Opportunity to learn in Norwegian and Finnish lower secondary mathematics textbooks

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Although both Finland and Norway are part of a common Nordic education culture, Finland have consistently outperformed the other Nordic countries in PISA studies. In this study, we compare Finnish and Norwegian textbook series. The results indicate that both textbook series largely facilitate skill efficiency, and most tasks are low cognitive demand. However, the Finnish textbook series facilitate conceptual understanding to a greater degree. The Finnish textbook series also introduce and develop connections between a greater number of mathematical ideas, and there is a greater number and proportion of high cognitive demand tasks in the Finnish textbooks.

Although many factors besides textbooks can mediate the relationship between the intended and the implemented curriculum (Van Steenbrugge et al., 2013), students and teachers often rely on the textbook to consider what is important in mathematics education (e.g. Schmidt et al., 2001; Vincent & Stacey, 2008; Schmidt et al., 2012). Furthermore, according to some researchers, textbooks are typically the main resource for teachers to make decisions about what to teach and how to teach (Fan et al., 2004), and what students learn (Stein & Smith, 2010). Mathematical topics that are not included in textbooks are unlikely to be taught, and how topics are presented set into motion different pedagogical approaches and different opportunities for student learning (Stein et al., 2007, p. 327). Other researchers have argued that textbooks have a more limited impact on instruction and students' learning (Freeman & Porter, 1989). The different views on the impact of textbooks arguably stems from a variation in the use of textbooks both at local level

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among individual teachers and at a national aggregated level. It is therefore more accurate to say that textbooks offer probabilistic rather than deterministic opportunities to learn mathematics (Mesa, 2004; Valverde et al., 2002). Textbook research can therefore be viewed as an enterprise that asks: "What would students learn if their mathematics classes were to cover all the textbook sections in the order given? What would students learn if they had to solve all the exercises in the textbook?" (Mesa, 2004; pp. 255–256). The conditional tense of these two questions acknowledges that textbook research is generally directed at a potentially implemented curriculum, and not the implemented curriculum (Valverde et al., 2002). Nevertheless, due to the strong potential for textbooks to influence both mathematics instruction and students' mathematical activities, researchers have concluded that cross-national textbook research can reveal similarities and differences in opportunities to learn mathematics, and provide partial explanations differences in student students' performance in international comparative studies (Fuson et al., 1988; Li, 2000; Charalambous et al, 2010; Son & Diletti, 2017).

In this study, we examine and compare mathematical textbooks from Norway and Finland to determine learning opportunities provided by mathematics textbooks in Finland and Norway. Both Norway and Finland are part of what is often referred to as the Nordic model of education (e.g. Blossing et al., 2014; Klette, 2018). Historically, the Nordic countries (Norway, Sweden, Denmark, Iceland and Finland) have stressed that the educational system should be inclusive, uniform, free-of-charge for children from all social strata, and based on an egalitarian philosophy where it is considered the state's duty to provide equal educational opportunities for all children (Blossing et al., 2014). Across the Nordic countries, this has led to the construction of a publicly funded school system without tracking or streaming students until the age of 16. The underlying motivation for this egalitarian *School for all* principle is closely related to the development of the welfare state, and it has both economic and social motives. First, quality education for all citizens has been considered a requirement for economic growth. Second, having students of different backgrounds come together in the same classrooms has been seen as measure to reduce social and class differences in general.

Not only does the education research literature often refer to a common Nordic model of education, but the mathematics education research literature often refers to a more specific common Nordic profile within mathematics education. Cluster analyses of international mathematics assessments have for example revealed meaningful and clear groupings of countries according to similarities in relative response patterns. Across multiple datasets and analyses, the Nordic countries have emerged as one such clear and distinct cluster of countries (Grønmo & Onstad, 2013). A key feature of the Nordic cluster, is that they tend to perform well on items closer to daily-life mathematics like estimation and rounding of numbers, while they score relatively low on items dealing with more classical, abstract mathematics like fractions and algebra (Grønmo & Onstad, 2013). This strong emphasis on real-world mathematics and a daily-life perspective on mathematics has also been identified in analyses of curricula in the Nordic countries (Grønmo et al., 2013). Finally, several studies have indicated that that mathematics instruction in the Nordic countries seem to generally be teacher-centred and focused primarily on procedural skill, with an emphasis on individual seat work (e.g. Klette et al., 2016; Krzywacki et al., 2016).

Despite the many similarities between the Nordic countries, Finland have consistently outperformed the other Nordic countries in international performance studies such as PISA and TIMSS (Jensen et al., 2019) – sometimes referred to as the Finnish "PISA miracle" (Simola et al., 2005). Of course, there are several observed differences between mathematics education in Finland and the other Nordic countries that could at least partially explain these trends. Although mathematics instruction seems to be generally teacher-centred and focused on procedural skill in the Nordic countries, some smaller studies have suggested that there is some variation to the deductive teaching approach. First, it seems mathematics classroom instruction in Finland is somewhat more teacher-centred than in other Nordic countries. Hemmi and Ryve (2015), for example, found that Swedish teacher educators emphasize interactions with individual children and building on students' ideas, while Finnish teacher educators highlight the importance of clear presentation and specific goals for every lesson. In a more recent study, Luoto et al. (2022) concluded similarly after systematic observations of 16 classrooms in Norway and Finland. While clear instructional explanations, connection of new knowledge to existing knowledge, and explicit learning goals were predominant in Finnish classrooms, students in Norwegian classrooms had greater opportunities to participate in classroom discourse and peer discussions. Second, some studies also indicate that mathematics instruction in Finland is influenced by the textbook to a greater extent than in other Nordic countries. For example, Finnish teachers seem to make us of textbook examples more than Norwegian teachers (Lepik et al., 2015; Kilhamn & Säljö, 2019), and Finnish teachers provide less variation in tasks than Norwegian teachers (Taajamo et al., 2014). Nevertheless, the Nordic countries are more alike than different, and they make up a clearly distinct cluster of countries in a global education context (Grønmo & Onstad, 2013; Klette et al., 2016; Krzywacki et al., 2016. The observed classroom nuances between Finland and the other Nordic countries seem insufficient to explain the noticeable differences in international performance tests. For example, although mathematics instruction in Finland seems to be somewhat more direct and teacher-centred, which is thought to be less conducive for learning problem solving and conceptual understanding (e.g. Hiebert & Grouws, 2007; Lester, 2013), Finland outperforms the other Nordic countries across cognitive and thematic domains within mathematics (e.g. Kjærnsli et al., 2014). It is therefore important to investigate other plausible causal factors for why Finland outperform the other Nordic countries in international performance tests in mathematics.

As the previous discussion demonstrated, textbooks can be highly important factors of students' learning – particularly in Finland and Norway, where mathematics textbooks are, according to teachers, the foundation of mathematical instruction for 90–95 % of Norwegian and 90–95 % of Finish students in elementary and lower secondary school (Mullis et al., 2012). Other, more recent, studies have similarly reported that both Finnish and Norwegian teachers use textbooks extensively (Lepik et al., 2015; Viholainen et al., 2015). Although there is a fairly extensive research literature on textbooks in general (Fan et al., 2013), there is considerably less research on mathematics textbooks in the Nordic countries. Furthermore, previous studies in Nordic contexts have had narrow analytic foci, and have been limited to a single Nordic country, such as: opportunities to learn proof in upper secondary integral calculus (Bergwall & Hemmi, 2017), how fractions are dealt with in elementary textbooks (Yang, 2018), opportunities to develop algebraic thinking in elementary textbooks (Bråting et al., 2019), how problem solving is represented in upper secondary textbooks (Brehmer et al., 2016) etc. No previous study to our knowledge has systematically and comprehensively examined and compared entire textbook series between multiple Nordic countries. In this study, we therefore analyze entire mathematics textbook series from Finland and Norway – as two representatives of the Nordic countries. Our aim is to map the opportunities to learn mathematics provided by the textbooks in the two countries, which could, at least potentially and partially, explain why students from seemingly similar educational systems perform differently in international performance tests. More specifically, we set out to determine the opportunities to learn mathematics provided by lower secondary mathematics textbooks in Finland and Norway. The choice of lower secondary textbooks is primarily based on that the PISA test determine mathematical proficiency of mathematics students in their final year of three years of schooling in lower secondary school. The following three research questions is posed.

RQ1. Which overall and structural differences can be observed between Norwegian and Finish mathematical textbooks?

RQ2. How do Finnish and Norwegian textbooks introduce and develop important mathematical ideas?

RQ3. What characterizes mathematical tasks in textbooks in Finland and Norway in terms of cognitive demand, mathematical features and contextual features?

