Social affordances in students' mathematical modelling with digital tools

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This study explores the social affordances that emerge from group interactions when students use digital tools in mathematical modelling activities. Drawing on video recordings and screen-capture data, we analyzed the collaborative work of four groups of 14-17-year-old students as they engaged with two mathematical modelling tasks. Using Gibson's Affordance Theory as an analytical lens, we identified three key social affordances of digital tools: common focus, observing and improving strategies, and authority of the digital tool. These social affordances shaped collaboration, though some were not completely actualized due to constraints that hindered the students' working process. The findings demonstrate how digital tools mediate social interaction in mathematical modelling, highlighting the interplay between affordances, constraints and the learning context.

Several studies have investigated the role of digital tools in mathematical modelling (Greefrath & Siller, 2017; Greefrath et al., 2018), a process that maps real-world situations in mathematical terms with the goal of finding a real-world solution (Niss & Blum, 2020). However, most research in this area has been conducted from a cognitive perspective, focusing on heuristics and the modelling process, often schematized in a cyclic diagram (Cevikbas et al., 2022). As Vos and Frejd (2022) note, an exclusive emphasis on cognitive aspects risks overlooking other important dimensions, such as metacognitive strategies, digital tools used, and social norms that play a role in mathematical modelling.

To address part of this research gap, the present study investigates two aspects that have not been addressed sufficiently: digital tools used and social interactions in mathematical modelling activities (English et al., 2016; Greefrath et al., 2018). While some studies have explored how digital tools shape the social dimensions of group work in math-

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ematical modelling (Afram, 2023, 2024; Geiger et al., 2010), this line of research remains limited. Previous studies have tended to investigate these aspects separately and have rarely examined them in combination. This study brings these strands together by analyzing how digital tools mediate students' group interactions during mathematical modelling. By focusing on interactional processes rather than solely on cognitive outcomes, it moves beyond a purely cognitive perspective to incorporate the social dimensions of students' engagement. In doing so, it highlights how social aspects in a digital environment are essential to understanding the broader classroom context of modelling, offering an integrated social and technological perspective on how digital tools can facilitate or constrain group interactions.

We subscribe to the views of Greefrath et al. (2018) and refer to digital tools as digital technologies, such as computers, tablets, or hand-held devices that can be used to support the learning and teaching of mathematics in some specific way. Furthermore, we do not limit the concept of digital tools to specific devices but also encompass how they are used or mediate the activities of individuals.

The structure of this paper is as follows: first, we review the literature on mathematical modelling with digital tools, and outline the theoretical framework used to analyze social affordances; next, we present the methodology for identifying and analyzing these affordances; finally, we discuss the findings and conclude with key insights and implications.

Literature Review and Theoretical Framework

This study examines the social affordances of digital tools in students' group interactions during mathematical modelling. To situate this focus, we first review research on digital tools in mathematical modelling and their potential to influence social interactions. We then outline an Affordance Theory perspective that underpins our analysis, clarifying how it guides the identification and interpretation of social affordances in this context.

Digital tools in mathematical modelling

Research on mathematical modelling outlines different perspectives and approaches (Blum, 2015; Kaiser & Sriraman, 2006; Stillman, 2019), with the modelling cycle being the most frequently used theoretical approach (Geiger & Frejd, 2015). Figure 1 shows a commonly cited version from Blum and Leiß (2007). Variations exist in the number of phases and terminology compared to the phases shown in figure 1 (Perrenet & Zwaneveld, 2012). Niss and Blum (2020) emphasize that it cannot be overstated that the depiction of the cognitive processes (phases 1-7 in figure 1) involved in performing modelling is an analytic reconstruction of what must happen in principle.

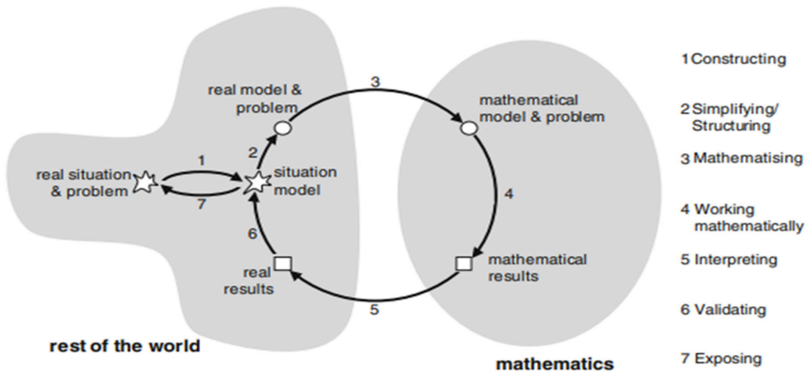

Figure 1. *The modelling cycle by Blum and Leiß (2007).*

Digital tools can be used in all the phases outlined in figure 1 (Greefrath et al., 2018) and can influence group interactions throughout the process (Afram, 2024). For example, in group work, digital tools may shape the approach students take while working mathematically (phase 4 in figure 1) or affect how solutions are validated (phase 6). Research shows that such tools not only impact students' modelling processes (Molina-Toro et al., 2019) but are also shaped by the social dynamic of group work. For instance, GeoGebra and other Dynamic Geometry Software can support sense-making and negotiation in group interactions as students solve mathematical tasks (Granberg & Olsson, 2015; Zengin, 2021). These interactions, however, depend on factors beyond the tools themselves, including the nature of the mathematical task (Geiger et al., 2010) and group composition, as questioning, challenging ideas (Goos et al., 2002), and dominance by high-performing students (Esmonde, 2009) can significantly influence outcomes.

