Investigating in-service Norwegian primary school teachers' modelling specific pedagogical content knowledge

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The article investigates the changes in modelling specific pedagogical content knowledge (MsPCK) among practicing Norwegian primary school teachers resulting from a single module intervention within a continuing education program. A measure was administered to a group of 15 participants, and pre- and post-test results report on growth in four dimensions of MsPCK knowledge about mathematical modelling theory, tasks, instruction, and diagnostics. An independent-sample Mann Whitney U-test showed that post-test scores were significantly higher than pre-test score ($p < 0.001$), both for total test scores, and scores for each of the four dimensions. On the item level, a significant change in scores was found for 33 items. Supported by this analysis, the findings of this study indicate a pattern of increased modelling specific pedagogical content knowledge with this group of practicing teachers, recommending the inclusion of a modelling module in future continuing education courses.

Keywords: mathematical modelling, primary school mathematics, modelling specific pedagogical content knowledge

Mathematical modelling in education is a process for connecting the real world and the world of mathematics that serves as a powerful tool for student learning (Borromeo Ferri & Blum, 2010; Borromeo Ferri, 2018). According to Turner et al. (2012), tasks that encourage the modelling process are inquiry based and open-ended, encouraging diverse solution strategies and connections to multiple mathematics content areas. Teaching with these types of tasks benefits learners in multiple ways, including fostering creativity, problem solving, sense-making, and communication (Chamberlin et al., 2022; Niss & Blum, 2020). It also requires teachers to have a sound knowledge of modelling itself, as well as a set of modelling specific teaching competencies to teach effectively (Ferri, 2019). One

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Helder, P. & Jakobsen, A. (2025). Investigating in-service Norwegian primary school teachers' modelling specific pedagogical content knowledge. *Nordic Studies in Mathematics Education*, 30 (4), 67–93.

facet of these teaching competencies is the pedagogical content knowledge (PCK) needed for teaching modelling (Ferri, 2019; Wess et al., 2021). PCK refers to the synthesis of pedagogical knowledge and content knowledge used by teachers to teach a given topic (Shulman, 1987). Given that the teacher's pedagogical knowledge and associated actions in the classroom can account for as much as 30% of variance in learner achievement (Hattie, 2008), researching the development of PCK specific to teaching of modelling can provide insights into improving teaching practices around modelling.

Mathematical modelling is often introduced in schools at the secondary or tertiary level (English & Watters, 2005; Niss & Blum, 2020), and a rich body of research into the teaching of modelling on learner achievement at these higher levels can be found (Borromeo Ferri, 2018). In comparison, research looking at many aspects of teaching of modelling at the primary level is under-represented. In their recent review of literature on modelling, Cevikbas et al. (2022) found that only 3% of the articles they reviewed focused on the primary school level, while a literature review of mathematical modelling research in the primary school by Wei et al. (2022) yielded fewer than 200 research articles for consideration. Though research discussions around modelling in the primary school have bloomed in the last decade (e.g., Chamberlin et al., 2022, Stohlmann & Albarracin, 2016), research exploring the modelling specific PCK (MsPCK) for both secondary and primary school teachers is sparse internationally, and none could be found in the Norwegian context. This supports the need for additional research around the different facets of teaching competencies for mathematics modelling (Wei et al., 2022), especially MsPCK.

The Norwegian discussion around modelling and its associated teaching competencies in the primary school has been fuelled by the inclusion of Modelling and Applications as a Core Element of mathematical learning in the most recent iteration of the national core curriculum (Norwegian Directorate of Education [UDIR], 2019). This curriculum, referred to as the Knowledge Promotion Reform (LK20), puts modelling at the forefront of teaching and learning at the primary level for the first time. With this inclusion comes the mandate for primary teachers to include modelling experiences in their teaching practices.

However, integrating modelling into the day-to-day practice of primary teachers presents a challenge, as many practising teachers do not have the personal modelling skills or modelling specific teaching competencies needed to plan, implement, assess, and productively reflect on modelling tasks used in their classrooms (Borromeo Ferri, 2018). In response to this challenge, and the inclusion of modelling in the curriculum, modelling has been added to the course syllabus for the Norwegian national continuing education program, Competence for Quality (KfK). This program, described later in this article, has the goal of increasing learning outcomes for learners by helping practicing teachers develop their teaching competencies (Norwegian Directorate for Education [UDIR], 2024).

Five years after the implementation of LK20, little is known about the development of modelling in primary schools. To add to our knowledge in this area, this small-scale quantitative implementation study focuses on the PCK facet of modelling specific teaching competencies, seeking to answer:

> What changes in modelling specific pedagogical content knowledge can be found among practicing primary school teachers in Norway before and after participating in a single module on mathematical modelling?

These changes will be investigated using a pre- and post-test design, measuring the MsPCK designed for use with primary school teachers.

Theoretical background

Mathematical modelling

Anhalt et al. (2018) describe mathematical modelling as "a process in which students consider and make sense of an everyday situation that will be analysed using mathematics for the purpose of understanding, explaining, or predicting something" (p. 202). It involves working with complex, open-ended, reality-based tasks, shown to develop learner' understanding, appreciation, and perception of mathematics as relevant and applicable to real life situations (Stohlman & Albarracin, 2016), as well as improve overall mathematics literacy skills (Organisation for Economic Co-operation and Development [OECD], 2023).

