Teachers' learning of ambitious mathematics teaching as changes in pedagogical discourse

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This study investigates changes in teachers' pedagogical discourse as they participate in a professional development project. Fourteen Norwegian elementary-school teachers collaborate in learning cycles where the overarching aim is to learn ambitious mathematics teaching. Data from interviews with two groups of teachers are analysed. The findings reveal significant changes in the teachers' pedagogical discourse: valuing new teaching practices, talking about more specific aspects of ambitious teaching and making references to student learning. The findings also reveal a shift in the teachers' pedagogical discourse around struggling students.

Ambitious mathematics teaching is grounded in students' emergent thinking and puts their sensemaking at the centre of instruction (Kazemi et al., 2016). Key principles of ambitious mathematics teaching are to provide students with equitable access to learning, position them as sense-makers and engage deeply with students' thinking. An ambitious teacher must know the students, develop positive relationships and be responsive to them in culturally appropriate ways (Ghousseini et al., 2015). As ambitious teaching is a complex and demanding endeavour, professional development (PD) needs to support teachers in learning how to practise it (e.g. Lampert et al., 2010; McDonald et al., 2013).

Ambitious mathematics teaching is supported by research that shows positive outcomes for student motivation and learning (e.g. Leinhardt & Steel, 2010: Jacobs & Spangler, 2017; Wæge, 2007). In mathematics education, the aim of PD has often been characterised as moving from "traditional" teaching to more "reform" practices (e.g. Hemmi & Ryve, 2015; Heyd-Metzuyanim et al., 2018). Developing teachers' practice towards an ambitious or reform mathematics teaching has been found to be a

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complex process. Researchers recommend that PD should aim to add to teachers' mathematical knowledge for teaching, improve their ability to use this knowledge in practice and support their development of new perspectives of teaching and learning (e.g. Charalambous & Delaney, 2020; Munter, 2014). Kazemi and Wæge (2015) argue that developing an ambitious teaching practice is not just a matter of developing technical skills. Teaching is value-laden work that requires both intellectual and emotional investments. Learning ambitious teaching includes developing a particular stance and identity as teachers who care about student thinking (Lampert et al., 2013). Lee and Francis (2018) and Stockero et al. (2020) call for research that explores how PD programs support teachers' development of understandings or views of mathematics teaching that align with the principles of ambitious teaching. The aim of this study is to shed light on this.

Related research

Teacher learning or professional growth as a result of PD has been a widely researched area (e.g. Borko, 2004; Kennedy, 2016). Exploring mathematics teachers' evolving visions of high-quality mathematics teaching, Munter (2014) developed an interview-based instrument for describing trajectories of the participants' visions and tracking changes in those visions over time. The instrument presents trajectories from less to more sophisticated visions along four dimensions: the role of the teacher; classroom discourse; the nature of classroom tasks; and student engagement in classroom activity. In developing different levels of sophistication, Munter differentiated between descriptions that "included both new *forms* and thorough descriptions of their *functions* in terms of supporting students' learning and those that promoted new forms but failed to indicate a shift away from old functions" (Munter, 2014, p. 598). He argues that teachers' ability to articulate a strong rationale for enacting specific ambitious teaching practices indicates a more sophisticated view of high-quality mathematics teaching.

Studies have investigated teachers' orientations and conceptions of key practices of ambitious teaching related to student thinking (e.g. Lee & Francis, 2018; Stockero et al., 2020). Stockero et al. (2020) explored teachers' orientations towards using student mathematical thinking during whole-class discussion. Identifying and characterising teachers' orientations according to their potential to support or impede the development of the practice of productively using students' emergent ideas, they distinguished between high-potential orientations, low-potential orientations and hindering orientations. They argue that more research

is needed to better understand how PD can support teachers to develop high-level orientations that position student thinking as a valuable resource in teaching.