Some studies have examined group interactions in modelling using socio-cultural perspectives such as Cultural-Historical Activity Theory (CHAT) (Afram, 2023, 2024; Hernandez-Martinez & Harth, 2015), based on Engeström's framework (Engeström, 1987). CHAT provides a lens for understanding how socio-cultural factors mediate human activity. Hernandez-Martinez and Harth (2015) highlight that ideas in group work gain value only when connected to a shared understanding of the problem. Similarly, Afram (2023) notes that one of the key factors influencing modelling outcomes is the way group members engage with and build on the outputs, representations, and feedback generated by digital tools during collaborative activities. Importantly, the use of digital tools is inseparable from both the users and the specific activity in which they are embedded (Borba & Villarreal, 2006; Goos et al., 2003; Jacinto & Carreira, 2017).

Although there is a small but growing body of research examining how digital tools influence the social dimensions of group work in modelling, this area remains underexplored—particularly when approached from an Affordance Theory perspective. Wertsch (1998) argues that the impact of mediational means (such as digital tools) can be understood in terms of actions they enable, aligning with Gibson's (2014) conceptualization of affordances. Yet, existing studies that reference "affordance" in the context of digital tools in mathematical modelling (English et al., 2016; Siller et al., 2023) rarely engage directly with Affordance Theory as a framework. Among those that do, the emphasis has often been on the role of digital tools in validating results (phase 6 in figure 1) (Hankeln, 2020), which leaves opportunities to explore their broader influence across different phases of the modelling process and on the social dimensions of group work.

To address this gap, the present study analyzes video and screen-capture data to identify the social affordances that influence students' group interactions during mathematical modelling activities. Specifically, it addresses the research question: *What social affordances of digital tools impact students' group interactions in mathematical modelling activities*?

Social affordances of digital tools: An Affordance Theory perspective

Gibson (1977) introduced the term *affordance* to describe the relationship between organisms—in this case, humans—and their environment. By challenging the traditional dichotomy between subjective and objective aspects that separated organisms from their environment, Gibson contributed to ecological psychology, which emerged as an alternative to the dominant behaviorist paradigm (Bærentsen and Trettvik, 2002). According to Gibson, affordances exist independently of the observer, but must be explicitly, directly and consciously perceived—without necessarily requiring conscious reflection—to be acted upon (Gibson, 1979, 2014). Here, *perception* refers to being attuned to relevant possibilities for action, and not necessarily conscious reflection. Affordances describe the action possibilities an object offers, given the capabilities of the observer. For example, a knee-height horizontal surface (e.g., a low bench) can function as a seat for a human but not for most animals, which lack the ability to sit in a similar manner. Thus, an affordance is not an inherent property of an object but emerges from the relation between the user and the object. Importantly, affordances are always coupled with constraints, which are complementary rather than opposite to them (Hadjerrouit, 2017, 2020).

Some studies have emphasized that affordances are perceived before being actualized (Anderson & Robey, 2017; Bernhard et al., 2013), in line with Gibson's original theory. In contrast, other studies—using perceived in a narrower, more explicit sense—have shown that affordances can sometimes be actualized without explicit and direct perception, for example, through imitation, routine, trial-and-error, or habit (Strong et al., 2014; Volkoff & Strong, 2013; Wang et al., 2018). Markus and Silver (2008) argue that affordances should be perceived by the individual(s) before they can be acted on or actualized. For instance, the affordances that emerge in students' mathematical modelling with digital tools (e.g., GeoGebra) can be categorized into two aspects: the students being aware of the existence of the action potential of GeoGebra (perceived affordances) and when the students turn the potential of GeoGebra into action (actualized affordances). In this study, we adopt this distinction—perceived affordances as explicit and direct perception in Gibson's sense, and actualized affordances as the resulting action—while acknowledging that the former often precedes the latter but is not strictly necessary.

Gibson's original formulation focused mainly on functional or operational aspects of the environment, without explicitly addressing the influence of the socio-cultural context. In response, a more recent approach has sought to integrate Gibson's Affordance Theory with Cultural-Historical Activity Theory. This perspective, as explored by Pedersen and Bang (2016) and Bærentsen and Trettvik (2002), considers affordances through the lens of Leontev's Activity Theory, which presents a three-tiered hierarchical model comprising operations, actions, and activity. Moreover, there exist a number of research studies that explore socio-cultural affordances of digital tools (Afram, 2024; Bærentsen & Trettvik, 2002; Chiappini, 2013; Hadjerrouit, 2017; Kirschner et al., 2004; Turner & Turner, 2002).

Chiappini (2013) explores the socio-cultural dimensions of affordances, introducing "cultural affordances" to describe the cultural objectives embedded in digital learning tools—for example, how a tool's design can reflect the mathematical practices and values of a specific educational context, such as for teaching algebra. Turner and Turner (2002), in their work on collaborative virtual environments, also define cultural affordances as features within an artefact that, through its creation or use, are imbued with cultural values. They note that such affordances are often recognizable only to members of the originating culture; for instance, many people today would not recognize the affordances of a slide rule for performing logarithmic calculations. While these concepts highlight the broader socio-cultural context in which tools are used, our focus narrows to affordances that directly shape student collaboration in mathematical modelling.

Kirschner et al. (2004) identify three relevant categories: technological affordances (usability and functional features that encourage specific learning behaviors), educational affordances (features that support specific learning activities, such as collaborative learning), and social affordances (possibilities for interaction and peer engagement facilitated by the tool). The prominence of each depends on factors like user expectations, prior experience, and the learning context. This categorization is closely aligned with that of Hadjerrouit (2019), developed specifically in the context of mathematics education. Hadjerrouit distinguishes between technological affordances (e.g., to draw graphs and functions), pedagogical affordances (particularly mathematical—e.g., linking representations between geometric, numeric, and graphical forms) (Pierce & Stacey, 2010), and socio-cultural affordances—the last of which aligns with Kirschner et al.'s social affordances and is central to our analysis.