The process of working with these tasks lends itself to a cyclic process of problem solving that supports the translation of real-world problems into mathematical language and back into reality (Blum, 2011; Lesh & Doer, 2003; Pollak, 2003). This process can be represented visually as a cycle, or model, of a learning path.

Evolving from a desire to explain a learners' modelling process from a cognitive perspective, Blum and Leiß (2007) developed a widely used representation of a modelling cycle. This representation considers the process of mathematical modelling as a series of seven phases, or subcompetencies, on which learners can focus and master to help them solve problems (See figure 1). These sub-competencies include understanding, simplifying, mathematizing, working mathematically, interpreting, validating, and exposing (Blum & Leiß, 2007). It is acknowledged that learners may use different modelling routes as they solve a problem (Borromeo Ferri, 2018), and, indeed, some researchers view a modelling cycle as a result of the modelling process, rather than a guide to teach learners how to model (Lesh & Doerr, 2003). Regardless, an understanding of this theoretical model of modelling sub-competencies can be useful for teachers to understand the thinking of their learners and how to help them progress, and is therefore an important concept for teachers to visit.

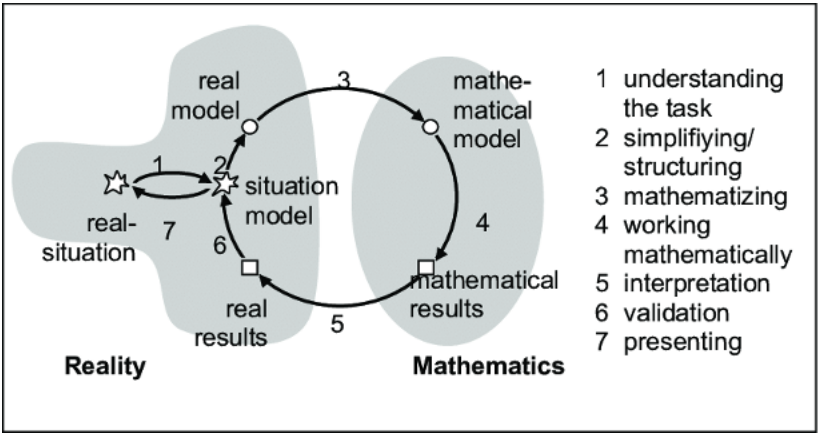

Figure 1. *Theoretical modelling cycle according to Blum and Leiß (2007, p. 225)*

Teaching competencies, modelling specific teaching competencies and PCK

In general, "competence" is defined as those latent dispositions that enable professionals to master their job-related tasks. These dispositions include cognitive abilities, professional knowledge, convictions, and values (Blömeke & Kaiser, 2014). In his seminal work describing teaching competence, Shulman (1987) identified seven facets of knowledge that professional teachers use simultaneously while teaching:(a) content knowledge; (b) general pedagogical knowledge; (c) curriculum knowledge; (d) pedagogical content knowledge; (e) knowledge of learners and their characteristics; (f) knowledge of educational contexts; and (g) knowledge of educational ends, purposes, and values. To thrive in a mathematics classroom, a teacher must develop all these facets of knowledge, including those extra-mathematical (not math content specific) skills that support learners gaining content knowledge (Blömeke & Kaiser, 2014).

Because teacher PCK has an impact on learners' outcomes, increasing PCK is a key goal for teacher education (Botha et al., 2023; Campbell et al., 2014; Greefrath et al., 2021). This is the facet of competency that is reported in this study. Shulman (1987) writes that PCK, "...represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction." (p. 8). Though researchers over the past 40 years have reconceptualized PCK according to different theoretical and research alignments, Greefrath et al. (2021) point out that the synthesis of content and pedagogical knowledge remains at the core of PCK.

Evolving from the overarching facets of teaching competency described by Shulman (1987) are those teaching competencies which support a specific topic. In the area of mathematics, a widely cited model for professional competencies in teaching mathematics grew from the COACTIV project in Germany (Baumert & Kunter, 2013). This model pays special attention to different areas of knowledge for mathematics teachers, including mathematics specific PCK (Wess et al., 2021). Influential in the creation of the COACTIV model for mathematics PCK was the work of Borromeo Ferri and Blum (2010) on modelling teaching competencies.

In this work, Borromeo Ferri and Blum (2010) introduced a four-dimensional construct of modelling specific teaching competencies composed of MsPCK in theoretical, task-related, instructional, and diagnostic areas. The theoretical dimension includes knowledge about the goals of modelling and modelling cycles, while the task dimension involves the teacher's ability to solve, analyse, and create modelling tasks. The instructional dimension involves a teacher's ability to plan and implement modelling lessons, as well as appropriate intervention during student modelling processes. The diagnostic dimension concerns the ability to identify phases in the modelling cycle and to diagnose student difficulties within this process. As seen in figure 2, each of these dimensions is broken down into subareas that can be aligned with the PCK needed to demonstrate competencies in each dimension.

