

# Studying how a mathematics teacher's professional identity shapes and is shaped by the use of digital resources in the classroom

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Teachers face many issues when trying to integrate digital resources (DR) into mathematics classes. This article applies an identity-based perspective to understand teachers' roles in the practices that evolve in such classes. We focus on Victor, a Greek mathematics teacher, who, when viewed from levels beyond the classroom, experienced to become a reform-oriented teacher and a designer of DR. We explore how these experiences of professional identity shapes and is shaped by his work with DR at the classroom level. We show how Victor's identity changed from "being a *mathematics* teacher who struggles with inquiry-based teaching" to "becoming a *mathematics teacher* who uses DR to support inquiry-based learning" and outline what fuelled these changes. Our results suggest that it is important to connect an identity perspective with classroom interactions and mathematics.

Since 2000, research on mathematics teachers' professional identity has increased (Darragh, 2016; Lytovac & Kaasila, 2018; Graven & Heyd-Metzuyanim, 2019). One reason for this is that teacher identity is perceived as closely intertwined with professional development (PD), as it "involves at least in part becoming a 'different' teacher and a 'different' person" (Hodgen & Askew, 2007, p. 474). However, the growth in identity research is not reflected in research on mathematics teachers' use of digital resources (DR) (Clark-Wilson et al., 2020). An abundance of studies in this field shows the complexities teachers face when trying to integrate and exploit DR' affordances in mathematics classes to support student learning in new and more profound ways (Chronaki & Matos, 2014; Clark-Wilson et al., 2020; Trouche et al., 2019). To better understand these complexities, recent studies suggest that an identity perspective on teachers is needed, particularly with respect to teachers' roles in the practices that unfold in such classes (Chronaki & Matos, 2014; Goos, 2013;

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Psycharis & C. K. Skott, 2023). This study aims to narrow this research gap by applying an identity perspective to exploring a Greek mathematics teacher's (Victor)<sup>1</sup> interactions with DR. This study also addresses calls in teacher-identity research to use observation of practice (Lutovac & Kaasila, 2018), and to foreground mathematics (Graven & Heyd-Metzuyanin, 2019) when studying identity.

Victor's case is extreme (Flyvbjerg, 2006), as he is one of the few successful cases of integrating DR' affordances to support students' deeper mathematics learning, at least as viewed from how he relates to questions of culture and power at the school (meso) level and at the political (macro) level. Recently, we showed how and why Victor's professional identity developed over 22 years (1998–2020) at these two levels (Psycharis & C. K. Skott, 2023). His identity changed dramatically from *being a traditional mathematics teacher who barely used DR* (as was usual in Greece that time), to *becoming a reform-oriented teacher and a national DR designer* who was recognised as such, and well-known in the Greek mathematics education community. In this study, we explore how and to what extent Victor's success at the meso- and macrolevels in becoming a reform-oriented teacher and a designer of DR shapes how he interacts with his students at the classroom (micro) level when he uses DR to teach mathematics. This study contributes to identity research as (a) we use classroom observation to investigate the nature and development of Victor's identity as it relates to the ways he interacts with his students, his colleagues and others related to his job as a mathematics teacher at North river school and beyond, and (b) we foreground mathematical aspects underlying Victor's identity.

We use *patterns of participation* (PoP), a participative framework for teacher identity (J. Skott, 2019) that focuses on teachers' actions and meaning-making in local practices (e.g. at a school or in a classroom), and the influence on these practices of broader social constellations (e.g. a reform world) and modes of reasoning (e.g. what are teachers' obligations according to parents, leadership and more generally). Using PoP, we define Victor's professional identity as his "fluctuating experiences of being, becoming and belonging" (J. Skott, 2022, p. 16), as a professional at North river school and beyond. We address two research questions.

How does Victor's fluctuating experiences of being, becoming and belonging evolve, as he uses DR in his mathematics teaching at North river school? What influenced his identity development?

Beyond these questions, we expect to address the potential of PoP for understanding macro-, meso- and microlevel changes in teacher identity.

## Mathematics-teacher identity and use of DR

Although the field of mathematics-teacher identity is growing, one criticism is the vagueness of the concept of teacher identity and the lack of clarity surrounding its use (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; J. Skott, 2022). To clarify the different approaches, we found two overviews of the field particularly relevant. The first is Darragh's (2016) development of five categories of commonly-used theoretical approaches.

- 1 Participative (focus on participation in social groups)
- 2 Narrative (focus on narratives people tell about themselves or others)
- 3 Discursive (focus on discourses at various levels)
- 4 Psychoanalytic (focus on individual characteristics)
- 5 Performative (focus on how people act at a specific time, or position themselves in broader discourses)

The other is J. Skott's (2022) identity triad of common aspects in definitions of teacher identity – situatedness, structure and agency (see figure 1) – which C.K. Skott & J. Skott (2023) suggest may be "used to distinguish between different approaches to Individualities in Context" (p. 220). Situatedness is about the local situation (e.g. local practices in a classroom or PD) and is concerned with "whether and how teachers negotiate and identify with normative identities" (J. Skott, 2022, p. 11) locally. Structure refers to matters and power relations external to the local situation (e.g. political discourses or new syllabuses), which are part of the conceptual framework from the start, whereas agency addresses teachers' professional decision-making space, and their willingness and capacity to act and influence their current situation. J. Skott's (2022) triad and most of Darragh's (2016) categories suggest that identities are often seen

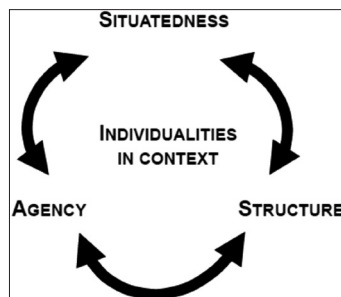


Figure 1. *The identity triad* (J. Skott, 2022, p.13)

as socially constituted (locally or structurally), and as multiple, dynamic and context-dependent.

