Second graders' problem solving in a playful inquiry-based mathematics activity

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This study investigates second graders' problem-solving process within a playful inquiry-based context by drawing on previous research on problem solving and a sociocultural theory specific for mathematics education, the theory of objectification. Inspired by design research, a playful inquiry-based mathematics activity was co-designed with two teachers and applied in small groups. With a focus on dialogue and actions, qualitative data was complemented with a quantification of the children's use of strategies. The study shows how the second graders were provided with opportunities to practice their problem-solving abilities through a process characterised by the development of and advancement in the students' use of strategies differentiated into four levels: the Initial, Emergent, Structured, and Comprehensive strategies.

Within mathematics education, there has been a change from focusing on students' factual and procedural knowledge to focusing on students' competencies (Lannin et al., 2011; Niss & Jensen, 2002; Schoenfeld, 1985). This change of focus is reflected in the large-scale assessment Trends in International Mathematics and Science Study (TIMSS) in year 4 and 8 (Mullis et al., 2020), as well as in the Norwegian primary school curriculum for mathematics. Problem solving is to be understood as a process with a focus on developing competent problem solvers by fostering critical thinking, creativity, and strategy development (Ministry of Education and Research, 2020).

However, to become competent problem solvers, students need to experience mathematical problems in different ways and in various contexts. The aim of the present study is to gain insights into groups of second graders' problem-solving process as they together inquire into

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mathematics problems within a playful inquiry-based context in lower elementary school. Despite that existing research, both theoretical and laboratory-based, provides a comprehensive foundation on students' problem-solving abilities (e.g., Lester, 2013; Liljedahl et al., 2016; Schoenfeld, 1985, 1992), there is limited research conducted in classroom settings. Particularly within the early years mathematics education, there is a need to know more about how children mathematise problems as few studies on problem solving have involved young children (Björklund et al., 2020; Palmér & van Bommel, 2018). Furthermore, as pointed out by Björklund et al. (2020) and Lester and Cai (2016), future studies should investigate how the foundations for problem solving can be laid for young children's reasoning and strategy development.

Since the teaching of mathematics should support students to apply the knowledge to real-life problems and be based on their experiences (Freudenthal, 1968), the present study builds on children's playful experiences following the broad construct of playful learning by Hirsh-Pasek et al. (2008). Research has shown that children can develop mathematical knowledge and skills through solving problems within a playful environment (e.g., Blinkoff et al., 2023; Hopkins et al., 2019; Palmér & van Bommel, 2018), but research on learning through play within primary school is limited (e.g., Jay & Knaus, 2018). Second graders' learning opportunities within a playful inquiry-based mathematics activity were identified in Flaten (2025). The present study extends that work and contributes to a broader understanding of lower primary students' mathematics learning within a playful inquiry-based context by using other data from the same project to explore students' problem-solving process in a different activity. Based on the identified gaps in the research literature, the present study pursues the research question: What characterises the second graders' problem-solving process as they participate in a playful inquiry-based mathematics activity?

Theoretical background

Problem-solving processes within a playful inquiry-based context

According to Schoenfeld (1985), a problem can be described as a non-routine task that children should be interest in solving. They should have adequate resources to solve the problem, i.e., mathematical knowledge, and should not know the answer or have a predetermined solution strategy. Building on key features from the literature (e.g., Polya, 1945; Schoenfeld, 1992), the mathematical behaviour of a problem solver is a complex four phase process. As elaborated by Lester (2013), in the first phase, the individual simplifies the problem by identifying the concepts, or processes, that appear to have most direct relevance. This involves identifying the important parts of the problem and requires "decisions about what should be attended to and what can be ignored" (Lester, 2013, p. 256). The problem solver must develop a sense of how the fundamental concepts are interrelated. As a result, the problem solver develops, what Lester (2013) terms, a practical representation of the problem, which serves as a model that is easier to analyse, modify, and grasp. Mathematical concepts and symbols are introduced in the second phase which is directed by how the representation can be used in the computation phase (Lester, 2013). In the third phase, the students manipulate the representation by applying their prior knowledge, e.g., mathematical concepts and procedures. The fourth and final phase consists of comparing the results with the original problem and the representation. However, Lester (2013) emphasises that such an "act of comparing" (p. 257) is not reserved to the last phase. All phases may involve continuous comparing of the work so far, which distinguishes a problem from a routine task (Lester, 2013; Schoenfeld, 1985).

