from being INACTIVE to RINGING. This can only happen if there is a REQUEST $(u_1, u_2) \in U \times U$, which then advances to be a CALL.

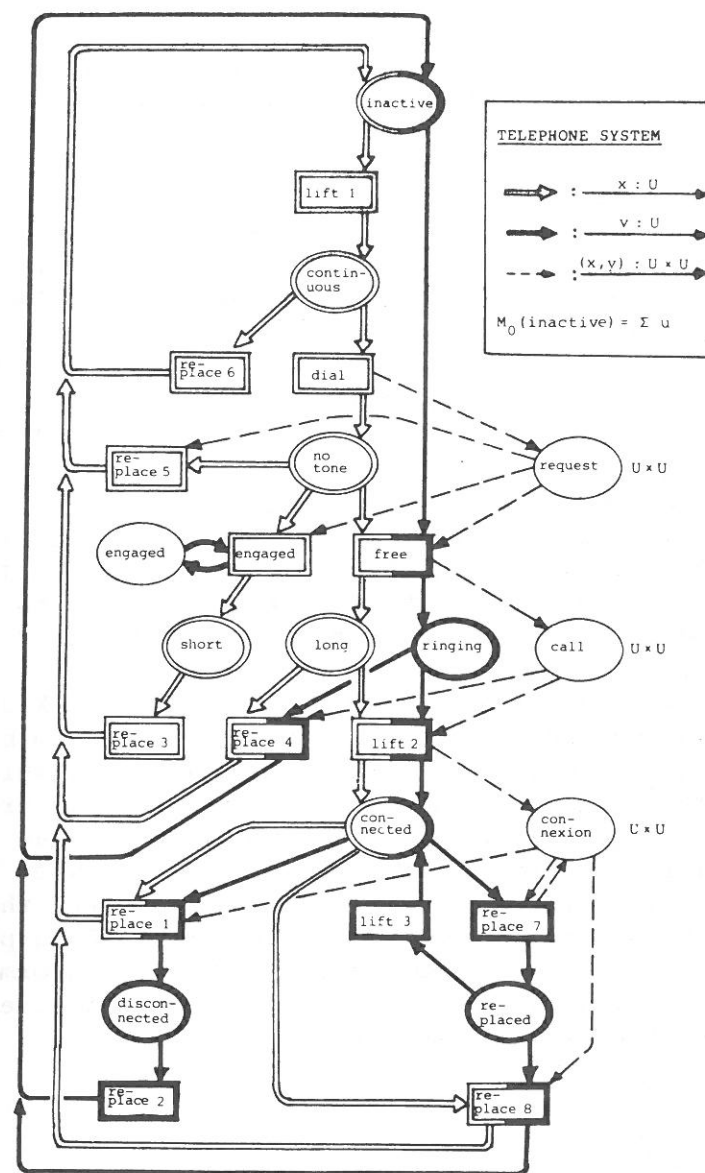


Figure 1: HL-net describing a telephone system

ENGAGED is the complement of INACTIVE. This means that ENGAGED is marked with a colour $u \in U$ iff INACTIVE is not. We have omitted the arcs which update ENGAGED. It is now possible to interpret the HL-net in Figure 1 as follows. Originally all phones are INACTIVE. The status of a phone may change from INACTIVE to the situation where the receiver has been LIFTed and you hear a CONTINUOUS tone. Next a number may be DIALled and you hear NO TONE, until you either hear a tone with SHORT intervals (indicating that the dialled phone is ENGAGED) or a tone with LONG intervals (indicating that the dialled phone is RINGING). In the latter situation the receiver may be LIFTed at the called phone and the two phones are CONNECTED until one of the two receivers are REPLACed. When a number is DIALled, a REQUEST is set up at the telephone exchange. If the dialled phone is INACTIVE, the REQUEST may be transformed to a CALL, and next to a CONNEXION, if the receiver is LIFTed at the called phone.