This study focuses on social affordances as they pertain to students' mathematical activities during their interactions with peers (student-student) and digital tools (e.g., student-GeoGebra). In this view, digital tools act as socio-contextual mediators relevant to the student's social interactions (Kirschner et al., 2004). For example, when a group member steps onto the social stage and solves a task with a unique strategy, the digital tool acting as a socio-contextual mediator may invite, allow, encourage, or even guide another member to initiate or suggest another strategy—either to repair divergences or to improve the previous one—within the ongoing interaction. While shared artefacts such as paper-and-pencil can also foster collaboration, our focus is on how digital tools uniquely do so by enabling simultaneous access to shared representations, rapid testing of ideas, and structuring group engagement in ways less feasible with paper-and-pencil methods. We also acknowledge that the affordances and constraints that emerge may depend on the student's characteristics, the type of tasks, the classroom setting, fellow students, and other factors.

Building on this, the present study was conducted in a setting where students worked collaboratively on a single computer. This arrangement meant that certain affordances of the digital tool were simultaneously accessible to all group members. We refer to these as *shared* affordances—

affordances perceived and actualized collectively by several participants in similar ways (Leonardi, 2013; Volkoff & Strong, 2017)—which enable students to coordinate their actions and pursue a common modelling goal.

Given this collaborative setup, several types of social affordances can emerge in mathematical modelling, depending on the specific socio-cultural context. In this study, we focus on three that have been described in prior research: *common focus* (shared knowledge and creating a shared goal); *observing and repairing divergencies* (Granberg & Olsson, 2015; Roschelle & Teasley, 1995); and *authority of the digital tool* (leading to personalizing problem-solving in group situations) (Lowrie, 2011). While these categories are not explicitly labelled as "social affordances" in the literature, they align with our definition and are applied as such in this study.

To create a shared goal (common focus), the students have the facility (provided by the digital tool) to look at the same thing as they negotiate and agree on the appearance of the mathematical representation generated by the digital tool. Granberg and Olsson (2015) argue that students might use digital tools as reference tools to visually demonstrate their ideas to one another in group interaction. For instance, a student might suggest a function to their peers and use GeoGebra to represent this function graphically.

To observe and repair divergences—in the group's solution strategy or process—digital tools can help maintain shared knowledge and ideas in group interactions. In some instances, students might find themselves in a situation marked by uncertainty and divergences (among others), which might cause their solution process to cease. However, digital tools could be used to verify ideas or settle disagreements by performing tests and referencing, among others (Granberg & Olsson, 2015).

The authority of digital tools describes situations where students only accept an answer from the digital tool as correct. Personalizing problem-solving (Lowrie, 2011) might be another way of addressing the authority of the digital tool. Personalizing problem-solving is based on an individual's interest, such as the adopted problem-solving strategies (Yerushalmy, 2000) or the choice of mathematical representation and representational types offered by digital tools. There are also situations where students uphold their strategy or results from digital tools and do not accept other strategies when they think they are close to finding an answer (Afram, 2024).

Methodology

Context of the study

This paper results from a PhD study on secondary school students' mathematical modelling with the aid of digital tools, comprising four secondary schools in southern Norway (Afram, 2024). Mathematical modelling has been a component of the Norwegian curriculum for decades. However, it has been regarded as a compulsory component in the core elements of the mathematics subject at all levels, implemented in autumn 2020 (Berget, 2022). Following the current curriculum (particularly on mathematical modelling), students are expected to have insight into how models are used to describe everyday life and to undertake mathematical modelling themselves in creating such models. This study investigates four groups of secondary school students tackling mathematical modelling tasks using digital tools.

Research design

This study adopts a qualitative case study approach, focusing on four groups (Groups A, B, C and D) of students aged 14—17 from four different schools. All names used are pseudonyms. Groups A (Thea, Rolf and Kåre), B (Emil, Thor, Ella and Tore) and C (Nils, Anna and Jørn) attended upper secondary school (12th, 11th and 11th grade, respectively), while Group D (Olga, Hege and Lena) attended lower secondary school (9th grade). The 11th and 12th graders were under the program for general studies, taking 1T (theoretical mathematics) and R1 (mathematics for science), respectively. Groups A, B and D were mixed-achievement groups, whereas Group C was a same-achievement group (but also a group of high-performing students). Achievement levels were based on teacher-assigned grades using the Norwegian grading scale (1—6), where 5—6 indicate high performance, 3—4 indicate average performance, and 1—2 indicate low performance. This grading scale served as the basis for classifying students into mixed- and same-achievement groups.

The students were selected based on geographical accessibility, prior experience with digital tools, and a mathematics curriculum that supports mathematical modelling. The students in each group were randomly selected (forming the focus group) from among the students who volunteered. The study took place during regular lesson hours. Before the students solved Tasks 1 and 2 without any help (main activity), they solved similar tasks with the help of the teacher and the first author (introductory activity). In both activities, each student group shared a single computer. During the introductory activity, GeoGebra was the primary tool, though other digital tools were occasionally used. The unit of analysis in this study is "social affordances in a group of students' interactions with digital tools during a mathematical modelling activity".

The tasks

The tasks used in the students' activities were purposely chosen as they highlight all the aspects of the modelling process (Berget, 2022). These tasks have been used elsewhere for different purposes (Mousoulides, 2011). The tasks involve the entire modelling cycle in one way or another, even though different phases (in the cycle) manifest themselves with varying weights across the set of tasks. For instance, Task 1 below involves equations and mathematical methods, whereas Task 2 emphasized logical reasoning, analysis, and the application of real-life experiences. No supporting materials, such as maps or links, were provided for Task 2; it was designed as an open task in which students selected and used their own resources, including any digital tools they deemed useful. This meant that while no restriction was placed on tool use, students' choices could be informed by the similar tools used in the introductory activity. The tasks were administered by the teacher in whole-class settings during regular lesson hours, with a 20-minute time allocation per task. Students were not stopped/interrupted if they exceeded this time, though such limits can still impose time-related constraints (Caviola et al., 2017).

Solar power car task (Task 1): A car making company is launching a new solar powered car. Recent market research showed that one hundred people would buy the car for a selling price of €5000. Further, the market research showed that for every €100 price increase, people's interest in buying the car would decrease by one person. Find the best-selling price for the car, so as to maximize the company's sales revenue. Send a letter explaining how you solved the problem to the company's sales manager.

