Commonalities and differences between Chinese and Swedish teachers' noticing in third-grade mathematics classrooms

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This study contributes to understanding mathematics teachers' noticing based on teacher-selected moments through a case comparison of two third-grade mathematics teachers in China and two in Sweden. The data includes written and oral reflections, and students' worksheets. A four-dimensional (mathematics, mathematics teaching, mathematics learning, and general pedagogy) coding framework was used to analyse commonalities and differences between the teachers' noticing. The results revealed both commonalities but also important differences. The biggest differences were shown in mathematics teaching and general pedagogy. The study brings some first insights into Chinese and Swedish teachers' noticing and provides practical implications on how teaching in the East and West can learn from each other.

Keywords: Mathematics teaching, teacher noticing, Eastern and Western educational paradigm, elementary school

Teacher noticing is seen as an essential component of professional teachers' competency and expertise (Kaiser et al., 2017) and informs teachers' instructional decisions (Jacobs et al., 2010). Moreover, teacher noticing influences the likelihood of desirable teacher actions, such as monitoring student behaviour, responding to student mistakes, and making a variety of other pedagogical choices (Copur-Gencturk & Rodrigues, 2022), and consequently determines the quality of mathematics teaching. Over the last two decades, teacher noticing has become a focus of research due to its increasingly central role in the pedagogical unfolding of a lesson, particularly in mathematics teaching and mathematics teacher education (Schack et al., 2017). Research in this area has addressed both preservice

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and in-service mathematics teachers' noticing from various perspectives by using different conceptualizations of noticing and different research methodologies, and the underlying goal of these studies is to support mathematics teachers in developing their competency in noticing and therefore enhancing mathematics instruction (Cai et al., 2022). Until recently, research has critiqued and expanded the mathematics teacher noticing framework to consider how ideologies influence how teachers notice. However, there has now been a sociopolitical turn in research on mathematics teachers' noticing, emphasising that the teachers and learners histories and cultures are at play in classroom interaction (König et al, 2022). Since teacher noticing highly depends on teachers' knowledge base and context (Ding et al., 2023; Kaiser et al., 2015) and noticing is socially and culturally constructed (Louie, 2018), there is a strong need among the prominent teacher noticing communities for research to understand how cultural, contextual, and ideological factors influence teacher noticing (Santagata et al., 2021). One line of inquiry for such endeavours is to investigate cross-cultural mathematics teachers' noticing and explore the differences and commonalities from the perspective of different educational cultures and teacher training systems (Wei et al., 2023).

Our present joint work follows this line of inquiry and focuses on Chinese and Swedish teachers' noticing in third-grade mathematics classrooms, bringing in one country from the Chinese Confucian Heritage Culture tradition and one from the Greek/Latin/Christian tradition (Peng et al., 2018). We chose to focus on the third-grade mathematics classroom because the authors have ongoing research projects in their countries with mathematics teachers working with this group of students, and also because this is a year with a strong focus on basic mathematical skills. In the fast-growing body of research on mathematics teachers' noticing, few studies have been conducted from a cross-country perspective. This study fills a gap and aims to further enhance our understanding of the cultural dependency of teacher noticing from the Chinese and Swedish perspectives. Specifically, we address two research questions: (a) What are the commonalities between Chinese and Swedish elementary mathematics teachers' noticing in third-grade mathematics classrooms? (b) What are the differences between Chinese and Swedish elementary mathematics teachers' noticing in third-grade mathematics classrooms? By addressing these two questions, we aim to understand what Chinese and Swedish mathematics education can learn from each other through mathematics teachers' noticing.

Theoretical framing and literature review

Conceptualization of teacher noticing

With the increasingly widely recognized significance of teacher noticing in the mathematics education community, teacher noticing is used to capture different foci in complex teaching scenarios, ranging from different domain-specific content areas (e.g., mathematical modelling, Cai et al., 2022; argumentation, Ayalon & Nama, 2025) to different general pedagogies (e.g., equity-based teaching practices, Warshauer, et al., 2023). In existing studies, teacher noticing has been conceptualized in different ways, which can be identified as four main perspectives: expertise-related, discipline-specific, social-cultural, and cognitive-psychological (König et al., 2022; Chen et al., 2025). The cognitive-psychological is the dominant perspective among existing studies and derives from the work of van Es and Sherin (2002) and Jacobs et al. (2010). Van Es and Sherin (2002) stated that teacher noticing consists of three processes: a) identifying important events in the classroom, b) reasoning about these events using knowledge of the context, and c) making connections between specific events and broader teaching and learning principles, which was expanded recently with the subprocesses attending, interpreting, and shaping (Zeeb et al., 2023). Jacobs et al. (2010) more specifically conceptualized teacher noticing of students' mathematical thinking as involving three interrelated skills: attending to student strategies, interpreting students' mathematical understanding, and deciding how to respond based on students' understanding. Similar distinctions have also been proposed. For instance, Kaiser et al. (2015) identified perception, interpretation, and decision-making as facets of noticing, and conceptualized decision-making as anticipating a response to students' activities or proposing alternative instructional strategies. Furthermore, Kaiser et al. (2015) emphasized that teacher noticing should reflect the situated and contextualized nature of teaching demands.