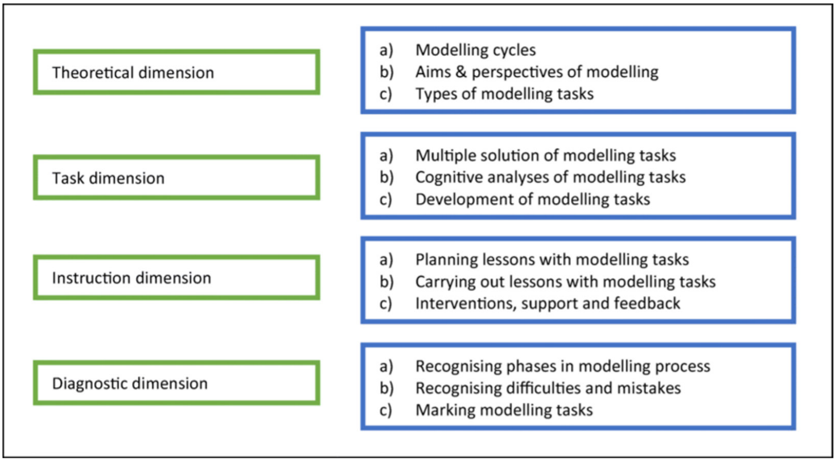

Figure 2. *Modelling specific teaching competencies (Borromeo Ferri, 2018, p. 5)*

Literature review

Mathematics and MsPCK

Mathematics PCK refers to the knowledge that teachers possess to effectively teach mathematics. This integration of the what and how of teaching mathematics in general may be central to explaining effective teaching and as such has been studied extensively. For example, Venkat and Adler (2020) point to several major research areas within mathematics PCK, including sharpening theorizations of PCK (Blömeke et al., 2015), measuring PCK (Baumert et al., 2013), and using concepts of PCK to build practical skills within teacher education (Cardoso et al., 2023; Tan & Ang, 2012). A literature review by Sakaria et al. (2023) reported that research on the development of mathematics PCK through professional development was the primary topic for research in mathematics PCK in the years 2018–2022, with qualitative research dominated this publication period. This research supports the importance of gaining knowledge about teachers' attainment and use of mathematics PCK through teacher professional development through teacher education programmes like the Norwegian KfK programme

Similar in import, the development of MsPCK is less well researched and often focuses on PCK as an element of modelling specific teaching competencies (Wess et al., 2021). There is a growing body of work that focuses on small scale, qualitative research focused on one dimension of MsPCK. For example, Wess and Greefrath (2019) studied the development of the competency in the task dimension in preservice teachers during a teaching lab experience. Results showed increased MsPCK in the task dimension after using self-led tasks. Tan and Ang (2012) used lesson images to describe the teaching moves used by preservice teachers in Singapore. They concluded that lesson images could be used as a powerful starting point for adaptive development of teachers' PCK in modelling. In Indonesia, Kurniad et al. (2022) worked with pre- service teachers through a learning module designed to optimize MsPCK and received positive feedback on their learner survey. In a case study with an in-service secondary teacher in Japan, Saeki et al. (2024) discussed their observations on aspects of the teacher's advances in MsPCK, and the value of teacher educator support. These small studies indicate that there remains much research around MsPCK and the professional development for teachers yet to be done. For example, a study with secondary teachers in Singapore showed that educators were lacking in two main domains: teacher knowledge about mathematical modelling (theoretical domain), and how to teach and intervene while students are modelling (instructional domain) (Chan et al., 2019). Although these studies are small scale, Adler et al. (2005) consider this as an indicator of an emerging research field, where results from small scale studies can culminate in generalizations. With this evolution may come the creation of standardized measures of MsPCK that can be used to better understand the field.

Measures of MsPCK

Working towards this, Wess et al. (2021) restructured the work of Borromeo Ferri and Blum (2010) into an alternate four-dimensional framework for PCK in modelling for teachers in secondary education. This new framework focuses more on the teacher-knowledge elements of each dimension presented by Borromeo Ferri and Blum (2010), making paper and pencil measurement more straightforward. Their dimensions of MsPCK include knowledge about interventions, modelling processes, modelling tasks, and aims and perspectives. These dimensions were both content- and construct-validated (Greefrath et al., 2021; Wess et al., 2021). Borromeo Ferri (2019) is also in the process of refining a four-dimensional framework based on modelling specific teaching competencies that measures MsPCK, for secondary teachers using a similar format.

Recognising the differing teaching competencies required for primary teachers and building on the work of Greefrath et al. (2021) and Wess et al. (2021), Nehrkorn et al. (2022a) adapted items for primary teachers. The adapted measure—the ProMoPri measure—comprised of 42 multiple-choice items. The measure is divided into four modelling specific teaching competencies dimensions derived from the work of Borromeo Ferri (2018), and the items are designed to measure the modelling-specific PCK of primary school teachers. Nehrkorn et al. (2022a) were unable to statistically validate the four dimensions as distinct constructs, as they were found to be interrelated. However, these four dimensions are commonly used in research (e.g., Alwast & Vorhölter, 2022; Greefrath et al., 2021). This unique measure provides feedback in th area of growth of MsPCK overall at the primary level, and as such is one tool to be used when evaluating the professional development of primary school teachers.

Methodology

Participation

Participants in this study were in-service teachers enrolled in a digital, two semester KfK course for primary school teachers in Norway. The participating teachers ranged from 2–24 years teaching experience, and all teachers used English as their language of instruction through an International Baccalaureate curriculum into which the Norwegian core curriculum is folded. Though this group of teachers may have professional identities that differ from a public-school teacher in Norway (Walker & Bunnell, 2024), they are held accountable to the same learner standards, teacher qualification requirements, and participate in the same continuing education course as their public-school colleagues. Some of the teachers participated in the course with a group of other teachers from their school, while other teachers were the only participating teacher from their school. At the beginning of the second semester, course attendees were invited to participate in this study via email from the course instructor. Fifteen attendees agreed to become participants.