The first research question (RQ1) is directed at the overall structure of the textbooks, and how the mathematical content is organized within each textbook and across each textbook series. The main objective is to determine which mathematical topics and ideas are covered by the textbooks, and in what way topic sequence and relative emphasis is communicated to the reader. This is commonly referred to as a macroanalysis (Li et al., 2009) or horizontal analysis (Charalambous et al., 2010) of textbooks, and represents a basic measure of what kind of mathematics content students have an opportunity to learn (Li et al., 2009). However, a macroanalysis do not reveal how the mathematical content is presented. An analysis of specific content topics, at a micro level, is therefore necessary to determine how the mathematical content is presented and how students are expected to engage with the mathematical content (Li et al., 2009).

The second research question (RQ2) shifts the investigation of the textbooks to a micro level. More specifically, RQ2 is directed at how specific mathematical concepts and ideas are treated in the textbooks. Charalambous et al. (2010) refer to this aspect as how the mathematics is "communicated to students" within a vertical analysis, while Li et al. (2009) refers to this aspect "introduction and development of concepts and procedures" within a micro analysis. The main objective is to determine how the mathematical concepts or ideas of textbooks is presented didactically and mathematically, and moreover what kind of environment for knowledge construction the textbooks facilitate (Charalambous et al., 2010).

Research question 3 is also part of a microanalysis of the textbooks. However, while the focus of RQ2 was on how the mathematical content is treated and presented to the students, the focus of RQ3 is how students engage and work with mathematics themselves (Charalambous et al., 2010). More specifically, RQ3 is directed at the learning opportunities mathematical tasks in the textbooks provide. According to Hiebert and Wearne (1997), it is primarily mathematical tasks in mathematical textbooks that influence students' learning. NCTM (2000) similarly claim that mathematical tasks are essential to students' learning, as they communicate what mathematics is and what it means to "do mathematics". Drawing on the recommendations of Pepin and Haggarty (2007), the main objective is to determine to what extent tasks in the textbooks highlight relational rather than instrumental understanding, make connections with underlying concepts and relations within mathematics, make high cognitive demand and are embedded in real life connections. To do this we attempt to determine the cognitive demand of mathematical tasks, due to its importance for students' potential learning trajectories (Stein & Smith, 1998). However, in order to capture auxiliary and potentially other important aspects of students' mathematical experiences, we also investigate mathematical and contextual features of tasks (Charalambous et al., 2010; Son & Diletti, 2017). Together, the three research questions entails a systematic and comprehensive analysis that takes into account both the micro- and macrolevel of textbooks.

Background

Textbook research

According to Fan et al. (2013), textbook research can broadly be placed in four categories: 1) Role of textbooks in mathematics teaching and learning, 2) Textbook analysis (and comparisons), 3) Textbook use by teachers and/or students, and 4) Other areas such as electronic textbooks and the relationship between textbooks and achievement. Furthermore, research related to textbook analysis usually emphasize one or more of five distinct areas: mathematics content and topics presented to the reader; cognitive aspects of textbook tasks; gender, ethnicity, equity, culture and value; comparison of different textbooks; and conceptualization and methodological matters (Fan et al., 2013). Although each of the aforementioned categories and areas of textbook research are important to understand the wider role and impact of textbooks (Pepin & Haggarty, 2001), we focus here on textbook analysis and comparisons related to content, topics, and cognition due to the specific aims of this study.

Most studies on textbook analysis have concentrated primarily on issues related to mathematical content and topics in textbooks, such as content distribution on textbook pages, content presentation, content-topic coverage and page space devoted to each topic (Delil, 2006; Grishchenko, 2009; Li, 2000; Törnroos, 2005). An early and often cited study by Flanders (1987), found for example that the amount of new content in US textbooks tended to decrease from grades 3 to 8. The study questioned whether students would be less motivated when much of the content was old and much of the new content would be repeated in the future. In a more recent and larger study, Baker et al. (2010) carried out a content analysis of 141 elementary mathematics textbooks from 1900 to 2000. Over the course of the century, mathematics textbooks changed in several important manners: 1) textbooks grew in size, and in number of content categories and pages devoted to each content category; 2) the use and importance of textbooks increased; 3) more advanced topics were introduced; 4) and new topics were introduced for ever-earlier grades. However, the two aforementioned studies focused on US textbooks. Large scale international studies have identified substantial cross-national differences between mathematics textbooks. For example, Schmidt et al. (1997) and Valverde et al. (2002) compared and contrasted mathematics textbooks across nearly 40 countries, and concluded that the textbooks exhibit substantial differences in presenting and structuring mathematical content and pedagogical situations. Similar conclusions were drawn in a recent review of the literature on mathematics textbooks in the USA and East Asian countries. Son and Diletti (2017) found that mathematical topics tend to appear earlier in Asian education systems than they do in the USA. Furthermore, US textbooks tend to spend more time on specific content areas, as well as on revisiting previously taught material. Cross-national differences have not only been observed between larger cultural spheres. Smaller studies have found important differences – both in terms of what content covered and how mathematical ideas and concepts are presented – among textbooks used in European countries (Pepin et al., 2001; Charalambous et al., 2010). Regarding content and topics in mathematics textbooks, it is therefore difficult to identify general trends other than that there seem to be substantial historical, cross-cultural and cross-national variation. As for the Nordic countries, comparative research on the mathematical content of textbooks is scarce. However, a few studies indicate certain characteristics. First, there seems to be a greater emphasis on placing mathematics in a real world context in Nordic countries than in other countries (e.g. Yang et al., 2017; Yang, 2018). Second, Nordic textbooks seem to emphasize practical activities and application of mathematics over theoretical properties and formal mathematical reasoning (e.g. Pepin et al., 2013; Bergwall & Hemmi, 2017). Unsurprisingly, these trends are also in line with the strong emphasis on real-world mathematics and a daily-life perspective on mathematics in the mathematics curricula in the Nordic countries that was mentioned earlier (Grønmo et al., 2013).

Although there has been not as much focus on the cognitive aspects of textbook tasks as on the mathematical content and concepts of textbooks, some studies have addressed the former using various analytic schemes to identify the cognitive processes required to solve textbook tasks (Fan et al., 2013; Son & Diletti, 2017). The overarching cross-cultural trend in the research literature seems to be that most textbook tasks emphasize procedural skill and memorization over conceptual understanding and problem solving (e.g. Vincent & Stacey, 2008; Hong & Choi, 2014; Polikoff et al., 2015; Son & Diletti, 2017). This also seems to be the case in a Nordic context, as Brehmer et al. (2016) found for example that only five percent of tasks in Swedish upper secondary textbooks can be defined as mathematical problem solving tasks. According to several researchers (e.g. Palm et al., 2011; Schoenfeld, 2012; Stacey & Vincent, 2009), textbooks also generally steer students into skill-based task solving through extensive and frequent use of pointers, key words, step-by-step worked examples etc. However, although most textbooks seemingly prioritize procedural skill, several studies have found noticeable differences between countries (Valverde et al., 2002). In a more recent study, Jäder et al. (2019) analyzed geometry and algebra tasks in textbooks from twelve countries. In line with most of the related literature, they found that most tasks in textbooks in all twelve countries could be solved using a template as guidance. However, there were also noticeable differences between countries, with for example a higher percentage of procedural tasks in textbooks from Australia and Scotland, than in textbooks from Finland and India. Interestingly, Jäder et al. (2019) also found that there was a similar percentage of tasks in the textbooks from Finland and Sweden that could be classified as problem solving tasks that could not be solved using an explicit template.

Opportunity to learn and textbooks

The most well-known definition of Opportunity to learn (OTL) can be found in the report on the First International Mathematics Study as "whether or not ... students have had the opportunity to study a particular topic or learn how to solve a particular type of problem" (Husén, 1967, pp. 162–163). In general, OTL is considered the inputs and processes within a school context necessary for producing student achievement of intended outcomes, and it has been used to account for differences in students' mathematics performance from different countries (Hiebert & Grouws, 2007). It is also widely considered the single most important predictor of students' achievement" (National Research Council, 2001, p. 334). As previously discussed, mathematics textbooks are a significant determinant of students' opportunity to learn. Studying textbooks can therefore reveal similarities and differences in students' opportunities to learn mathematics between countries (Charalambous et al., 2010).

Researchers have traditionally used a variety of approaches to study how mathematics textbooks in different countries provide learning opportunities for their students. However, literature reviews have suggested that there are two main tendencies in textbook analysis studies (Charalambous et al., 2010; Son and Diletti, 2017). The first trend is broadly related to the overall structure of textbooks and its focus is often topics, placement of topics and ideas, textbook size etc. Charalambous et al. (2010) has referred to this type of analysis as *horizontal analysis*, while Li et al. (2009) referred to it as a *macroanalysis*. In a horizontal analysis the textbook is examined as a whole to get a sense of to what extent, what kind of and in what order mathematical content is presented. The second trend, which can be seen as complimentary to the first trend, is related to in-depth analysis of the mathematical content and in particular the characteristics of the mathematical tasks. Charalambous et al. (2010) referred to this aspect as a *vertical analysis*, while Li et al. (2009) referred to it as a *microanalysis*. By combining a horizontal and vertical analysis it may therefore be possible to identify disparate features of textbooks that might contribute to structuring student learning opportunities (Charalambous et al., 2010).