Building a shopping centre (Task 2): The authorities of three towns (Kristiansand, Lillesand and Vennesla) are planning to build a mega shopping center that will serve the needs of their citizens. Identify the optimal place for the shopping center location so that the needs of the three towns are served in a fair way. Send a letter to the ministry in charge explaining and documenting your solution.

Figure 2. *Task 1 and 2 used with the students.*

Data collection method

Data was collected during the main activity, which in total consisted of 3 hours and 30 minutes of recorded conversations (via video recordings) and computer activities (captured using screen-capture-software) from all the groups combined.

The video recordings of the students' interactions during the activities revealed their working process and their interactions with each other and the digital tools.

The screen-capture-software complemented the video recordings as it provided information about how the students solved the task on the computer. The groups' approaches to tackling the tasks differed (although some groups' approaches were similar to others).

This paper does not focus on the students' results (although it is indeed a relevant issue on its own) or other topics concerning the students' activities. Instead, our primary focus is investigating the social affordances of digital tools that impacted the students' mathematical modelling activities. We did not probe into the students' views (by interviews during the activity on why they took a particular action), as that would distort the flow of the activity and, perhaps, influence the emergence of social affordances to some extent.

Dialogue between the students was transcribed verbatim, and interactions with GeoGebra, Excel/spreadsheet, calculator, Google Maps, Google Search, and gestures (such as pointing to the screen) are described below using square brackets (e.g., [Hege plots a point 'A = (100, 5000)' in GeoGebra]).

Data analysis and interpretation

This study uses thematic analysis, as outlined by Braun and Clarke (2006), based on the combination of inductive and deductive approach to coding. Specifically, the process involves generating codes directly from the data, identifying, defining and naming the themes, drafting a report, and categorizing the themes within predefined categories. In essence, thematic analysis serves as a method for identifying, analyzing, and reporting patterns (themes) within the data. The coding process is based on interpretations of the data. However, because of the theory-driven (deductive) approach, the interpretations are influenced by predefined categories within the theoretical framework. The process of searching for, defining, and naming themes emerge both from the empirical data (and the codes derived from it) and from the predefined categories. Thus, thematic analysis involves an iterative process between the codes and the empirical data, allowing new patterns (themes) to emerge and generating additional codes as the process evolves.

The data was organized based on the phases of the modelling process (see figure 1). Each phase was then analyzed according to the social affordance categories. In this study, the predefined categories are the social affordances: *common focus (CF)*, *observing and repairing divergencies (ORD)*, and the *authority of digital tools (ADT)* (defined and explained in the theoretical background section). In the analysis process, ORD could not explain a section of the data; hence, a new category, *observing and improving strategies (OIS)*, was introduced. We define the social affordance OIS as the process where the digital tool allows students to view or follow the solution process and improve the strategy adopted due to the affordances of the digital tool perceived by the students. For instance, one student might input numbers into a function/equation to examine changes, while another student might suggest using a slider as a more efficient way to track those changes based on what he/she perceived of the tool.

This new category, OIS, resulted from allowing the categories to emerge from the data, although we started with theory-informed categories. We also invited another researcher to code a section of the data, and we compared, discussed and modified our codes/categories (intercoder reliability). The codes for the social affordances were analyzed along with other codes, resulting in an overall intercoder reliability of 98.44 % (Afram, 2024). The analysis focused on students' language (suggestions, questions, answers, arguments, and others) and actions (gestures and interaction with digital tools). The analysis followed an interpretative perspective, considering that providing a detailed and in-depth description of the students' activities would encompass a conjunction of the students' perception of their activity through their actions (data from video recordings), the analysis of their solution (data from screen capture) and our perspective informed by the theoretical background of this study.

Results

Table 1 summarizes the actualized social affordances identified in the activities of Groups A–D for Tasks 1 and 2, while table 2 (non-group-specific) shows how these affordances appeared across the phases of the mathematical modelling process (see figure 1). In both tables, an "X" indicates that the affordance was identified, while an empty space means it was not. These identifications are based on our categorization of social affordances actualized through students' interactions with peers and digital tools.

As shown in table 1, the recorded social affordances were CF, OIS and ADT; no instances of ORD were identified. CF was the most prevalent, emerging in all groups and both tasks. Table 2 shows that CF was also the only affordance that emerged in every phase of the modelling process. OIS emerged exclusively in phase 4 (working mathematically), while ADT emerged in phases 4 and 6 (validating). Phase 4 was also the phase in which the greatest variety of social affordances emerged, followed by phase 6.

Together, the tables provide complementary perspective: table 1 links social affordances to specific groups and tasks, whereas table 2 situates them within the broader modelling process. The following subsections illustrate these patterns with excerpts from video transcripts and screen recordings as evidence, focusing on qualitative insights into how each social affordance manifested rather than on statistical analysis.

Table 1. *The social affordances identified in the students' activities.*

		Groups			
Social Affordances	Task	A	B	C	D
Common focus (CF)	1	X	X	X	X
	2	X	X	X	X
Observing and repairing divergencies (ORD)	1				
	2				
Observing and improving strategies (OIS)	1	X			
	2				
Authority of digital tools (ADT)	1	X	X		
	2				X

Table 2. *The social affordances identified in the phases of the mathematical modelling process.*

	Phases of the modelling process (see figure 1)						
Social affordances	1- Constructing	2- Simplifying/ Structuring	3- Mathematising	4- Working mathematically	5- Interpreting	6- Validating	7- Exposing
CF	X	X	X	X	X	X	X
ORD							
OIS				X			
ADT				X		X	

The solution strategies employed by the groups shared both similarities and differences. Consequently, the actualized social affordances were alike but manifested in different ways. To illustrate this, we present an example of the actualized social affordances for each task.