Modelling module content

The KfK is a national initiative designed to increase the teaching competencies of practising teachers in Norway through enrolment in free continuing education courses run by 13 Norwegian institutions of higher education (UDIR, 2024). Nationally prioritised subjects include mathematics, English, Norwegian, and Sami and Norwegian sign language. This digital course was run through a university in southern Norway and participants met weekly for two hours across two semesters in the 2022–2023 school year. This study took place during the second semester of the course. The course organized according to topics, including geometry, probability and statistics, and counting and formation of number concepts. These topics were to be taught with a "particular emphasis" on problem solving, modelling, creativity, discussion and argumentation. During this continuing education course, modelling was taught as a stand-alone topic during a single module dedicated to teaching modelling

The elements of the single modelling module included preparatory readings, a two-hour lecture with small group discussion and activities, handouts and guides, as well as a follow-up assignment involving lesson plan writing, implementation, and reflection. In line with suggestions from Wei et al's (2022) literature review of effective professional development practices for teaching of mathematical modelling, the content of the lecture module included elements from three of the dimensions of modelling teaching competency: theoretical, task, and instructional. The diagnostic dimension was mentioned, but due to lack of time, it was not discussed or practiced. The theoretical elements covered included defining modelling, a modelling cycle, the aims of modelling, and types of modelling tasks. Within the task dimension, participants were introduced to identifying elements of a modelling task, how students use modelling cycles, and how to modify a standard word problem into a modelling task. Within the instructional dimension, participants were introduced to modelling planning guides, discussed the extra-mathematical features of teaching with modelling that they expected to meet (i.e., grouping, classroom arrangement, differentiation), and practiced using sentence starters that encourage discussion.

Measure of MsPCK for primary teachers

The ProMoPri measure (Nehrkorn et al., 2022a) was used to investigate changes in MsPCK. Though it is challenging to assess the complexities of MsPCK with a paper-and-pencil test, the measure items were designed to address both declarative and conceptual knowledge that translates into MsPCK. The 42 items comprising the measure had already been translated into English by the German researchers, and the English version were content validated against the Norwegian context. Both the German and Norwegian educational systems are based on competency-based curricula with overlapping content and context. According to Buchholtz et al. (2022), curricula developed in both Germany and Norway have evolved from the same notion of mathematical competencies and emphasise the importance of nurturing critically mature citizens. After carefully reviewing all items, and at the request of the German research team who authored the measure, no changes to the translated English version of the items were made

Although Nehrkorn et al. (2022a) were not able to construct validate the four dimensions, we opted to group the items into the conceptual validated four dimension based on the work of Borromeo Ferri (2018):

i) Knowledge about mathematical modelling (Items 1–12, 12 items),

ii) Knowledge about mathematical modelling tasks (Items 13–28, 16 items),

iii) Knowledge about mathematical modelling instruction (Items 29–36, 8 items), and

iv) Knowledge about diagnostics of mathematical modelling (Items 37–42, 6 items).

The Norwegian sample ($N = 15$) was too small to conduct any meaningful construct analysis to document the four dimensions.

Using a sample of $N = 676$ prospective teachers in Germany, Nehrkorn et al. (2022a) fitted a one-dimensional Rasch model to the data, with items represented according to item difficulty. The scale ranged from –3 to +3, with the majority of items located around the mean difficulty (zero) (Nehrkorn et al., 2022b). Figure 3 presents the distribution of the items' relative difficulty, where –3 represents the easiest and +3 the most difficult items.

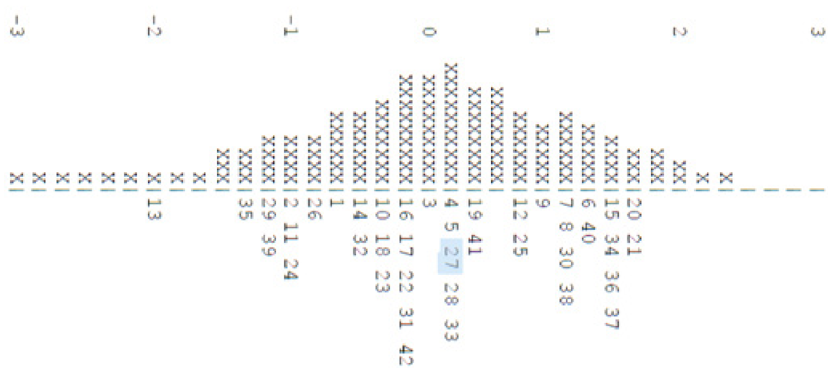

Figure 3. *Distribution of items (numbers) along the difficulty (Nehrkorn et al., 2022b)*

The English version was administered to the 15 study participants at the beginning of the second course semester (pre-test) and at the end of the second course semester—after the module of the teaching of mathematical modelling was taught (post-test). Each item consisted of statements (e.g., Item 1: "Modelling is based on real questions") and participants were given choices of answering "True", "False" or "I don't know". The items were of varying degree of item difficulty. Item with correct answer was coded 1 and 0 otherwise. For the Norwegian sample, a cumulative score for each participant were calculated for both pre- and post-test results across the four dimensions and for the whole measure. The test scores were compared at group level as the test were taken anonymously online and it was not possible to compare on individual scores.

Data analysis

The quantitative data were analysed using the Statistical Package for the Social Sciences (SPSS), version 29.0.1.0. Since it was not possible to pair individuals' pre- and post-test scores, an independent-sample Mann-Whitney U-test was used to compare MsPCK scores between the pre- and post-test groups, both for the total score and for the scores within each of the four dimensions of MsPCK. A Fisher's exact test was conducted for each of the 42 items to investigate whether there was any association between the number of participants answering the item correctly and the pre- and post-test.