Theoretical framework

In this study, we compare Finnish and Norwegian textbooks using a constructed theoretical framework – often referred to as a conceptual framework – that incorporates both a micro- and macro-perspective of textbook analysis. Combining both dimensions of analysis can help reveal characteristics of textbooks that would otherwise be lost and provides meaningful and integrated basis for evaluating the OTL of textbooks (Charalambous et al., 2010).

Macro perspective

In this study, the macroanalysis was first directed at background information and the overall structure of the textbooks, such as page size, number of pages per topic, and topics covered by the textbooks. Afterwards, the macroanalysis shifted to what specific mathematical ideas and concepts were presented explicitly in the textbooks and the sequencing of the ideas and concepts. As mentioned earlier, this approach is a basic indicator of what kind of mathematics content students have an opportunity to learn (Li et al., 2009).

Micro perspective

The microanalysis was carried out in two phases, both of which focused on how the mathematical content is represented in the textbooks. The first phase of the microanalysis was directed at how mathematical concepts and ideas were introduced and developed in the textbooks. According to both Son and Diletti (2017) and Charalambous et al. (2010), the introduction and development of ideas and concepts refer to the explicit exposition and communication of mathematics in textbooks – unlike tasks and exercises the students are expected to work on themselves. Most research takes a qualitative approach to investigate this subcomponent, and the focus is usually related to mathematics specific factors, worked examples and explicit thinking models (Charalambous et al., 2010; Son & Diletti, 2017). The second phase of the microanalysis was directed the textbook tasks, and the learning opportunities they provide for the students (Charalambous et al., 2010; Son & Diletti, 2017). Three key task dimensions that have effect on students' mathematical learning were investigated: cognitive demand, mathematical features, and contextual features (Charalambous et al., 2010; Son & Diletti, 2017).

Methods

Educational systems and curriculum in Norway and Finland

Education systems and curriculum are similar in both Finland and Norway. In Norway, students begin elementary school at age six. Compulsory school lasts from grade 1–10. Elementary school consists of grades 1–7. After elementary school, students begin lower secondary school that last from grade 8–10. In Finland, students begin elementary school at age seven. However, according to the *Finnish ministry of education and culture*, most children attend a pre-school at age six that include some formal learning of mathematics. Compulsory schooling lasts from grade 1–9. Elementary school in Finland consists of grades 1–6, and lower secondary school consists of grades 7–9. The intended curriculum is also in many ways similar in the Nordic countries in general, and in Finland and Norway in particular (Carlgren & Klette, 2008). Both education systems have a national written core curriculum with clustered grade differentiated learning goals. In both Norway and Finland, the core curriculum provides the basis for local curriculum work, and the more detailed curricula are constructed locally, in collaboration between actors in the municipalities and educational practitioners in the schools (Mølstad & Karseth, 2016).

Table 1. *Overview of the theoretical framework*

Analytical perspective	Subcomponent	Features
Macroanalysis	Content coverage	Page size, number of pages per topic, topics covered, and sequence of topics
	Introduction and development of ideas and concepts	Ideas and concepts covered, and sequence of ideas and concepts
Microanalysis	Introduction and development of ideas and concepts	Mathematical characteristics such as the use of definitions and representations, and didactical characteristics such as worked examples and thinking models.
	Task analysis	Task characteristics such as cognitive demand, mathematical features, and contextual features

Note. Table 1 provides an overview of the framework used in this study. A more detailed exposition of the analysis is provided in the methods section.

Selection of textbooks

According to publishing companies and educational authorities in both Finland and Norway, the textbook series Faktor (Hjardar & Pedersen) was widely used in Norway. In Finland, the textbook series Pi (Heinonen et al.) was widely used. Pi was also extensively used by Swedish speaking students in Finland, and was therefore available in already translated versions that were understandable to the researchers of this study.

The Norwegian textbook series Faktor consist of three textbooks (Faktor 1–3 "Grunnbok") and three supplemental task collections (Faktor 1–3 "Oppgavebok"). Each textbook is divided into seven chapters, each named after a particular mathematical topic. At the beginning of each chapter there is a short collection of 4–6 explicit learning goals. Each chapter is then separated into 7–11 subsections, each devoted to a key mathematical idea related to the overall topic of the chapter. At the end of each textbook there are solutions to all regular tasks. For each of the textbooks in the Faktor series there is a corresponding task collection. Each task collection book is separated and sequenced into the same chapters and subsections as the corresponding textbook (see appendix A).

The Finnish textbook series consist of four textbooks (Pi 7–9 Matematik and Pi Statistik och Sannolikhet). Each of the three textbooks Pi 7–9 consist of three chapters that are named according to the main mathematical ideas in the chapter. The Finnish textbook series also include a separate textbook on statistics and probability (Pi Statistik och Sannolikhet) which is intended to be used intermittently across all three lower secondary years. Each chapter is separated into 12 subsections devoted to a key mathematical idea related to the overall topic of the chapter, and 2 subsections devoted to repetition of the preceding six subsections. At the end of each textbook there are solutions to all regular tasks (see appendix A).

Analysis

Macroanalysis

The macro-analysis of the textbooks was carried out using a qualitative content analysis approach (Mayring, 2015). The analysis was performed in three steps. First, background and structural information such as publisher, authors, pages, chapters etc about each textbook series was collected and systematized. Second, we compared the mathematical topics covered by each of two textbook series, and created a sequence of topics for each of the textbooks using comparable labels. Third, and similar to the second step, we identified the mathematical ideas and concepts presented in each of the textbooks, and created a detailed sequence for each textbook.

In order to outline the sequences of mathematical topics in the two textbook series, a comparative procedure was required. Mathematical topics in each of the textbook series was mainly categorized according to chapter titles in Faktor, as these were both consistent across individual textbooks and highly descriptive of content. However, certain chapters were overlapping in terms of content. These chapters were merged together. The two chapters on fractions and percentage were merged into a single topic called Fractions and Percentages. The chapters on algebra, equations and functions were similarly merged into a single Algebra topic. The Pi textbooks were divided into three main chapters that varied across individual textbooks and the topic names were less precise (see appendix B). However, the content of each chapter in the Pi series was similar to the content in the Faktor series. We were therefore able to classify the mathematical topics of each chapter in the Pi series using the categories formed on the basis of the Faktor series. There were three exceptions that did not fit into any of the mathematical topic categories. The first exception was a *Mixed* category in the Pi series that did not match any of the topics in the Faktor series. The second exception was a *Problem solving* category that consisted of problem solving tasks that were highlighted and isolated from the regular tasks in textbooks. The third exception was a set of tasks related to the use of digital tools at the end of each chapter in the task collection books in the Faktor series. In order to outline the sequences of mathematical ideas in the two textbook series, an analytical procedure similar to the previously explicated comparative procedure on mathematical topics was carried out. However, instead of focusing on the mathematical topics in the textbook, we now concentrated the analysis on mathematical ideas such as concepts, subconcepts and procedures explicitly defined and/or explained in the text.

Microanalysis

The microanalysis was carried out in two phases. First, we investigated how mathematical ideas, concepts and procedures were introduced and communicated to students. We then analyzed each task in the textbooks according to cognitive demand, mathematical features and contextual features.

Introduction and development of mathematical ideas: To determine how important mathematical ideas were introduced and developed, we employed a mixed content analysis (Mayring, 2015) in two steps. First, we reviewed the relevant research literature to identify common features that had been used to classify literature (e.g. Li et al., 2009, Charalambous et al., 2010; Fan et al., 2013; Son & Diletti, 2017). The review resulted in four overarching categories: 1) mathematical general content (such as overall structure and sequencing of ideas, use of definitions and proofs, different representations, ideas presented in formal or informal mathematical language, and operational and/or structural approach to objects), 2) worked examples (frequency, completeness, and context), 3) use of explicit thinking models to support student reasoning and understanding, and 4) explicit and implicit connections to other mathematical ideas. We then worked through each chapter in each textbook, writing down characteristics of each chapter within each of the four categories. After working through each textbook, we grouped together characteristics that were seen across all individual chapters – except for special chapters such as a practical instruction on the use of digital tools. The main objective of this analysis was to identify general features of how ideas were introduced and developed in each textbook, and to determine in what way the textbooks were different.