This describes how to establish a CONNEXION, while the lower part of the net tells how to break it. The HL-net constitutes a formal model, which allows us to determine even the more subtle properties of the specified telephone system. As an example, we can investigate what happens when a phone is calling itself; and it can be seen that a CONNEXION can be removed only by the calling phone and not by the called phone. If only an informal description was given, it would be easy to overlook some of these special cases.

3. ANALYSIS BY MEANS OF INVARIANTS

A PT-net can be represented as an incidence-matrix, I . The matrix contains a row for each place and a column for each transition, and the matrix-element $I(p,t)$ describes the effect at place p when transition t is fired. This is done by an integer (positive when tokens are added and negative when removed). For HL-nets the situation is more complicated since we have to record, not only the number of moved tokens, but also their colour. This means that the matrix-elements now become functions which indicate how to calculate the moved token-colours from the firing colour. If place p has $C(p)$ as possible token-colours and transition t has $C(t)$ as possible firing-colours, the corresponding matrix element becomes a function $I(p,t) \in [C(p) \rightarrow C(t)]$.

The functions are deduced from the arc-expressions used in the graph-representation of the HL-net. As an example we get

$$\begin{aligned} I(\text{LIFT1}, \text{INACTIVE}) &= -\text{ID} \\ I(\text{LIFT1}, \text{CONTINUOUS}) &= \text{ID} \end{aligned}$$

where ID is the identity function on U . This describes that when LIFT1 fires with some firing-colour $u \in U$, we remove one token with colour u from INACTIVE (since $I(\text{LIFT1}, \text{INACTIVE})(u) = -\text{ID}(u) = -u$) and we add one token with colour u to CONTINUOUS (since $I(\text{LIFT1}, \text{CONTINUOUS})(u) = \text{ID}(u) = u$). Analogously

$$\begin{aligned} I(\text{LIFT2}, \text{LONG}) &= -P1 \\ I(\text{LIFT2}, \text{RINGING}) &= -P2 \\ I(\text{LIFT2}, \text{CALL}) &= -\text{ID} \end{aligned}$$

where P_1 and P_2 are the projections mapping an element $(u_1, u_2) \in U \times U$ into u_1 and u_2 , respectively, while ID now is the identity-function on $U \times U$. This describes that, when $LIFT2$ fires with some firing-colour (u_1, u_2) , we remove a token with colour u_1 from $LONG$, a token with colour u_2 from $RINGING$, and a token with colour (u_1, u_2) from $CALL$.

In general, it is possible for a transition to remove or add several tokens at the same place, and thus $I(p, t)$ must be a function mapping the firing-colours $C(t)$, not only into the set of token-colours $C(p)$, but into $BAG(C(p))$, which denotes the set of all bags (multisets) over $C(p)$. Moreover, it is possible for a transition to fire concurrently with itself. As an example $LIFT1$ may fire concurrently in two different firing-colours, since two receivers may be lifted at the same time. This means that we now demand the matrix-elements to be functions $I(p, t) \in [BAG(C(t)) \rightarrow BAG(C(p))]_L$ where the L -subscript denotes linear functions (with respect to normal addition and scalar-multiplication of bags). In the examples above ID becomes the identity-function on $BAG(U)$ or $BAG(U \times U)$, while $P_1, P_2 \in [BAG(U \times U) \rightarrow BAG(U)]_L$ are the unique linear extensions of the projections.

The incidence-matrix for the telephone-net is shown in Figure 2 (to the left of the first vertical double line). The set of token-colours is shown immediately after each place name, while firing-colours are shown immediately below the transition names. Between the two horizontal double lines we find the initial marking, where we have used Σu to denote the bag containing exactly one occurrence of each element in U .