Results and analysis

The independent-sample Mann-Whitney U-test showed that pre- and post-test results were significantly different: U = 225, $p < 0.001$, $N_1 = N_2 = 15$, with a large effect size r = 0.85. This means that the overall scores of the post-test group are significantly higher than the scores of the pre-test group. This is also visible by looking at the frequency plot for pre- and post-test total scores in figure 4.

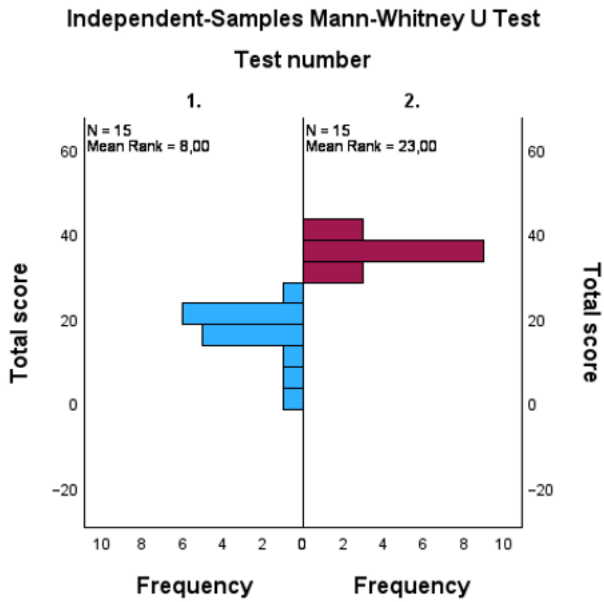

Figure 4. *Frequency plot for pre-test scores (1) and post-test scores (2)*

An independent-sample Mann-Whitney U-test comparing pre- and post-test total scores for each of the four teaching competency dimensions was also conducted. Figure 5 shows the frequency distribution for both pre- and post-test for all the four dimensions:

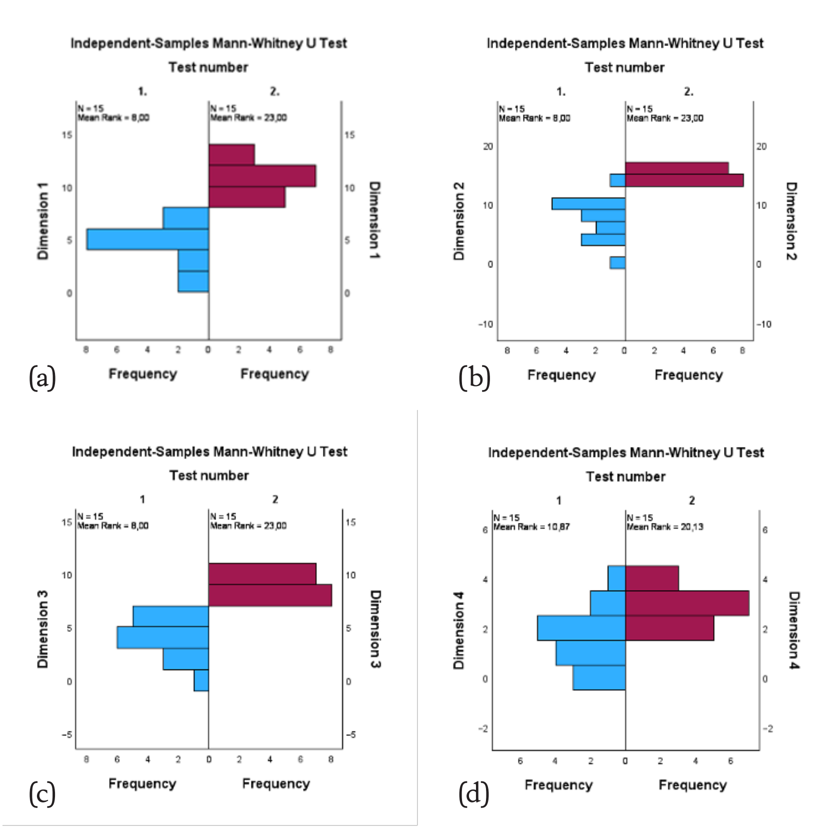

Figure 5. *Frequency plot for pre-test scores and post-test scores for the four MsPCK dimensions: a) Knowledge about mathematical modelling; b) Knowledge about mathematical modelling tasks; c) Knowledge about mathematical modelling instruction; and d) Knowledge about diagnostics of mathematical modelling*

From figure 5, we can observe that the frequency distribution of test scores is highest for the post-test group for all four dimensions, but less distinct for dimension 4 (Knowledge about diagnostics of mathematical modelling). This is documented by running an independent-sample Mann-Whitney U-test for each of the four dimensions and comparing the scores for the pre- and post-test groups. The test showed a significant difference between pre- and post-test scores for all four dimensions, with scores for post-test group significantly higher than scores for the pre-test group. The effect size is also large, except for dimension 4, which has a medium effect size (see table 1).

Table 1. *Result from independent-sample Mann-Whitney test, comparing scores for pre- and post-test groups for each of the four modelling specific dimensions*

	Dimension 1	Dimension 2	Dimension 3	Dimension 4
Mann-Whitney U	225	225	225	182
p	< .001	< .001	< .001	= .003
r	.85	.86	.86	= .55

The lower effect size for dimension 4 can probably be attributed to the content of the continuing education course. Dimension 4—Knowledge about diagnostics of mathematical modelling—had a minor role in the course.