Task analysis: Each task the textbooks was classified according to three dimensions: cognitive demand, mathematical features, and contextual features. Tasks were defined as the textbook exercises that required students to answer one or more questions, apply a procedure, or solve one or more problems. The general analytical procedure was documented in a spreadsheet and carried out in the following steps.

1. Analysis of the task: Each task was first analyzed according to possible solutions and answers. We considered possible solution strategies and algorithms that could solve the task, and identified possible correct answers.
2. Analysis of the textbooks: For each task, we searched for theory, examples and exercises presented in the textbooks prior to the task that had a similar structure and could be solved with the same answer or algorithm.
3. Cognitive demand categorization: Each task was categorized according to the Task analysis guide (Stein et al., 2000). The framework is built on the premise of how mathematical tasks in the classroom shape and determine students' learning in mathematics (Doyle, 1988). Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking, while tasks that require students to think conceptually and that make connections lead to a different set of opportunities for students (Stein & Smith, 1998). The framework is broadly separated into low and high cognitive demand tasks. Low cognitive demand tasks are further separated into two categories: *Memorization* and *Procedures without connections*. High cognitive demand tasks are also further separated into two categories: *Procedures with connections* and *Doing mathematics*.

Memorization tasks ask students to reproduce previously learned facts, rules, formulas, or definitions and can be solved without using procedures. They are not ambiguous, as they involve the exact replication of prior material (Smith & Stein, 1998, p. 348). We classified a task as memorization if it could be solved immediately using previously presented theory, examples, and exercises in the textbooks.

Procedures without connections tasks can be solved using previously learned algorithms and have no connection to the concepts or meaning that underlie the procedures being used. These tasks focus on only finding the correct answer and require no explanation or mathematical understanding (Smith & Stein, 1998, p. 348). If a task could be solved directly using an algorithm or procedure presented previously in the textbook, and focus was exclusively on getting the right answer, we classified the task as procedure without connections.

Procedures with connections tasks highlight the use of procedures in order to develop a students' deeper level of understanding of math concepts and ideas. These tasks suggest pathways to follow that are broad general procedures with close connections to the fundamental

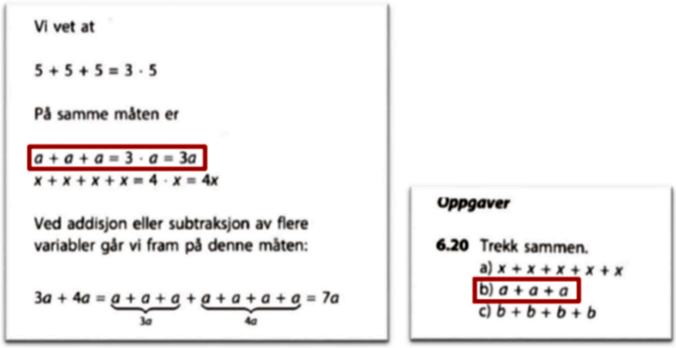


Figure 1. *Example of task classified as memorization*

conceptual ideas, as opposed to narrow algorithms that are opaque with respect to underlying concepts (Smith & Stein, 1998, p. 348). If the task required use of algorithms, but also contributed to a deeper understanding of concepts and ideas, we classified it as procedures with connections. Furthermore, these tasks required the use of either multiple algorithms or a close connection to underlying ideas and concepts.

Doing mathematics tasks require complex and non-algorithmic thinking because of the unpredictable or not easily discernible nature of their solution process. Students need to explore, grapple with and understand the nature of mathematical concepts, processes and relationships to solve the tasks (Smith & Stein, 1998, p. 348). We classified tasks as doing mathematics if they couldn't be solved using a previous algorithm or procedure, and required substantial originality and creativity to be solved.

An example helps illustrate the coding procedure. Figure 1 shows task 6.20b in Faktor 8 which was classified as a Memorization task. The task is posed as an instruction, as the reader is told to combine like terms. It is easily solved by identifying like terms, in this case the variable a, and simply count or add the number of a. Figure 1 also shows an excerpt from the textbook on page 190. The excerpt is placed immediately ahead of task 6.20, and is an explanation of how to simplify algebraic expressions by combining like items. The encircled section of the explanation is exactly the same as task 6.20b [$a+a+a$] and task 6.20b can therefore be solved as 3a using exact replication of prior textbook material. There is no need for the use of any procedure, algorithm or connections to the underlying ideas of algebra, variables and terms.

4 Mathematical features: Mathematical features is related to the mathematical complexity of a task and involve the number of steps required to answer a problem (Son & Diletti, 2017). A

multi-step problem is more complex than a single-step problem, and according to Stiegler et al. (1986), the number of steps of has the potential to affect the development of students' mathematical thinking. We determined a minimal path solution for each task, which was represented as a behavior graph. From the graph the number of steps was identified (see Leung & Silver, 1997). Tasks were categorized as zero-step if they could be solved immediately from the information provided in the task. Tasks were categorized as one-step if they required a single computation and that couldn't be solved directly from the information in the task. Multi-step tasks required several computations and several sub-goals in order to finally reach the main goal of the task.

5 Contextual features: Contextual features of tasks has been the most common theme of task analyses in the literature (Son & Diletti, 2017). This term generally refers to the setting of a task, and whether it is presented with illustrations including representations and/or real-life contexts or presented purely mathematically. According to NCTM (2000), the contextual features of mathematical tasks are a significant determinant of students' opportunity to learn, and a lack of experience with real-world problems may result in difficulties solving this type of problems and acquire a flexible and deep conceptual understanding of mathematical ideas (Wijaya et al., 2015). In this study we classified tasks that were purely mathematical and had no representations or connections to any real life as no context, while tasks that were related to personal, occupational, societal or scientific contexts were all classified as context.

Coding reliability

A selection of 100 problems from both textbook series was first assessed independently by three of the authors of this article. The resulting coding scheme was then compared and discussed qualitatively. The main discrepancies were related to cognitive demand of the tasks, and in particular the categories procedures without connections and procedures with connections. The discussion revealed that it was difficult to draw a clear distinction between procedural tasks that "have no connection" to underlying concepts, and procedural tasks that implicitly have close connections to underlying conceptual ideas (Stein et al., 2000). To improve reliability of the coding scheme, and construct a robust coding procedure, we decided that the category procedures with connections had to satisfy the following two criteria: 1) explicit connections to mathematical ideas (concepts or procedures) other than the specific called upon procedure, and 2) require some cognitive effort as the task could not be solved by applying a procedure straight forward. All problems in the textbooks were then coded independently by three of the authors of this paper using the analytical framework. Interrater reliability was assessed using Cohen's kappa (k). Reliability was excellent for mathematical features and contextual features ($k \geq .85$), and substantial for cognitive demand ($k \geq .75$) (Landis & Koch, 1977).

Results

RQ1. Which overall and structural differences can be observed between Norwegian and Finish mathematical textbooks?

The two textbook series are structurally similar. The Norwegian and Finish textbooks generally cover the same mathematical topics, and each subsection is structured similarly with theoretical explanations and worked examples first and then a collection of related tasks for the students to solve. However, probing the textbooks deeper reveal many important differences. First, although the number of pages devoted to each topic is in general comparable, there are nearly 150 more pages devoted to algebra in the Pi series than in the Faktor series. This could indicate that the Pi series place a greater emphasis on algebra than the Faktor series. Second, there is also some variation (see appendix B) regarding topic placement in the textbooks. Unlike the Pi series, general mathematical topics, such as for instance algebra in Faktor 10, are split up into smaller subtopics and spaced out within each textbook in the Faktor series. Third, although important mathematical ideas are placed and sequenced similarly with some repetition across textbooks, there is a larger extent of repetition in the Faktor series than the Pi series (appendix B). Important mathematical ideas introduced in one textbook in the Faktor series, are usually repeated in later textbooks when related new ideas are introduced. Fourth, there are also differences between the Faktor and Pi series in terms of which ideas are introduced and emphasized. In general, there are more ideas presented in the Pi series than in the Faktor series (appendix B). Within algebra, for instance, the Pi series includes concepts such as monomials and polynomials when dealing with algebraic expressions and variables. The Faktor series, on the other hand, refer to algebraic expressions simply as expressions with variables or letters across all three textbooks. These differences indicate that the two textbook series have somewhat different approaches to both sequencing of topics and ideas , and which topics and ideas are emphasized.

RQ 2. How do Finnish and Norwegian textbooks present and explain important mathematical ideas?

The content analysis identified certain trends according to the four categories: mathematical general content, worked examples, connections between mathematical ideas, and explicit thinking models.

