LOWER BOUNDS ON THE COMPLEXITY OF SOME PROBLEMS CONCERNING L SYSTEMS

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ABSTRACT

This is the second of two papers on the complexity of deciding membership, emptiness and finiteness of four basic types of Lindenmayer systems: the EDOL, EOL, EDTOL and ETOL systems. For each problem and type of system we establish lower bounds on the time or memory required for solution by Turing machines, using reducibility techniques. These bounds, combined with the upper bounds of the preceding paper, show many of these problems to be complete for $\mathcal{NP}$ or PSPACE.

1. INTRODUCTION

In this paper we complete the program started in [7], of determining tight bounds on the complexity of several problems concerning L systems. We establish lower bounds in this paper; with the results of [7] it is shown that most of the problems are complete, either for non-deterministic polynomial time or for polynomial space. Consequently it is unlikely that efficient algorithms can be developed to solve them.

We use the terminology of [7]. The following table summarizes the known results (ours and previous ones), with the context-free and context-sensitive cases included for comparison.

GRAMMAR CLASS	PROBLEM: MEMBER (FIXED G)	MEMBER (GENERAL)	NONEMPTY	INFINITE	BOUNDS
CONTEXT SENSITIVE	NSPACE(n)	NSPACE (n log n)	UNDECIDABLE	UNDECIDABLE	UPPER
		NSPACE(n)			LOWER
ETOL, EPTOL	$\mathcal{NP}$	NSPACE (n log n)	NSPACE(n)	NSPACE(n)	UPPER
		NSPACE $(n^{1-\epsilon})$	NSPACE $(n^{1-\epsilon})$	NSPACE $(n^{1-\epsilon})$	LOWER
EDTOL, EPDTOL	$\mathcal{NL}$	NSPACE (n log n)	NSPACE(n)	NSPACE(n)	UPPER
		NSPACE $(n^{1-\epsilon})$	NSPACE $(n^{1-\epsilon})$	NSPACE $(n^{1-\epsilon})$	LOWER
EOL, EPOL	DSPACE($\log^2$ n) DTIME(n^4)	$\mathcal{NP}$	DSPACE(n)	NSPACE(n)	UPPER
	$\mathcal{NL}$		$\mathcal{NP}$	$\mathcal{NP}$	LOWER
EDOL, EPDOL	$\mathcal{L}$	$\mathcal{P}$	$\mathcal{NP}$	$\mathcal{NP}$	UPPER
	$\mathcal{L}$	$\mathcal{L}$			LOWER
CONTEXT FREE	DSPACE($\log^2$ n) DTIME(n^3)	$\mathcal{P}$	$\mathcal{P}$	$\mathcal{P}$	UPPER
	$\mathcal{NL}$				LOWER

The results of the top and bottom rows and the leftmost column are known, and may be found in [4],[5],[6],[9],[10],[13],[14],[15],[16],[17], and [18].

Let $M \subseteq \Sigma^*$. We say that M is <u>$\mathcal{NP}$-hard</u> if any set in $\mathcal{NP}$ is (polynomially) reducible to M. We use polynomial-time many-one reducibility - namely L is reducible to M just in case there is a polynomial-time-computable function f such that for all x, $x \in L$ if and only if $f(x) \in M$. M is <u>complete</u> for $\mathcal{NP}$ if M is $\mathcal{NP}$-hard, and M is in $\mathcal{NP}$. To show that a problem M is $\mathcal{NP}$-hard it suffices to show that some other problem already known to be $\mathcal{NP}$-hard is reducible to M (this follows since reducibility is transitive). Hardness and completeness can also be defined for PSPACE, in the same way. An introduction to these topics may be found in [1].

More refined notions of reducibility are needed to formulate and study completeness for NSPACE(n), $\mathcal{P}$ and $\mathcal{NL}$, but will not be defined here, since we prove no new results for these classes.

2. SYSTEMS WITHOUT TABLES

Theorem 1 There is an EPD0L-system G, such that if L(G) is in DSPACE (S(n)), then

$$\sup_{n \to \infty} \frac{S(n)}{\log n} > 0$$

Proof $L = \{ a^n b c^n \mid n \geq 0 \}$ is an EPD0L-language. By Alt and Mehlhorn [2], if L is in DSPACE(S(n)) then S must satisfy the condition above. □

Theorem 2 NONEMPTY^{EPD0L} is $\mathcal{NP}$ hard.