On the item level, a Fisher's exact test was conducted for each of the 42 items. For 33 of the items, the test showed a significant association between the number of participants answering the item correctly ($p \leq .05$, one-sided) and the pre- and post-test. The difference in total score on these 33 items (sum of scores from all 15 respondents) between post- and pre-test ranged from 4 (min) to 15 (max) with a median score change of 8, and mode 10(6). This means that for the items where the change in total item score was significant, the median improvement in number of correct answers on these items was 8. For example, the one item with the most significant change in answers from incorrect to correct from pre- to post-test was Item 3, which discusses the phases of a schematic representation of modelling. This item had an increase of 14 correct answers (represented by the right-hand bar in figure 6).

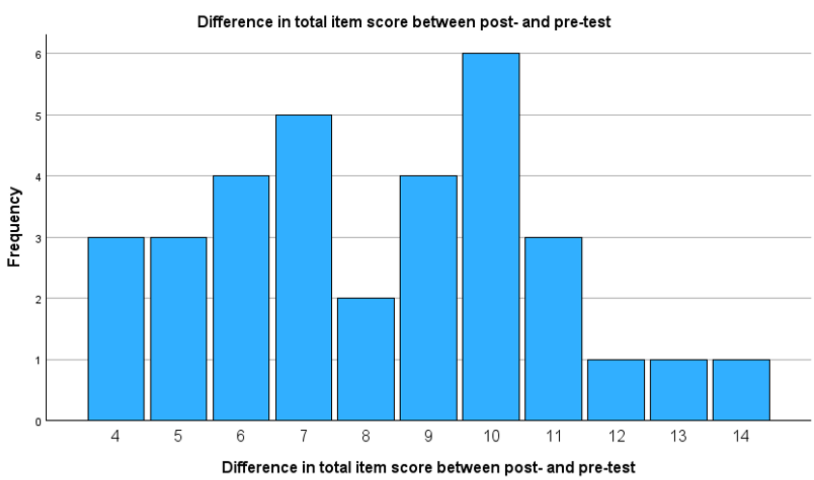

Figure 6. *The x-axis represents the difference in total item scores (sum of scores for all participants) between the post-test and pre-test, and the y-axis shows the number of items (frequency). This plot includes the 33 items for which Fisher's exact test showed a significant difference (p ≤ .05, one-sided) in total item scores (median: 8; mode: 10 (8)).*

Perhaps more interesting are the nine items where the Fisher's exact test did not show any association between item score and pre- and post-test ($p > .005$, one-sided). These items are presented in table 2, including the dimension to which these items belong.

Table 2. *Items where there is no association between the number of correct answers on the item and pre- and post-test, identified from Fisher's exact test (total number of items in parentheses)*

(42)	Dimension 1 (12)	Dimension 2 (12)	Dimension 3 (8)	Dimension 4 (6)
Item number	1, 5	13, 16, 26	31, 32	40, 41

Again, we observe that compared to the number of items in each dimension, dimension 4 has the largest number of items (two out of six) where there is no association between the total number of correct answers and the pre- and post-test. This can again be attributed to the fact that the content of dimension 4 was mentioned but not discussed and practiced in the continuous education course.

After a closer look at the nine items listed in table 2, we find that two types of characteristics can explain this pattern. Firstly, relatively easy items (items with a relatively high number of correct answers at pre-test) had small changes in the number of correct answers at post-test. Table 3 below lists these items:

Table 3. *Number of correct answers for items with no significant association as identified from Fisher's exact test (table 2)*

Item	1	5	13	16	26	31	32	40	41
Pre-test	12	6	12	13	12	13	13	7	4
Post-test	15	11	15	15	15	15	15	12	6

An example is Item 1 from dimension 1, for which 12 out of 15 participants answered correctly in the pre-test, and all 15 answered it correctly in the post-test, indicating a relatively small change in the total number of correct answers from the pre- to the post-test. Item 5—also from dimension 1—showed a difference, with 6 answering correctly at the pre-test and 11 answering correctly at the post-test, although this difference was not significant. The next group of items with no significant change in the number of correct responses from the pre- to the post-test consists of item 40 and 41 which are included in dimension 4. It should also be noted that these two items were classified as more difficult items (positive item difficulty) in the German validation study (figure 4).

Another area of interest involved the quantity of items answered ”I don't know” between pre- and post-test. Though no analysis was run describing the change in this quantity, on the pre-test, ”I don't know” was the given answer 325 times comparted to 30 ”I don't know” answers on the post-test – a decrease of roughly 91%.

Discussion

Initial MsPCK

As found in many studies looking into gaining MsPCK (e.g. Alwast & Vorhölter, 2022; Greefrath et al., 2021; Quarder et al., 2025), the pre-test results from this study offered a baseline from which growth could be measured. They additionally offered several thought-provoking patterns regarding the pre-existing MsPCK of participants. The participants demonstrated general awareness and appreciation for modelling tasks but lacked competence in key aspects of teaching with modelling.