As such, problem solving is a demanding process requiring more than recalling facts as students need to reflect on and regulate their sense making during the entire problem-solving process. As noted by English and Sriraman (2010), students need to know which strategies to apply, how to apply them, and at what time, as the choice of strategy varies with the type of problem. Lesh and Zawojewski (2007) argue that it is precisely the development of "a more productive way of thinking" (p. 782) that distinguishes a problem from a routine task. Iterative cycles of trying and re-trying are essential, as what characterises effective problem solvers is the ability to adapt and learn from previous attempts (Lester, 2013; Schoenfeld, 1985, 1992). This requires children to practice their collaborative and critical thinking skills, as well as ways of communicating their thinking.

However, efforts to teach problem-solving strategies and heuristics are shown to have little effect on students' problem-solving performance (Cai, 2010; English & Sriraman, 2010; Lesh & Zawojewski, 2007; Schoenfeld, 1992), whereas to teach through problem solving with a focus on understanding can improve students' problem-solving abilities (Hiebert & Wearne, 2003; Lambdin, 2003). There is a consensus that problem solving should be taught as an integral part of mathematics, dependent on, and not isolated from, mathematical knowledge and competencies (Cai, 2010; Lester, 2013; Lester & Cai, 2016; Schoenfeld, 2013). Moreover, to educate successful problem solvers, problem solving should be integrated in mathematics teaching in all grades and content areas so children from an early age get to explore problem situations and invent strategies to solve problems (Lester & Cai, 2016).

Problem posing is fundamental to processes of problem solving (Cai, 2022). According to English (2003), problem posing takes place in all kinds of investigative activities, including young students' play, when they: "(a) distinguish the important from the unimportant data, and (b) detect the mathematical structure" (p. 190). Attention is directed towards problem solving through the importance of students asking questions and expressing their thinking when using mathematical knowledge to inquire into and solve the presented problems (Artigue & Blomhøj, 2013; Lester, 2013).

As such, problem posing is closely connected to the students' inquiry and sense making in problem-solving processes. Building on Dewey (1933) and key researchers to inquiry like Jaworski (2005) and Wells (1999), inquiry is considered as a way the students and teachers collaborate and willingly engage in the process of solving problems the students themselves find interesting, that appeal to their curiosity, to questioning and to further inquiry. However, questions remain as to how students can be encouraged to pose mathematical questions based on their own curiosity and interest (Cai & Leikin, 2020).

Following van den Heuvel-Panhuizen and Drijvers (2020), 'real' problems are real when perceived as such by the students and can come from the real world or the fantasy world. For instance, as in the present study the problems are embedded in situations of imaginative play. Children's learning through play is argued by Fleer (2011) to be most effective when imagination and cognition are intertwined. According to English (2003), problem posing can be seen in all investigative activities, including play. For instance, when students create "what if" scenarios (English, 2003, p. 188), opportunities to critically analyse problems and make sense of the mathematics may occur.

In the present study, the intent is to engage the second graders in activities where they get to explore, pose problems, give reasons for their solutions, and make generalisations, which according to Cai (2010), can give students ownership of the knowledge and strategies devised in the process. Groups of second graders are presented to a mathematics problem in situations of guided play, i.e., a type of teacher-led and student-directed playful experience (Weisberg et al., 2015; Zosh et al., 2018). Adult guided play is advocated as the most effective for attaining learning outcomes, e.g., for learning mathematical skills (Pyle et al., 2017) and within number sense (Weisberg et al., 2015).

A multimodal approach to investigate second graders' collaborative problem-solving processes

Given the need to recognise the central role of dialogue and active learning to students' problem solving (Lester & Cai, 2016; Liljedahl, 2016), the participants' dialogue and actions are focused on through a multimodal approach. A multimodal approach is emphasised as especially important in early years mathematics education (Björklund et al., 2020; Nordin & Boistrup, 2018), including in active learning through a playful approach (e.g., Nergård & Wæge, 2021; Sumpter & Hedefalk, 2015), amongst others for effective communication and to gain insights into young children's competencies. Björklund et al. (2020) emphasise the importance of providing opportunities for children to make use of tools and to capture how they mediate knowing through speech, gestures and other symbolic systems.