However, outside the European and North American context, there are only a few and less well-known teacher noticing frameworks. For instance, in the Chinese context, a three-point template, which is composed of the Key Point, the Difficult Point, and the Critical Point, has been used for investigating teacher noticing in a lesson. The Key Point refers to a mathematical concept targeted in the lesson; the Difficult Point refers to a cognitive obstacle that students often confront while learning the Key Point, which can be a persistent error or a misconception commonly associated with the concept; and the Critical Point refers to the approaches teachers use to help their students overcome the Difficult Points and learn the Key Points (Yang & Ricks, 2012; Wei et al., 2023). As there are different purposes for teachers to attend to classroom events, which may depend on different overarching instructional goals and beliefs (Colestock & Sherin, 2015), what characterizes quality mathematics teaching may differ for teachers in different cultures and shape what the teachers notice in the classroom. Thus, what teachers notice in different cultures may be indicative of their different pedagogies and choices for different ways of orchestrating classroom resources that are impacted by their cultural norms (Yackel & Cobb, 1996). In summary, research on teachers' noticing falls into what and how teachers are noticing. In this article we focus on mathematics teachers' noticing based on teacher-selected moments, namely, teachers' identification of important events in the classroom, which is built on van Es and Sherin's (2002) conceptualization of teacher noticing.

Cultural norms and their influences on teachers' noticing

Although recent research on teacher noticing has highlighted the importance of social or cultural factors and Dreher et al. (2021) also made a first step toward "proof of existence" of cultural norms on teacher noticing, there are few international cross-cultural studies (Wei et al., 2023; Santagata et al., 2021) that elucidate how cultural norms influence teacher noticing. One exception is Yang et al. (2019), which compared the noticing expertise of a group of middle school mathematics teachers in Germany and China via a video-based test instrument and reported that German teachers focused more than the Chinese teachers did on noticing aspects related to general pedagogy, whereas Chinese teachers focused more on noticing aspects connected to mathematics instruction. The study suggested that societal and cultural factors, such as different philosophical paradigms, traditions of teacher education, and teaching and mathematics curriculum traditions, are the main influencing factors on teachers' professional noticing. In another study, Ding et al. (2023) investigated the differences between sampled US and Chinese elementary teachers by inviting them to comment on cross-cultural mathematics videos online, and reported both commonalities and differences between US and Chinese teachers' noticing in the teaching domain, including representations, communication, and teacher questioning/guiding. The study also suggested that these differences are caused by cultural norms related to effective teaching in Eastern and Western cultures. Damrau et al. (2022) examined three middle school teachers' noticing in contrasting contexts (Australia, China, and Germany) as part of their classroom teaching. The researchers monitored the teachers through the process of planning and teaching lessons when provided with a researcher-developed lesson plan and reported that there were many commonalities across the cases despite the different cultural and individual backgrounds of the teachers. Furthermore, they reported that the specific topic and individual lessons as well as the teachers' expectations based on their lesson planning seemed to influence what the teachers noticed in their teaching process.

Primary school mathematics teaching in China and Sweden

Primary school education in China is divided into three stages: grades 1–2 (low), grades 3–4 (medium), and grades 5–6 (high). Primary school mathematics teaching is largely impacted by a national steering document, the Mathematics Curriculum Standards for Compulsory Education, issued in 2022. The curriculum aims to develop students' three core mathematics competencies: using mathematical viewpoint to observe the real world; using mathematical thinking to think about the real world; and using mathematical language to describe the real world, including number sense, quantity sense, symbolic awareness, computational ability, geometrical intuition, spatial concepts, reasoning awareness, data awareness, modelling awareness, application awareness, and innovation awareness (Ministry of Education of the People's Republic of China, 2022). Primary mathematics teaching is characterized as an active teacher?student interaction model of questioning–responding with a mathematical knowledge-package summary at the end of each lesson in which students reflect to consolidate their knowledge, the integration of the history of mathematics into teaching, and the development and implementation of well-structured lessons (Peng & Cao, 2021). In the third grade, the teaching content covers calculating time, addition and subtraction (up to 10 000), multiplication, fractions, and geometrical figures (square and rectangle). Chinese primary school mathematics teachers are regarded as content (mathematics) specialists.