Common focus (CF)

From table 1, the social affordance CF was identified in Groups A, B, C and D's activities as they worked on Tasks 1 and 2. The students had the facility to look at the same thing as they agreed on a shared goal through a flow of turn-taking, dialogue and action. However, CF was not identified individually when the students only had the device (digital tool) to themselves (for example, hand-held calculators and mobile phones). Below, we will present an example of CF regarding Group A's activities in Task 1. Group A's solving activity started outside the computer. Thus, the students recognized and classified the variables in the problem (the people and the car price) and the things they needed to do. Group A used the trial-and-error method by analyzing patterns of numbers after searching for a function that represents the number of people buying the car and the price at which they buy it. Below is part of the transcription of Group A's activity regarding Task 1, illustrating the CF category:

Kåre: Like this [Points to the x and y axis in GeoGebra, draws a graph with paper-and-pencil and writes $f(x) = 100x$ representing the graph].

Thea: Erm no, then you say that erm... its going down with a 1000... If you understand.

Kåre: So, it will be naturally in there, right? 'Konstantledd' [constant term] or something?

[...]

Thea: Yeah, it's going to be on the x-axis, it's not a constant. Do you have any ideas? [Thea asks Rolf if he has any idea] ... If we try... I just try something [Draw the graph of the function $f(x) = -x + 100$ in GeoGebra, see figure 3]. Erm, it goes down by one person, if we just try, I don't think this is the right...

Rolf: It could be true.

Thea: Yeah, if we think that 5000 is zero then when [writes $x = 1$ on the graph, see figure 3]

Rolf: Should be 99 or something.

Kåre: So, what you are showing here is erm... you lose one person per 100.

From the dialogue above, the students negotiated and agreed on the function representing the number of people who bought the car. They made a linear graph with the number of people on the y-axis and the price of the car on the x-axis (see figure 3). From figure 3, if $y = 99$ (people), it intersects the function f at $x = 1$, meaning 99 people will buy the car at 5100 euros (since the x-axis starts from 5000 and every one point represents 100). The students then used a hand-held calculator to find the total revenue by multiplying the number of people by the corresponding car price (they repeated this procedure until they arrived at an answer). GeoGebra was used as a reference tool to visualize one's reasoning during the mathematical discourse. For Kåre to visually demonstrate his reasoning to his peers, he used GeoGebra as a reference tool by pointing to the coordinate axis and sketching with paper-and-pencil ($f(x) = 100x$) in relation to the coordinate axis. Thea, responding to Kåre's proposed function, used GeoGebra to visually demonstrate her suggested function $f(x) = -x + 100$.

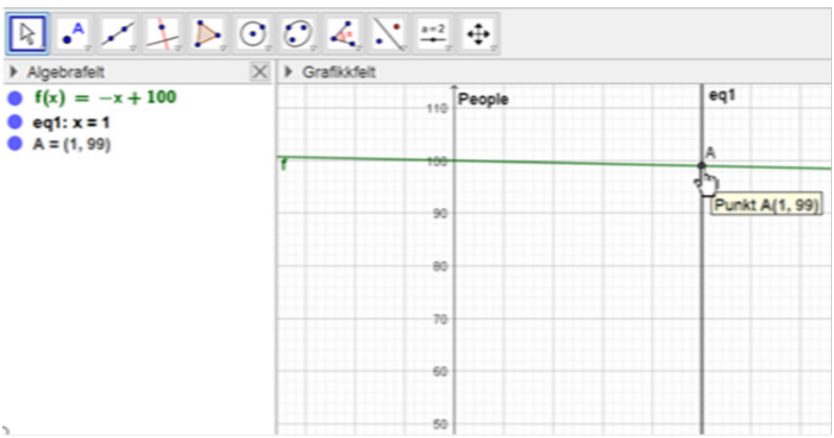

Figure 3. *Illustration of Group A's initial function regarding Task 1.*

Below, we present another example of CF from Group A's engagement with Task 2. After reading the task, the students suggested the optimal location and located the three cities on Google Maps, and the following dialogue occurred:

Kåre: Yeah, so it should be like above Vennesla somewhere. Can you show me where Lillesand is on the map?

Thea: Yes, a moment. Should we find the map, do you agree?

Rolf: Yes.

Thea: [Opens Google Maps, and searches for Lillesand, see figure 4]. Lillesand, my cousin has a cabin there.

Kåre: So, this is Kristiansand and there is Vennesla [Pointing to the map].

Thea: We save it [Saves Lillesand and the other cities on Google Maps].

Rolf: It's like a triangle then [Joins the three points on the map by hand].

In this dialogue, Rolf visualizes his reasoning by joining the points of the cities by hand and concluding that it will form a triangle. This helps with the interaction between the students as they have the tools to look at and follow the same things in their interactions.

The activities of Group A illustrated above (regarding both tasks) occurred in the first three phases of the modelling process (see figure 1). Thus, the students recognized and classified their initial variables, searched for the position of the three cities on the map, drew an initial function with paper-and-pencil, and created a model in GeoGebra, among others. This does not mean CF only emerged in the first three phases of the modelling process. It emerged in the other phases of the modelling process (see table 2), but we only reported two instances due to the scope and focus of the study.

Figure 4. *Illustration of Group A's search of the three cities on Google Maps.*

As shown in table 1, the social affordance ORD was not identified in the students' activities for either task. Therefore, we will proceed to report on the next category, OIS.

Observing and improving strategies (OIS)

As shown in table 1, the social affordance OIS was only identified in the activities of Group A as they worked on Task 1. We will report two instances of this category as they occurred in Group A's activities. While Group A repeated the procedure highlighted under CF, the students found themselves in a situation where they wanted an efficient way to find the company's maximum revenue. The excerpt below describes Group A's interaction:

Rolf: But isn't it like a faster way to find that out. I feel like there is, but I don't have any idea how to do it.

Thea: We can make sliders, I think... We can try.

Kåre: I don't know.

Thea: Erm [Makes a slider a = 100, but the slider has no effect on the graph, see figure 5] ... We just try something else [writes y = 60 on the graph and found the point of intersection with the line f(x) = - x + 100, see figure 6]. Here 40 multiplied with 100, 4000 so it's not more. So, I think we should try ...