To understand how the second graders solve the mathematics problem while being supported by social interactions with peers and the teacher, the study adopts a sociocultural perspective to learning and development (Vygotsky, 1978) and particularly Radford's (2013) theory of objectification. Radford's (2013) theory is found useful to account for the contextual and social factors influencing the students' sense making as the focus is on the knowing that emerge in dialogue through the participants' use of different semiotic means of objectification in the problem-solving process.

Following Radford (2013), knowledge cannot be seen but exists in culture and is understood as potentiality or capacity to do something or to think in a certain way. The students and teachers must put knowledge into motion by making their actions and reflections concrete (Radford, 2009). This is described as the materialisation of knowledge into "something perceptible, sensible, concrete" (Radford, 2021, p. 40) termed knowing. The second graders and teachers must work together and do something to make the encountered mathematics cohere with the students' experiences, which Radford (2013) describes as "the transformation of 'in itself' knowledge into 'for itself' knowledge" (p. 27). Such encounters, i.e., learning processes or objectification processes, can take place when the students solve the presented problem using prior arithmetic knowledge and various semiotic means of objectification, like speech, signs, objects, tools, linguistic devices, gestures, and artefacts. As the students and teacher work together, their joint activity is stimulated by the collective effort to progressively become aware of, make sense of, and solve the presented mathematics problem. Following Radford (2003, 2009), various semiotic means of objectification are used in meaning-making processes, not only to materialise knowledge and make the thinking apparent, but also as part of thinking, since thinking and activity are intertwined.

Methods

The empirical basis for this qualitative research study is a Norwegian public elementary school. Two teachers and 38 of their six-and-seven-year-old students agreed to participate. Since a lack of experience can affect practice (Jay & Knaus, 2018), the teachers were a strategic sample recruited based on having an interest in a playful approach to mathematical problem solving. The study has similarities to design-based research (Cobb et al., 2003). Playful inquiry-based mathematics activities were designed in collaboration with the teachers based on their insights into the students' prior knowledge and the competence aims in the Norwegian curriculum (Ministry of Education and Research, 2020).

Following Radford (2003), to gain insight into the students' problem-solving processes and the collective sense making that takes place, it is important to capture the dialogue and actions that emerge through the participants' joint activity. Therefore, the participants were video recorded and, following Bryman (2016), the author's role was overt, fluctuating between a passive and an active participating observer role, without being a full member. Since these methods come with risks (Wellington, 2015), e.g., intrusiveness and ethical considerations, it was maintained throughout the study to respect and uphold the participants' well-being and privacy by conducting a pilot study and anonymising the participants with 'T' for teachers and pseudonyms for students. It was intended for the students to work in groups of four, which according to Lester (2013) is a suitable group size when the emphasis is on the problem-solving process and exploration of mathematical ideas. However, to adjust for absentees and to ensure equal educational opportunities for consenting and non-consenting students, the group size varied, and the data comprises a total of 35 students.

The present study is based on empirical data from one activity that was implemented two times with three weeks in between each implementation. The duration varied from 10 to 15,5 minutes per group. The teachers continued their regular teaching in between rounds of data generation, focusing primarily on the numbers up to 100, addition of one and two digit numbers, and reflection symmetry. Following a guided play approach (Weisberg et al., 2013) and the concept of joint activity (Radford, 2021), the teachers were meant to interact in a dynamic way with one group at a time to support the students' sense making in the problem-solving process.

Presented to the students within a playful narrative was the problem of figuring out the combination of coins locked inside a chest (see the Appendix). The problem contains no formulas or equations, but mathematically speaking, it may be modelled as a Diophantine equation problem. In the role as princesses and princes of the Number King's Kingdom, the students are to discuss the possible combination of coins, e.g., equal to the amount of 37 Norwegian kroner (NOK), which contains coins of the value 1 NOK, no coins of the value 20 NOK, and eight coins in total. Through the narrative, one by one criterion is revealed which eventually limits the possible solutions to one, making the problem solvable by revising the combination and exchanging coins to fulfil all criteria.