Proof By Stockmeyer & Meyer [12] the following problem is $\mathcal{NP}$-hard:

Given a regular expression R of the form

$0^{p_1} (0^{q_1})^* + \dots + 0^{p_r} (0^{q_r})^*$

to determine whether $L(R) \neq 0^*$.

Construct an EPD0L system $G = (V, P, Z_1^0 \dots Z_r^0, \Sigma)$ where

$V = \{ Z_i^j \mid 1 \leq i \leq r,\ 0 \leq j \leq p_i + q_i - 1 \}$, $\Sigma = V - \{ Z_1^{p_1}, Z_2^{p_2}, \dots, Z_r^{p_r} \}$

and P consists of the productions $(i = 1, \dots, r)$:

$Z_i^j \to Z_i^{j+1}$ for $j = 0, \dots, p_i + q_i - 2$, and $Z_i^{p_i + q_i - 1} \to Z_i^{p_i}$.

Now $L(R) \neq 0^*$ iff $L(G) \neq \emptyset$ iff $\overline{G} \in$ NONEMPTY^{EPD0L}.

Clearly $\overline{G}$ can be constructed from R in polynomial time, so NONEMPTY^{EPD0L} is $\mathcal{NP}$-hard.

□

<u>Corollary 3</u> INFINITE^{EPD0L} is $\mathcal{NP}$ hard.

<u>Proof</u> Obtain a new grammar G' from G by replacing $Z_i^{p_i + q_i - 1} \to Z_i^{p_i}$ by $Z_i^{p_i + q_i - 1} \to Z_i^{p_i} Z_i^{p_i}$.

Then L(G') is infinite iff $L(R) \neq 0^*$

□

<u>Corollary 4</u> The following problems are $\mathcal{NP}$-complete:

NONEMPTY^{ED0L}, INFINITE^{ED0L}, NONEMPTY^{E0L}, INFINITE^{E0L},

and their restrictions to propagating systems.

<u>Proof</u> Immediate from the above and theorems of [7]

□

<u>Theorem 5</u> MEMBER^{EP0L} is $\mathcal{NP}$ hard.

<u>Proof</u> Let $G = (V, P, w, \Sigma)$ be an EPD0L-system.

Construct an EP0L-system $G' = (V \cup \{g, 0\}, P', w, \{0\})$ where P' consists of all productions in P, $a \to 0$ for $a \in \Sigma$, $0 \to g$, and $g \to g$. Now L(G) contains words of length i iff $0^i \in L(G')$.

The theorem follows then by observing that in the proof of Theorem 2 $L(R) \neq \emptyset$ iff $L(G) \neq \emptyset$ iff L(G) contains a word of length r.

□

From this and Theorem 7 of [7] we have:

<u>Corollary 6</u> MEMBER^{E0L} and MEMBER^{ED0L} are $\mathcal{NP}$-complete.

3. SYSTEMS WITH TABLES

Theorem 7 $\text{MEMBER}^{\text{EPDT0L}} \notin \text{NSPACE}(n^{1-\varepsilon})$ for any $\varepsilon > 0$

Proof Let $Z = (K, \Sigma, \Gamma, \#, \delta, q_0, \{q_f\})$ be an arbitrary 1 tape Turing machine which operates in space n ($\#$ is an end marker). For any $x = a_1 \ldots a_n$, construct the EPDT0L system $G_x = (V_n, \mathfrak{T}_n, w_x, \{0\})$ where

$$V_n = \{g, 0\} \cup \{A^i \mid A \in \Gamma \text{ and } 0 \leq i \leq n+1\} \cup K$$
$$w_x = p\,\#^0\, a_1^1 a_2^2 \ldots a_n^n\, \#^{n+1}$$

For each $(p, a) \in (K - \{q_f\}) \times \Gamma$ there will be a table $T_{p,a}$ in $\mathfrak{T}_n$ defined as follows:

If $\delta(p,a) = (q, b, R)$ then

$$T_{p,a} = \{p \to q,\ a^0 \to b^{n+1}\} \cup \{c^i \to c^{i-1} \mid c \in \Gamma,\ 0 < i \leq n+1\} \cup G_{p,a}$$

where $G_{p,a}$ contains $d \to g$ for every $d \in V_n$ other than p, a^0 or c^i for $c \in \Gamma$, $0 < i \leq n+1$.