Rolf: Try 100.

GeoGebra could be used to maintain and improve shared ideas in group interactions. In the dialogue above, Rolf reviewed the solution strategy and felt there was a faster way to find the maximum revenue, but he could not visually demonstrate his ideas. This could be that Rolf was not confident enough to put forward his thoughts, or did not know how to actualize what he perceived GeoGebra could afford them. However, this triggers Thea to come up with the idea of making sliders (see figure 5). Thea made a slider ($a = 100$), but it did not affect the graph as it has no link with the function ($f(x) = -x + 100$). The tool has a constraint that the slider must be well-defined to have any effect on the function. When unsuccessful with the sliders, the students reverted to their initial strategy. Considering the students' activities, there was no divergence in the initial strategy, as the strategy adopted only needed improvement to be more efficient. However, their interactions and solution process might have changed if they had successfully created the slider. In this case, GeoGebra might have afforded the possibility of "observing and improving" solution strategies in group interactions if the function and slider under construction were mathematically linked. Thus, if the students inserted the function $f(x) = -x + 100$ in the algebra view in GeoGebra with $x = a$ (forming a slide $a = 1$ and an equation eq1: $x = 1$). Then, intersecting f and eq1 with the intersection point A (i.e., A = Intersect (f, eq1,1)) might help to regulate the number of people buying the car and the price at which they buy the car (using the slider).

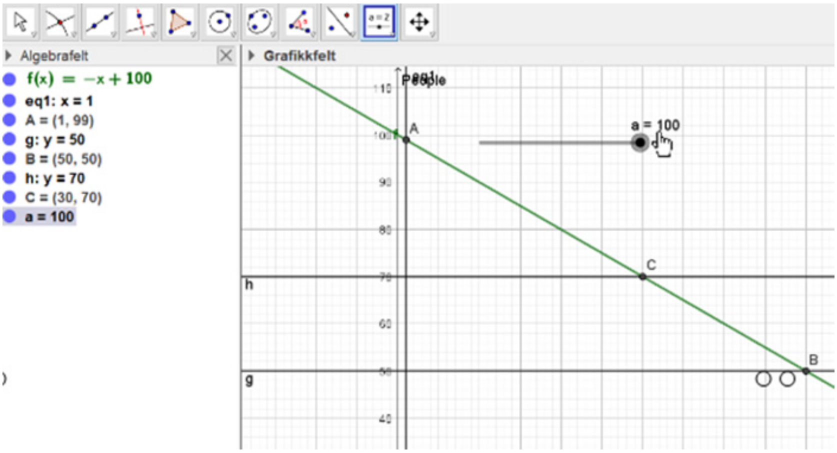

Figure 5. *Illustration of Group A's attempt to make a slider to regulate their function.*

In another instance, the category OIS was identified in Group A's activities as they shifted to a different strategy for efficiency. In this case, a group member reviewed their strategy and came up with the idea of introducing another function. The excerpt below describes this situation:

Rolf:	[Writes the function $y = 100x + 5000$ in the algebra section and reduces the size of the graph, see figure 6].
Thea:	So, we should go over 50 and... or between 100 people and 50 people apply ...What have you done?
Rolf:	I just wanted to draw a new graph so that we can maybe take erm... I don't think it's right, cos... it can be over the border, I mean go over 100. There might be more money... I just forgot it actually.
Thea:	I don't understand the graph.

Group A attempted to manipulate their function with the function $a(x,y) = xy$ (see figure 6), but they were unsuccessful. Hence, they tried another method. In the dialogue above, Rolf made a new equation

$y = 100x + 5000$ (see figure 6), representing the price of the car (while the initial function $f(x)$ represented the number of people buying the car). Rolf seemed confident enough to demonstrate his ideas compared to the previous attempt. However, the group did not attain the desired results as no function was defined to combine the two functions (e.g., $g(x) = f(x) \cdot h(x)$, where $h(x) = y$). Again, Group A reverted to their initial strategy when unsuccessful.

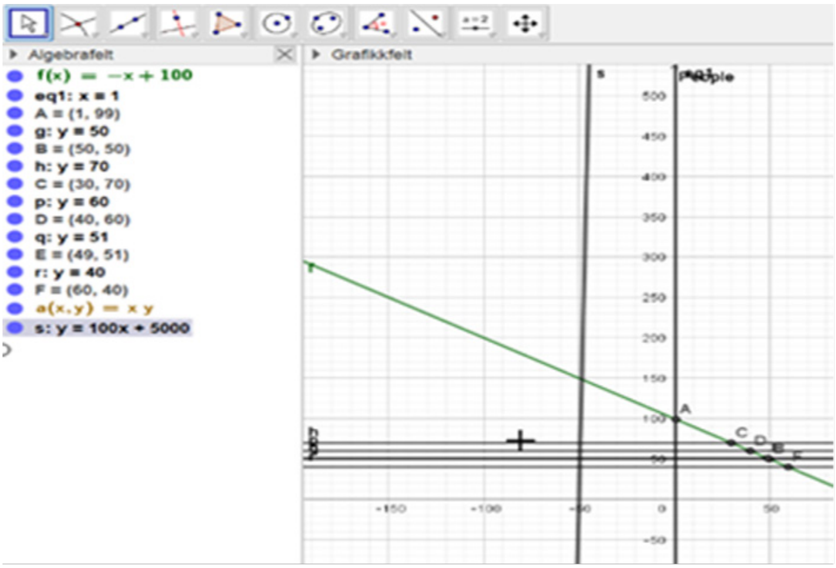

Figure 6. *Illustration of Group A's attempt to draw a new graph.*

The activities of Group A, as illustrated above (regarding Task 1), emerged in phase 4, "working mathematically" (see figure 1), of the modelling process (see table 2). Based on the analysis, their activities in phase 4 might have changed if they had actualized what they perceived of the digital tool. In this case, their new strategy might have improved their initial strategy. In summary, GeoGebra affords OIS in group interactions; however, there are constraints of the tool that hinder this process.