Data processing and analysis

Derived from the epistemological stance of dialogism, Linell's (1998) dialogic approach emphasises how humans ascribe meaning to and appropriate knowledge, in line with a sociocultural perspective on learning and development. Since dialogues are jointly constructed by the participants and dependent on the activity, the meaning of the utterances are based on the contributions from others and must be seen in connection (Linell, 1998). Therefore, as exemplified in table 1, the recordings were transcribed line by line and divided into smaller building blocks to structure and gain a preliminary understanding of the data.

Table 1. *Example from the transcripts.*

T:	And what did you take at the end? You took...? (looks at Ole)
Ole:	Two one's (slides two coins of 1 NOK to the grid, forming a 2x4 grid, using both hands to create a small gap between four and four coins, see pictures below).

In line with a multimodal approach and Radford's (2021) conceptualisation of doing and thinking as intertwined, the transcripts were supplemented with descriptions of actions and pictures of students' use of artefacts to enable an in-depth analysis of the participants' verbal and nonverbal communication.

Informed by a preliminary understanding of the theoretical foundations, but without a predetermined set of codes, the dialogues were analysed through an inductive analysis to coding, with the phases of analysis resembling a reflexive thematic analysis (Braun et al., 2019). Codes that seemed to relate to similar phenomena were grouped into categories. The students' use of strategies was examined by going back and forth between groups of both classes by continuously watching the videos and reading the transcripts to look for themes.

For example, codes like *compare*, *simplify*, *adapt* were categorised and related to the problem-solving phases by Lester (2013). In the first implementation, most students started to *simplify* the problem in phase one, by only considering two of the criteria, e.g., using solely coins of the value 1 NOK to fulfil the criteria of the total sum. The artefacts served as a representation of the problem that the students could explore in phase two and further *modify* and *adapt* in phase three, e.g., by applying arithmetic knowledge to make their combination of coins fit with more of the criteria. In phase four and throughout all phases, students *compared* and evaluated their suggestions according to peers' suggestions and the given criteria. The preliminary code *structuring the coins* were specified into *place value* and *grid* and, when revisiting Radford's (2003, 2009) theory, grouped as the students' use of semiotic means in the category *structuring of artefacts*.

Since the students' use of strategies seemed to differ in the two implementations, a quantification of the qualitative data was conducted. The frequencies of the strategies in the first and second half of each implementation were registered to provide two measuring points to trace the students' strategy use, within and between groups, in both implementations. When a student explained the steps of a strategy to peers, the strategy was counted according to the number of students applying it to their coins. If peers solely listened and watched, the strategy was counted once.

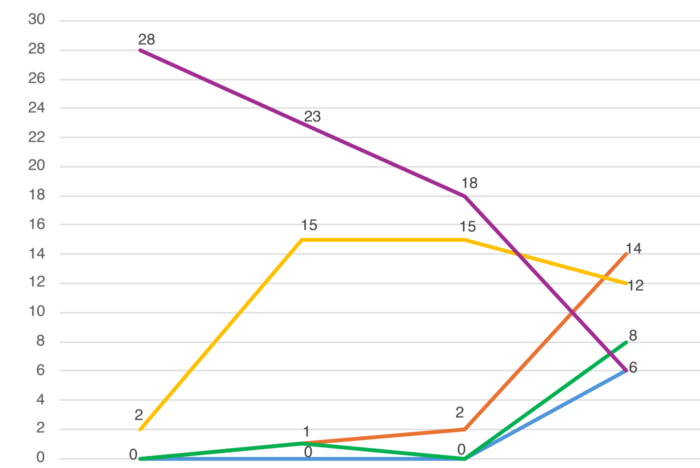
Results

The identified strategies differentiated into four levels (0-3) are summarised in table 2. No other strategies were identified beyond those presented. As reflected in the level, naming, and description, the strategies are differentiated according to the students' justifications with an increase in their consideration of the criteria of the problem and structured use of the artefacts.