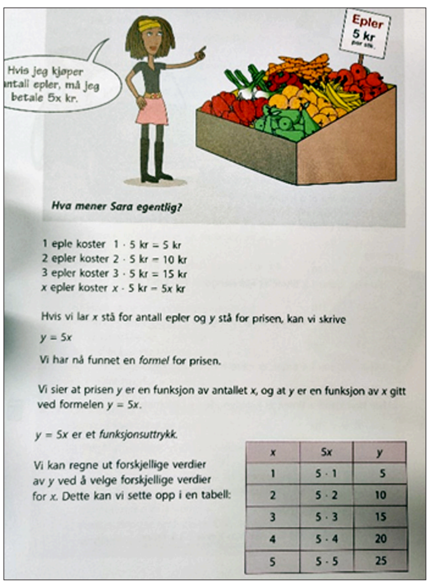
Mathematical general content: Both textbook series present definitions of important concepts, propositions and procedures similarly in terms of structure and sequencing. Usually, the textbooks would first explain what the concept, proposition or procedure is or means, using both formal and informal language, and then provide one or several example(s). Furthermore, neither textbook series presented formal proofs of propositions, but relied instead on illustrations, general explanations, specific examples, or no justification at all. However, our analysis revealed several important conceptual differences between the two textbook series. Although both textbook series presented mathematical ideas through both formal and informal language, the Finnish textbook series involved formal mathematical language to a greater extent than the Norwegian textbook series. Figure two, for example, illustrates how the concept *monomial* is presented in the Pi series in the same chapter as polynomials are introduced.


Monom
Produkten av ett tal och en bokstav kallas **monom.** Ett monom som är en del av ett uttryck kallas **term.** Ett monom består av en **koefficient** och en **bokstavsdel.** En term som saknar bokstavsdel kallas **konstantterm.**

Figure 2. *Example of concept presentation in Pi series*

Monomial is described in the Pi textbook as both the "product of a number and a letter", and that it "consists of both a coefficient and a letter". Here, the Pi series combines formal aspects of mathematical language, such as monomials and coefficients, with informal aspects of mathematical language. Polynomials are then presented in the textbook in a similar manner. In the Norwegian textbooks, on the other hand, polynomials are referred to simply as *algebraic expressions* or *letter expressions*, and they are described as expressions that contain "numbers and letters" as opposed to *numerical expressions* that contain only numbers. We found similar differences between the two textbook series across numerous different mathematical ideas such as arithmetic, number, geometric figures and their characteristics, equations, linearity, symmetry, functions etc. In general, the transition from informal everyday mathematical language to formal mathematical language is an important aspect of students' mathematical learning. Although it is unclear to what extent formal or informal language should be emphasized vis-à-vis mathematics learning, it is commonly accepted that is important to scaffold students from the use of everyday language to the use of technical mathematical language (e.g., Leung, 2005).

The two textbook series were also different in terms of operational and/or structural approach to mathematical objects. While the Norwegian textbooks has a clear emphasis on applications, and the process aspects of mathematical objects, the Finnish textbooks tend to present mathematical objects both structurally and operationally. The introduction of the function concept in both textbooks illustrates this difference. While both textbook series describe functions as process that, given an input value, returns an output value, only the Finnish textbooks emphasize mathematical functions as an object in itself and a relationship between two sets. In the Norwegian series, the function concept is first introduced in Faktor 9. Here, functions are first introduced through a practical application (the price of apples) and the textbook describes functions as formula that can help us calculate the total price, given the price of each apple and the total number of apples (figure 3). Faktor 9



Hva mener Sara egentlig?

1 eple koster $1 \cdot 5$ kr = 5 kr
2 epler koster $2 \cdot 5$ kr = 10 kr
3 epler koster $3 \cdot 5$ kr = 15 kr
x epler koster $x \cdot 5$ kr = $5x$ kr

Hvis vi lar x stå for antall epler og y stå for prisen, kan vi skrive

$y = 5x$

Vi har nå funnet en *formel* for prisen.

Vi sier at prisen y er en funksjon av antallet x, og at y er en funksjon av x gitt ved formelen $y = 5x$.

$y = 5x$ er et *funksjonsuttrykk*.

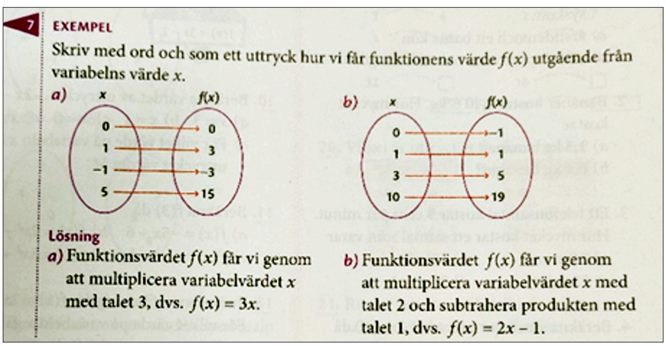
Vi kan regne ut forskjellige verdier av y ved å velge forskjellige verdier for x. Dette kan vi sette opp i en tabell:

x	$5x$	y
1	$5 \cdot 1$	5
2	$5 \cdot 2$	10
3	$5 \cdot 3$	15
4	$5 \cdot 4$	20
5	$5 \cdot 5$	25

Figure 3. *Introduction of the function concept in Faktor 9*

then defines function as a "*y* is a function of *x* when each value of *x* gives a value of *y*". The verb "give" stress the process aspect of the function as you provide an input value to compute an output value

In the Finnish textbooks, the function concept is, as in the Norwegian textbooks, presented as a process, or a function machine as the textbook explains, that computes and returns an output value according to some rule and an input value. However, unlike the Norwegian textbooks, the Finnish textbooks also present functions as a relationship between sets – or as an object in itself. Pi 9 defines functions as a "rule that connects each value of a variable with a particular value of another variable, the function value. The rule can be expressed using words, pairs of numbers, mathematical expressions, equations, or a graph. The rule must satisfy the property that for each variable value there is only one function value". Furthermore, the Finnish textbooks visualize this definition using specific examples of mathematical functions as a relationship (rule) that associates each element of one set with exactly one element of another set (figure 4).


7 **EXEMPEL**

Skriv med ord och som ett uttryck hur vi får funktionens värde $f(x)$ utgående från variabelns värde x.

a) x f(x): 0 → 0, 1 → 3, –1 → –3, 5 → 15

b) x f(x): 0 → –1, 1 → 1, 3 → 5, 10 → 19

Lösning

a) Funktionsvärdet $f(x)$ får vi genom att multiplicera variabelvärdet x med talet 3, dvs. $f(x) = 3x$.

b) Funktionsvärdet $f(x)$ får vi genom att multiplicera variabelvärdet x med talet 2 och subtrahera produkten med talet 1, dvs. $f(x) = 2x - 1$.

Figure 4. *Examples of functions as relations between sets*

Similar differences between the Norwegian and Finnish textbooks were found in the presentation of fractions, equations, algebraic expressions, geometric constructions, negative numbers etc. According to several researchers (e.g. Sfard, 1991; Gray & Tall, 1994), emphasizing both the structural and operational aspects of mathematical ideas can support the development of a flexible understanding of mathematical ideas.

There was also a clear difference in use of representations in the two textbook series. The Pi textbook series consistently used more representations and more varied types of representations than the Faktor textbook series. For instance, when explaining the concept of the variable, the Pi series used geometrical representations (square, rectangle, circles etc), table representation, purely symbolic representations, and everyday language. The Faktor series, on the other hand, used only purely symbolic and everyday language when explaining the variable concept. According to Duval (2006), mathematical objects are not directly accessible and learners have no choice other than using representations when dealing with those objects. Multiple representations, which can complement each other, are therefore usually helpful for the development of an appropriate understanding of mathematical objects and ideas (e.g. Tall, 1988; Ainsworth, 2006).

Worked examples: Both textbook series used worked examples extensively to illustrate and demonstrate every new mathematical concept or procedure. The worked examples in both textbook series were complete, as they presented complete solutions to problems or required no additional work from the reader. However, there were more than twice as many examples in the Pi series as there were in the Faktor series. We identified three reasons for this. The first reason was that the Pi series split concepts and procedures into a greater number of subconcepts or subprocedures – each with their own corresponding examples. For example, both the Pi and Faktor series explicitly demonstrated adding or subtracting terms on each side of a linear equation as a step in a procedure for solving linear equations. However, the Pi series also explicitly demonstrated adding and subtracting multiple terms on each side of a linear equation. The two other reasons were related to connections to other mathematical ideas and models to support student thinking.

Connections between mathematical ideas: As mentioned, there were more examples in the Pi series. One reason for that was that the Pi textbooks explicitly connected concepts and procedures to other mathematical ideas to a greater extent – each connection accompanied by its own set of worked examples. For example, in the Faktor series, the variable concept is connected to the symbolic notation as a referent of a set, and to formulas and mathematical expressions – which they refer to as letter expressions. In the Pi series, the variable concept is also connected to symbolic notation as a referent of a set, and formulas. However, it is also connected to monomials, polynomials, terms, coefficient, and constant. Later, when equations are introduced, the concept of variable is also discussed in relation to the concept of unknowns as solutions of equations.