TELEPHONE SYSTEM	PLACES									
	U	U	U	U	U	U	U	U	U	U
inactive	U	U	U	U	U	U	U	U	U	U
continuous	U	U	U	U	U	U	U	U	U	U
no tone	U	U	U	U	U	U	U	U	U	U
long	U	U	U	U	U	U	U	U	U	U
short	U	U	U	U	U	U	U	U	U	U
ringing	U	U	U	U	U	U	U	U	U	U
connected	U	U	U	U	U	U	U	U	U	U
disconnected	U	U	U	U	U	U	U	U	U	U
replaced	U	U	U	U	U	U	U	U	U	U
engaged	U	U	U	U	U	U	U	U	U	U
request	U	U	U	U	U	U	U	U	U	U
call	U	U	U	U	U	U	U	U	U	U
connexion	U	U	U	U	U	U	U	U	U	U

TRANSITIONS	PLACES									
	U	U	U	U	U	U	U	U	U	U
Lift 1	U	U	U	U	U	U	U	U	U	U
dial	U	U	U	U	U	U	U	U	U	U
free	U	U	U	U	U	U	U	U	U	U
engaged	U	U	U	U	U	U	U	U	U	U
Lift 2	U	U	U	U	U	U	U	U	U	U
repl 1	U	U	U	U	U	U	U	U	U	U
repl 2	U	U	U	U	U	U	U	U	U	U
repl 3	U	U	U	U	U	U	U	U	U	U
repl 4	U	U	U	U	U	U	U	U	U	U
repl 5	U	U	U	U	U	U	U	U	U	U
repl 6	U	U	U	U	U	U	U	U	U	U
repl 7	U	U	U	U	U	U	U	U	U	U
repl 8	U	U	U	U	U	U	U	U	U	U
Lift 3	U	U	U	U	U	U	U	U	U	U

Figure 2: Incidence-matrix for the HL-net in Figure 1 and six invariants

In PT-nets a weight-function W is a vector attaching to each place an integer as weight. For a marking M we may then talk about the weighted sum of tokens, where we before summation multiply the token count on each place with the corresponding weight. The weighted sum can be calculated as the matrix product $W \cdot M$ (where W is a matrix containing only one row and M contains only one column). In PT-nets we can represent a firing sequence by a vector X , which for each transition records the number of its firings. The total effect of the firing sequence can be calculated as the matrix product $I \cdot X$ (where I is the incidence-matrix). For PT-nets it is now easy to prove the following theorem, originally formulated by Lautenbach.

Theorem If a weight-function W satisfies $W \cdot I = 0$, we have $W \cdot M = W \cdot M'$ for all markings M and M' reachable from each other. \square

Thus we can, by solving a homogeneous matrix-equation, calculate weight-functions which have the property that the firing of transitions do not change their weighted sum. Such weight-functions are called place-invariants. By using the linear algebra of matrices with integer elements, we have got a tool to analyse PT-nets. It turns out that the method can be generalized also to apply for HL-nets. But in HL-nets the incidence-matrix contains functions and thus we shall also demand weight-functions to be vectors containing functions as elements. As an example consider the column W_3 in Figure 2. This is a weight-function attaching $-ID$ as weight to NO TONE and $P1$ as weight to REQUEST (while the other places have a zero-function as weight).

When we multiply the two matrices W and I containing functions, we do exactly as for matrices containing integers, except that each multiplication of two integers is replaced by a composition of two functions. Analogously, when we multiply the two matrices W (containing functions between bags) and M (containing bags) we replace each integer multiplication by a function-application. With this generalization, it is straightforward to prove that Lautenbach's theorem is also valid for HL-nets.