Table 2. *An overview of the identified strategies.*

	Name and description
Level 3	**Comprehensive** Structured arrangement of artefacts with all being visible. Few or none counting or calculation errors. Considering and fulfilling all the criteria. Strategic arrangement to solve the problem and visualise the solution through combining aspects of structured (a) and (b). Providing valid explanations and justification.
Level 2	**Structured** Structured arrangement of artefacts. Strategic arrangement to solve the problem and visualise the solution. Few counting or calculation errors. Fulfilling the less complex criteria. Attempting to keep track of one complex criterion while adjusting for another, or the other way around, termed the structured strategy (a) and (b), respectively. Providing valid explanations and justification.
Level 1	**Emergent** Attempts to structure and use the artefacts to visualise the solution. Considering and fulfilling the less complex criteria. Mentioning the more complex criteria, without fulfilling them. Trial-and-error efforts. Fewer counting errors. Fewer nonvalid explanations or justifications.
Level 0	**Initial** Engaging in the problem in an unstructured way. No indication of strategic use or arrangement of artefacts. Considering and fulfilling the less complex criteria of the problem without indications of considering the more complex criteria. Multiple trial-and-error efforts. Often resulting in counting and calculation errors. Providing nonvalid explanations or justifications.

The tracing of the strategies presented in figure 1, revealed that 28 of 35 (80%) of the students' problem-solving process involved an advancement in strategy use where all the students who utilised the structured and comprehensive strategies advanced from using the initial and emergent strategies.



	Imp. 1 1st half	Imp. 1 2nd half	Imp. 2 1st half	Imp. 2 2nd half
Comprehensive	0	0	0	6
Structured (b)	0	1	2	14
Structured (a)	0	1	0	8
Emergent	2	15	15	12
Initial	28	23	18	6

Figure 1. *Frequencies of the strategies in the first and second half of the two implementations*

In the first implementation, the initial and emergent strategies were prominent. Students using the initial strategy tended to focus on one or two of the criteria. They were observed using many coins of the lowest value typically arranged in a big pile without all coins being visible. They were observed making counting and calculation errors, also when finger pointing. Lack of valid arguments when attempting to explain, justify, or compare solutions was observed, e.g., answering ”I don’t know”, ”I just think so”, or arguing for their suggestion by claiming they could ”hear” or ”feel” the coins by shaking the treasure chest.

Students using the emergent strategy also tended to consider only a few criteria, often the less complex criteria. However, efforts to consider the other criteria were observed. Students using the emergent strategy used coins of different values, but like with the initial strategy, they still tended to use too many coins of the lowest value. They still arranged the coins in piles or stacks, but now according to coin value. Counting errors were fewer, but multiple trial-error-efforts were still observed needed and there were little indication of the students using the arrangement of the artefacts to their advantage in solving the problem.

During the first implementation, there were at least one student in all the groups who came up with a solution to the problem. Of those students who did, only two kept using the initial strategy. The others advanced, as indicated by the increase in the frequency of the emergent strategy and two observed cases of the structured strategy (see figure 1).

In the second implementation, the frequencies of the initial and emergent strategies decreased, and the structured and comprehensive strategies increased. Compared to the first implementation, there were fewer students who solely used coins of the lowest value and who stacked coins or otherwise sorted them in unstructured ways using multiple trial-and-error attempts. Instead, they advanced to use the structured and emergent strategies which have some commonalities.

Students who used the structured and comprehensive strategies evaluated the suggestions of coins by arguing according to whether the suggestion fit the criteria or not, e.g., they provided valid justification, e.g., they referred to all criteria or they argued against a suggestion by stating ”because it is too many (coins)” or ”it amounts to just 32”. They made equivalent exchanges of coins, made fewer counting and calculation errors, and they structured the artefacts in a strategic way to solve the problem and visualise the solution. What differs between the two strategies is the way in which the students structured the artefacts.

Students who used the structured strategy sorted coins according to value, made equivalent exchanges, and either kept the sum fixed and exchanged coins to adjust for the total number of coins, or the other way round, i.e., termed the structured strategy (a) and (b), respectively. Students using the structured strategy (a) arranged coins according to place value, with designated areas for the units and tens. The arrangement made it possible to keep control of the sum while adjusting according to the total number of coins. Students using the structured strategy (b) grouped the coins in grids to keep track of the total number of coins while adjusting according to the total sum (see pictures in table 1 for an example).

The comprehensive strategy was only observed used by six students during the second implementation (see figure 1). The strategy is ranged higher than the structured strategy because these students combined the arrangements of artefacts of the structured strategy (a) and (b), e.g., by arranging coins according to place value and, at the same time, arranging the coins in the tens place in a grid.

In addition to solving the presented problem, students posed other problems. For instance, how they could share or spend the money as they imagined going to a store and envisioned what they could afford to buy, which resulted in other arithmetic problems to be solved using arithmetical procedures such as counting, adding, and subtracting.