Swedish mathematics teaching prioritizes the importance of individual interaction, builds on students' ideas, and derives mathematics from everyday situations. In Swedish classrooms, whole-class discussions are common but students also regularly work with their textbooks independently at their own individualized pace. Moreover, teachers are given considerable autonomy in terms of the choice and implementation of the curriculum (Hemmi & Ryve, 2015; Star et al., 2022). Primary school education in Sweden is divided into two stages: pre-school class to grade 3 (F–3) and grades 4 to 6. The school mathematics curriculum standard in Sweden was issued in 2022. The curriculum aims to develop students' abilities to: use and describe mathematical concepts and relationships between concepts; choose and use appropriate mathematical methods to perform calculations and solve routine tasks; formulate and solve problems using mathematics and evaluate chosen strategies; make and follow the reasoning behind mathematical inferences; and use mathematical expressions to discuss and explain questions, calculations and conclusions, including number sense, algebra, geometry, statistics, relationships and problem-solving (Skolverket, 2022). In the third grade, there is a national mathematics test, and students are expected to know numbers (up to 10 000), perform multiplication and division, exhibit datalogical thinking, tell time, and do measurements. Swedish primary school teachers are regarded as generalists.

Methodology

Case study paradigm

Similar to the study conducted by Bruns et al. (2020), which examined kindergarten teacher students' situational perception of mathematics in a comparison between Austria and Norway, this study uses a cross-country design between Sweden and China. The design allows for an analysis of similarities and differences between the teachers' noticing, while also considering the cultural and educational contexts that may influence interpretations. As participants' perceptions and expressions are shaped by contexts, the cultural differences may affect both data collection and interpretation. This requires a context-sensitive approach to ensure that findings are interpreted regarding context variations rather than being directly compared. In relation to the cross-country design, we therefore use a case study methodology to discover essential characteristics of a specific entity, building a more general understanding of the studied phenomena, and to avoid validity issues of research instruments encountered in international comparative research (Dreher, et al., 2021). According to Stake (2005, p. 445), cases can be anything that can be identified as a "specific, unique, bounded system". We chose a cross-case paradigm, within which single cases are understood to be part of a collection of cases, sharing common and related conditions and characteristics. Two Chinese and two Swedish teachers teaching third-grade students participated in the study, and each country was viewed as a case. The four teachers, all competent and committed mathematics teachers, have ongoing research collaborations with the authors in their respective countries, and they consented to participate in the study. Table 1 shows the participants' information, using pseudonyms. The Chinese teachers Tian and Li work as mathematics teachers at a school with 1700 students located in a large city in central China. The Swedish teachers Sara and Lena work as class teachers and teach several subjects, mainly mathematics and Swedish, in a school with 400 students located in the central area of a medium-sized Swedish city.

Table 1. *Participants' information.*

Teacher	Country	Gender	Years of teaching experience	Number of students	School type (grades)
Tian	China	Female	16	44	Primary school (1–6)
Li	China	Female	8	46	Primary school (1–6)
Sara	Sweden	Female	5	15	Primary school (F–6)
Lena	Sweden	Female	17	17	Primary school (F–6)

Data collection

We collected data from October 2023 to January 2024. Instead of using targeted prompts and facilitators to guide teachers to attend to certain aspects of classroom events, we chose to investigate teachers' noticing by asking teachers to retrospectively self-report in writing about what they thought was noteworthy during their own teaching (Copur-Gencturk & Rodrigures, 2022) and why these were noteworthy. Previous studies on teacher noticing have generally been conducted in clinical settings, such as professional development programmes, which hindered understanding classroom environments and accordingly influenced teacher noticing. There are rather few studies investigating teacher noticing based on teacher-selected moments of instruction with reflection notes (Taylan, 2017), which allow the researcher to gain insights into the teacher's basis for action during teaching. Overall, exploring teacher noticing based on teacher-selected moments has the potential to capture the uniqueness and complexity of building instruction based on student thinking. We investigated what the teachers were attending to, how they understood and explained these events, and how they described their attunement as related to their teaching.

The data from China comprised four written teaching noticing reflections from Tian on the lesson topics Decimal (2), Reading Calendar, and Problem-Solving, and from Li on the topics Area, Reading Calendar (2) and Problem-Solving. From Sweden, the data included four written and oral teaching noticing reflections from Sara and Lena on the topics of Number Sense, Arithmetic Operations, Calculation Methods, and Mathematical Concepts. Students' worksheets were collected from each lesson. The reflections were guided by the same noticing questions: "What do you notice in this lesson?" and "Why do you notice these?". The oral reflections were conducted with the Swedish researchers present and were audio-recorded and subsequently transcribed verbatim and then translated into English using translation software. All data were collected with participants' permission and in compliance with both Swedish Ethical Research Standards (Swedish Research Council, 2017) and Chinese research ethics rules. Table 2 shows the data sources.