Initially, the group entered the course with a basic understanding of the foundational aspects of mathematical modelling. Specifically, 12 out of 15 participants correctly answered items 13, 16, and 26, which highlight the importance of translating between mathematics and real-life contexts. This understanding suggests a strong starting point for teachers, as real-life contexts are essential for modelling tasks. These three items also were rated easier on the difficulty axis in figure 4. The participants also presented a shared understanding that primary school learners at all grades are ready and able to tackle modelling tasks, as 13 of 15 participants answering correctly to Item 32, which asks if learners at all grade levels can engage in modelling tasks. This suggests an understanding that modelling tasks can have value for leaners at all grade levels, increasing the likeliness that participants see value in using modelling with their classes.

When questions addressed more specialized knowledge about modelling cycles and tasks in dimensions one and two, participants' answers indicated mixed knowledge. Though nine participants were aware of modelling cycles, they were uncertain about the sub-competencies of modelling, including mathematizing (Item 8) and the relationship between situational and mathematical models (Items 5–8). One exception to this lack of knowledge involved the importance of validation in mathematical modelling, where there were 12 of 15 correct responses to Item 31, ”Validating the results of modelling tasks must be practised from the beginning”. Most answered that the tasks needed to come from the learners' world (items 13, 16 and 26), but were unsure of the features of a modelling task. For instance, 3 of 15 participants answered incorrectly to Item 15, "A modeling task can be created by omitting data from a word problem". Of special note is that most participants (2 out of 15) answered incorrectly to Item 17, "Modeling tasks are suited for heterogeneity-sensitive mathematics lessons". This could indicate a lack of appreciation for the inclusivity of modelling experiences—one of the most compelling arguments for its use in the classroom. Alternatively, it could also indicate unfamiliar vocabulary.

One could assume that because participants did not express clear knowledge of modelling cycles (dimension one), modelling cycles could not be used as an instructional or diagnostic tool as indicated in dimensions three and four. This idea is supported by these findings, where among the six items asking about diagnostics knowledge, a cumulative total of 10 participants answered "I don't know" to these questions. It was also interesting that the participants shared more uncertainty—as indicated by "I don't know" answers, than misconceptions, as indicated by wrong answers, about mathematical modelling.

The pre-test results also highlighted a general lack of familiarity with modelling specific vocabulary. For instance, questions involving theoretical terms such as "schematic representation" (Item 3), "enactive activity materials" (Item 28), and "overdetermined" or "underdetermined" tasks (Items 20 and 21) saw many participants answer with "I don't know." While the underlying concepts might have been familiar, the terminology itself posed a challenge, reflecting a gap in shared vocabulary for discussing modelling. This absence of a common language could lead to misinterpretations of modelling theories and practices, making it difficult to set clear goals for student learning (Helder, 2024). It is also worth noting that through all of the vocabulary discussed is relevant to MsPCK, all vocabulary may not be equally important. For example, one could argue that a teacher could effectively teach modelling with a limited understanding of the vocabulary 'enactive', but not without an understanding of modelling cycles.

Overall, the pre-test scores indicated that while participants had a general knowledge of mathematical modelling, their understanding of the theories and practices required for competent classroom implementation was underdeveloped. This was not surprising, given that the focus on mathematical modelling is relatively new in the Norwegian curriculum (LK20), and it is unrealistic to expect teachers to have in-depth knowledge of content they have not yet encountered. Therefore, it can be predicted that, based on their initial level of MsPCK, classroom instruction using modelling would likely be ineffective without further professional development.

Though no research could be found using the ProMoPri measure, Greefrath et al (2021) reported baseline information for the MsPCK in each dimension from a related measure for three groups of participants in their quasi-experimental comparison of German pre-service teachers learning of mathematical modelling. Their findings suggested a similar level of knowledge before the intervention, and allowed for analysis and comparison of post-intervention test results.

Patterns of change of MsPCK

After engaging in the modelling module and completing a modelling specific lesson planning assignment, participants' post-test scores demonstrated significant growth across all four dimensions. This correlation was expected, as the four dimensions measured build on one another and are connected (Ferri, 2019). Greefrath et al., (2021) and Quander et al. (2025) also found growths across all of the measured dimensions in their studies looking at MsPCK and MsTPACK, respectively. In this study, items answered correctly during the pre-test remained thus on post-test results. Consequently, for items for which there was a high number of correct answers in the pre-test, the Fisher's exact test found no significant association between the number of correct scores on the pre- and post-tests. These items included knowledge about the importance of the translation between mathematics, and reality (e.g., Item 13), the appropriateness of using modelling at all grade levels (e.g., Item 27), and validating task results (e.g., Item 12).

When looking at results from dimension one, the theoretical dimension, a pattern showing change involved the increased understanding of the role of modelling cycles in working with modelling tasks emerged. For example, the whole group answered correctly on Item 2 ("Modeling processes can be illustrated though modeling cycles.") and 4 ("Simplified modeling cycles are an effective metacognitive tool for learners.") indicating a recognition that modelling cycles can be a useful tool for understanding and learning through modelling tasks. Knowledge of the roles of the individual sub-competencies of modelling cycles (i.e., interpreting, simplifying and mathematising) did increase overall, but the change was not significant. For example, correct item responses referring to the roles of situational and mathematical models improved, though the change was not significant (p = .07). This may be explained by the abundance of modelling specific vocabulary found in this dimension, with which they remain unfamiliar even after the module. This suggests that more emphasis on teaching this vocabulary, may be valuable, especially if the associated concepts are not addressed.