Authority of digital tools (ADT)

From table 1, the social affordance ADT was identified in Groups A, B (regarding Task 1) and D (regarding Task 2) activities. We will provide an example of this category as it occurred in Group A's activities related to Task 1 and another from Group D's activities concerning Task 2. In the case of Group A, while testing various numbers to determine the maximum revenue, a student suggested using spreadsheets to generate their data. The excerpt below illustrates this situation:

Rolf: Oh! we could have done all of it with the 'regneark'(spreadsheet).

Thea: Yeah, that's right.

Rolf: And then just try with the

Thea: We didn't think about it.

Rolf: Or we just… I mean we can do it now; it might take a shorter time.

Thea: Do you think?

Rolf: I think so.

Thea: But we are already done, though.

To provide further context for the dialogue above, we will first offer a brief background on Group A. This group is a mixed-achievement group, and their teacher notes that Thea consistently outperforms her peers. In Task 1, Thea often assumed a leading role, dominating the conversation and guiding the group's focus on her input at the computer (Afram, 2024). Prior to the dialogue above, Thea started with a problem-solving strategy that she was comfortable with, starting with a graphical representation and then analyzing patterns of numbers and observing the increment in revenue. In the dialogue above, Rolf proposed an efficient way to generate data; however, Thea, having personalized the problem-solving strategy, dismissed Rolf's suggestions and reverted to the existing idea, thinking they were already close to finding the answer (which might also be influenced by time constraints). Subscribing to Rolf's suggestions might have helped the group generate their data with the spreadsheet and find a function representing it. However, the features of GeoGebra afford multiple problem-solving strategies, and the approach used by the group depends on the representational choice of the students taking the leading role, especially when they think they are close to finding the answer. It is possible that Thea hesitated to accept the new idea because she might have felt it would require restarting the entire solution process.

In Task 2, we identified an example of the category ADT in Group D's activities. The group took a screenshot from Google Maps and inserted it in GeoGebra, where they constructed a theoretical middle point (using the circumcircle/circumcenter of a triangle approach) without factoring in population, roads, or other considerations. One student noted equal distances, but another insisted on measuring them in their solution process. The excerpt below describes this situation:

Olga:	Now you can see it is equally long between all points.
Lena:	We can measure.
Hege:	[Searches for the distance between each city and the middle point, see figure 7].
Olga:	Why are you measuring? It is the same length. Aha! It is not as long as that, is it? That is the fairest.
Hege:	Oh, yeah [Finished measuring the lengths, see figure 7].
Lena:	Yes.
Olga:	To have it there, are we certain?
Hege:	Yeah.

From the dialogue above, Olga suggested that the distances from the cities to the optimal/middle point are equal since the circle passes through all the cities (see figure 7). The other students suggested they still measure these distances (see CF, FE and FD in figure 7). Thus, these students would rather accept the answer/outcome from the digital tool than their peers or measure these distances to ensure their final results.

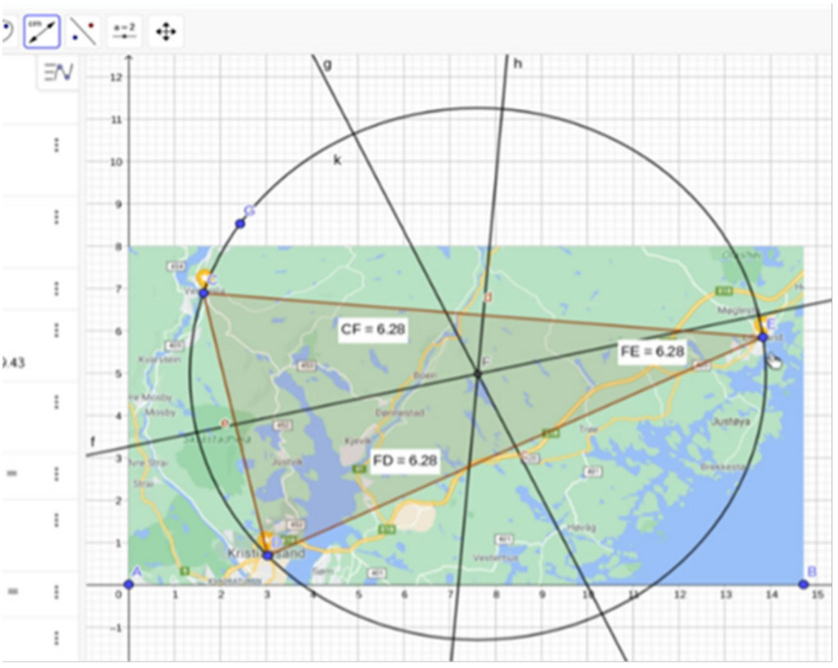

Figure 7. *Illustration of Group D's measurement between the optimal point and the cities.*

The activities of Group A illustrated above (regarding Task 1) occurred in phase 4, ”working mathematically”, while those of Group D occurred in phase 6, ”validating”, of the modelling process (see table 2).

Discussion

The results indicate that the social affordances of digital tools impact group interactions in mathematical modelling activities. The analyzed social affordance categories are common focus (CF), observing and improving strategies (OIS), and authority of digital tools (ADT). These social affordances were actualized in the students' activities. The discussion below follows the order of social affordances listed above.

Digital tools (not hand-held devices) provide a platform where students can create a shared goal by looking at the same element as they negotiate and agree on the appearance of mathematical representations generated by the tool (Granberg & Olsson, 2015). For example, a student used GeoGebra to demonstrate her suggested function graphically while responding to the suggestions of another student during the mathematical modelling activities (see figure 3). Our study reinforces findings made by Granberg and Olsson (2015) that students use digital tools as reference tools to visualize their reasoning or demonstrate a mathematical representation. The interaction example reported in this study occurred in the first three phases of the modelling process (see figure 1). However, the category CF is not limited to these three phases but might emerge in other phases (see table 2). Our main explanation for this might be that the students had the facility to look at the same things during their interactions throughout the modelling process. However, aside from table 2, the study does not provide empirical data to support this claim. Instead, we can only offer a plausible explanation based on our insights into the students' activities. On that basis, we emphasize that digital tools can provide affordances in the different phases of the modelling process (Greefrath et al., 2018), and the category CF emerged in these phases as the students engaged with the tool.