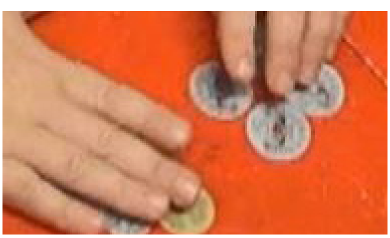
Two classroom episodes

Excerpts 1 and 2 are chosen as they exemplify the structured and comprehensive strategies, and the students' use of different semiotic means to convey their thinking. Prior to excerpt 1, Ole has come up with a solution to the problem using the structured strategy (b) when the teacher asks him what he started with.

The teacher, in the role as the Number King, engages with the students and invites Ole to present his solution. The strategy is coded 'grid' since he arranges the coins together in two rows by four columns, i.e., a 2x4 grid. Despite a gap between four and four coins, potentially helping him to keep track of eight coins in total, there is no indication of arrangement according to place value. Leah and Nico follow the steps. Nico compares his own and Ole's coins, notices a difference, and adjusts before he continues. As a semiotic means of objectification (Radford, 2003), the arrangement of artefacts, i.e., coins in grids, is adopted by the other students.

Excerpt 1

T:	What did you start with?
Ole:	First, I took... (reaches for coins). Five fives
T:	Oh, five fives, can you find five fives? (Looks at Leah and Nico)
Nico:	Five fives (Nico and Leah reaches for coins)
T:	Yes, find five fives
Ole:	Then we have twenty-five (lays down five fives in two rows and three columns)
T:	And then you said? You have...?
Ole:	A ten (lays down a ten to form a 2x3 grid, Leah reaches for a ten)
T:	But how much did you say you have when you had those? (Holds right hand over the five fives, Ole picks up the ten coin again)
Ole:	Twenty-five
T:	Okay (looks at Leah and Nico who lay down five fives in a grid). And then you took a ten (coin)?
Ole:	Yes (puts down a ten expanding to a horizontal 2x3 grid, Leah does the same vertically)
T:	Then you got...?
Ole:	Um... (Furrows the forehead, looks at the coins, pause 7,5s). Twenty... No. Thirty-five.
T:	Okay, thirty-five. Then you took? (Nico looks at Ole's coins and his own)
Nico:	I haven't got a ten coin yet (T turns and looks at Nico's coins)
T:	Oh yes, you didn't get a ten... (Slides a ten coin to Nico. Both Leah and Nico arrange coins in a 2x3 grid)
Ole:	Like this (Looks at the 2x3 grid of coins in front of him), and then...
T:	And what did you take at the end? You took...? (Looks at Ole)
Ole:	Two one's (slides two coins of 1 NOK to the grid, forming a 2x4 grid, using both hands to create a small gap between four and four coins, see pictures below)




T:	Two one's (Leah and Nico adds two one's to their grids as well). Can you count them? Princess Leah, how much money do you have? (Leah looks at the coins, looks at T, looks down on the coins, finger counts the coins)
Leah:	Eight! (Elevated tone of voice)
T:	Eight coins! (Elevated tone of voice)

As exemplified in excerpt 2, the students also built on each other's contributions during the problem-solving process, before they came up with a solution. Selma has tried to solve the problem using too many tens. Chris has yet to come up with a solution. He arranges four fives in a 2x2 grid. Selma watches Chris, states "He gave me a hint", puts coins from earlier attempts back, lays down four fives in a 2x2 grid, adds a ten to the grid, and lays down an additional five and two one's separate from the grid. Selma builds on a peer's contribution when arranging coins in a grid for the first time and using the comprehensive strategy to solve the problem. She reconstructs the steps as follows.

Excerpt 2

T:	Princess Selma, can you elaborate on what you were thinking? (Selma stares in the air. Pause 4,5s)
Selma:	A bit like Chris
T:	A bit like Chris, yes. I saw that Prins Chris helped you a bit along the way. Prins Chris, can you see how she has approached this? (Chris is leaning over looking at the coins in front of Selma)
Selma:	First, I did this (points to four fives in a 2x2 grid). Then I did it like this (points to the ten with the right index finger) and then I took a five and... (Holds left hand over a five and two one's which she slides away from the grid and next to the left of the grid, see the three pictures below).

The teacher directs the group's attention and asks Chris if he can see, i.e. make sense of, how Selma has solved the problem. Selma extends her elaboration and uses artefacts and pointing gestures as semiotic means to convey her thinking. She builds on Chris's 2x2 grid but extends it by adding a ten. She separates these coins from one five and two one's, making it a total of 37 NOK with eight coins arranged according to place value. Thus, the comprehensive strategy differs from the structured strategy used by Ole in excerpt 1.

Discussion

I set out to investigate: What characterises second graders' process of solving a mathematical problem encountered when participating in a playful inquiry-based mathematics activity? In line with the emphasis on problem solving as a process and how a problem is defined (Lesh & Zawojewski, 2007; Lester, 2013; Schoenfeld, 1985, 1992), also in the Norwegian curriculum (Ministry of Education and Research, 2020), none of the students immediately knew the solution or how to solve the problem. However, they had the prior mathematics knowledge, i.e., arithmetic knowledge, necessary to do so. Following van den Heuvel-Panhuizen and Drijvers (2020), the students active engagement shows that the problem was real to them. They spent time in analysing the problem through several trial-and-error attempts using the initial and emergent strategies before eventually advancing to using the structured and comprehensive strategies. As will be discussed, the students' development of and advancement in use of the strategies characterised their problem-solving processes and provided opportunities for the students to practice their problem-solving abilities.

By identifying the problem-solving phases by Lester (2013), insights into the students' problem-solving abilities are provided. The classroom examples illustrate how the use of the artefacts added value to the students' sense making and creative thinking, as was observed in all phases of the problem-solving process. The students' critical thinking is shown in the planning-phase as the students selected important details of the problem to pay attention to and reflected in the descriptions of the strategies in table 2. The students translated the problem by using relevant information to create an adequate representation using the artefacts. They devised and monitored a solution plan as they exchanged coins and executed adequate calculations to make the necessary adjustments to fulfil all the problem's criteria.

Following Radford's (2003) notion of semiotic means of objectification, the students made use of the artefacts through which they detected the mathematical structure. With the advancement in strategy use, whether consciously or unconsciously, the students made decision and distinguished important from unimportant information, as Lester (2013) emphasises. They did so by advancing in strategy use through focusing on the two most important or complex criteria, i.e., the sum and total number of coins to be used. As such, the students using the structured and comprehensive strategies spatially organised the artefacts in a manner that seemed to be useful and efficient to keep track of the unknown quantities. Following the concept by Radford (2003), not only did they use the artefacts as semiotic means of objectification. They also structured the coins, e.g., in grids or according to place value, which in itself is a semiotic mean, through which they also made the steps of how they came up with a solution, and thus, their strategy, apparent to peers.

In line with English (2003) and Lester (2013), the students interpreted the situation mathematically by identifying the structure and goal of the problem. As emphasised as important in the literature (e.g., Lesh & Zawojewski, 2007; Lester, 2013; Schoenfeld, 1985), in doing so, their capacity to reason and think in a certain manner improved as they developed their strategies and new ways of thinking to solve the problem. By advancing to higher levelled strategies, they made the problem more manageable to solve, e.g., by holding some criteria fixed and varying the other while exchanging coins. Also, they seemed decisive in making these exchanges as their actions were more rapid compared to students using the unstructured and time-consuming trial-and-error efforts of the initial and emergent strategies. Critical thinking and self-regulation skills were shown as the students exchanged coins and compared their suggestion to the problem's criteria to evaluate the validity of their suggestion. When advancing to using the structured and comprehensive strategies, no instances of invalid arguments were observed. Instead, the students strived to provide valid arguments by explicitly or implicitly referring to all criteria as they evaluated their contributions and tried to convince the others. Moreover, the above argumentation shows the importance of focusing on both dialogue and actions to gain insights into young children's collaborative problem-solving processes, as emphasised by other researchers as well (Björklund et al., 2020; Nergård & Wæge, 2021; Nordin & Boistrup, 2018; Sumpter & Hedefalk, 2015).

I hold, that the identified strategies and the registered 28 of 35 students advancing in their strategy use, was a result of how the participants built on and refined their contributions in collaboration with others. Ideas and strategies were shared in interaction. Following Radford (2013), what was observed was students engaged in processes of clarifying their own thinking, who took on and considered peers' perspectives, and who contributed to each other's ways of thinking. Through dynamic multimodal participation, the students used prior knowledge and showed some of their emergent mathematical insights, e.g., arithmetic procedures like counting, adding, subtracting. Prompted by the teacher, the students recalled and presented the steps of solving the problem which became useful for their peers who applied the strategy. Peers who had been stuck on wrong or ineffective ways of solving the problem could be inspired to advance by attending to other criteria and to strategically arrange the artefacts according to more effective strategies and more productive ways of thinking. In joint activity with peers and the teacher, the students practiced their collaborative, communicative, critical thinking and self-regulation skills. Through iterative cycles with several trial-and-error efforts, by building on peers' contributions and advancing to using the artefacts in a more strategic way, the students practiced skills and abilities that are subject independent and emphasised in the research literature to characterise effective problem solvers (Lester, 2013; Schoenfeld, 1985, 1992).

For educators and practitioners, it is worth noticing how the students explained the strategies "in action", without labelling them. The mathematics involved, i.e., arithmetic knowledge, was the same regardless of the strategy used. Following Radford's (2003) notion of semiotic means of objectification, the differences and advancement in use of strategies were related to the participants' use of the artefacts by advancing to performing correct calculations and equivalent exchanges coupled with a strategic arrangement of the artefacts, in addition to their use of words in the arguments and justifications provided. Thus, the study supports the importance of paying attention to various semiotic resources in early years mathematics education (e.g., Björklund et al., 2020; Nordin & Boistrup, 2018) and provides teachers with relevant insights which may serve as a realisation for them to become more sensitive to how young children convey their thinking.

The study shows that taking a playful inquiry-based approach to the teaching of mathematics can provide lower elementary students with experience in solving problems and opportunities to practice important problem-solving skills. The study aligns with findings in previous research on children's learning in playful environments (e.g., Blinkoff et al., 2023) and provides needed insights relevant to the primary school context (Jay & Knaus, 2018). The study shows how integrating problem solving into mathematics teaching through a playful inquiry-based approach can be one way to frame learning environments that engages young students in problem solving and strategy development, of which more research is needed (e.g., Björklund et al., 2020; Lester & Cai, 2016).

Despite not directly examined, the study provides implications for the approach to have the potential to be one way for teachers to position students as initiators to pose their own problems, as called for Cai and Leikin (2020).

Concluding remarks

Although the results obtained are insufficient to draw firm conclusions regarding the reasons for the observed advancement in strategy use, the teaching in between rounds of data collection was oriented towards other subject matters. No observations were made of the teachers presenting the second graders with a strategy, a preferable way to utilise the artefacts, or otherwise solve the problem. Only when students had provided suggestions, did the teachers refer to specific contributions and potentially emphasise one over another. All of which support that the strategies were self-invented and made sense to the students, which, following Cai (2010), may provide students with ownership.

To combine qualitative and quantitative data was valuable as the quantification complemented the depth and details from the qualitative analysis through identifying patterns in the students' strategy development and in revising the descriptions in table 2 to make sure the strategies did not overlap. Still, it should be noted that the study's results and implications are highly content- and context dependent, based on few participants, and lack generalisability. In future research, it would be interesting to dwell deeper into students' multimodal reasoning through various semiotic means, and to study early and long-term support of young children's problem-solving process and what affects the development of their problem-solving abilities, which this study does not address.

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Appendix

The playful inquiry-based activity

Dressed up as the imaginative character called the "Number King", the teachers co-played along with the students in need of their help in solving a mathematical problem. The underpinning activity entails a treasure chest and coins of 20, 10, 5, 1 Norwegian krone (NOK). Each student was given a doll and greeted as princes and princesses of the Number King's Kingdom. The Number Queen had locked coins inside the chest and the Number King, played by the teachers, was ignorant of the combination of coins. The first criterion introduced through the narrative of the activity is the sum of the coins, e.g., 37 NOK. Eventually three other criteria are revealed through the narrative by the teachers in the role as the Number King, e.g., there are no coins of the value 20 NOK, there are coins of the value 1 NOK, and there are a total of eight coins inside the chest. The students, in the role as princesses and princes, are told that they will earn the money if they can figure out the combination of coins before they get to open the chest to verify their suggested solution.

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