Participant answers in the task dimension (dimension two) also showed overall growth, especially in the area of general task characteristics. This pattern is exemplified by significant increases in correct answers around task openness (Item 18), task complexity (Item 23) and the necessity of assumption-making (Item 25). Items 14 and 15, which were also answered correctly by all participants, are related to ideas around task identification and modification, emphasising the relationship between word problems and modelling tasks. An additional pattern of knowledge about the use of modelling with diverse learners showed when all participants affirmed that modelling tasks could be used in heterogeneity-sensitive classes (Item 17). Improvement in these areas indicates a deeper understanding of the role and importance of the features of good modelling tasks.

Patterns found in answers provided in dimension three, Instruction with modelling, also illustrated positive outcomes when comparing pre- and post-test results. However, correct answers were more common among items that rated easier on the difficulty scale (figure 4), such as, Item 32 (”Modeling can be introduced in all grades”). Items that rate higher on the difficulty scale, such as those that asked about specific pedagogical strategies saw improvement, but did not show enough change to be significant. For example, Item 30 (”Learning to model is successful through closely guided, questioning-developing lessons”) and Item 34 (”All seven sub-competencies of the modeling cycle can already be addressed in the very first lessons”) saw improvement from 1 to 9 and 1 to 8 respectively. It is unclear whether this level of MsPCK can translate into effective teaching practices overall. This pattern may also be explained by the limited duration of the teaching module.

The diagnostic dimension, dimension four, received the least attention in the modelling module, and saw the smallest amount of change among the dimensions measured. One could credit increases in correct responses to the interrelatedness of the dimensions. Item 37 (”Surveys are conducted in the process-oriented diagnostics of modeling”) showed an unexpected growth, as the vocabulary-specific challenge of the statement was not discussed in the teaching module. As Bloom (1956) points out, change in learning occurs in areas that are taught. Given the lack of attention to this dimension, the lack of change overall makes sense

An important, albeit expected, finding was that participant knowledge of modelling specific vocabulary improved after the modelling module. For example, Item 3 asked participants about a ”schematic representation”. In the pre-test, one participant answered correctly, while all 15 responses were correct on the post-test. It is likely that exposure to the terms allowed participants who were unsure of possible modelling specific meaning to feel more secure in their interpretations, thus revising their ”I don’t know” answers. There continued to be uncertainty regarding some terms, however, including ”underdetermined”, ”overdetermined” and ”interpreting” in relation to modelling as indicated by minimal improvements in post-test results. This can likely be attributed to lack of prior knowledge of the vocabulary words themselves, in addition to the limited exposure offered in the module.

Implications for future development of education courses

The positive impact of professional development aimed at improving mathematics PCK has been noted by several researchers in the last decade (Anhalt et al., 2018; Wei et al., 2022). In line with this, this study illustrates the change in MsPCK that can result from a single module during a primary level continuing education course. Though little research on the effectiveness of single session modelling interventions could be found, pre-test/post-test research by Asempapa and Love (2021) found similar growth patterns with their participants in a full day modelling and 3/D printing seminar. It has been the consensus view in research that practicing teacher education initiatives are more likely to improve participant attainment if they are sustained, collaborative, have teacher buy-in, are subject-specific, draw on external expertise and are practice based (Desimone, 2009; Wei et al., 2022), some characteristics of which this module did not uphold. Though this view is well supported by research, recent interpretations of past research have challenged this consensus view, suggesting that there is little evidence supporting that the individual elements listed affect learner achievement (Sims & Fletcher-Wood, 2021). This challenge leaves room for discussion around the impact of alternate professional development features, such as one-on-one instruction, insider perspective of experts, philosophical alignment, and opportunity for mechanising practice—several of which were provided in the module discussed in this study. Given the broad content requirements of the KfK programme, focusing on the comparatively new and complex strategy of teaching with modelling in primary school classrooms brings a challenge for teacher educators. Results from this study suggest that providing practicing primary teachers with an introductory module on modelling may increase their modelling specific teaching competencies, thereby providing a foundation for introducing modelling in their class.

This report on the MsPCK of practicing primary school teachers has several limitations to note. First, the significant gains shown in the results may be a result of confounding variables, including the small sample size (N = 15) and the reliability and validity of the new ProMoPri measure itself. These ideas represent challenges to generalization of these results to continuing education. These limitations also beg other significant questions about the integration of modelling into a continuing education course as a single module of learning, including, the long-term retention of MsPCK, planning and implementation skills for the classroom, and the effectiveness of teaching with modelling on student learning.

Conclusion

This study explores the development of mathematical modelling knowledge among 15 primary school teachers in Norway, focusing on changes in their MsPCK after participating in a modelling module as part of a continuing education course. Pre-test results indicated that while participants were familiar with the concept of mathematical modelling, they lacked knowledge on how to use modelling tasks to engage students effectively, highlighting the initial gap in understanding of modelling cycles and the sub-competencies involved. After completing the module, participants showed significant improvement in their MsPCK across the three competency dimensions covered in the course: theoretical, task, and instructional. There was also a significant change in the fourth dimension (diagnostics) that was not explicitly covered by the course although the effect size was moderate. This indicates that the four dimensions correlate as reported by Nehrkorn et al. (2022a). These results suggest that even limited exposure to modelling instruction may enhance teachers' MsPCK and that this group of teachers may be better prepared to implement modelling in their classroom after engaging in the modelling module than before.

Given the emphasis on modelling in Norway's latest national curriculum, the study underscores the importance of ongoing professional development for primary teachers. Future training initiatives will be essential to support teachers in learning how to integrate and teach mathematical modelling effectively in their classrooms.

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