From Figure 2 it follows that $W_3 \cdot I = 0$ (check that the inner product of W_3 and an arbitrary column in I yields zero). As a consequence, the theorem now says that each marking M reachable from the initial marking M_0 satisfies the equation $W_3 \cdot M = W_3 \cdot M_0 = 0$, i.e.

$$-ID(M(\text{no tone})) + P1(M(\text{request})) = 0$$

or more straightforward

$$M(\text{no tone}) = P1(M(\text{request})).$$

Thus a phone u is in the state NO TONE if and only if there is a REQUEST containing u as its first element. Analogously the rest of W_1 - W_6 can be checked to be invariants. W_1 tells us that each phone is in exactly one state, while W_2 tells us that INACTIVE and ENGAGED are complementary places. We leave the interpretation of W_4 - W_6 to the reader.

We now know how to interpret invariants in HL-nets, but unfortunately it is not as easy to find them as for PT-nets, where we can use Gauss-elimination to solve the homogeneous matrix-equation in the theorem. In HL-nets we deal with much more complicated matrix-elements and no general method is known to calculate all invariants. This

may seem to be a serious problem for the use of HL-nets, but fortunately the situation is not at all that bad, due to several reasons. First of all, in practice you are never in a situation where you have an HL-net and no idea about its invariants. On the contrary, you normally have a very good idea about the invariants which the HL-nets is supposed to fulfil. What you really want to do is to check invariants, not to find them. Secondly [3] defines a set of transformation rules, which can often be used to obtain a matrix being much smaller but guaranteed to have exactly the same invariants as the original incidence-matrix. By inspection of the small matrix invariants can be guessed. Thirdly, a lot of current research is devoted to obtain methods which allow you to calculate all invariants of certain restricted but useful types.

PT-nets also have the concept of a transition-invariant, which is a firing sequence X with no effect, i.e. satisfying $I \cdot X = 0$. In [6] it is shown how to generalise transition-invariants to "relation-nets", which are equivalent to HL-nets.

4. ANALYSIS BY MEANS OF REACHABILITY TREES

The basic idea of a reachability tree is to organize all reachable markings in a tree-structure where each node has attached a reachable marking, while each arc has attached a transition and a firing-colour (which transform the marking of its source-node into the marking of its destination-node). Such a tree contains all reachable markings and all possible sequences of transition-firings. By inspection of the tree it is possible to answer a large number

of questions about the system. However, in general, the reachability tree described above will be infinite. For practical use it is necessary to reduce it to finite size. For HL-trees this is done by means of covering markings and equivalent markings. Covering markings are already well-known from the PT-trees defined by Karp & Miller and by Hack. Equivalent markings is a new concept for HL-trees, which generalizes the duplicate (or identical) markings used in PT-trees.

A marking m_1 in an HL-tree may have a successor m_2 which covers m_1 (in the sense that the bag-inclusion $m_2(p) \supseteq m_1(p)$ is satisfied for all places, and $m_2 \neq m_1$). We can then repeat the firing sequence leading from m_1 to m_2 an arbitrary number of times, and thus obtain an arbitrarily high occurrence-value for those colours which have increased from m_1 to m_2 . In the HL-tree we record this by modifying m_2 , such that each of the increasing colours now get the occurrence-value ∞ (indicating that they may occur an unlimited number of times). This reduction may result in a loss of information.

In the telephone example all phone numbers are considered to behave in an analogous way. Thus we say that the marking where two phones u_1 and u_2 are CONNECTED (while the rest are INACTIVE) is equivalent to the marking where two other phones u_3 and u_4 are CONNECTED (while the rest are inactive). More generally, we will consider two markings of the telephone-net to be equivalent if there exists a

permutation (renaming of phone numbers) which maps one of the markings into the other. When a reachability tree contains several equivalent markings, only one of them is further developed, while the rest become leaves of the tree. This reduction will not result in a loss of information, because we can construct the missing subtrees from the one developed.

The HL-trees turn out to be considerably smaller than the corresponding PT-trees. However, it is still possible to prove the same kind of properties about the system, as could be done by PT-trees.

To use HL-nets and PT-nets for practical purposes, it is necessary to develop edp-systems which can assist the user in the construction, editing and analysis of nets. A variety of such tools is described in [7-9].

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