If $\delta(p,a) = (q, b, C)$ then

$$T_{p,a} = \{p \to q,\ a^0 \to b^0\} \cup \{c^i \to c^i \mid c \in \Gamma,\ 0 < i \leq n+1\} \cup G_{p,a}$$

If $\delta(p,a) = (q, b, L)$ then

$$T_{p,a} = \{p \to q,\ a^0 \to b^1\} \cup \{c^i \to c^{i+1} \mid c \in \Gamma,\ 0 < i \leq n\} \cup \{c^{n+1} \to c^0 \mid c \in \Gamma\} \cup G_{p,a}.$$

In addition, $\mathfrak{T}_n$ contains the table

$$T_f = \{q_f \to 0\} \cup \{c^i \to 0 \mid c \in \Gamma,\ 0 \leq i \leq n+1\} \cup \{a \to g \mid a \in K \cup \{g, 0\} - \{q_f\}\}$$

It is easily verified that Z yields an I.D. $\alpha = b_0 \ldots b_{i-1}\, p\, b_i \ldots b_{n+1}$ iff G derives the string $p\, b_0^{n-i+2} \ldots b_{i-1}^{n+1}\, b_i^0 \ldots b_{n+1}^{n-i+1}$. Consequently $L(G) = \{0^{n+3}\}$ if Z accepts x, and $L(G) = \emptyset$ if Z does not accept x. Further, $|\overline{G}| = 0(n \log n)$.

Now suppose $MEMBER^{EPDT0L} \in NSPACE(n^{1-\varepsilon})$ for some ε, $0 < \varepsilon < 1$. Let $L \in NSPACE(n) - NSPACE(n^{1-\varepsilon/2})$; such sets are known to exist by [11]. Let Z as above recognize L in space n. Then we can decide whether an arbitrary $x \in \Sigma^*$ is in L by first constructing G as above, letting $n = |x|$ and $y = 0^{n+3}$, and then deciding whether $< \overline{G}, \overline{y} > \in MEMBER^{EPDT0L}$. Now $| < \overline{G}, \overline{y} > | = 0(n \log n)$, so this process works in space $0((n \log n^{1-\varepsilon})) = 0(n^{1-\varepsilon}(\log n)^{1-\varepsilon}) = 0(n^{1-\varepsilon} n^{\varepsilon/2}) = 0(n^{1-\varepsilon/2})$, a contradiction.

□

Corollary 8

None of the following is in $NSPACE(n^{1-\varepsilon})$ for any $\varepsilon > 0$: $MEMBER^{EDT0L}$, $NONEMPTY^{EDT0L}$, $INFINITE^{EDT0L}$, $MEMBER^{ET0L}$, $NONEMPTY^{ET0L}$, or $INFINITE^{ET0L}$, or their restrictions to propagating systems.

Proof

The construction is easily modified so that L(G) is infinite if and only if Z accepts x, giving the result for $INFINITE^{EDT0L}$. The other results are immediate.

□

Corrolary 9

Each of the problems just mentioned is complete for polynomial space.

Proof

Each is recognizable in polynomial space by [7]. It is well known that there is a context-sensitive language L which is complete for polynomial space. By the construction above, $L \leq MEMBER^{EPDT0L}$.

□

Remark

The following somewhat simpler construction yields the same results except for $\text{MEMBER}^{\text{EPDT0L}}$ and $\text{MEMBER}^{\text{EPT0L}}$, and may be interesting in its own right. Given a nondeterministic finite automaton $M = (K, \Sigma, \delta, q_o, F)$, define the EDT0L-system $G = (K, \{ P_a \mid a \in \Sigma \}, q_o, K-F)$, where for each $a \in \Sigma$,

$$P_a = \{ p \rightarrow q_1 q_2 \dots q_k \mid \delta(p, a) = \{ q_1, q_2, \dots, q_k \} \}$$

it is easily seen that $L(G)$ is nonempty and infinite just in case $L(M) \neq \Sigma^*$.

The NSPACE $(n^{1-\varepsilon})$ lower bound obtains from the fact that $\{ R \mid L(R) \neq \{0, 1\}^*$ and R is a regular expression $\}$ is known to be in NSPACE (n) and no smaller class [9]; given any R, a nondeterministic finite automaton is easily constructed to accept $L(R)$, so an EDT0L system G can be built as just described satisfying $L(R) \neq \{0, 1\}^*$ just in case $L(G) \neq \emptyset$. If λ productions are allowed it is easy to modify G so $L(G) = \{\lambda\}$ just in case $L(G) \neq \emptyset$, giving the result for $\text{MEMBER}^{\text{EDT0L}}$.

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