Most identity studies are small-scale, empirical and based predominantly on interview data (e.g. autobiographical or narrative) (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; Lutovac & Kaasila, 2018). Lutovac and Kaasila (2018) suggest that greater emphasis should be placed on observation of practice, to gain deeper insight into teacher identity, and Graven and Heyd-Metzuyanim (2019) take this point further, and call for "methodologies that extract identity from classroom activity" (p. 374). These authors also call for studying identity in a closer relationship with mathematics, as studies tend to background mathematics, or address it only tangentially, severing it from the discussion of teacher identity.

According to the above-mentioned literature, teacher identity and DR are rarely studied together. This is confirmed by studies of teacher use of DR (e.g. Clark-Wilson et al., 2020). In our literature study, we found only four studies of teacher identity and DR, which we have categorised in relation to the identity triad: two studies focused on structure and agency, one focused on agency and one on situatedness and agency. This categorisation reflects the general tendency of studies of mathematics-teacher identity, with the fewest studies focusing on situatedness and agency (J. Skott, 2022).

The two studies that emphasise structure and agency – Chronaki and Matos (2014) and Goos (2013) – took different approaches to identity. Chronaki and Matos (2014) used a discursive approach and focused on a group of Greek teachers who participated in a PD programme "as agents who struggle to make sense and shape their realities" (p. 110) and on their ways of negotiating and identifying with positions and often-competing discourses, offered at the institutional, educational and political levels. Goos (2013) used a participative approach (Lerman, 2000) to focus on the long-term interactions between Brian, an Australian secondary-school teacher, and his professional environment (e.g. organisational structures and cultures, access to resources) "while still emphasising individual agency" (p. 523). Neither study used classroom observation to construct teacher identity, but Goos used observation to document the successful changes in Brian's classroom teaching with DR.

The study that emphasised agency (Elbaum-Cohen & Tabach, 2020) applied a narrative approach based on commognition, and concerned Yosef, an Israeli secondary-school teacher, who participated in PD focused on reflection on, and experimentation with DR. The authors analysed Yosef's and the other participating teachers' narratives about Yosef and his classroom teaching. These narratives defined Yosef's identity. The

other two aspects of the triad – situatedness and structure – seemed to play no role in the construction of his identity.

The fourth study focused on situatedness and agency, and applied PoP, a participative approach. Based on interviews, C. K. Skott et al. (2021) identified three identity markers to describe significant aspects of a Danish elementary teacher's (Sofia) identity, while she was involved in PD. The authors used the markers to relate emerging practices of Sofia's classroom teaching to aspects of her identities.

The four studies of teacher identity and DR also reflect the variety of theoretical approaches and the dominance of the participative approach in mathematics-teacher identity research (Lutovac & Kaasila, 2018). The present study shares similarities with C. K. Skott et al.'s (2021) but differs from most of the other identity studies in that it focuses on the role of DR, considers mathematical aspects and draws on classroom observation to understand identity. Hereby we aim to contribute to the emerging sub-field of teacher identity and DR, and the abovementioned call to connect teacher identity to classroom interactions and mathematics.

### Theoretical approach

In line with most mathematics-teacher-identity studies, we apply a participative approach based on social practice theory (Lutovac & Kaasila, 2018). Hence, our framework, PoP, draws on the ideas of practice and figured worlds (J. Skott, 2019), where practice "connotes doing [but] doing in a social and historical context that gives structure and meaning to what we do" (Wenger, 1998, p. 47), and figured worlds are "socially and culturally constructed realm[s] of interpretation, in which particular characters and actors are recognised, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p.52).

J. Skott (2022) argues that studies that apply PoP tend to focus on situatedness and agency in the identity triad, and not on situatedness and structure, as do most other participative approaches that apply social practice theory. These other approaches tend to focus on predefined social structures or local practices, for instance, examining how teachers move from peripheral to more extensive participation in predefined practices or figured worlds, such as practices of a PD programme or a world of reform-oriented teaching. Although acknowledging that individual identity is largely shaped by social context, a strong focus on predefined structures or practices risks losing sight of the individual (Lutovac & Kaasila, 2018). To re-centre the individual, PoP focuses on "how different social constellations and practices come to play a role for" (J. Skott, 2019, p. 469) a teacher's participation in local practices.

PoP networks social practice theory with symbolic interactionism. Thus, identity becomes "an emergent phenomenon [...] one that may be in flux, for instance as classroom processes unfold" (J. Skott, 2022, p. 17). A teacher's participation in social interactions is understood as influenced by their interpretation of the immediate situation and simultaneous meaning-making. This means that teachers "continuously interpret others' actions symbolically, including their actual or possible reactions to one's own behaviour" (J. Skott, 2019, p. 472), and assume others' attitudes to themselves. The others may be colleagues or parents, but may also be a social group or community, in which case teachers assume the attitudes to themselves of generalised others. In PoP, practices, characters and figured worlds are interpreted as possible generalised others, and we will refer to the ones that describe significant aspects of a teacher's identity as *identity markers*. For example, C. K. Skott et al. (2021) identified three identity markers significant for Sofia's identity two of which were characters related to teaching mathematics – *a modern teacher* and *a digital spearhead* – and the third was a contrasting figured world, *the traditions of mathematics teaching*.