The category ”observing and repairing divergencies” (ORD) was not identified in the student's activities (see table 1). Granberg and Olsson (2015) describe this category as using digital tools to maintain shared knowledge and ideas through verifying ideas or settling disagreements by performing tests, among others. On the other hand, a new category emerged from the data, ”observing and improving strategies” (OIS). Thus, digital tools provide a platform where students can view or follow their solution/working process and improve the strategy adopted during group interactions. For example, a student felt there was a faster way to find the maximum revenue for the car-selling company but could not visually demonstrate his ideas. A possible explanation for this issue is that the student was not confident enough to express his thoughts. However, his suggestion triggered another student to develop the idea of making sliders to improve their adopted strategy. Even though the students made a slider, it did not affect the graph as it has no link with the function (see figure 5). Hence, the group reverted to the initial strategy they had begun with. Drawing students' attention to such constraints might help them improve their problem-solving strategies. This finding shows that affordances and constraints are complementary (Hadjerrouit, 2020). In another example, the students introduced a correct function that might help improve their strategy but could not combine their initial and current functions (see figure 6). From the above examples, there was no divergence in the strategy they began with, as the strategy adopted only needed improvement to be more efficient. In this situation, the digital tool might allow observing and improving solution strategies in mathematical modelling activities if the function and the slider under construction are mathematically linked. In this study, the category OIS mainly emerged in phase 4, "working mathematically" (see figure 1), of the modelling process (see table 2). "Working mathematically" is a cognitive barrier in the modelling process that entails manipulating the algebraic formulas, calculating, comparing and others (Blum, 2015). However, other aspects, besides the cognitive aspects, play a role in the modelling process (Vos & Frejd, 2022). For instance, from an Affordance Theory perspective, the students' perception of the digital tool influences how they work mathematically (see figures 5 and 6). Again, affordances and constraints result from not only what the students perceive of the tool but also the educational environment (nature of the task, characteristics of the students, and others) in which the students engage with the tool (Hadjerrouit, 2020).

The social affordance category authority of digital tools (ADT) describes situations where students might only accept an answer from the tool as the correct one. For example, a student suggested that the distances between the middle point and the triangle's vertices are the same since the circle passes through all the vertices. Another student insisted they measure these distances with the tool to ensure the answer (see figure 7). This happened in phase 6, "validating" (see figure 1) (Hankeln, 2020), of the modelling process (see table 2). On the other hand, personalizing problem-solving strategies could result from ADT. Thus, personalizing problem-solving is based on an individual's interest, such as the adopted problem-solving strategy or choice of mathematical representation and representational types offered by digital tools. For example, the dominant student (Thea in Group A) had a problem-solving strategy similar to what Yerushalmy (2000) reports as starting with graphical representation and analyzing patterns of numbers. This student had personalized the problem-solving strategy and dismissed the comments from another student (returning to the existing idea), thinking they were already close to the answer. This happened in phase 4 (see figure 1) of the modelling process (see table 2). Subscribing to the new suggestions might have changed their solution strategy. This echoes previous research that points out that personalizing problems might hinder the potential for sophisticated sense-making (in group interactions) that could lead to a better outcome (Lowrie, 2011). Lowrie (2011) argues that the more personalized the students might want the problem to be, the more likely these students might complete aspects of the problem individually (and not consider the ideas of others). If we draw students' attention to not personalizing the problem-solving strategy but considering input from peers, it might benefit their learning and achievement in mathematical modelling with digital tools (Afram, 2023). Rejecting new ideas could be that the new ideas are not clear enough to connect with the group's current thinking (Hernandez-Martinez & Harth, 2015). Alternatively, as reported in this study, a new idea would involve restarting when a solution is imminent. Rejecting a new idea could result from time constraints, although our study does not provide empirical data to support this claim. However, we share Caviola et al.'s (2017, p. 7) views that time constraints might "interfere with decision making by altering strategy selection" in problem-solving.

Conclusion

We now revisit the research question of this study, namely: *What social affordances of digital tools impact students' group interactions in mathematical modelling activities?* This question is addressed from an Affordance Theory perspective. Our study has contributed novel insights for research on mathematical modelling with digital tools. Firstly, Affordance Theory has been shown to be helpful in investigating the social dimensions and impact of digital tools on students' mathematical modelling activities, in contrast to the cognitive approach in the research field. Again, from Gibson's Affordance Theory perspective, affordances are relational and emerge from the students' interactions with the educational environment of the mathematical modelling activities. Secondly, utilizing the categories of common focus (CF), "observing and improving strategies" (OIS), and authority of digital tools (ADT) has further shown to be an appropriate methodological approach for exploring social affordances that emerge in school educational settings. It is reported in this study that CF mainly emerged in all the phases of the modelling process, OIS emerged in phase 4 (working mathematically), while ADT emerged in phases 4 and 6 (validating). Thirdly, the approach outlined in this study is intended to map other social affordances beyond the ones presented in this paper. Moreover, from a practical point of view, affordances and constraints of digital tools in mathematical modelling activities need to be critically examined. They might be an opportunity for students' learning of mathematical modelling by enabling collaborative problem-solving, fostering communication, enhancing group dynamics, and providing diverse perspectives, including those of the teacher, which can enrich the collaborative learning process with digital tools. For instance, how teachers can help students make sliders to manipulate their function or combine two functions while using GeoGebra, among others. Finally, in terms of the study's limitations, the smaller number of participants does not warrant a generalization of results. Thus, further research is needed to achieve more reliability and validity in broader modelling contexts.

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