Researchers have used the concept of *framing* in exploring teachers' views and conceptions in ambitious mathematics reform efforts, particularly when studying conceptions of students' capabilities (e.g. Bannister, 2015; Horn, 2007, Jackson et al., 2017). Horn (2007) explored the way teachers' conceptions were embedded in their daily work, focusing on the ways in which they framed the problem of differential success. She found that in one department some teachers framed the problem in terms of varying degrees of ability, using such labels as "slow" and "fast" learners. In another department, however, the teachers framed the problem in terms of learning opportunities provided by the teacher. The findings showed that the way teachers framed the problem of differential success had implications for how they engaged in ambitious mathematics reform. Horn suggests that a key aspect of enacting ambitious teaching entails framing student difficulty as a problem of instruction. Jackson et al. (2017) examined teachers' views on students' mathematical capabilities in relation to the principles of ambitious teaching. The findings revealed that most teachers did not view all of their students as capable of participating in rigorous mathematics activities and lowered the cognitive demand of the problem (e.g. showing students how to complete a similar problem) to support students who faced difficulties. Their findings suggest that PD needs to support productive shifts in teachers' views of students' capabilities along two dimensions: how they explain the source of students' difficulties and how they address students who face difficulties in mathematics. Jackson et al. (2017) argue that we need more research on how PD supports teachers in reorganising their views on students' capabilities.

Recent studies have demonstrated the usefulness of adopting a discursive approach to examining teachers' learning of explorative teaching practices (e.g. Nachlieli & Heyd-Metzuyanim, 2022; Heyd-Metzuyanim, 2019). Heyd-Metzuyanim et al. (2018) differentiate between *Delivery pedagogical discourse* (DPD) and *Exploratory pedagogical discourse* (EPD). Heyd-Metzuyanim and Shabtay (2019) point out that teachers' pedagogical discourse can be located on a continuum between these two pedagogical discourses. The current study follows Heyd-Metzuyanim and colleagues (e.g. Heyd-Metzuyanim et al., 2019) in using a discursive approach to study teachers' learning of ambitious teaching. This framework affords the examination of delicate differences in the extent to which the teachers adopt the DPD or EPD (Nachlieli & Heyd-Metzuyanim, 2022). Another strength of viewing learning to teach ambitiously

as changing alignment from DPD to EPD is that it places a new emphasis on "the fact that teachers do not come to a PD 'tabula rasa' (or as empty slates)" (Heyd-Metzuyanim, 2019, p. 10). This means that teachers have to unlearn their old practices while they learn new ambitious practices.

Learning ambitious mathematics teaching

Our work is informed by sociocultural views on teacher learning. A commognitive approach views learning as becoming a participant in a certain discourse (Sfard, 2008; Heyd-Metzuyanim, 2019). Previously, this theoretical framework was mostly applied to student learning. More recently, studies have applied this framework to teacher learning (e.g. Heyd-Metzuyanim et al., 2019). Like these studies, we conceptualise the learning that takes place as changes in the teachers' pedagogical discourse. Heyd-Metzuyanim et al. find that DPD values actions that assume a delivery model of teaching in which the teacher "delivers" knowledge that students "acquire", mostly by listening to explanations and repeating procedures demonstrated by the teacher. The EPD values student exploration and mathematical discussions, which involves students' constructing their own knowledge. This means that they engage with meaningful problems and develop understanding. The DPD and EPD value different teaching actions and learning outcomes, and differentiating between them can help us understand changes in the pedagogical discourse of teachers who undergo PD for ambitious teaching. In the present study, pedagogical discourse refers to *"what* is to be taught and learned, *how* this should be done and *who* can (or cannot) learn" (Heyd-Metzuyanim, 2019, p. 3).

In our study, the overarching aim of the PD was for the teachers to learn the principles and practices of ambitious mathematics teaching and develop identities as teachers who care about student thinking (Wæge & Fauskanger, 2021). The aim of ambitious mathematics teaching is to develop students' conceptual understanding, procedural knowledge, adaptive reasoning and engagement in mathematics (Lampert et al., 2010). Ambitious teaching involves creating classroom communities where the students are seen as sense-makers, are given access to key mathematical ideas, and are positioned as capable of engaging with each other's ideas (Kazemi et al., 2016). We organised the PD according to the development of the following set of ambitious teaching practices: launching problems, eliciting and responding to students' ideas, using representations, facilitating student talk and aiming towards a mathematical goal (Wæge & Fauskanger, 2021, 2022). Ambitious teaching also involves careful planning (Lampert et al., 2013), and the PD included work on five practices (5Ps) for orchestrating mathematical discussions

listed by Stein et al. (2008): anticipating, monitoring, selecting, sequencing and connecting. Within the present study, we conceptualise both the principles and practices of ambitious teaching and the 5Ps as specific teaching actions valued within the EPD. The following research question is addressed:

> How do the teachers' pedagogical discourse change as they participate in a community of learners focused on ambitious mathematics teaching?