With PoP, we define Victor's professional identity as his "fluctuating experiences of being, becoming and belonging" (J. Skott, 2022, p. 16) as a professional at North river school and beyond. This may include experience of being (in)competent (e.g. when collaborating with colleagues), of becoming better or worse (e.g. at using DR) and of belonging or not (e.g. to the group of teachers at North river school).

### The trajectory of Victor's identity: meso- and macrolevels

We studied Victor's long-term PD at the meso- and macrolevels by combining two theoretical frameworks and constructing his professional trajectory for each of them (Psycharis & C. K. Skott, 2023). Here, we focus only on one, Victor's identity trajectory, defined as "the development of [his] professional experiences of being, becoming and belonging as a teacher" (J. Skott, 2019, p. 470). We showed how Victor's identity developed over 22 years, as he drew on six different identity markers, and how the roles of these markers shifted in significance for his experiences over three periods (see figure 2).

### The six identity markers

1. *Para-education* (figured world) (or auxiliary schooling): Educators (not necessarily certified to teach primary or secondary education) prepare students for the entry exams to tertiary education through a traditional teaching approach.

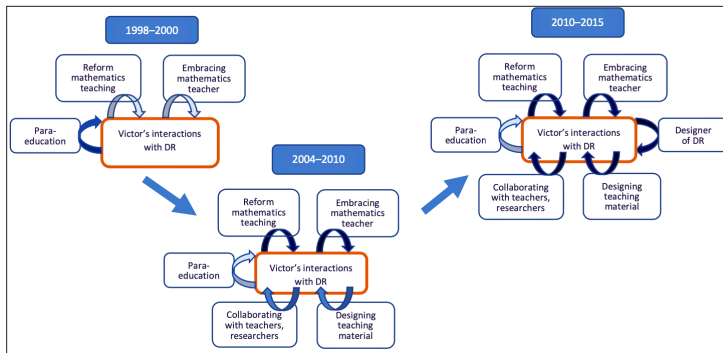


Figure 2. Victor's identity trajectory at the meso- and macrolevels, over three periods. The darker the arrow, the more prominent the identify marker's role.

2. *Reform mathematics teaching* (figured world): Primarily conveyed in the literature, teacher education and PD programmes. Generally, Greek teachers find it difficult to teach in line with this world, with its focus on involving students in mathematical processes and inquiries, drawing on student contributions in classroom discussions and using a variety of representations to support their mathematical thinking.
3. *Collaborating with teachers, researchers etc.* (practice): Victor aimed to explore his classroom teaching by collecting data and using these and research results to improve it.
4. *Designing teaching material* (practice): Victor aimed to develop teaching material that could support him (and others) in solving the problems he faced in his classes.
5. *Embracing mathematics teacher* (character): For Victor, this character reified his preoccupation with student-centred teaching approaches and his political ideology of empowerment through education.
6. *Designer of DR* (character): For Victor, this character meant designing challenging mathematical tasks for all students by exploiting DR' affordances.

At the school and national levels, Victor's identity developed from being a traditional mathematics teacher who drew on *para-education* with no DR, to becoming a recognised *reform-oriented teacher* and a *designer of DR*. For our present purpose, two of his experiences are particularly important. First, throughout his career, Victor strove to become the *embracing*



*teacher*, which motivated him to continuously seek and take new opportunities for PD at his school and beyond. Second, in the final period, Victor experienced to become and be widely recognised as a *designer of DR* and belong to an elite group of such designers, who exploited DR' affordances to support reform-oriented teaching. We explained Victor's identity development by emphasising the roles of affective and contextual aspects (e.g. motivation, social recognition) and broader modes of reasoning (e.g. political ideologies). However, it remains an open question, how these dramatic changes shaped and are shaped by his interactions at the classroom (micro) level. Thus, in this article, we explore Victor's experiences and the role of the markers at this level.

## Methodological approach

### *Study setting*

The Greek educational structure is highly centralised, and the Ministry of education introduces innovations through top-down initiatives. Auxiliary schooling (private or para-education) is widespread; built on a traditional teaching approach, its purpose is to help students to acquire the skills needed for examination success. Like most novice teachers, Victor worked in auxiliary schools before entering public secondary education (years 7–12) in 2000. Since then, he has worked at North river school, a multicultural public school near the Turkish border, with huge language and cultural challenges. He has also participated in several national projects, the two most important of which were (1) *New school*, aimed at modernising the national mathematics curriculum (years 1–12) in a reform-oriented way, by drawing on learning trajectories as a framework for design based on three components: mathematical goal, learning activities, expected learning paths (Potari et al., 2019), and (2) *Digital school*, aimed at developing DR in the form of interactive applets, called *micro-experiments*, and of *generic scenarios* promoting project-work through the use of micro-experiments (see Barcode scenario below) to be integrated into the new curriculum and enrich the online version of the current national textbooks (Kynigos, 2020). The two design teams consisted of educational consultants, teachers (including Victor) and mathematics and mathematics education researchers (including the second author). Their work was guided by a set of principles, such as combining multiple mathematical representations and promoting inquiry.

The second author is well-acquainted with Victor, personally and professionally. Their collaboration began with *New school* and continued with *Digital school*. Although the developed DR were piloted at selected



schools, Victor decided to implement some of them in his classes, to explore their potential for reform-oriented teaching. This was the first time that he used DR systematically in his classes (i.e. to cover a whole topic). He invited the second author to work with him, and their collaboration continued over the next years as a teacher–researcher partnership, satisfying all the required ethical conditions (e.g. permissions from educational authorities). Victor gave informed consent for the current analysis.

### *Data generation*

Our data relates to (a) two of Victor's designs and classroom implementations (in 2010–11 and 2011–12) of a teaching unit that introduces linear functions to 8th grade students by using a generic scenario, *Barcode scenario*, and (b) Victor's reflections on these implementations. We chose these two implementations as both included the Barcode scenario and Victor's approach to teaching functions changed significantly from the first implementation to the second. Before each implementation, Victor and the second author systematically collected his teaching resources and discussed his objectives, decisions and choices at the design level. Victor video-recorded all the lessons, which they analysed separately by selecting incidents they considered critical in terms of a gap between intended and realised teaching, student understanding or difficulties, or on-the-spot teacher decisions. They then discussed the selected incidents. They conducted two pre-lesson and two post-lesson discussions per year. In this article, we draw on the following data, which were transcribed; selected parts were translated to English.

- Victor's resources: lesson plans, worksheets, digital files etc.
- Video recordings: the first implementation consisted of 12 lessons and the second implementation consisted of 17 lessons.
- 20 (of 40) critical incidents (10/year).
- 4 pre-lesson discussions (pd1<sup>2</sup>, pd2).
- 4 post-lesson discussions (pod1, pod2).

### *Barcode scenario*

The problem presented in the Barcode scenario is, "A salesclerk sells three products – A, B and C – at different prices. After each sale, the clerk records the weight,  $x$ , and the amount of money received,  $y$ . After 20

completed sales, the clerk enters the values in two columns in the Function probe (FP)<sup>3</sup> table window to check (a) number of sales of each product and (b) whether there were mistakes in computing prices”. Phase 1 of this scenario, which we use in the analyses, is described below.

**Phase 1. Exploring proportional relations**

A table containing pairs (weight, price) of 20 sales in two corresponding columns is provided along with one empty column defined by the ratio  $z=y/x$  (figure 3). Students are expected to note that this ratio equals three specific values (0.7, 1.4, 2.3) – unless a value corresponds to a calculation mistake. These values indicate the existence of  $y = ax$  relations. Students send the points  $(x,y)$  to the graph window (figure 4, left), and they are expected to note that the three values of the ratios correspond to three different groups of collinear points. They are encouraged to construct the graph  $y=x$ , and stretch it with the stretch tool<sup>4</sup> to coincide with the existent groups of points (figure 4, right). The tool provides the corresponding formula for each transformation of the graph.

*Analytical approach*

We used the six identity markers identified by Psycharis and C. K. Skott (2023) as an analytical lens to examine Victor’s experiences at the microlevel. We focused on his design of the two implementations and his participation in classroom interactions when enacting the Barcode scenario. We analysed the transcriptions of the critical incidents, and the pre-lesson and post-lesson discussions. Inspired by grounded theory (Charmaz, 2014), we initially coded Victor’s utterances related to actions

Αρχείο	Επεξεργασία	Αποστολή
x	y	$z=y/x$
weight	price	
0.8	1.68	
0.2	0.24	
1.6	1.92	
1.3	0.78	
5.5	11.55	
1.9	3.99	
11.5	6.9	
3.8	4.56	
3.4	2.04	
6.2	4.96	
2.2	2.64	
3.5	7.35	
5.3	3.18	
6.8	4.08	
5.7	6.84	
4.7	9.87	
15	9	
3	6.3	
8.1	9.72	
5	3	

Figure 3. Weight-price pairs of 20 sales

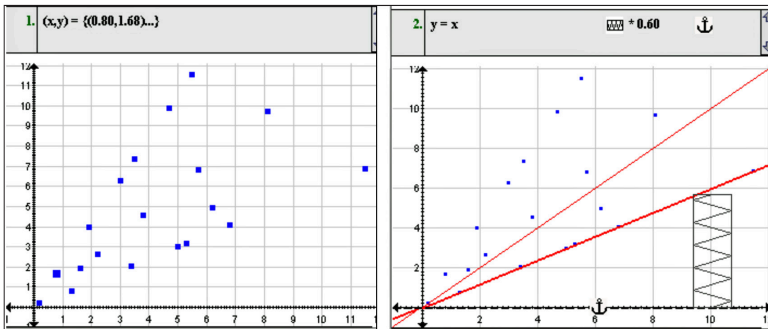


Figure 4. (left) Points sent from table to graph. (right) Stretching vertically  $y = x$  to pass through the three groups of points. Point (6.2, 4.96) will be omitted.

and meaning, line-by-line, using gerunds. For example, we assigned the initial code "focusing on specific answers" to the utterance "I will ask in another way. You are not far off. What kinds of quantities are these?" which was part of a longer class discussion, in which students gave different answers, but not the desired one. We then compared and assessed our initial codes, and assembled them into focused codes, such as "bypassing students' answers" Finally, we compared and related our focused codes with the six identity markers; in the example, we related the codes to *para-education*. As we could relate all focused codes to the markers, we did not identify new markers, but only refined and changed the status of some of them. Importantly, *collaborating with teachers, researchers etc.* and *designing teaching materials* crystallised into two new markers, the *New school team* and the *Digital school team*.

The result of our analysis was that Victor's identity had changed from *being a Mathematics teacher who struggles with inquiry-based teaching* during the first implementation to *becoming a mathematics Teacher who uses DR to support inquiry-based learning*, during the second. In the next section, we outline how these changes occurred and what influenced them, by relating them to Victor's professional experiences of being, becoming and belonging.

## Analysis and results

### *Being a Mathematics teacher struggling with inquiry-based teaching*

When Victor initially enacted the Barcode scenario, he was clearly in deep water, and characterised the enactment as "moving in the opposite direction" (pod1) of the targeted reform approach. During the first implementation, he struggled to balance his usual teaching approach

with his new orientation to the *reform world*, but his experience from *para-education* tended to dominate. We stress three ways in which this happened.

In line with *para-education*, Victor's design of the first implementation built on a formalised and abstract approach to mathematics teaching, with a strong focus on the content. Apart from the Barcode scenario, all the lessons resembled Victor's usual approach to functions (see table 1).

Table 1. *Victor's design of the first implementation (based on his lesson plans, worksheets, digital files etc.)*

Lessons	Topics	Teaching material
1–4	Definition of function as a correspondence, the coordinate system and function-graphs	Chapter 3 (national textbook) Worksheets: definition of function and multiple representations of functions
5–6	Familiarisation with FP	Worksheet: technical aspects of FP
7–10	Proportionality and functions	Barcode scenario
11–12	Linear functions $y = ax + b$	Textbook exercises

The Barcode scenario presented a new teaching approach. Thus, Victor's use of the scenario in his otherwise traditional design indicates that he viewed reform-oriented teaching as a separate activity that could supplement his usual classroom teaching, not as a teaching approach. Drawing on *para-education*, Victor designed to start by defining a function: "This [was] an influence of teaching in upper secondary schools for years" (pd1). He justified his choice of the correspondence-definition from a mathematical perspective (not based on his experience with student difficulties or a learning perspective): "[the] mapping between sets helps students understand what a function is, and most importantly, to clarify what is not a function" (pd1). At the design level, Victor thus tended to draw on his experience from *para-education*, primarily focusing on the mathematics and its abstract nature, not on student understanding.

When enacting the Barcode scenario, Victor divided the inquiry into a sequence of questions, insisting that each question needed to be answered in a specific way before continuing to the next one (see the incident below). He described this way of contributing to classroom interactions as related to *para-education*: "We are used to cutting mathematical knowledge into small pieces [procedures, definitions] to be easily digested [by students] ... [they] prefer true–false questions or multiple-choice tasks with clear didactic aims and simple answers" (pod1). Accordingly, the enactment alternated between brief group work to address these questions, and classroom discussions of their answers. Furthermore, Victor

felt pressured "when you teach a material for the first time you unconsciously expect to cover a specific number of questions" (pod1), which reinforced his usual teacher-centred practices: "[in case of] unexpected students' difficulties that cause delays to the flow of the lesson, I feel that I ha[d] to conclude and move to the next questions" (pod1). In addition, when "students' answers or questions provide[d] opportunities for exploration, I g[a]ve the answer. I d[id] this ... not because I appreciate[d] this practice but because I did not understand" (pod1) that this practice was inappropriate. Victor realised that his ways of interacting with the students "cancel[led] the process of inquiry" (pod1).

The interactions between Victor and his students, during class discussions and group work, tended to have an IRE (Initiation Response Evaluation) nature. Although Victor invited students to contribute, consistent with the *reform world*, there was generally little openness in what he treated as acceptable answers to his questions. A critical incident (8th lesson) from a class discussion shows this.

- Victor: My question is why is the formula  $y=0.7x$ ? Why are these points located on this line?
- Maria: Because they were sold at this price.
- Victor: Why are they located on the same line?
- Eleni: Because they have the same price, sir.
- Victor: I want you to explain it to me in mathematical terms. How is belonging to the same line explained mathematically?

It is clear from this and other exchanges that the students struggled to connect the entities (including the ratios) in the table to their graphical representations. However, in the incident Victor insisted on strict and specific mathematical answers, even when students provided meaningful ones that related to the scenario's real-world setting as did Maria and Eleni. During the enactment, Victor only rarely tried to understand the students' ideas, or use them to provide alternative explanations. His fixation on getting a specific answer led to reducing student answers to factual ones, or giving the answer himself, as in the continuation of this incident. Such IRE communication is typical of *para-education*, and Victor seems to draw primarily on this world when enacting the scenario.

Victor's general experiences of belonging to *para-education* dominated his design of the first implementation and his first enactment of the Barcode scenario. Victor tended to participate with an exclusive focus on the mathematics, foregrounding it, in particular its products, almost to the extent of ignoring students' incorrect or unexpected ideas and understanding. Thus, despite his efforts and intention to teach

reform-oriented his students were offered hardly any opportunities to engage in inquiry, and the enactment tended to be like his usual classroom teaching, which dominated the first implementation. We interpret his microlevel identity as primarily *being a Mathematics teacher who struggles with inquiry-based teaching*.

### *Becoming a mathematics Teacher who uses DR to support inquiry-based learning*

Victor's experiences of professional being, becoming and belonging changed dramatically in the period leading up to the second implementation. We show how his design and scenario-enactment changed, compared to the first implementation, and outline what influenced his identity development.

Victor's design was still inspired by the national textbook, but now he and the *New school* and *Digital school teams* developed most of the resources he used (e.g. micro-experiments) (see table 2). The way he integrated these resources indicates a shift from regarding reform-orientated teaching as a separate activity, to seeing it as a teaching approach: "I used many micro-experiments because they promote inquiry [...] They focus on specific concepts through different representations in ways that cannot be addressed by ordinary teaching" (pd2).

Table 2. *Victor's design of the second implementation (based on his lesson plans, worksheets, digital files etc.)*

Lessons	Topics	Teaching material
1-2	Patterns	Worksheets: figurative, arithmetic and geometric patterns, including one micro-experiment
3-8	Relations between two variables	Worksheets: covariation, points and graphs in the coordinate system, including 6 micro-experiments
9-13	Proportional relations $y = ax$	Worksheets: basic FP functionality via mathematical tasks. Barcode scenario enriched with 3 micro-experiments
14-17	Linear functions $y = ax$ and $y = ax + b$	Barcode scenario Textbook exercises

Victor's new design showed three mathematics-related shifts: first, a shift from dividing the content into knowledge "pieces", to engaging with it as coherent sequences of concepts, ideas and procedures "The learning trajectories we used in the design helped me a lot. Now when I teach

functions I think of the links of this topic to proportional relations, covariation, and students' knowledge from the primary school" (pod2); second, a shift from defining functions as correspondences from the start, to involving students in exploration and generalisations of patterns, and then providing a covariant definition; third, a shift from focusing primarily on mathematical products (e.g. definitions, procedures) to involving students in processes (e.g. generalisation, reasoning). Victor's experience from the *New school* and *Digital school teams* played prominent roles in these transformations. The *New school team* introduced him to the idea of a learning trajectory and the pattern-based approach to teaching functions, and the *Digital school team* built their DR designs on this approach: "Pattern recognition and expression of general rules were seen as an introduction to the idea of function. This was a dominant view in the design team [...] This influenced my [teaching] approach and prompted a lot of changes" (pod2). Regarding the use of DR, Victor recounted that the *New school* and *Digital school teams* helped him "underst[and] more deeply what using technology means ... the role of representations, the idea of exploration, the dynamism of dragging a slider and observing what happens" (pod2), which helped him change his design and teaching with DR. Even today, he seems to profit from these experiences "I observed what [the team] designed, and I took ideas at the technical level but also at the didactical one ... even now ... I take what I need, and I connect it to other things. I put it in my own design" (pod2).

When enacting the Barcode scenario, Victor mostly interacted with students in an open way. This is shown in the critical incident (10<sup>th</sup> lesson) below, where Victor invited students to share their ideas about how to group the points in the graph window.

Anna: If we add more points from the table, it would be easier for us to see where these points belong on the graph.

Victor: Try to show your idea on the projector. Are there some points that you believe are in the same group?

Anna: Yes. These [traces a straight line with her finger over the points belonging to an imaginary  $0.7x$  line]. And these [traces another imaginary line].

Victor: Why did you show them that way? Do these points have a specific feature?

Anna: They have these ratios here [points to the table]. They take the same number from the table.

Victor: Thank you. Are there other ideas?

Adonis: The dots that are very close are the same.

Victor: What do you mean?



- Adonis: If one dot is 3.2, the next one is the same. I refer to those that are very close to one another [In the subsequent dialogue it became clear that Adonis also tried to link the proximity of points to the ratios in the graph].
- Victor: Are there any other suggestions apart from these two? [Victor then proceeded to show the students how the graph of  $y = x$  could be transformed by using the stretch tool]. Now let's see if these two ideas are correct, using this special tool. [By using the tool, the students realised that the points belonged to straight lines  $y = ax$ ] Thus, Anna's and Adonis' suggestions were correct.

Victor characterised the incident as critical, because it related to his aim of helping students to connect functional concepts when using DR. In it, Victor tried to make sense of Anna and Adonis' suggestions by posing clarifying questions and relating to the class' previous work: "I recognise[d] in the students' ideas influence of the preceding task" (pod2). Also, he tried to involve more students in the discussion by repeatedly inviting more ideas. In contrast to the first enactment, he seems genuinely interested in understanding his students' thinking, and in foregrounding their ideas. These concerns are important aspects of the *reform world*, and also relate to Victor's desire to become the *embracing teacher*, which he often stressed in interviews: "the school should mitigate the impact of social and class differences on learning, and not magnify them, as now occurs [...] My goal is for all students to improve, not only in terms of learning".

The foregoing incident also reveals other central aspects of the *reform world*, such as using student ideas to structure classroom discussions (e.g. Victor invited the class to explore Anna and Adonis' ideas), scaffolding students' exploration with mathematical ideas and DR (e.g. he introduced the stretch tool), closely relating the discussion to the content and DR (e.g. he supported students to use the tool as an intrinsic part of their meaning-making) and gradually guiding students towards the lesson's goals. There were few instances of IRE communication in the second enactment of the Barcode scenario.

Victor's experience of collaborating with the *New school* and *Digital school teams* seems also to play a prominent role in his transformed participation in classroom interactions. One part was his collaboration with a team member with whom he thoroughly analysed the classroom incidents and dug deeply into his interactions with the students. Another part was the reform-oriented way that the *Digital school team* designed new DR. Victor recounted that he thereby learned to understand "what kind of questions I select for the students to characterize a micro-experiment as inquiry-based. This understanding was not immediate. It took time" (pod2). He stressed that Greek teachers in general

struggled with reform-oriented ideas "there were all these ideas circulating around, but the question was always among the teachers "Who can do all these?" (pod2), but that he learned to manage it, among other things, by "focus[ing] on the kind of questions I need to pose to facilitate students' exploration. I learned about all these things through this collaboration" (pod2).

To summarise, during the second implementation, it is remarkable how Victor transformed his way of designing the function unit and interacting with students, and how he gained confidence in reform-oriented teaching at North river school. He no longer divided the content into knowledge "pieces" or fixated on getting specific answers. On the contrary, he shifted his attention to students' ideas, and used these to structure classroom interactions around central concepts, while using DR to structure the teaching and involve students in mathematical processes. We interpret his new identity as *becoming a mathematics Teacher who uses DR to support inquiry-based learning*. This identity foregrounds his role as a teacher (with a capital letter) of mathematics rather than foregrounding mathematics as in this first identity. Our analysis shows that Victor's experience of belonging to the *reform-world* played a prominent role in his identity development as well as his experiences from outside the school setting of belonging to the *New school* and *Digital school teams* and being a valued collaborator in these teams. His experience of becoming more like the *embracing teacher* played a secondary role and *para-education* seemed to play only a marginal role.

### Discussion and concluding remarks

This study presents several insights into Victor's microlevel identity-development by analysing how his fluctuating experiences of being, becoming and belonging evolved as he used DR to teach mathematics at North river school. It shows that Victor's identity changed from *being a Mathematics teacher who struggles with inquiry-based teaching* to *becoming a mathematics Teacher who uses DR to support inquiry-based learning* – a change that teachers find difficult (e.g. Chronaki & Matos, 2014; Clark-Wilson et al., 2020). Our analysis shows that these changes were influenced by Victor's experiences of belonging to the *reform-world*, of belonging to and being a valued collaborator in the *New school* and *Digital school teams*, and to some extent of becoming more like the *embracing teacher*.

As indicated by the titles of his identities, Victor's experiences of mathematics and DR played prominent and shifting roles for his ways of contributing to classroom interactions – as shown in the analyses of the two enactments – and hence for his identity formation. Compared

to our previous study, this analysis shows that Victor's microlevel transition into the *reform world* lagged with respect to his meso- and macrolevel development. At these levels, he was already widely recognised as a reform-oriented teacher. Such discrepancies between what teachers identify themselves with at meso- and macrolevels, and what they do in class are well-known. This study thus shows that it is important to connect teacher identity with mathematics and classroom interactions, as also noted in the literature (e.g. Graven & Heyd-Metzuyanim, 2019).

Although this study adds further evidence that Victor's case is successful in terms of exploiting DR' affordances to teach in a reform-oriented way, the two interpretations of his identity – at the microlevel and at the meso- and macrolevels – do not tell the exact same story. First, as mentioned above, Victor experienced to belong to the *reform world* at the meso- and macrolevels before he did at the microlevel. Our previous analysis showed that Victor wanted to become a *designer of DR* and quickly accomplished this, due to his technical readiness to design DR and the means provided by *Digital school* to design in line with the *reform world*. However, when he used the designed DR in class, he was confronted with a complex situation that required considerable effort and where success could not be predetermined. Second, Victor's experience of collaborating with a member from the *New school* and *Digital school teams* proved significant to his microlevel identity development, whereas this experience seemed almost unimportant at the meso- and macrolevels. Third, the *embracing teacher* played a significant role for Victor at the meso- and macrolevels: becoming this character motivated him to launch new initiatives. The character played a less prominent role at the microlevel, but Victor sometimes drew on it indirectly, as shown in the second enactment. The same apply to the marker *designer of DR*. Despite these differences, from an overarching perspective, the two interpretations do not conflict: both describe Victor's identity-development as shifting from being a traditional mathematics teacher who did not use DR to become a reform-oriented teacher who exploited DR' affordances. Each of the two interpretations contributes important nuances and adds supplementary affective and contextual aspects to Victor's identity development. This suggests PoP's potential for helping to understand and explain teachers' identity-development at both the micro-, meso- and macrolevels.

Regarding research on teacher identity and use of DR, our study shows the importance of analysing classroom interactions when studying teacher identity. Of the four articles we identified in our literature study, only ours did so. For example, the teachers mentioned by Chronaki and Matos (2014) negotiated a "becoming a new teacher" identity (p. 108), which they portrayed as being at "the cutting edge" (p. 115) and

as exploiting DR' pedagogical novelty. However, the teachers negotiated these identities at levels beyond the classroom, so the extent – if any – to which their new identities contributed to changes in their classroom teaching, is unclear. By analysing classroom interactions, we were able to clarify such discrepancies between what Victor identified himself with at meso- and macrolevels, and what he did in class. The analyses also enabled us to show that Victor used DR as a structural element for developing a new approach to teaching functions and that his use of DR proved to be an important mechanism in his microlevel identity development. Both types of insights require classroom analyses, and they are important, if we aim to better understand the complexities teachers face when integrating DR in teaching.

We studied only one teacher, in a specific setting. This is a clear limitation of this study with regard to teacher identity-development in general, but not with regard to our theoretical and methodological tools. Another problem is the close relationship between Victor and the second author. We did a double analysis to avoid bias, but the relationship may have made it easier for Victor to draw on the *reform world*. Whether the results would be the same without this partnership is an open question.

By focusing on situatedness and agency in the identity triad, PoP enabled us to study one teacher's identity development by analysing his lesson design and classroom enactments, while considering his agency to change his current ways of teaching. PoP also acknowledges the importance of structure (the third aspect of the triad) and allowed us to explore which identity markers beyond the local setting influenced the teacher's actions and meaning-making. Those characteristics of PoP, especially its focus on situatedness, differ from most other approaches to teacher identity, and therefore PoP may suggest one way to "extract identity from classroom activity" (Graven & Heyd-Metzuyanin, 2019, p. 374). Our study suggests two directions for future research: to study further teacher identity and use of DR in classroom settings, and to explore other theoretical ways of studying teacher identity that consider the triad's situatedness without losing sight of agency and structure.

## References

- Chronaki, A. & Matos, A. (2014). Technology use and mathematics teaching: teacher change as discursive identity work. *Learning, Media and Technology*, 39(1), 107–125. <https://doi.org/10.1080/17439884.2013.776076>
- Charmaz, K. (2014). *Constructing grounded theory. A practical guide through qualitative analysis* (2nd ed.). Sage.

- Clark-Wilson, A., Robutti, O. & Thomas, M. (2020). Teaching with digital technology. *ZDM*, 52(7), 1223–1242.  
<https://doi.org/10.1007/s11858-020-01196-0>
- Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, 93(1), 19–33.  
<https://doi.org/10.1007/s10649-016-9696-5>
- Elbaum-Cohen, A. & Tabach, M. (2020). A possible path from teachers' collaboration towards teachers' change in practice. In H. Borko & D. Potari (Eds.), *Proceedings of the twenty fifth ICMI study on Teachers of mathematics learning and working in collaborative groups* (pp. 118–125). University of Lisbon.
- Flyvbjerg, B. (2006). Five misunderstandings about case-study research. *Qualitative Inquiry*, 12(2), 219–245.  
<https://doi.org/10.1177/1077800405284363>
- Goos, M. (2013). Sociocultural perspectives in research on and with mathematics teachers: a zone theory approach. *ZDM*, 45(4), 521–533.  
<https://doi.org/10.1007/s11858-012-0477-z>
- Graven, M. & Heyd-Metzuyanim, E. (2019). Mathematics identity research: the state of the art and future directions. *ZDM*, 51(3), 361–377.  
<https://doi.org/10.1007/s11858-019-01050-y>
- Hodgen, J. & Askew, M. (2007). Emotion, identity and teacher learning: becoming a primary mathematics teacher. *Oxford Review of Education*, 33(4), 469–487. <https://doi.org/10.1080/03054980701451090>
- Holland, D., Lachicotte Jr., W., Skinner, D. & Cain, C. (1998). *Identity and agency in cultural worlds*. Harvard University Press.
- Kynigos, C. (2020). Half-baked constructionism: the challenge of infusing constructionism in education in Greece. In N. Holbert, M. Berland & Y. Kafai (Eds.), *Designing constructionist futures: the art, theory, and practice of learning designs* (pp. 61–72). MIT Press.
- Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Ablex Publishing.
- Lutovac, S. & Kaasila, R. (2018). Future directions in research on mathematics-related teacher identity. *International Journal of Science and Mathematics Education*, 16(4), 759–776. <https://doi.org/10.1007/s10763-017-9796-4>
- Potari, D., Psycharis, G., Sakonidis, C. & Zachariades, T. (2019). Collaborative design of a reform-oriented mathematics curriculum: contradictions and boundaries across teaching, research, and policy. *Educational Studies in Mathematics*, 102(3), 417–434. <https://doi.org/10.1007/s10649-018-9834-3>

- Psycharis, G. & Skott, C. K. (2023). Studying a mathematics teacher's documentational and identity trajectories over time. *Journal of Mathematics Teacher Education*. Published online. <https://doi.org/10.1007/s10857-023-09605-3>
- Skott, C. K. & Skott, J. (2023). A novice teacher's identities – from losing her balance to regaining her confidence. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel & M. Tabach (Eds.), *Proceedings of PME 46* (Vol.4, pp. 219–226). PME.
- Skott, C. K., Psycharis, G. & Skott, J. (2021). Aligning teaching with current experiences of being, becoming and belonging: an identity perspective on the use of digital resources. In A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, J. Trgalová & H. Weigand (Eds.), *Mathematics education in the digital age. Learning practice and theory* (pp. 213–227). Routledge.
- Skott, J. (2019). Changing experiences of being, becoming, and belonging: teachers' professional identity revisited. *ZDM*, 51 (3), 469–480. <https://doi.org/10.1007/s11858-018-1008-3>
- Skott, J. (2022). Conceptualizing individual-context relationships in teaching: developments in research on teachers' knowledge, beliefs and identity. In J. Hodgen, E. Geraniou, G. Bolondi & F. Ferretti (Eds.), *Proceedings of CERME 12* (pp. 34–60). Free University of Bozen-Bolzano.
- Trouche, L., Gueudet, G. & Pepin, B. (Eds.). (2019). *The "resource" approach to mathematics education*. Springer. <https://doi.org/10.1007/978-3-030-20393-1>
- Wenger, E. (1998). *Communities of practice: learning, meaning, and identity*. Cambridge University Press. <https://doi.org/10.2277/0521663636>

## Notes

- 1 All names of persons and schools are pseudonyms.
- 2 The number refers to implementations 1 and 2, respectively.
- 3 FP is multi-representational software with three windows: table, graph and calculator.
- 4 The stretch tool allows mouse-driven horizontal and vertical stretching of the graph.

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