The collected data from the two cases were not physically identical as the mathematics teaching cultures in China and Sweden are different, but they did provide a description of how teachers from the two countries make sense of significant moments of mathematics teaching and therefore had the potential to offer insights for exploring teacher noticing. The Chinese written noticing reflections were more extensive than the Swedish ones, but the oral reflections provided supplementary information to the Swedish noticing reflections, which all together provided rich data.

Table 2. *Data sources.*

Teacher	Data types/number	Lesson topics
Tian	Written teaching noticing reflections from 4 lessons Student worksheets from 4 lessons	Decimal (2), Reading Calendar, and Problem-Solving
Li	Written teaching noticing reflections from 4 lessons Student worksheets from 4 lessons	Area, Reading Calendar (2), and Problem-Solving
Sara	Written and oral teaching noticing reflections from 4 lessons Student worksheets from 4 lessons	Number Sense, Arithmetic Operations, Calculation Methods, and Mathematical Concepts
Lena	Written and oral teaching noticing reflections from 4 lessons Student worksheets from 4 lessons	Number Sense, Arithmetic Operations, Calculation Methods, and Mathematical Concepts

Data analysis

Data analysis began with reading and rereading the written teaching noticing reflections and the transcripts of the oral teaching noticing reflections. Thematic analyses were used to identify patterns and refine shared meanings and themes from the data (Braun & Clarke, 2006) to answer the research questions, and we integrated the four categories from earlier studies of teacher noticing (Yang et al., 2019; Ding et al., 2023) – mathematics, mathematics teaching, mathematics learning, and general pedagogy– as our initial coding guide. Table 3 shows an overview of the coding categories and their operational definitions. The specific analysis included four steps. First, the first author familiarized herself with the Chinese data by reading individual teachers' reflections thoroughly and then labelling segments directly from raw data, which generated descriptive subcategories from meaningful segments. For example, the coding of mathematics was further categorized into mathematical competency, knowledge, methods, and attitudes, and in such a way, a full sub-codebook was developed. This provided initial analytical strategies and was further tested to determine whether all the categories were applicable to other raw data. Second, each Chinese teacher's noticing was coded using participants' statements and expressions and then combined as group noticing. Third, the codebook from the Chinese data was used for the Swedish data analysis, and a data-driven inductive analysis was used to understand unexpected aspects of the data, generating additional sub-categories in the Swedish dataset. Fourth, similarities and differences across preliminary categories were continuously compared to cluster similar patterns and refine higher-order themes across groups in the two countries. To enhance the study's reliability, the authors worked collaboratively to clarify and discuss categories and themes at regular meetings, and agreement was reached on the final categories. This step in the analysis was facilitated by the first author being fluent in Chinese and knowing basic Swedish, so that possible differences in the analyses could be made visible.

Table 3. O*verview of coding categories of teacher noticing.*

Mathematics	The primary content of teacher noticing must be related to mathematics, including mathematical competency, knowledge, methods, and attitudes.
Mathematics teaching	The primary content of teacher noticing must be related to teachers' mathematics teaching, including mathematics textbooks and resources, teaching approaches, teaching strategies, and teaching skills.
Mathematics learning	The primary content of teacher noticing must be related to students' mathematical learning, including student traits, engagement, and learning processes.
General pedagogy	The primary content of teacher noticing must not be related to mathematics teaching and learning; instead, it should be relevant to general education issues.

Results

The main results of this study, in terms of Chinese and Swedish teachers' noticing in the subcategories of the four main categories –mathematics, mathematics teaching, mathematics learning, and general pedagogy– are presented in table 4. The results are reported in relation to these four categories.

Table 4. *Chinese and Swedish teachers' noticing in third-grade mathematics classrooms.*

Category	Subcategories	Chinese	Swedish
Mathematics	Mathematical competency in line with the curriculum and the lesson taught	x	x
	Mathematical knowledge in line with the curriculum and the lesson taught	x	x
	Mathematical methods in line with the curriculum and the lesson taught	x	x
	Mathematical attitudes	x	
Mathematics teaching	Teaching approach – conscious choices	x	x
	Teaching approach – whole-class and individual	x	x
	Teaching approach – pair and group		x
	Textbooks and resources	x	x
	Exactness of language	x	
	Speed in solving problems and calculations	x	
	One-to-one support		x
Mathematics learning	Engagement	x	x
	Student traits – prior knowledge and student thinking	x	x
	Student traits – working memory		x
	Learning processes – communication and analysing shortcomings	x	x
	Learning processes – building on students' strengths		x
General pedagogy	Time	x	x
	Student background		x
	Substitute teachers		x
	Special needs		x

Mathematics category

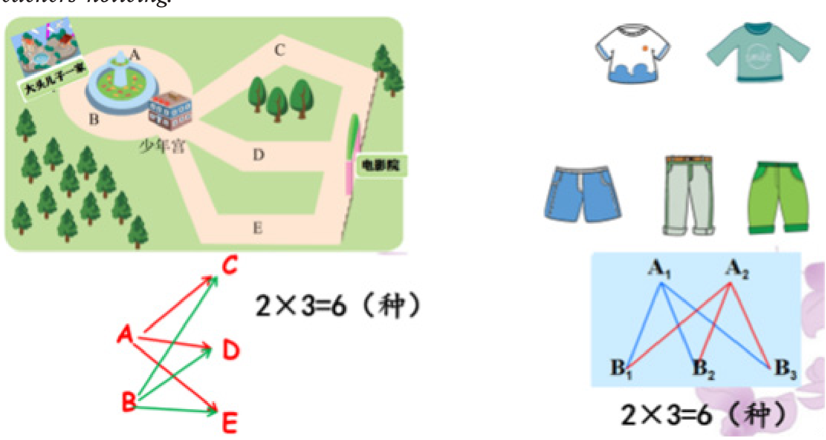
The Chinese teachers' noticing in this category was consistently more varied and detailed than the Swedish teachers' noticing. The Chinese teachers also noticed a subcategory that the Swedish teachers did not.

Commonalities in noticing

Mathematical competency, *mathematical knowledge*, and *mathematical methods* were noticed by both Chinese and Swedish teachers. In mathematical competency, the Chinese teachers noticed competency connected to the lessons taught–mathematical modelling and reasoning–and the Swedish teachers noticed using and understanding number sense. In mathematical knowledge, the teachers' noticing was connected to concepts associated with the lessons taught. Both Chinese and Swedish teachers noticed mathematical concepts and algorithms; for mathematical methods, the method of calculation was a common noticing. For example, Li paid special attention to whether the students understood the concept of "area" by checking whether they could distinguish between "perimeter" and "area". Accordingly, Li organized the teaching activity to ask students to measure the length of the four edges of their mathematics textbook covers, which was the perimeter of the book cover, and then asked the students to paint the book cover to gain a preliminary understanding of the spatial conception of the plane.

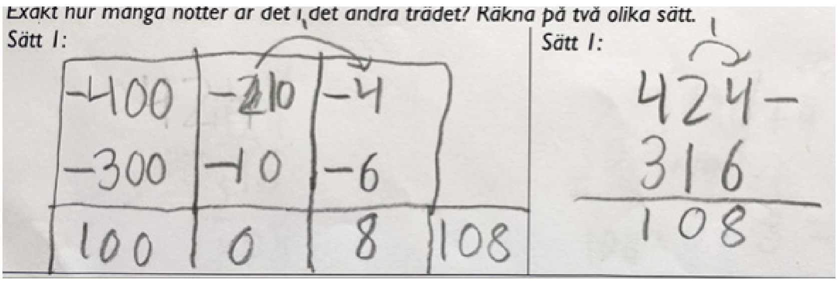
With respect to Chinese teachers' noticing of simple combinatorics during the problem-solving lesson, Li introduced two problems with different situations (routine and clothing matching) and discussed how to use mathematics as a tool to solve the problems (see figure 1).

Figure 1. *Example of how to use mathematics as a tool to solve problems in Chinese teachers' noticing.*



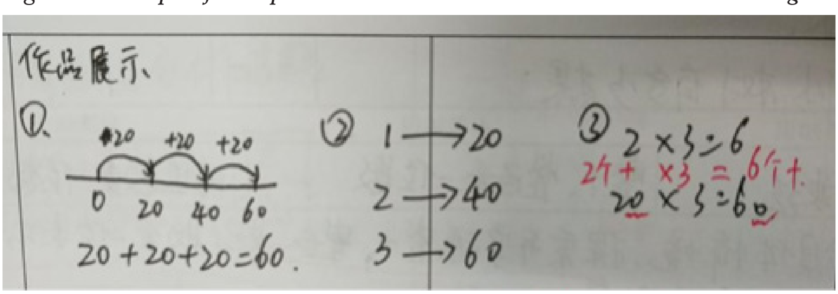
On the Swedish side, the Swedish teachers discussed the importance of being able to express written calculations in different ways and that students need to fully understand the mathematics they are working on and not just learn an algorithm that gives the right answer. Figure 2 below shows an example of student calculations of 424 – 316 = __. This example is a continuation of the task in figure 5 below. The written instructions are as follows: "Exactly how many nuts are there in the second tree? Calculate in two different ways. Way 1. Way 2." The word "exactly" is used here as a continuation of the students first being asked to calculate the approximate answer.

Figure 2. *Example of Swedish students' written calculations in different ways in Swedish teachers' noticing.*



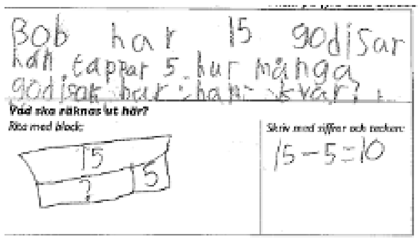
For the mathematical methods subcategory, the Chinese and Swedish teachers noticed methods in line with the lessons taught. With respect to Chinese teachers' noticing, in the lesson Problem-Solving, for example, Tian invited the students to understand the calculation principle by using drawing, enumeration, and calculation by "cover 0 and fill 0", and paid special attention to students' understanding that 2 tens times 3 = 6 tens (see figure 3).

Figure 3. *Example of multiple calculation methods in Chinese teachers' noticing.*



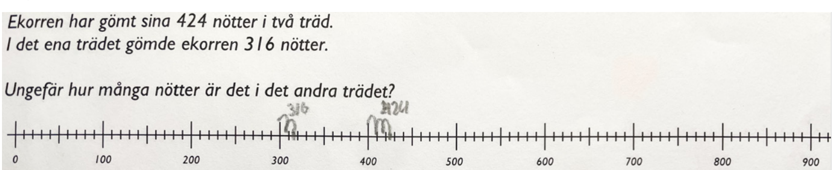
In terms of calculation methods, the Swedish teachers referred mainly to the ”number rectangles” method and number line calculation. The ”number rectangle” method involves visualizing the calculation via number blocks and could be compared to number bar models and Singapore bar diagrams. An example of this is shown in the lower left box in figure 4, where the calculation 15 – 5 =__ is represented. The student's text reads ”Bob has 15 candies. He loses 5. How many does he have left?” The written instruction reads ”What is to be calculated here? Draw with blocks.” and ”Write with numbers and symbols.”

Figure 4. *Example of the "number rectangles" method in Swedish teachers' noticing.*



Another example of calculation methods noticed by the Swedish teachers is number line calculation, used to visualize how the two numbers in a calculation are related to each other. This is seen in the example below (see figure 5), where the two numbers 424 and 316 are marked on the number line. The written instructions were as follows: ”The squirrel has hidden 424 nuts in two trees. In one tree, the squirrel hid 316 nuts. Approximately how many nuts are hidden in the other tree?”. However, here, the student misunderstood and marked the two terms instead of marking the approximate difference on the number line.

Figure 5. *Example of the number line calculation in Swedish teachers' noticing.*



Differences in noticing

Mathematical attitudes were noticed by the Chinese teachers. This concerned mathematical viewpoints deriving from mathematics curriculum standards in China. In their noticing, they emphasized the use of mathematical viewpoints to observe the real world, the use of mathematical thinking to think about the real world, and the use of mathematical language to describe the real world.

Mathematics teaching category

This category shows large variation between Chinese and Swedish teachers' noticing. Three similarities and four differences were noted.

Commonalities in noticing

Concerning *teaching approach*, all teachers' noticing revealed *conscious choices*, where the Chinese teachers talked about flipped learning and the Swedish teachers about traditional and nontraditional teaching. Both Tian and Li highlight the importance of using flipped learning to support students' learning. Sara and Lena described traditional teaching as when the teacher has an introduction, and the students then work individually in their textbooks. They highlighted the importance of challenging traditional teaching as, according to them, it does not benefit students' learning. Lena noted, "So, it's more challenging for them [the students] than if you follow a traditional mathematics textbook where you sort of mimic what's there". Here, the teacher meant that nontraditional teaching allows students to think on their own and not just copy and replicate the examples in the textbook.

Both Chinese and Swedish teachers also emphasized *whole-class and individual work* as a basis for their teaching. According to this, the Chinese teachers mention whole-class discussions and the Swedish the importance of adapting the student's perspective and letting them describe their knowledge. For *textbooks and resources*, both manipulatives and teaching materials were common noticing for Chinese and Swedish teachers. The teachers from both countries described working with manipulatives and other concrete teaching materials as successful in supporting student learning. Sara provided an example of this, stating, "So I think yes, but it would be interesting to do the same task with multilink [differently coloured plastic cubes that can be linked together] and see how the reasoning goes then". Thus, this teacher saw potential in offering manipulatives to visualize specific mathematical content. Whole-class discussions, which aim to allow students to put their knowledge into words and to listen to other students' descriptions, were described as a central part of the teaching. The Swedish teachers also described dealing with student errors as a starting point for future lesson planning and wished that they could learn more from students' incorrect calculations but that this opportunity was often missed. Lena described this as follows: "You can also see that the students erase so you don't have the possibility to see how they think". Therefore, guided by a student's incorrect calculation, the teacher gains important insights into students' misconceptions and difficulties.

Differences in noticing

Four differences were noted. The Swedish teachers' noticing included a discussion about *pair and group work* and *one-to-one support*. The teachers described the need to talk to students individually when difficulties for students were discovered in order to understand their thinking and to provide individual support. Something noticed by the Chinese teachers was the *exactness of mathematical language* and *speed in solving problems and calculations*. Both Tian and Li emphasize the importance of both teachers and students using correct mathematical terms and making an effort for students to achieve speed in their work to support student learning.

Mathematics learning category

In this category, the Swedish teachers' noticing had more variation than the Chinese teachers'. The Swedish teachers noticed two subcategories that the Chinese teachers did not.

Commonalities in noticing

Commonalities in noticing concerned students' *engagement*, *student traits such as prior knowledge*, and *students' thinking and learning processes*. In the engagement subcategory, students' interest in mathematics and students' experience were common noticing elements and all teachers reported that students' interests in and experience with mathematics affect the outcome of the teaching and thus the students' mathematical learning. Both Chinese and Swedish teachers emphasized the importance of starting with each student's prior knowledge. Concerning learning processes, both Chinese and Swedish teachers talked about learning as a process and highlighted communication and analysing *students' shortcomings* to support the process of learning.

Differences in noticing

Two differences were shown concerning mathematics learning: *working memory* as an aspect of students' traits and *building on students' strengths* as an aspect of learning processes, both addressed by the Swedish teachers. They described working memory as an important factor when students faced difficulties and discussed methods to address it. An example of building on students' strengths is exemplified when Sara described "We've worked with algorithms before, which made this easier. They had prior knowledge". Another example is when analysing the students' worksheets and Lena recognized that two students had the correct thinking at the beginning of the calculation but then failed to complete it: "But then, what have they done here? In the beginning, it's great. It is divided into 3. $30+30+30=90$. And then I think that here they have tried to show that". Therefore, even if a student showed shortcomings, the teachers noted what the student had understood.

General pedagogy category

This category shows a large variation between Chinese and Swedish teachers' noticing, where Swedish teachers' noticing was more varied. The Swedish teachers noticed three subcategories that the Chinese teachers did not.

Commonalities in noticing

Time was a common noticing for both Chinese and Swedish teachers. All teachers expressed that students need time to consolidate knowledge, and that teaching and planning for teaching is time-consuming for teachers.

Differences in noticing

The Swedish teachers noticed *student background*, *substitute teachers* and *students' special needs*, which did not appear in Chinese teachers' noticing. The teachers expressed that students' background affects their opportunities for learning and that, for example, students with a different mother tongue have greater challenges in school. When it came to substitute teachers, the noticing concerned that they do not have the required competence and could thereby guide the students too much and almost give them the right answers to the tasks, instead of allowing the students to think for themselves. The Swedish teachers also noticed the special needs aspect; namely, specialized teaching and that students had received good help from special needs teachers, which contributed positively to students' development and learning.

Discussion

Using the four-dimensional (mathematics, mathematics teaching, mathematics learning, and general pedagogy) framework, we found commonalities and differences between Chinese and Swedish teachers' noticing in the third-grade mathematics classroom. We discuss these results and their implications below.

First, the results show that the Chinese teachers' noticing had a larger variation in the mathematical category, whereas the Swedish teachers had a larger variation in mathematics learning and general pedagogy. This result is in line with that of Yang et al. (2019), who reported that German teachers' noticing was related to general pedagogy, whereas Chinese teachers' noticing was related to mathematics. An explanation for this could be that Chinese teachers are specialized mathematics teachers, whereas Swedish teachers are general teachers and teach several different subjects. As teacher noticing is informed and shaped by teachers' professional knowledge and views (Taylan, 2017), it is reasonable to assume that the Chinese teachers and the Swedish teachers would notice different aspects in the same teaching situation. This could be related to Damrau et al. (2022), who reported that the teachers' expectations seemed to influence what the teachers noticed. On the basis of this reasoning, students receive different types of support in Chinese and Swedish classrooms. The Chinese teachers in this study have a greater focus on mathematics content, whereas the Swedish teachers in this study have a greater focus on individual students receiving more individualized support. In the other two categories, there were both similarities and differences between the Chinese and Swedish teachers' noticing, although mathematical learning had greater variation in the Swedish teachers' noticing. Perhaps this is related to the fact that learning aspects in mathematics can be linked to pedagogy and learning in general.

Second, there were commonalities found in all four categories among Chinese and Swedish teachers' noticing. The commonalities were: mathematical competency, knowledge, methods, teaching approaches, textbooks and resources, engagement, students' traits, learning processes, and time. Thus, even in mathematics classrooms from completely different educational cultures there are many commonalities, implying that Chinese and Swedish teachers have a similar basis regarding pedagogical and didactic choices for mathematics teaching. This result is supported by Damrau et al. (2022), who reported many commonalities among teacher noticing in Australian, Chinese, and German contexts. These commonalities show that there are some shared values about teaching and learning mathematics around the world that promote teachers' noticing.

Third, the results also show differences in all four categories. Our results point to that the Chinese teachers focus on mathematical attitudes, exactness of language, and speed in solving problems and calculations. The Swedish teachers focus on pair and group work, one-to-one support, working memory, building on students' strengths, student background, substitute teachers, and special needs. These differences could be related to the mathematics teaching culture in each country. For instance, speed in solving problems and calculations in the Chinese teachers' noticing is emphasized in Chinese mathematics teaching in primary schools, which is rooted in a key belief that "practice makes perfect" (Li, 2006) and is related to exactness of language. Meanwhile, both substitute teachers and special needs in Swedish teachers' noticing are related to the educational system in Sweden regarding how to deal with the issue of when teachers are absent and how to help students in special needs. Furthermore, some of the differences still reflect the identified general distinctions in mathematics education between East and West, such as valuing the pedagogy placing authority in the teachers' knowledge in the East, as opposed to valuing being individualistic with associated pedagogies in the West (Cai et al., 2024), which is demonstrated in the one-to-one support in Swedish teachers' noticing but not in Chinese teachers' noticing.

Results from this case study show that the comparison between Chinese and Swedish teachers' noticing have implications for both mathematics teacher education and classroom practice. What is "the best of both worlds" in terms of mathematics teachers' noticing from the study? As the results revealed that, in the Chinese context, there was more focus on mathematical content, and in the Swedish context, there was more focus on individual students and their learning, the two countries have different things to learn. In the Chinese context, although the mathematics curriculum standard has the aim of "mathematics teaching for all students" (Ministry of Education of the People's Republic of China, 2022), it is not easy to achieve such an aim. Thus, Chinese teachers could have a greater focus on individual students and special needs so that it is more customized to suit each student rather than focusing on the whole group. In the Swedish context, as Swedish students are showing a decline in mathematics results in international tests such as the TIMSS study and PISA in recent years (Swedish National Agency for Education, 2019; 2020), a greater focus on mathematical content and methods could improve the quality of mathematics teaching by giving the subject content a more prominent position. Altogether, this raises the question of whether teachers can focus on both a coherent mathematics instructional routine and student discourse and mathematical thinking (Nilsson, 2019). When it comes to classroom teaching, the question of how teachers can develop mathematics collaboratively with students while simultaneously maintaining a coherent and structured learning trajectory should be a common pursuit in both Eastern and Western classrooms.

Conclusions and limitations

This case study is one of the few studies in mathematics education conducted in both China and Sweden, two countries with distinctive educational cultures and mathematics teacher training systems. As such, it not only provides some first insights into teachers' noticing in Chinese and Swedish classrooms but also contributes to understanding mathematics teachers' noticing from a cross-cultural perspective. The revealed commonalities remind us that there are shared values in mathematics classrooms worldwide that deserve special attention from teachers. The differences show the cultural dependency of teacher noticing and provide practical implications for mathematics teacher education and classroom teaching in the two researched countries, as well as in Eastern and Western countries more broadly. Despite the value and contribution of this study, we need to keep in mind its limitations. First, owing to the case study nature, the results are not generalizable to the broader population of Chinese and Swedish mathematics teachers. Second, teacher noticing and the lessons for researching teachers' noticing were chosen by the participating teachers themselves, and while this approach provides autonomy and flexibility to teachers, it may also have an impact on our research results. Moreover, the discrepancies between data sources from China and Sweden enable us to discuss general issues in teacher noticing but limit us from drawing direct comparisons between specific lessons and content. Third, this study scratches the surface of Chinese and Swedish mathematics teachers' noticing. It addresses "what" is noticed by the teachers, but not the deeper questions about how teachers' knowledge, beliefs, and cultural factors shape teacher noticing and, by extension, classroom practices.

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