Our work provides insight into changes in teachers' pedagogical discourse as they participate in a PD project with the aim of learning ambitious mathematics teaching.

Methods

To answer the research question we use a qualitative research design to explore teachers' pedagogical discourse. The discourse-analysis approach taken in this study enables us to explore teachers' pedagogical discourse through the way they frame what it means to do, learn, and teach mathematics (Heyd-Metzuyanim, 2019). We use data from a Norwegian PD project, *Mastering ambitious mathematics teaching* (MAM), where teachers collaborate in cycles of enactment and investigation (Lampert et al., 2013).

Participants

All public elementary schools in one Norwegian urban municipality were invited to apply for MAM. Ten schools applied and all were approved. Three generalist teachers were selected at each school to serve as mentors for their colleagues. The 30 teachers were divided into four groups. One group did not want to be part of the research study. We randomly picked two out of the three remaining groups, thus the teachers in our study are 14 elementary school teachers (years 1–7). Their teaching experience varied: five had 1–5 years of experience, three had 6–10 years' experience and six had more than 10 years' experience. Our informal conversations with the teachers and school leaders indicated that none of the teachers had extensive experience of ambitious teaching before the project.

Setting

The MAM project consisted of 12 sessions held over a two-year period. To help the teachers learn ambitious teaching, we gave them repeated opportunities to work on a set of intentionally selected instructional activities

embedded in a cycle of enactment and investigation. The project began with a session where the teachers were introduced to the principles and practices of ambitious mathematics teaching and the instructional activities they would be learning (Wæge & Fauskanger, 2021). The work in nine of the sessions was organised according to a cycle of enactment and investigation with six phases: 1) preparation; 2) collective analysis; 3) co-planning; 4) rehearsals; 5) classroom co-enactments; and 6) collective analysis (see Wæge & Fauskanger, 2021). In the two remaining sessions, the participants worked on the challenges of ambitious teaching and on plans for implementation in their own schools. The mathematical content of the project focused on number sense.

Data collection

We draw on a subset of our data from the MAM project. Semi-structured group interviews (Kvale, 1996) were conducted with each of the two groups of teachers when the project started and at the end of the project, giving a total of four video-recorded group interviews. The first interview, conducted with each group after the second session, lasted about 70 and 65 minutes, respectively. The second interview, conducted after the final (12th) session, lasted about 80 and 75 minutes, respectively . In both interviews, we asked questions relating to two themes: the "good" lesson and a "normal" mathematics lesson. We asked questions such as: "What characterises a good lesson?", "What is the teacher's role?", "What characterises the students' activity?", "Could you describe a normal lesson?", "What do you emphasise in your teaching?". We also asked the teachers to describe the challenges they face and how they support students who face difficulties in mathematics.

The analysis

The video-recorded interviews were transcribed and analysed. The first interviews were analysed by using a constant comparative method (Corbin & Strauss, 2008), with open coding and the writing of memos to help us look for patterns in the teachers' pedagogical discourse. After completing this, we analytically reviewed the data many times, compiling data segments according to their codes, writing analytical memos and reviewing the entire corpus of data to capture the teachers' pedagogical discourse. As we worked through the data, we identified eight categories: student learning and engagement; differentiating; structure; mathematical content; activities; use of resources; the role of the teacher; and mathematical discussions. We then started the analysis of the second interview

by coding these eight a priori categories. No additional codes emerged. Finally, we analysed the first and second interviews in relation to each other to identify similarities and differences in the teachers' discourse.

We arranged data segments according to the codes and reviewed the entire data set qualitatively. Tables that helped us to look for common features across the two interviews were constructed. Throughout the process we wrote analytical memos and reviews concerning similarities and differences in the data (Corbin and Strauss 2008). As we worked through it we found that the initial eight codes were too unwieldy to code for changes between the first and second interview. We then looked across the analytical memos from all the interviews and found it possible to combine some of the categories, ending up with five main categories: role of the teacher; mathematical discussions; student capabilities; activities and resources; and structure and mathematical content. Note that the five categories are closely connected and partly overlap. We selected examples from the data that illustrate the conceptions that were voiced in the teachers' discussions and that were representative of their category.

Findings

In this article we focus on three of the categories: the role of the teacher; mathematical discussions; and student capabilities. These categories were the most prominent in the interviews, and were also where we found the most significant changes. Moreover, they represent dimensions that are highlighted by other studies on teachers' views of ambitious teaching (e.g. Jackson et al., 2017; Munter, 2014). We present and elaborate on each of these categories and the changes we identified in the teachers' pedagogical discourse for each category.

Role of the teacher

First interview

In the first interview, the teachers stated that students' thinking was important and talked about this in relation to planning the next lesson. They also said they could use this productively to support the students' understanding:

> But it isn't the teacher standing and lecturing, you know. You put an activity out there, and then you play it out using the thoughts and understandings of the students ... It requires quite a lot of knowledge from us teachers so we know where the students actually are, and "how can I help each student to progress"?

The teachers also claimed that knowing what the students are thinking is necessary to support their learning:

> And so I think that as a teacher I depend on knowing what the students are thinking to help them in some way to progress and be able to plan what to do next.

The teachers described the teacher as a facilitator, saying that the teacher should spend less time talking and provide more time for student work:

> I really feel that for my own sake, because I have stepped back from the teacher role, I've become more of a facilitator. Instead of my standing there by the board and talking fifty per cent of the time I have cut down on my time and rather let the students have their say.

One of the teachers said that it is challenging for her when some of her students fail to understand the way she explains things, no matter how "simple" the explanation is:

> Really, they don't have any strategy. No matter how simply I present it, they're not, they don't connect. I think this is difficult.

As the extracts above illustrate, the teachers found that using students' thinking was important in their teaching, which is highly valued within ambitious teaching and the EPD. They talked about the importance of students' thinking in relation to lesson planning and supporting student learning, but nowhere in the interview did they explicate how the teacher could do this or how it promotes student learning. While teachers positioned the teacher as a facilitator, which may seem to be aligned with the EPD, they did not elaborate on what they meant by this. Although the teachers' pedagogical discourse tended to align with the EPD, it was very general and vague. As the last extract illustrates, we also identified instances where they talked about teacher actions that are valued in the DPD, such as explaining ideas or procedures.

Second interview

In the second interview the teachers also talked about the importance of using students' thinking, describing how they used the students' contributions to highlight key mathematical ideas. Moreover, they talked about the importance of formulating learning goals, how they orient students to each other's thinking and orchestrate mathematical discussions:

> ... I'm very focused on this bit about how to create student activity and talk among the students and with the students. And I also focus on the types of questions we ask, and then I think the talk moves are really important. And then I have or try to have a very clear goal

> for what I'm teaching, I have to think about where I'm trying to take them. And then I believe that I'm now much more focused on making them explore and share ideas with each other and achieve a good summary of the mathematical ideas I want them to work with.

The teachers emphasised the importance of planning the lesson, and especially the practice of anticipating student responses. They stated that if they anticipated students' responses, they could use the students' thinking to promote a deeper understanding of mathematical ideas:

> But if you're going to be able to guide them in the process, then you have to go through the task yourself first and consider possible strategies. If it's just a task you pick and hand out, then there's no guarantee you'll accomplish anything.
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> [...]
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> I believe that my role, obviously, is to know where we're going and to know what comes next. And then what I can expect that they're going to say ... And if I hadn't been clear about all this, if I hadn't known anything about what could be answered to this, what are possible incorrect answers, how we can take this further, then I would not have been a good teacher. Then I couldn't have managed this role here, so that's an important part of it.

The teachers mentioned that the practice of being responsive to student thinking and using their emergent ideas to achieve the lesson goal might be challenging for the teacher, but that it is crucial for students' learning:

> ... You have to know how students think, which strategies they're expected to find, what are common ways of thinking in the age group you're teaching, how do you ask the good questions to help them progress without herding them all the same way. That may be what I notice as the greatest challenge.

The teachers highlighted the importance of attending to students' thinking, which aligns with the EPD. Unlike the first interview, the teachers provided thorough descriptions of the teacher's role and of key practices of ambitious teaching, such as orchestrating discussions and orienting the students towards the lesson goal. They suggested that the teacher should formulate clear learning goals and select problems that supported these goals. The teachers also pointed out the importance of planning the lesson and of anticipating student responses, and they described how the teacher could use the anticipated strategies to support students' understanding of key mathematical ideas. These practices are highly valued within the EPD.

Mathematical discussions

First interview

In the first interview, the teachers talked about mathematical discussions, mentioning that they listen to the students' mathematical talk and argumentation when they walk around the classroom. They also stated that mathematical talk can help them to better elicit and understand students' thinking:

> And then, at the teacher station the quality of the talk with the students becomes very good, then it works very well. There are three or four in a group, and then it's easy to see what the students are thinking, how they think, and that works very well.

The teachers mentioned that sometimes it is difficult to find good questions to elicit students' thinking and initiate discussions. This is related to the role of the teacher:

> I think it's a bit challenging to ask the right questions every time, ... sometimes it goes very well, and you ask a good question so that lots of good thinking emerges, but other times you can ask a question and then, [pauses] nothing comes, and then you ask yourself why don't I get anything here.

As the first interview took place after the second session, the teachers also mentioned aspects of mathematical discussions that they had worked on in the two previous sessions. They mentioned their experiences of trying out a talk move called repeating, that is, asking a student to repeat what someone said, and that it worked well:

> There's one thing I think has been a lot of fun, and that's making them explain each other's strategies. So they sit there afterwards and "Okay, that's another way to think about it". And also that they have to, they need to explain the way others think. That's when it clicked for me, because the way it was, I noticed that I didn't do that so much, I would simply say the usual "explain what you were thinking here", but perhaps not "explain how he was thinking", and I have used this a lot now, and it works very well.

As the extracts illustrate, the teachers stated that it is important that the students talk about and discuss mathematics, which is highly valued within the EPD. They appreciated talk moves (Chapin et al., 2009) for orienting the students to each other's ideas, a practice that is highly valued in EPD. They also talked about the practice of questioning, and that eliciting student thinking is challenging. These teacher actions seemed to be

aligned with EPD. However, the analyses show that the teachers' descriptions focused more on teacher decision-making than on student learning. They did not include thorough descriptions of mathematical discussions or how they might support students' learning. Thus, the underlying purposes of mathematical discussions were not an explicit part of the pedagogical discourse.

Second interview

In the second interview the teachers mentioned that there is a lot of mathematical talk in their classrooms, and that students' explaining and arguing are integral parts of the learning process – and that this characterises a good lesson:

> It's [a good class] when you hear the kids discuss mathematics. It doesn't matter what you've presented and what you're working with, you hear: "Yes, no, I believe so", and then they're sitting and arguing with each other to conclude their process. I experience this quite a lot now ...

They described how the students' focus has changed, from a focus on the answer to a focus on strategies and why the strategies work. They mentioned that the students always come up with multiple strategies for a problem:

> Earlier there have been many questions about the answer. And now they [the students] dare, they don't think about the answer at all, they really don't. I can give them a task, and then ask "how do you solve this?", and then they suggest a lot of different solution strategies. Nobody gives an answer.

The teachers emphasised the importance of using talk moves in the mathematical discussions, describing how using them supported the students' participation in discussions, and in taking risks and understanding that making mistakes and revising their thinking is a natural part of the learning process:

> And then the talk moves are important, where everybody dares to speak and dares to make mistakes, and dares to say things, and state arguments for the answer. What could be right here, I wonder? So they dare to ...

The teachers mentioned that if they orchestrate the discussions, orient students to each other's ideas and connect different student strategies, they can support students' learning and understanding:

> Then you [the teacher] sum up at the end, and you ask the student, "How have you done it?" "Well, I did it this way and used a column diagram." And then the next student, "Well, no, we actually used a sector diagram, and I think we displayed it in a totally different way." Some have used the number line, and then you get a discussion about that, and then you [the teacher] start the next round. Then the student might start to think, "That sector diagram was really good." And then he starts to use it like that.

The teachers pointed out that they have an important role in the discussions by representing students' mathematical ideas in writing and connecting different kinds of representations and highlighting key mathematical ideas: "Then you also need to summarise it and help them translate representations and all that too".

The extracts illustrate how the teachers pointed out the importance of mathematical discussions, which aligns with the EPD. Unlike the first interview, the teachers gave thorough descriptions of what is involved in discussions and their functions in terms of supporting students' learning. The teachers stated that students' mathematical talk, argumentation and explanations were an integral part of the mathematics classroom. These student actions are highly valued in the EPD. They explained how the students' focus was on the process and on finding multiple strategies, and not on the answer, which is another aspect of EPD. The teachers mentioned how talk moves supported the students' participation in discussions and their learning and understanding of mathematics. Moreover, they included detailed descriptions of the important role the teacher had in orchestrating the discussions and in supporting students' learning, for example by representing their strategies. As some of the extracts in the previous section illustrate, the teachers emphasised the importance of planning the discussions and anticipating possible student strategies, which are highly valued within the EPD.

Student mathematical capabilities

First interview

In the first interview, the teachers talked a great deal about the pronounced gap between students in mathematics. They labelled the students, where some were described as "talented" or "good", and others were described as "not good" or "weak". They experienced that it is difficult to satisfy the needs of all the students, but a good lesson is when they succeed in doing this:

> I think the time in the classes goes fast, and there's so much you need to cover in mathematics, and there's the skills gap. It's much

> more pronounced compared to many other subjects, there are major differences between the strong and the weakest students, so having good classes when you're all together all the time, that can be very difficult ...
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> [...]
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> And, mathematics, that's a subject where there are large gaps between the best and the weakest. The classes that are very good are when you see those who often drop out, often not managing very much, when you see that they take part and participate in the class, then I at least feel that this is a good class. When you see that both the best and the weakest are active in class.

The teachers did not say much about how they can help all the students to develop, and they talked about how the "most talented" students do not get enough attention. The teachers plan their teaching to fit most of the students, and also consider "the very weakest":

> I think it's difficult, very difficult for the very smart students. They might get less [attention]. I honestly think I can say that. Because if you take care of the very weakest and then cover the majority, then those at the top get a bit lonely.

Most teachers stated that it was demanding to have a large spread in the student groups. One of the teachers stated, however, that the spread among the students could be a resource.

> But then I think that it's also really good to have a large spread in a student group because when the students give their input, then it helps to get some students to see that "Oh, I didn't think of that".

As the extracts illustrate, the teachers labelled the students. They called them "strong" and "weak", and using terms like that may indicate that the teachers see these as inherent qualities in the students, which aligns with the DPD (Nachlieli & Heyd-Metzuyanim, 2022). Much was said about the large gaps between the students' abilities. However, little was said about how the teacher can support all the students' learning. The teachers experienced that they plan for the majority of the students, giving special consideration to the "weakest", but they found it difficult to have time to follow up the "most talented" students. There was an expectation that these students had to do something else than the rest of the students. Such teacher actions seem to belong to the DPD. However, one of the teachers pointed out that this gap may also be positive, which aligns more with the values of EPD.

Second interview

In the second interview the teachers did not talk about the large gap between the students at all. The labelling of students was gone. The teachers stated that all students can contribute and manage something, which aligns with ambitious teaching. They also pointed out that the students should develop from their level, giving examples of strategies for achieving this goal, such as tasks, talks and teacher input, and that this is important for creating mastering and motivation.

> They have something, everybody has something they can contribute. Everyone can manage something.
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> [...]
>
> I agree completely with everything that's been said, but I believe that it's very important that the kids are allowed to develop, on their level, through the tasks and through varied teaching, through talks, through input from the teacher. In the teaching, they should be able to develop on their level where they feel they master something and are motivated and engaged in the mathematics subject.

The teachers expressed greater awareness of how to deal with the students. They acknowledged incorrect answers as part of the learning process and were aware of how to respond to different student answers.

> And then I think it's our intention to make them good, to help the kids feel that they're mastering, that they are good at what they're doing, instead of no, that's wrong. This is new, they feel that, okay, I can do this, I can manage this. I think this is important.
>
> I'm thinking a lot about how to address the things they say. At least I feel I'm thinking much more about it. How to respond to things they say. We can't just shut them down, or dismiss their solutions even if they're incorrect, rather address them. And also what to address to progress.

The analysis indicates development in how the teachers talk about the students from the first to the second interview. We see that the student labels are gone. Now the teachers pointed out that everyone can learn mathematics, and that they expected all the students to contribute something, views that are highly valued within the EPD. The focus has been shifted from considering the qualities of the students as the cause of the problems, to how the teacher can support the learning of all students, which aligns with the EPD. The teachers explained that they need to

accept the students where and as they are, and that the choice of tasks, the mathematical talk and the teacher's leadership of the learning activities are vital elements to succeed in this work. The expressed teacher actions are highly valued in the EPD. Incorrect answers were seen as important both for acknowledging the input of various students and for using them to develop the students' understanding – another aspect of the EPD.

Discussion

In this article, we have examined changes in the teachers' pedagogical discourse while participating in a community of learners focused on ambitious mathematics teaching. Our findings show that there were significant changes in the teachers' pedagogical discourse between the interviews. The first interview included elements from both the *Delivery pedagogical discourse* (DPD) and *Exploratory pedagogical discourse* (EPD) as described by Heyd-Metzuyanim & Shabtay (2019), whereas the second interview was mostly aligned with the EPD, which is the discourse aligned with ambitios mathematical teaching. Most of the valued EPD actions that the teachers talked about in the first interview were related to teacher actions. There were few references to student learning, the teachers' talk was general and vague, and they described themselves as aiming at building on student thinking and facilitating mathematical discussions. Yet, they failed to come up with any specific examples of such actions. Neither did they talk about the purposes or functions of the EPD aligned teacher actions. The findings show that the teachers' discourse did not include specificities of EPD in the beginning of the project. In the second interview, the teachers' pedagogical discourse drew more consistently on the EPD and was more detailed and specific. The changes in the discourse entailed using new words, valuing new ambitious teaching practices, talking about more specific aspects of ambitious teaching, providing rationales for teacher actions, and making specific references to student learning. The findings indicate that the specificities of EPD were more accessible to the teachers, and the teachers' discourse was much more detailed. A study by Heyd-Metzuyanim and Shabtay (2019) revealed similar results. They showed how a teacher's discourse became more detailed and how his interpretations of teacher actions changed as he became more familiar with the specificities of the EPD. Our findings are also supported by research on teachers' noticing, and more specifically, on teachers' learning to notice specific aspects of reform teaching (e.g. Sherin & van Es, 2009). These studies show that the teachers started by noticing elements of teacher actions and moved to talk more about specific aspects of student learning.

Previous research (e.g. Heyd-Metzuyanim et al., 2018; 2022) has shown that one of the challenges of drawing teachers into the EPD is that some aspects of the discourse are easily adopted by the teachers, while others remain hidden, and this might result in misalignments between the teachers' narratives. In the first interview, we identified misalignments between the teachers' narratives. On the one hand, they highlighted the importance of student thinking and discussion, which are valued within the EPD. On the other hand, the discourse differed from the EPD in relation to who can or cannot learn. They talked a great deal about the problem of differential success, using such labels as "weak" and "strong" students, and stated that it was challenging to support all the students in the class, which aligned with the DPD (Nachlieli & Heyd-Metzuyanim, 2022). This DPD-aligned discourse indicates a "fixed mindset" (Dweck, 2006). According to Horn (2007) and Jackson et al. (2017), a key aspect of enacting ambitious teaching involves framing the problem of differential success as a problem of instruction. Our findings show that there was a shift in the teachers' pedagogical discourse around struggling students between the two interviews. In the second interview, labels were not used, and the teachers talked about the problem of differential success in terms of learning opportunities provided by the teacher. They emphasised that all students can learn mathematics, and focused on how they could build on students' thinking to support everyone's understanding, a highly valued part of the EPD. Thus, our study indicates that the *Mastering ambitious mathematics teaching* project (MAM) supported the teachers in framing the problem of differential success in terms of learning opportunities provided by the teacher, which is a key principle of ambitious teaching (e.g. Horn, 2007; Lampert et al., 2013).

Our study provides insight into changes in teachers' discourse while participating in a community of learners focused on teaching ambitious mathematics. The analysis is an operationalization, showing what changes in teachers' pedagogical discourse about ambitious mathematics teaching might look like. Teacher educators can use these insights to identify and support changes in teachers' pedagogical discourse towards an EPD that aligns with the principles of ambitious teaching. While this study contributes knowledge on changes in teachers' discourse on mathematics teaching, more research is needed. One limitation of this study is that it only included volunteering teachers, and that may have affected the results. Another limitation is that it did not address the relationships between the teachers' discourse and their teaching practices (Heyd-Metzuyanim et al., 2019). Studying how the teachers' participation in MAM might lead to changes in their classroom practice should be a focus of future research.

References

Bannister, N. (2015). Reframing practice: teacher learning through interactions in a collaborative group. *Journal of the Learning Sciences*, 24 (3), 347–372. doi: 10.1080/10508406.2014.999196

Borko, H. (2004). Professional development and teacher learning: mapping the terrain. *Educational Researcher*, 33 (8), 3–15.

Carpenter, T. P., Fennema, E. & Franke, M. L. (1996). Cognitively guided instruction: a knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, 97 (1), 3–20.

Chapin, S. H., O'Connor, C. & Anderson, N. C. (2009). C*lassroom discussions: using math talk to help students learn, grades K-6* (2nd ed.). Math Solutions.

Charalambous, C. Y. & Delaney, S. (2020). Mathematics teaching practices and practice-based pedagogies: a critical review of the literature since 2000. In D. Potari & O. Chapman (Eds.), *International handbook of mathematics teacher education* (Vol. 1, pp. 355–390). Brill.

Corbin, J. & Strauss, A. (2008). Q*ualitative research*. Sage Publications.

Dweck, C. S. (2006). *Mindset. The new psychology of success*. Ballantine Books.

Ghousseini, H., Beasley, H. & Lord, S. (2015). Investigating the potential of guided practice with an enactment tool for supporting adaptive performance. *Journal of the Learning Sciences*, 24 (3), 461–497. doi: 10.1080/10508406.2015.1057339

Hemmi, K. & Ryve, A. (2015). Effective mathematics teaching in Finnish and Swedish teacher education discourses. *Journal of Mathematics Teacher Education*, 18 (6), 501–521.

Heyd-Metzuyanim, E. (2019). Changing teaching practices towards explorative mathematics instruction – the interweaving of teacher identity and pedagogical discourse. *Teaching and Teacher Education*, 86, 102862.

Heyd-Metzuyanim, E., Munter, C. & Greeno, J. (2018). Conflicting frames: a case of misalignment between professional development efforts and a teacher's practice in a high school mathematics classroom. *Educational Studies in Mathematics*, 97 (1), 21–37.

Heyd-Metzuyanim, E. & Shabtay, G. (2019). Narratives of "good" instruction: teachers' identities as drawing on exploration vs. acquisition pedagogical discourses. *ZDM*, 51 (3), 541–554.

Heyd-Metzuyanim, E., Smith, M., Bill, V. & Resnick, L. B. (2019). From ritual to explorative participation in discourse-rich instructional practices: a case study of teacher learning through professional development. *Educational Studies in Mathematics*, 101 (2), 273–289.

Horn, I. S. (2007). Fast kids, slow kids, lazy kids: framing the mismatch problem in mathematics teachers' conversations. *The Journal of the Learning Sciences*, 16 (1), 37–79. doi: 10.1080/10508400709336942

Jackson, K., Lynsey, G. & Sharpe, C. H. (2017). Teachers' views of students' mathematical capabilities: challenges and possibilities for ambitious reform. *Teachers College Record*, 119 (7), 1–43. doi: 10.1177%2F016146811711900708

Jacobs, V. R. & Spangler, D. A. (2017). Research on core practices in K–12 mathematics teaching. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 766–792). NCTM.

Kazemi, E., Ghousseini, H., Cunard, A. & Turrou, A. C. (2016). Getting inside rehearsals: insights from teacher educators to support work on complex practice. *Journal of Teacher Education*, 67 (1), 18–31. doi: 10.1177/0022487115615191

Kazemi, E. & Wæge, K. (2015). Learning to teach within practice-based methods courses. *Mathematics Teacher Education and Development Journal*, 17 (2), 125–145.

Kennedy, M. (2016). How does professional development improve teaching? *Review of Educational Research*, 86 (4), 945–980.

Kvale, S. (1996). *Interviews: an introduction to qualitative research interviewing*. Sage Publications.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E. & Franke, M. L. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M. K. Stein & L. Kucan (Eds.), *Instructional explanations in the disciplines* (pp. 129–141). Springer.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., et al. (2013). Keeping it complex: using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, 64 (3), 226–243. doi: 10.1177/0022487112473837

Lave, J. (1991). Situating learning in communities of practice. In L. Resnick, J. Levine & S. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 63–82). APA.

Lee, M. