Teaching ordinality to 6-year-olds using the number track

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This study examined the teaching of ordinality to 6-year-olds, using the number track as a teaching artefact. In observations of teaching situations in 95 Swedish preschool classes, 52 episodes showed to use the number track in teaching about numbers. The framework of variation theory was used to analyse the observations. In the analysis, we identified several aspects of ordinality and could draw conclusions of what was made possible to learn. The result indicated that the number track is primarily and almost exclusively used to arrange various number representations (number words, number symbols, objects and pictures) in sequence. The study further shows a broad range of teachable aspects of ordinality in addition to mere sequential placement of numbers.

Keywords ordinality, number track, teaching, variation theory

In early mathematics education research, many studies have contributed to knowledge about what students need to learn to make use of numbers in solving arithmetic problems. Beyond reciting numbers in the correct order, learning about numbers is a complex endeavour that involves a conceptual understanding of numbers as composite sets and related to other numbers in both ordinal and cardinal sense (Nesher, 2018). Many studies on early number learning focus on children's awareness of cardinality in numbers (see Sarnecka & Wright, 2013; Paliwal & Baroody, 2020) or cognitive processes such as subitizing (see Wynn, 1992; Macdonald & Shumway, 2016), but less have paid attention to ordinality as an equally important feature for learning the meaning of numbers (see Fuson, 1988a; Coles, 2014; Sinclair & Coles, 2017; Askew & Venkat, 2020).

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This study aims to identify and discuss how aspects of ordinality appear in teaching when using a number track. Ordinality involves numbers' (stable) order, size ordering and consequently also the (base-10) number system and place value (Coles, 2014; Coles & Sinclair, 2018; Harju & Van Hoof, 2025), making it an important aspect to become aware of in order to use numbers in basic arithmetic problem-solving. As little is known about how ordinality is taught to young students, the challenge is then, how to teach this aspect of numbers to facilitate learning that goes beyond the ordering of numbers.

A large variety of content related to numbers, and their features was observed while mathematics was taught to 6-year-olds in 95 Swedish classrooms (Elofsson et al., 2024). Many teachers were observed using the number track as an artefact, with number symbols arranged in a row from 0 or 1 to 10 or 20 to teach students about numbers. This artefact became the starting point to put other number representations in sequence, that is number as words, number (as quantity) of objects or pictures. It has been shown that teaching numbers in a sequence can support young students learning of numbers. For example, an awareness of ordinality may be supported when structured number sequences of the number system are chanted (e.g. 1 2 3 ... 10 20 30 ...100 200 300 ...) (see Coles, 2014; Sinclair & Coles, 2017; Coles & Sinclair, 2018). Searching for the missing number in a sequence (e.g. 2 4 ? 8) has also been shown to support students' understanding of number sequences (Pasnak, et al., 2016). However, how to utilize the potential, and be aware of the challenges of using the number track as an artefact in teaching, remain underexplored.

The artifact that we refer to in this study as a number track is, like the number line, a representation of numbers in sequence. The difference between them may be explained in terms of continuity, different sets of numbers and measurement. "The number track does not possess the underlying feature of continuity that is the governing feature of the number line. Its replicated units represent the discrete aspect of number and not its continuous aspect" (Doritou, 2006, p. 39). This means that on the number track, we will find the number 5 after 4 and before 6 (ordinal aspect). The number line shows the number 5 as a point on the line (ordinal aspect) but also as five units, a measurable distance between the points 0 and 5 on the line (cardinal aspect) (Lakoff & Núñez, 2000). This study deals with the number track, more specifically those that appeared in observations in preschool classes.

Teaching is the focal point of this study, particularly the aspects of ordinality highlighted in teaching acts using the number track. Observed preschool class lessons were analysed with a focus on the enacted object of learning, which refers to what is made possible to discern (and learn) in an actual teaching situation when using the number track. Although ordinality is considered a key aspect of learning numbers, it remains a relatively underexplored topic in mathematics education (Baccaglini-Frank et al., 2020). Hence, this study aims to identify and discuss how aspects of ordinality appear in teaching when using a number track by answering the following research questions:

– In what ways do aspects of ordinality appear when using the number track as an artefact in teaching about numbers?

– What opportunities to learn about ordinality are offered in the enacted teaching?

Learning about numbers

The field of mathematics education research has long directed interest towards the development of number sense, that is, what numbers mean to young students and how they are able to use numbers in quantitative problem-solving. In a study of preschoolers' learning of number meaning (Björklund et al., 2021), ordinality, together with cardinality, numbers' part-whole relationships and representations of numbers (e.g. number words, number symbols, objects and pictures) were highlighted as necessary aspects to discern in order to make use of numbers in basic arithmetic problem solving.

Early on, Piaget (1965) proposed that the concept of number involves a relationship between the cardinality and ordinality of numbers. When a child determines the quantity of a set of objects, often by pointing and saying the number word sequence, the ordinal features of the numbers appear as the child utters one word for each object to be counted in a stable order. The child stops counting (and pointing) once the last object of the set has been assigned a number word. That is, seeing the objects as a sequence and labelling them with corresponding ordinal numbers. If this procedure is done to determine the number of objects counted, the child also uses the number words in terms of cardinal numbers, that is, answering to the question "how many" (Jungic & Yan, 2020). Thus, the same number can usually, but not necessarily, refer to both cardinal and ordinal meanings simultaneously. By mastering a one-to-one correspondence between number words (stable order) and the objects of a group or set (enumerating), the students are in the situation of ordinality and at the same time enclosed in the situation of cardinality (Nesher, 2018).

According to Gelman and Gallistel (1978), ordinality is considered to precede cardinality in terms of the development of young students' number sense, and, according to some studies, it may even be considered necessary for learning cardinality (see e.g., Nesher, 2018; Askew & Venkat, 2020).

Sasanguie and Vos (2018) demonstrated that cardinal features of numbers represented by number symbols seem to precede 6-year-olds attention to ordinal features, showing that younger children recognise the relationship between number symbols in a cardinal sense (7 is a larger quantity than 4), however, over time a shift occurs to students' ordinal processing, and they rely on their memory of the order of number (7 comes after 4). Introducing numerical symbols in the process of learning numbers has two advantages: they represent large quantities accurately and they may facilitate the learning of the complex relationship between numbers (Lyons & Beilock, 2011). Older students' and adults' ability to estimate quantities, along with an understanding of number symbols and their skills to arrange them in sequence, has been shown to contribute to their advanced mental arithmetic competence. Thus, the ordering of numbers in relation to other skills and abilities (rather than an isolated skill) plays a significant role in the development of young students' numerical knowledge (Lyons & Beilock, 2011). Nevertheless, teaching numbers in the early years appears to largely overlook various aspects of ordinality, assuming that such a relationship between numerical symbols and quantities can be used as soon as students become familiar with the symbols (Coles & Sinclair, 2018). Although these and other studies do not reveal the origin of number sense – that is, whether ordinality, cardinality or some other features of numbers are foundational to understanding them – they do highlight the complexity of learning the meaning of numbers.

Ordinality – a multifaceted aspect of numbers

Sinclair and Coles (2017) argue that ordinality has a fundamental role in learning about numbers. Ordinality, along with language use (e.g. number words), symbols and number relations, seems to support the understanding of numbers in early childhood. According to Baccaglini-Frank et al. (2020), ordinality is a key aspect of numbers for children to develop their number sense. They describe knowledge of ordinality as "Associating number symbols to number words; knowing the number symbols sequence; knowing the number words sequence; knowing what number comes before or after a certain one" (Baccaglini-Frank et al., 2020, p. 781), highlighting its strong connection to symbolic understanding and relationships between number symbols, including number words. With support of a one-to-one correspondence between number words (stable order) and symbols (or objects) a student can navigate ordinal situations of numbers and arrange number representations sequentially.

In general, the first representation of numbers that children often encounter is number words. Children learn to recognise number words in specific order during their early years (Fuson & Hall, 1983; Ahlberg, 1997), first without numerical meaning and later, including relationship between the number words. The sequence of such words forms a foundation for experiencing and learning their relational meaning, as expressed through ordinal number words such as *first*, *second*, *third*…. In addition to the ordinal number situation of ordinality, Fuson (1988b) also describes the sequence situation of ordinality, in which a sequence is produced from various elements. Fuson (1988b; see also Coles, 2014) also describes ordinality in terms of order relations, as in a relationship between two situations, such as *four is more than three* (cardinal situations), *the fourth comes later than the third* (ordinal situations), *four meters is longer than three meters* (measure situation) or the sequence situation in which *four comes after three*. This approach of explaining ordinality in numbers reveals the context in which the number is presented and highlights that ordinality seldom stands alone; it is often intertwined with cardinal, measure or sequential features. To discern the ordinal relationships between number representations such as number symbols is thereby considered an important aspect of learning arithmetic (Lyons & Beilock, 2011).

Ordinality is an abstract mathematical feature of numbers that must be represented in some way for the meaning to be discerned and communicated. Using artefacts to teach numbers makes their meaning *visible*, and different representations of the same meaning assist students in generalising it (Lesh, 1981; Askew, 2019). The pedagogical use of artefacts may thereby illustrate ordinal relationships between numbers (especially in terms of number symbols).

Theoretical framework

In this study, the variation theory of learning (Marton, 2015) was used as theoretical framework. The theory is built on insights gained from phenomenographic research on different ways of experiencing the same phenomenon (Marton, 1981). Learning is seen as taking place when aspects not previously discerned (so called critical aspects) of an object of learning (what is to be learned) are discerned. Experiencing variation is necessary for the aspects to become noticeable for the learner. Therefore, how variation is used in teaching to make aspects visible for learners is central, because this is what liberates or limits what is made possible to experience for the learner, and the learning opportunities provided (Kullberg et al., 2017; Ekdahl et al., 2025). Hence, this study examines the object of learning, understanding ordinality, in the enacted teaching constituted by teacher and children together. What the students learned (the lived object of learning) is, however, not analysed in this paper as we did not collect any data on students' learning outcomes. The variation theory of learning emphasises that learners must experience variations in critical aspects for them to be discernible; hence, they cannot only be told or pointed out. Using variation in terms of contrast (indicating difference) and generalisation (indicating sameness) are ways to open dimensions of variation regarding the critical aspects. However, to experience variation (contrast) in one aspect, other aspects must be invariant. Marton (2015) argues that contrast precedes generalisation in the process of learning a new phenomenon. Critical aspects are dimensions of variation that can be opened up in teaching. In enacted teaching, students can be provided opportunities to discern the (critical) aspects, that is, what the learner needs to discern in order to experience the object of learning in the intended way (Lo, 2012). Critical aspects may differ among students, as they are individually bound, depending on a student's prior experiences and knowledge. In this paper, how aspects of ordinality are enacted in teaching and what learning opportunities are offered are analysed.

Method

In the project SATSA[1], the content and methods of teaching are studied, as observed in practices involving teaching numbers to 6-year-olds. Observations were made in the form of field notes of 145 teaching situations in 95 Swedish preschool classes. The field notes were transferred to a protocol based on the Mediating Primary Mathematics (MPM) framework (Venkat & Askew, 2018). This protocol focuses on different features of teaching, including the ways in which artefacts[2] and inscriptions are used, and how teachers through their talk and gestures direct attention to and explain mathematical meaning and relationships. The purpose for using the MPM protocol was to sort and structure the observation data according to the three strands artefacts, inscriptions, talk and gestures. From the 145 teaching situations, there were 52 episodes in which the artefact number track (e.g. projected on the whiteboard, notations on the whiteboard, paper strips or paper cards) was used in the teaching of ordinality. These 52 episodes were sorted into groups according to the way in which ordinality was explored in the teaching situations. That is, how connections between numbers were made in the interactions between teachers and students, including inscriptions, talk and gestures and particularly in relation to the number track. Because the participating teachers chose different number representations to demonstrate ordinality, we first determined which number representations were used and then in what way the number representations were used in the enacted teaching. The number representations in sequence were used as follows: number symbols were linked to number words (1-one, 2-two, 3-three), quantity were linked to number words and number symbols (●-one-1, ●●-two-2, ●●●-three-3) and finally, multiple number representations were used to find a specific numbers position. This resulted into three ways: *(1) Number symbols linked to number words in a sequence* (20 episodes), *(2) Quantity linked to number words and number symbols in a sequence* (20 episodes) and, *(3) A specific number's position in a sequence* (12 episodes). By means of variation theory (Marton, 2015), the dimensions of variation that emerged during the enacted teaching were revealed. To find out which aspects of ordinality emerged in teaching when using a number track, the patterns of variation was analysed, that is what were kept invariant *(i)* and what varied *(v)* in the enacted teaching. For example, when the reference numbers remained invariant (10 _ _ 13), and the positions of the number symbols varied (10, *12*, *11*, 13 and 10, *11*, *12*, 13), the ordering of number symbols and the number words sequence were made possible to discern.

The project has been approved by the Swedish ethical review authority. Written consent from the teachers was obtained for observing and taking notes and photos of the enacted teaching for research purposes[3]. The protocols are stored securely in a fire-proved cabinet. The observations likely affected the teaching to some degree, but this was reduced by the fact that the teaching was planned by the teachers themselves, and the observer was taking notes in the back of the classroom.

The results of the analysis is presented with typical examples for each category. These examples are taken from three different preschool classes.

Results

The analysis shows that, to describe ordinality, the number track was used as an artefact in the following three ways: Number symbols linked to number words in a sequence, Quantity linked to number words and number symbols in a sequence, and A specific number's position in a sequence. These three ways and what learning opportunities were offered are described and further analysed in subsequent examples (six episodes) below.

Three ways of using the number track were found, generating different learning opportunities as described in table 1. Each learning opportunity will be described below using empirical examples.

Table 1. *Ways of using the number track, pattern of variation and learning opportunities*

Ways of using the number track	Invariant *(i)*	Varying *(v)*	Learning opportunity
(1) Number symbols linked to number words in a sequence	Reference numbers	Number symbol positions	number symbol sequence and number word sequence
	Number word sequence (stable order)	Direction	number relations and distinguishing the number words from the counting rhyme
(2) Quantity linked to number words and number symbols in a sequence	Number symbol sequence and quantities	Number representations	number words linked to quantity (enumeration)
(3) A specific number's position in a sequence	Specific number	Number relations	relationships (cardinal and ordinal) of a specific number

Number symbols linked to number words in a sequence

The observations indicate that teachers often use sequential arrangements of number symbols. In one-to-one-correspondence between number words (stable order) and number symbols, the number symbols are positioned in a sequence. The starting point to arrange these number symbols in a sequence differ. Either *all* number symbols were positioned in a disorderly manner, e.g. spread out on the floor, or only *some* of them were missing in an incomplete sequence (_1_3_5). Furthermore, the conditions for positioning the number symbols during the teaching also differed, depending on the reference numbers (number symbols already placed) and empty spaces (number symbols not yet placed) to which a student could relate when a particular number symbol had to be positioned in the sequence (see Episode 1a).

Episode 1a

A thin string is hanging between two chairs, and seven students are sitting in front of this arrangement. The teacher places 22 number cards (1-22) on the floor in a disorderly manner and at the same time talks about the greatest and the smallest numbers on this number track. The assignment is to place 22 number symbols (1-22) in a sequence on this string. The teacher marks the beginning and end of the sequence on the number track.

Teacher: Here the lowest number should be [placed] at the left chair, at this chair [right] the highest should be [placed].

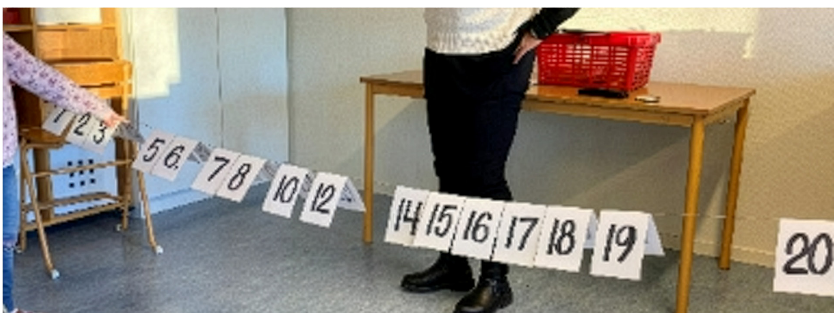
The teacher distributes the remaining number cards among the students and asks them to place the number card (one by one) into the expected number track. The teacher supports the students by pointing at the reference numbers and reminding them to use the counting rhyme.

Teacher: Sixteen, where should this hang? Is it higher or lower than ten? Is sixteen more than ten?

Teacher: Remember the counting rhyme, now we're going to count together.

When only a few numbers remain, a student is given the number 11.

Figure 1. *The number track in the final stage.*



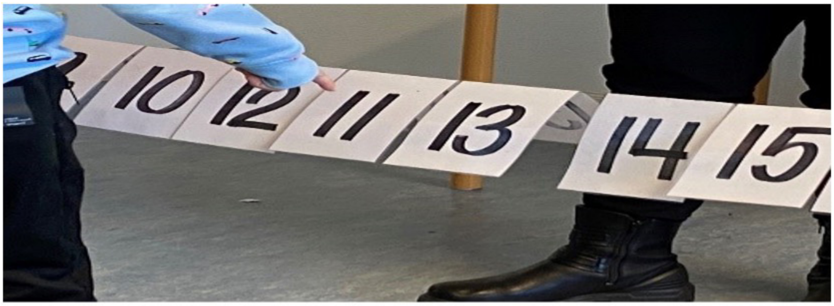
Student: I don't know [the student seems unsure of where to place 11].

Teacher: We walk from one (1), we count and point. [Suggest that they go together from 1, counting and pointing.]

Student: [The student placing 11 between 12 and 13.]

Teacher: [The teacher urges the student to count again from start to check the order.]

Figure 2. *Number track with misplaced symbols.*



Teacher: Do you say ten, *eleven*, twelve, thirteen *or* ten, twelve, *eleven*, thirteen? [Teacher and student change the position of 11 and then recite the number words sequence once again from the beginning.]

Positioning the number 11 symbol involves several reference numbers in the sequence. However, the uneven distances (space between 6 and 7 and no space between 10 and 12) obscure the order relations of the numbers, making the structure of the number sequence unclear (figure 1). When the student first positions 11 *after* 12, the teacher reminds the student of the counting rhyme, and they recite it together once more. By creating contrast between two number symbols, (10 *12 11* 13), the number words are also recited in the wrong order (ten *twelve eleven* thirteen) (figure 2). Thus, the student noted a variation of the stable order and detected the misplacement of 11, keeping reference numbers invariant *(i)*, while varying number symbols positions *(v)*, makes the *sequence of number symbols* and the *number words sequence* discernible.

Episode 1b

Once all the number symbols (1-22) are positioned on the string and the number track is complete, the teacher and students recite all numbers (stable order) from 1 to 22 first, followed by reciting the numbers in reverse order from 10 to 1 once and from 22 to 1 twice. As the number words are recited in the stable order, the teacher points, in a one-to-one correspondence to the corresponding number symbols.

When the teacher and students recite the number sequence in reverse order, starting with 10 or 22 rather than 1, the sequence is broken but implies that the numbers' relationship to other numbers are the same (in both directions) and each number always has the same *neighbours*. This also implies that the number words can be distinguished from the counting rhyme; for example, the number symbol 8 is called *eight* regardless of the direction in which the sequence is read out. In this episode, the number words sequence (stable order) is invariant *(i)*, and the direction is varying *(v)*. When starting with number 10 or number 22 and reciting the sequence in the reverse direction (from 10-1 or from 22-1) the pattern changes, making it possible for the students to discern the *number relations* and *distinguish the number words from the counting rhyme*.

Quantity linked to number words and number symbols in a sequence

In Episodes 2a and 2b, quantity (objects and pictures) is linked to number words and number symbols and arranged into a sequence. Students are supposed to represent each number from an already arranged number track (0-10) with corresponding quantities (objects and pictures) one student at a time. The objects are small plastic cubes (multi-links) of different colours but the same size. The pictures involve eleven paper cards displaying numbers (1-10) of sea animals (e.g. fish, turtles and sea-horses). Regardless of how this linking was carried out, that is starting with the symbol (5 – five – one two three four five – ●●●●●) or the quantity (●●●●● – one two three four five – 5), number words (stable order) and their one-to-one correspondence with objects and pictures determine the quantity of objects or the quantity in a picture.

Episode 2a

A number track of 0-10 is placed on the floor in front of nine students. The teacher asks each student three questions, one at a time.

Teacher: Which number symbol do you chose? What is the name of the symbol? Can you pick corresponding objects and place them next to the number? [The teacher demonstrates to the students how these instructions could be followed by carrying out the procedure with the numbers 1-4 and naming the symbol and enumerating the corresponding quantity.]

Teacher: What number symbol is this?

Student: Five.

Teacher: We count together, one, two, three, four, five. [The teacher points at the objects and they all count together.].

Student: [The student placing the five plastic figures next to the number symbol 5.]

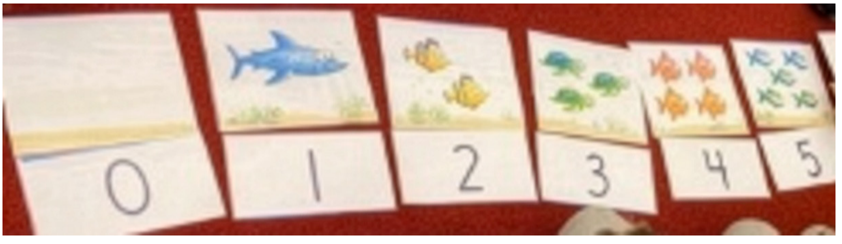
In this episode the task is to produce the corresponding quantity (number of objects) of a number symbol in the number track. Using the name of the number symbol (5), the number words (stable order) and a one-to-one correspondence (between number words and objects), the corresponding quantity is determined through enumeration (one, two, three, four, *five*). Naming the number symbol and counting objects until the same number word appears can help find the correct quantity. In this way of using the number track, the number selected (e.g., 5) is generalised through various number representations (words, symbols and objects). In Episode 2a, both number symbols and quantity are invariant *(i)*, while it is the number representations that vary *(v)*, making the *number representations (words and objects)* for the number symbols discernible.

Episode 2b

The teacher first removes all objects (plastic figures) from above the number track and then distributes 11 pictures (with different numbers of sea animals) to the students, who are supposed to match each picture with the corresponding number on the number track.

Student:	[Card with five fish, the student does not seem to know the quantity.]
Teacher:	If we count together.
Teacher:	[Points at one fish at a time [on the picture] and counts together with the student.] One, two, three, four, five.
Teacher:	How many were there?
Student:	Five.
Teacher:	Yes, good! Where is five?
Student:	[Points [at 5] and places [the picture] above 5.]

Figure 3. *A number track linked to corresponding numbers of sea animals.*



When the student seems somewhat unsure about the number of fish, the teacher suggests counting together using a stable order and one-to-one correspondence. As a result, quantity is determined through enumeration (one, two, three, four, *five*). The student answers the teacher's question regarding *how many* fish were there when saying the last enumerated number word (five) and find the corresponding number symbol (named five). The picture of the fish is then placed above the corresponding number symbol (figure 3). As in Episode 2a, both number symbols and quantity are invariant *(i)*, and the number representations vary *(v)* in this episode, making the *number representations (words and pictures)* for the number symbols discernible.

Hence, using the number track as an artefact, enumeration can help determine quantity and link with number symbols, highlighting aspects of cardinality over ordinality. When the students use the number word sequence (stable order) to determine the quantity, cardinality come to the fore and become the focal point (the object of learning).

A specific number's position in a sequence

The following episodes dealt with a specific number's position in a number track. The teacher provides several clues to 16 students to help them figure out *the secret number* by presenting different number representations within the classroom (e.g. objects and pictures). There is also a prepared box, with written number words, (memory) cards (a number symbol and corresponding numbers in a picture), a dice, an analogue clock, Lego™ blocks in different colours and sizes.

Episode 3a

The teacher unpacks the prepared box and asks questions about its contents, for example about a dice.

Teacher:	The second largest number you can get on a dice?
Teacher:	Which is the largest [number]?
Student:	Six. [The teacher shows six on the dice.]
Teacher:	Which is then the second largest [number]?
Students:	Five. [The teacher shows five on the dice.]

A similar situation regarding number relations arose in a discussion about number of schooldays.

Teacher: School days. How many days are you at school each week?

Student 1: Seven! [Some students protest against the answer.]

Teacher: How many days are there in a week?

Student 1: Seven!

Student 2: But we are off on Saturday and Sunday.

Teacher: Yes, that's right. The week has seven days, but we have two days off. So, we are at school five days. [The teacher counts Monday - Friday on five fingers.]

Teacher: Then it's the weekend for two days [shows two fingers on the other hand].

By asking questions about the *second largest number on a dice*, and the *number of school days per week* the teacher challenges the students to reflect on the number relations and in this way identify the secret number. In these episodes, the specific number is kept invariant *(i)*, while the relationship between numbers varies *(v)*, making *the cardinal relationship* of a specific number discernible.

Episode 3b

The teacher continues to review the clues to the secret number with the students, now using a 0-25 number track (a long paper strip) on the whiteboard.

Teacher: If we look at the number track. A number greater than three but less than seven. [The teacher points at the whiteboard and then places two magnets to mark 3 and 7.] Five is between three and seven, yes. [The teacher points to the number 5.] And what number comes before five [points to the number to the left of 5]? And what number comes after five?

Reducing the number range to *between three and seven* and then inquiring about the (before and after) neighbours, the ordinal relation of number five is revealed. Through clues such as *greater than and less than*, the search for the position for number five can be delimited and number relationships made visible. In this episode the specific number is kept invariant *(i)*, while the relationships between numbers varies *(v)*, making *the ordinal relationship* of a specific number discernible.

In sum, in this category a specific number's relation to other numbers, was explored in two ways. In episode 3a the specific number was explored by means of representations that emphasised differences in cardinality. On the other hand, when the teacher used the number track as an artefact in episode 3b, ordinal relationship was made discernible for the students.

Discussion

This study presents empirically based findings from analysing observations of teaching 6-year-olds about numbers using a number track as an artefact. It examined how ordinality appears in enacted teaching and which opportunities to learn about ordinality are offered in terms of teachers' use of number tracks. The result indicates that the number track is primarily and almost exclusively used to arrange various number representations (number words, number symbols, objects and pictures) into sequences. In this study, learning about number relations (see Fuson, 1988b) was not so prominent, as the teaching primarily focused on the numbers before and after a specific number. How ordinality appears in the observed teaching resembles the description by Baccaglini-Frank et al. (2020, p.781) as "Associating number symbols to number words; knowing the number symbols sequence; knowing the number words sequence; knowing what number comes before or after a certain one".

As a teaching artefact, the number track provides several opportunities to open up for students to discern necessary aspects of number, especially ordinality. The analysis showed that the sequence of number words presented in a stable order used in one-to-one correspondence with number representations (number symbols, pictures with numbers of figures and objects), supported students in positioning the number representations in sequence correctly (cf., Baccaglini-Frank et al., 2020). The coordinated use of different representations and contrasting and generalising aspects of ordinality, seems to have worked as facilitating means for the students', hence opened up for opportunities to learn about numbers' meaning. In many cases, the sequence of number words in stable order was observed to be used as a basis for determining quantity.

When enumerating, the students handle ordinality and cardinality simultaneously (Nesher, 2018). Thus, it seems to be common and perhaps a taken for granted meaning that the number sequence is used primarily as a tool for determining quantity. The key question here is then how the artefact could be used to support not only cardinal discernment (determining "how many") but also ordinality. In the analysis presented in this paper, ordinality is mainly made visible through the number word sequence (the stable order). The necessity of using the number words in the context of ordinality that is said to be significant when teaching young students, according to Fuson and Hall (1983) and Ahlberg (1997), is thereby evidently not as common. Nevertheless, the study highlights that it is possible to emphasise ordinality, specifically when visual number representations are arranged in a sequence and simultaneously related to the sequence of verbal number words (as the latter is usually better known to the students).

We suggest that when students only arrange symbols in sequence, the positions of the numbers on the number track are not discerned as easily as the counting rhyme (verbal number sequence) which they probably have learnt in their early preschool years. The known sequence contrasts with a new (sometimes false) sequence, highlights both the stable order of the sequence and the general structure of the ordered numbers presented in different modes of representation. In other words, when students have the possibility to experience variation in order (correct and incorrect order) this provides from the theoretical perspective a possibility to discern the significant aspect of order in numbers.

Using the number track to count forward and backward helps students separate rhyme from the order of numbers, thus differentiating number words as discrete parts within a structure that emphasises a stable order of elements (number words and number symbols).

Thus, this study demonstrated the use of the number track in enacted teaching and more significantly, described the complexity of learning ordinality and what becomes important to point out as aspects to become aware of. The study thereby has pedagogical implications as it shows that there is a broad range of aspects concerning ordinality beyond the ordering of numbers that by teaching can be made possible for students to discern. This involves discussing relations between numbers that are further apart than the neighbouring numbers and carefully using number representations to offer critical aspects through contrasts and generalisation.

There are certain limitations to this study that need to be considered. The observation protocol had limitations in regards to details of the teaching. Video recordings most likely would have contributed to a more detailed description of the enacted teaching.

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Notes

1 SATSA -Structural approach in teaching as foundation for sustainable arithmetic learning (2021- 2024).

2 "Artefacts are taken to mean physical equipment, manipulatives and material objects that have an enduring existence, before and after a lesson" (Askew, 2019, p. 216).

3 No photos or names of children appeared in the collected data.

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