Explicit thinking models: An extensive use of thinking models in the Pi series was the third reason for the greater number of examples. There were few explicit models that supported student thinking in the Faktor series, other than step by step explanations of procedures and illustrations of concepts – such as the often-used balance model to support understanding of linear equations, and bar models to support understanding of fractions. The Pi series, on the other hand, used explicit models to support student thinking extensively. These models were positioned next to additional worked examples, and provided students with visual representations that helped stimulate and structure student reasoning (e.g. figure 5). Visualization, as in the form of thinking models, have been shown to potentially support student understanding as it can help build a connection between an existing mental image and a given mathematical problem (Presmeg, 2014).

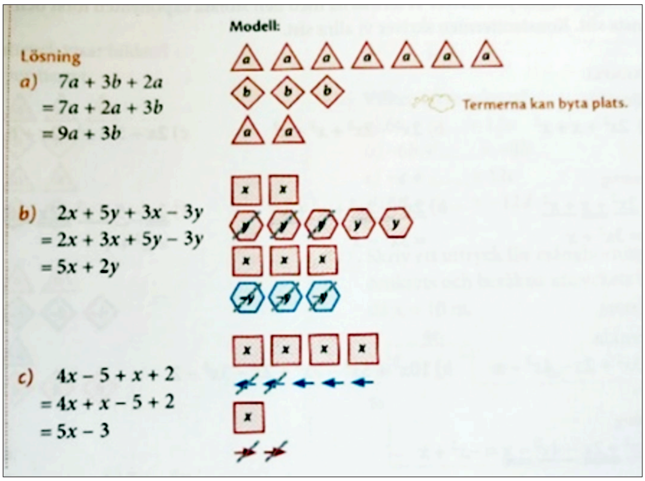


Figure 5. *Example of thinking model in the Pi series*

RQ3. What characterizes mathematical tasks in textbooks in Finland and Norway in terms of cognitive demand, mathematical features and contextual features?

A majority of the tasks in both textbook series are low cognitive demand tasks (table 2). In the Faktor series, 89 % of the tasks are either memorization tasks or procedures without connections. In the Pi series, 81 % of the tasks are either memorization tasks or procedures without connections. There is also a similar ratio of non-context and context tasks in both textbook series. In the Faktor series, 73 % of tasks are purely mathematical, while in the Pi series 80 % of tasks are purely mathematical. Finally, most tasks in both textbooks are one-step tasks – 72 % in Faktor and 75 % in Pi. Generally, it would seem as both textbooks emphasize memorization and the application of procedures over problem solving and conceptual understanding (Stein & Lane, 1996; Hiebert & Grouws, 2007).

Table 2. *Number of tasks by dimension and category across each textbook series*

	Cognitive demand					Context feature		Mathematical feature		
	Low			High						
Textbook	N	M	LP	HP	DM	NC	C	0-st	1-st	m-st
Faktor 1	3881	365	3222	244	50	3129	752	545	3029	307
Faktor 2	3248	232	2636	319	61	2220	1028	547	2389	312
Faktor 3	3461	251	2720	437	53	2412	1049	771	2267	423
Faktor series	**10590**	**848**	**8578**	**1000**	**164**	**7761**	**2829**	**1863**	**7685**	**1042**
Pi 7	4793	714	3245	751	83	4434	359	959	3731	103
Pi 8	4088	331	3066	628	63	3360	728	525	3336	227
Pi 9	3821	266	2845	632	78	2838	983	652	2721	448
Pi statistik	962	185	462	271	44	293	669	450	472	40
Pi series	**13664**	**1496**	**9618**	**2282**	**268**	**10925**	**2739**	**2586**	**10260**	**818**

Note. M = Memorization, LP = Procedures without connections, HP = Procedures with connections, DM = Doing mathematics, C = context, NC = no context, 0-st = zero-step, 1-st = one-step, m-st = multi-step.

However, although our analysis indicated similar overarching trends, there were certain differences that are worth pointing out. Although a majority of tasks in both textbooks are low cognitive demand tasks, there are more high cognitive demand tasks in the Pi series – both in absolute and relative numbers. In the Faktor series, only 11 % of the tasks are classified as high cognitive demand tasks. In the Pi series, ca. 19 % of the tasks are classified as high cognitive demand. Furthermore, our analysis also indicated large variation between the different mathematical topics. In the Fraction and Percentage category, only 2 % of the tasks in the Faktor series and 4 % of the tasks in the Pi series were high cognitive demand. In the Statistics category 15 % of the tasks in the Faktor series and 31 % of the tasks in the Pi series were high cognitive demand.

Discussion

Summary and conclusion

In this study, we aimed to map the opportunities to learn mathematics provided by mathematics textbooks series in Norway and Finland. A comprehensive horizontal and vertical analysis of the textbooks revealed both similarities and differences between them. Overall, both textbook series seem to align with a teacher-centred and deductive approach to mathematical instruction (e.g. Rocard, 2007). Each chapter in the textbooks first present and explain a particular concept, proposition or procedure, using both formal and informal language. The textbooks then give several worked examples to demonstrate the application of concepts, propositions or procedure. At the end of each chapter, the textbooks present numerous mathematical tasks that require the reader to answer one or more questions immediately related to the previous content exposition. A task analysis of all tasks in both textbooks strengthens this conclusion. First, most tasks in both textbooks provide little cognitive demand and can be solved relying on a previously given procedure or example in the textbooks. Second, most tasks in both textbooks had no representations or connections to real-life contexts. Our work therefore indicates that both textbook series seem to emphasize, and provide extensive opportunities to learn, skill efficiency through clear exposition, worked examples, substantial amounts of low-cognitive demand practice, and little real-life context (Hiebert & Grouws, 2007). Generally, our findings are in line with much of previous literature, which seem to posit that deductive teaching and direct instruction – often referred to as traditional mathematics education or an exercise paradigm (Skovsmose, 2001) – is a common trend in mathematics education in the Nordic countries (Grønmo & Onstad, 2013; Klette et al., 2016; Krzywacki et al., 2016).

However, despite the similar general instructional approach of both textbook series, the results of our analyses also provide important nuances to what opportunities to learn mathematics the textbook series provide. Although important mathematical ideas are placed and sequenced similarly in both textbook series, there is generally a stronger emphasis on repetition in the Faktor series. Ideas introduced in one textbook in the Faktor series are usually repeated in later textbooks, and explicitly connected to new ideas. Repetition distributed across time is one of the most widely substantiated cognitive principles of learning (van Merrienboer et al., 2003). Furthermore, the degree to which repeated practice improve memory depend in many ways on the way in which they are distributed over time, and, in general, longer intervals between study opportunities provide better results (Toppino & Gerbier, 2014). It may therefore seem as the Faktor series is structurally better aligned than the Pi series with the learning principles of spacing or distributed practice, which has been shown to be a highly efficient structuring of learning. On the other hand, the Finnish textbook series contained more than twice as many worked examples and more than 3000 more tasks as the Norwegian textbook series. Both worked examples and the amount of practice opportunities have been found be generally positive mechanisms of learning as transfer of information from short-term memory to long-term memory (Sweller et al., 1998; Rohrer & Pashler, 2007). The two textbook series therefore seem to be built upon somewhat different learning principles.

Delving deeper into the specific mathematical content of the textbooks revealed further important differences in terms of opportunities to learn. First, although both textbook series combined informal and formal mathematical language, the Finnish textbook series involved formal language to a greater extent. Second, while the Norwegian textbooks generally emphasized the process and operational aspects of mathematical objects, the Finnish textbooks tended to emphasize mathematical objects both operationally and structurally. Third, the Finnish textbooks included more connections between mathematical ideas and more representations of individual mathematical ideas. Fourth, the Finnish textbooks often included explicit and visual models to support student thinking when introducing new mathematical ideas. Overall, our findings seem to suggest that, even though both textbook series align with a deductive instructional approach, the Pi series provide quantitatively more opportunities to learn and qualitatively better opportunities to learn. Quantitatively, the Pi series contain more ideas, more tasks, more worked examples, more connections, and more representations than the Faktor series. For example, the concept monomial, as mentioned earlier, is presented and explained in the Pi series, but not mentioned at all in the Faktor series. As monomials are not mentioned in the Faktor series, one could conclusively say that the use of the textbook itself would provide no opportunities to learn the concept monomials. Simply by presenting more ideas, tasks, worked examples etc, the Pi series provides the reader with a greater number of opportunities to learn. Furthermore, the greater variety of ideas, connections, and representations, as well as the inclusion of visual thinking models and both operational and structural aspects of mathematical objects, suggest that the Pi series provide the reader with qualitatively better opportunities to learn. The reasoning behind this conclusion is as follows. We know that learning occurs in context, and that effective or powerful learning require transfer of knowledge and skills across different contexts (Sousa, 2011). By presenting mathematical ideas across a multitude of similar and dissimilar contexts, representations, examples, semantic variation, and through both operational and structural aspects, the Finnish textbooks might facilitate a depth, flexibility and adaptability in students learning (Luciarello et al., 2016). The greater number of ideas, connections, contexts, representations, semantic variation, and conceptions of mathematical objects in the Pi series also indicate a stronger emphasis on conceptual understanding. In a review of the literature on the relationship between teaching and learning, Hiebert and Grouws (2007) concluded that explicit attention to connections among mathematical objects, ideas, representations etc was a key characteristic of teaching that facilitates conceptual understanding. By facilitating a greater variation in the opportunities to learn mathematics, it is reasonable to conclude that although both the Finnish and the Norwegian textbook series generally align with a skill-focused and deductive instructional approach, the Finnish textbook series place a greater emphasis on conceptual understanding than the Norwegian textbook series.

Theoretical implications and future studies

Stigler and Hiebert (1999) were astonished at how much teaching varied across cultures and how little it varied within cultures. Building on the work of Stigler and Hibert (1999), Charalambous et al. (2010) proposed the term textbook signature as an idea that would represent distinctive features of textbooks within a particular educational culture. According to much of the literature, both Finland and Norway can be viewed within a Nordic profile of education and mathematics education (Blossing et al., 2014; Klette et al., 2016; Krzywacki et al., 2016). Based on these premises, we could expect mathematics textbooks in Finland and Norway to have similar textbook signatures. And, overall, both textbook series analyzed in this study aligned with a deductive approach to teaching. However, our comprehensive analyses revealed substantial differences in terms of general opportunities to learn mathematics and gain a conceptual understanding of mathematics. This leads to several important theoretical implications – some of which should be investigated further.

First, employing both a vertical and horizontal analysis of the textbook series revealed aspects of the textbooks that would otherwise have been unknown. For example, although both textbook series included relatively few cognitive demanding tasks, the Finnish textbooks facilitated a greater opportunity to learn and acquire a conceptual understanding than the Norwegian textbooks, primarily through a more comprehensive content presentation. Furthermore, including the entire textbook in our analyses, and not just sampling from one particular mathematical topic, revealed a large variation in terms of cognitive demand of tasks. Cognitive demanding tasks were much prevalent within statistics than within fractions and percentage. Based on these findings we caution against generalizations based on either just vertical or horizontal analyses, or analyses limited to individual mathematical topics of textbooks.

Second, the substantial differences between the Norwegian and Finnish textbooks observed in this study leads us to conclude that one of the following situations could be true: 1) there is no distinctive textbook signature within the Nordic mathematics education culture; 2) there is a distinctive textbook signature from both Norway and Finland, but the two countries are not part of the same mathematics education culture and have therefore different textbook signatures; 3) there is no distinctive textbook signature within the Nordic mathematics education culture, but the "lesson signature" – the teaching that actually happens in the classroom – is common to the Nordic countries, including both Norway and Finland. Deciding which of the three situations is actually occurring requires further investigations. To accomplish this one would have to: 1) analyze multiple textbook series from the Norway and Finland; and 2) investigate how mathematics is actually taught in classrooms in Norway and Finland and how teachers make use of textbooks.

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Appendix A

Background information of the Faktor and Pi textbook series

Country	Author	Textbook	Publisher	Published	Pages
Norway	Hjardar, E. & Pedersen, J-E.	Faktor 8. Grunnbok. Matematikk for ungdomstrinnet.	Cappelen Damm	2014	287
		Faktor 9. Grunnbok. Matematikk for ungdomstrinnet.	Cappelen Damm	2014	296
		Faktor 10. Grunnbok. Matematikk for ungdomstrinnet.	Cappelen Damm	2015	351
		Faktor 8. Oppgavebok. Matematikk for ungdomstrinnet.	Cappelen Damm	2014	205
		Faktor 9. Oppgavebok. Matematikk for ungdomstrinnet.	Cappelen Damm	2014	230
		Faktor 10. Oppgavebok. Matematikk for ungdomstrinnet.	Cappelen Damm	2015	284
Finland	Heinonen et al.	Pi 7. Matematik.	Schildts & Söderströms	2010	312
		Pi 8. Matematik.	Schildts & Söderströms	2011	344
		Pi 9. Matematik.	Schildts & Söderströms	2012	364
		Pi. Matematik. Statistik og sannolikhet.	Schildts & Söderströms	2013	136

Appendix B

Topic and idea sequence in the Faktor textbook series

Textbook	Chapter	Topic	Pages	Sequence of subsections and main ideas
Faktor 8	Number and number sense	Number	40	1) Natural numbers, odd and even numbers, place value, prime numbers, factorization, rounding; 2) mental arithmetic; 3) decimal numbers, 4) estimation; 5) negative numbers, number line; 6) exponentiation; 7) order of operations; 8) roman numerals.
	Fractions	Fractions and percentage	38	1) Fractions, equivalent fractions; 2) expand and simplify fractions; 3) addition and subtraction of fractions with equal denominators; 4) addition and subtraction of fractions with different denominators, least common multiple, factorization; 5) improper fractions, mixed number; 6) fractions and decimal numbers; 7) multiplication of fractions, fraction of a number; 8) division of fractions.
	Percentage	Fractions and percentage	22	1) Percentage; 2) percentage and fractions; 3) percentage and decimal numbers; 4) percentage of a number; 5) percentage and ratio.
	Geometry	Geometry	46	1) Lines, points, line segment, parallel and intersecting lines; 2) angles, obtuse angles, acute angles, right angles, complementary angles, corresponding angles, draw and measure angles with protractor; 3) triangles, equilateral triangles, isosceles triangles, right triangle; 4) quadrilaterals, rectangles, squares, parallelograms, rhombus, trapeziums; 5) circumference, circumference of polygons, circles, radius, diameter, arc, circumference of circle, π; 6) normal, geometric construction (normal); 7) geometric construction (60°angle, 90°angle, bisect angles, combinations); 8) geometric construction (triangles).
	Statistics	Statistics	28	1) Frequency distribution; 2) bar chart, 3) central tendency and dispersion, mean, median, mode, range; 4) line chart; 5) survey.
	Number and algebra	Algebra	26	1) Simplify numerical expressions, order of operations, 2) expressions with variables; 3) evaluate expressions with variables; 4) addition and subtraction of variables; 5) linear equations, solve linear equations, verify solution of linear equations.
	Measurement and units	Geometry	37	1) Length, units of length; 2) scale; 3) units of area, area of square; 4) units of volume, volume of cube; 5) units of mass; 6) units of time; 7) relationship between distance speed and time.
Faktor 9	Number and number sense	Number	30	1) Exponentiation, multiplication and division of powers, powers of 10, scientific notation; 2) square number, square root; 3) negative numbers, order of operations; 4) ratio; 5) number sequences.
	Algebra	Algebra	30	1) expressions with variables, evaluate expressions with variables, addition and subtraction of variables, multiplication and division of variables, exponentiation and variables, multiplication of powers, division of powers, parentheses and variables; 2) solve linear equations, quadratic equations, solve quadratic equations, verify solutions of equations; 3) inequalities, solve inequalities.
	Geometry	Geometry	56	1) Polygons, regular polygons; 2) circumference and area of rectangles, circumference and area of parallelogram, circumference and area of triangles, circumference and area of trapeziums; 3) π, circumference and area of a circle, 4) Pythagorean theorem; 5) geometric construction (normal, bisect angle, 60°angle, polygons); 6) geometry in nature and art, tessellation; 7) golden ratio.
	Statistics and probability	Statistics	50	1) Frequency distribution, relative frequency, relative frequency and percentage ratio decimal numbers; 2) pie chart; 3) bar chart and line chart, 4) critical use of charts; 5) central tendency and dispersion, mean, median, mode,

				range; 6) combinations, rule of product; 7) probability, favorable outcomes and possible outcomes, permutations; 8) multiple events, rule of product; 9) independent events.
	Measurement and calculations	Geometry	28	1) Measurement uncertainty; 2) scale; 3) prism, volume of cuboid, surface area of cuboid, volume of cylinder, surface area of cylinder.
	Functions	Algebra	24	1) Cartesian coordinate system; 2) formulas, function expressions, functions; 3) linear functions and graphs; 4) linear functions, equation of a straight line, straight line and coordinate system.
	Economics	Economics	29	1) Percentage, permille, permille of a number; 2) value-added tax, percentage of a number; 3) discounts, percentage of a number; 4) sale, percentage of a number; 5) loan interests, percentage of a number, interest per day; 6) payment in installments, percentage of a number.
Faktor 10	Number and algebra	Algebra	36	1) Numeral systems, decimal numeral system, binary numeral system, scientific notation; 2) equations and problem solving; 3) ratio, proportionality and equations, cross multiplication and equations; 4) expressions with variables, addition subtraction multiplication and division of variables, variables and parenthesis, binomial formulas, variables and factorization, variables and fractions, evaluate expressions with variables.
	Geometry and calculations	Geometry	62	1) Right triangle, Pythagorean theorem; 2) isosceles right triangle, $30^\circ 60^\circ 90^\circ$triangle; 3) geometrical construction (normal, bisect angle, 60°angle, polygons, circle centre), chords, tangent; 4) similarity, congruence, 5) congruence, reflection symmetry, rotational symmetry, geometric constructions (reflection, rotation, translation), perspective drawing; 6) geometry in technology, art and architecture.
	Functions	Algebra	36	1) Function, function expression, functions and graphs; 2) linear functions, slope, intercept, equation of a straight line; 3) functions and graphs, quadratic equations, parabola, extrema; 4) proportional relationships; 5) inverse proportional relationships, linear relationships.
	Equations and inequalities	Algebra	40	1) Equations, solve linear equations, equations with parentheses, equations with fractions; 2) equations and problem solving; 3) solve linear equations using graphs; 4) systems of linear equations, solve systems of linear equations using substitution method, solve systems of linear equations using graphs, systems of linear equations and problem solving; 5) inequalities, solve inequalities; 6) transforming formulas.
	Spatial geometry	Geometry	38	1) Right prisms, cylinders, volume of right prisms, volume of cylinders, surface area of right prisms, surface area of cylinders; 2) volume of pyramids, surface area of pyramids; 3) volume of cones; 4) volume of spheres, surface area of spheres; 5) density; 6) geometry formulas and problem solving .
	Statistics, combinatorics and probability	Statistics	44	1) Surveys, questionnaires, frequency distribution; 2) statistical uncertainty; 3) line chart; 4) permutations, combinations, rule of product; 5) probability, favorable outcomes and possible outcomes; independent events, multiple events, rule of product, 6) experiments, relative frequency, probability; 7) misconceptions.
	Economics	Economics	30	1) Salary, taxes; 2) bank loans, interests, installments, 3) insurance; 4) budgets, accounting; 5) currency.

Topic and idea sequence in the Pi textbook series

Textbook	Chapter	Topic	Pages	Sequence of main ideas
Pi 7	From digit to number	Number	78	1) Natural numbers, place value, roman numerals; 2) multiples, divisibility, least common multiple; 3) prime numbers, factorization, greatest common divisor; 4) negative numbers; 5) absolute value, additive inverse; 6) repetition; 7) addition, subtraction, number line; 8) simplify numerical expressions; 9) multiplication, division; 10) exponentiation; 11) order of operations, mental arithmetic; 12) decimal numbers; 13) rounding; 14) repetition.
	From point to figures	Geometry	110	1) points, curves, lines, line segment, polygon, parallel and intersecting lines, normal; 2) circles, radius, chord, diameter, arc, sector, segment, geometrical constructions (translations, normal); 3) angles, obtuse angles, right angles, acute angles, draw and measure angles with protractor, geometrical constructions (translation, bisect angles); 4) complementary angles, corresponding angles,; 5) Cartesian coordinate system; 6) repetition; 7) polygons, circumference of polygons, regular polygons, tessellation, geometric construction (regular polygon); 8) triangles; 9) isosceles triangles, equilateral triangles, geometric constructions (triangles); 10) quadrilaterals, rectangles, squares, parallelograms, rhombus, trapeziums, area of rectangle, circumference of rectangle; 11) units of length, units of area, rounding; 12) area of parallelogram, area of triangle, area of trapezium; 13) cuboid, cylinder, tetrahedron, volume of cuboid, surface area of cuboid, units of volume; 14) repetition.
	From numbers to letters	Algebra	80	1) number sequence; 2) expressions with variables, monomial, polynomial; 3) evaluate expressions with variables, evaluate polynomials; 4) addition and subtraction of variables; 5) multiplication and division of variables, variables and parenthesis; 6) addition and subtraction of polynomials; 7) repetition; 8) equivalence; 9) linear equations; 10) solve linear equations; 11) solve linear equations, 12) solve linear equations; 13) equations and problem solving; 14) repetition.
Pi 8	Number and percentage	Fractions and percentage	83	1) Rational numbers, fractions, equivalent fractions, expand and simplify fractions, fractions and decimal numbers; 2) addition and subtraction of fractions with equal denominators, addition and subtraction of fractions with different denominators, least common multiple, improper fractions; 3) fractions and multiplication, mixed numbers fraction of a number, exponentiation and fractions,; 4) inverse fractions, division of fractions, Egyptian fractions; 5) negative fractions, fractions and time; 6) repetition; 7) percentage, percentage and fractions, percentage and decimal numbers; 8) percentage and ratio, percentage of a number; 9) percentage and problem solving; 10) percentage and increase and decrease; 11) percentage and original value; 12) percentage change, percentage comparison; 13) loan interests, percentage points, percentage and problem solving; 14) repetition.
	Computation with variables	Algebra	91	1) Exponentiation and variables, multiplication of powers; 2) division of powers, zero power, negative power; 3) powers of 10, scientific notation; 4) polynomials; 5) addition and subtraction of polynomials; 6) multiplication of polynomials; 7) repetition; 8) linear equations; 9) ratio, cross multiplication and equations; 10) proportionality and equations; 11) inverse proportionality and equations; 12) equation of a straight line, solve linear equations

				using graphs; 13) straight lines and coordinate system, slope, intercept; 14) repetition.
	Figures and their properties	Geometry	106	1) Congruence, geometric construction (congruent triangles); 2) symmetry, reflection symmetry; 3) rotational symmetry; 4) translation, rotation; 5) similarity, scale, golden ratio; 6) scale; 7) repetition; 8) square root, quadratic equations; 9) right triangle, Pythagorean theorem; 10) Pythagorean theorem; 11) circumference of circles, π, arc; 12) sector and segment of circle, area of circle, area of segment, area of sector; 13) central angle, inscribed angle, tangent; 14) repetition.
Pi 9	Graphs and equations	Algebra	110	1) functions, argument and output, function expression; 2) functions and graphs, linear functions, quadratic functions, parabola; 3) linear functions, slope, intercept; 4) linear equations with two variables, straight line and coordinate system, solve linear equations using graphs; 5) proportional relationships, inverse proportional relationships, linear relationships; 6) functions and graphs, extrema, increasing and decreasing functions; 7) repetition; 8) solve equations, linear equations, equations with fractions, equations with parenthesis, equations without solutions, equations with infinite solutions; 9) proportionality and equations; 10) systems of linear equations; 11) solve systems of linear equations using graphs; 12) solve systems of linear equations using substitution method and elimination method; 13) systems of linear equations and problem solving; 14) repetition.
	Figures and shapes	Geometry	101	1) Right triangle, Pythagorean theorem, similarity; 2) tangent (trigonometry); 3) tangent (trigonometry); 4) sine, cosine; 5) tangent, sine, cosine; 6) repetition; 7) right and oblique prisms, cylinders, cones, pyramids, cuboids, units of length area and volume; 8) cylinders, cuboids, volume of cuboids, volume of cylinders,, surface of cylinders, volume of prisms, surface area of cuboids; 9) right cylinder, oblique cylinder, volume of cylinders, surface area of cylinders; 10) cones, pyramids, volume of pyramids and cones, surface area of pyramids; 11) volume of cones, surface area of cones; 12) volume and surface of spheres; 13) composite shapes; 14) repetition.
	From parts to whole	Algebra	50	1) Exponentiation and variables, multiplication of powers, division of powers, negative exponents; 2) addition subtraction and multiplication of polynomials, binomial formulas; 3) polynomials, evaluate polynomials; 4) division of polynomials, factorization and polynomials; 5) expand and simplify fractions, addition and subtraction of fractions, multiplication and division of fractions, variables and fractions; 6) proportionality and equations; 7) Repetition.
		Mixed, economics	38	8) Work context; 9) personal economy context; 10) construction and environmental context; 11) travel context; 12) leisure and sport context; 13) home and school context; 14) national and global context.
Pi statistik och sannolikhet		Statistics and probability	102	1) Line chart, bar chart, histogram, population pyramid; 2) pie chart; 3) frequency distribution, population and sample; 4) critical use of charts, 5) charts and bins; 6) central tendency, mean, median, mode; 7) dispersion, range; 8) repetition; 9) combinations, permutations, rule of product; 10) probability, random events; 11) probability, favorable outcomes and possible outcomes; 12) Problem solving; 13) surveys.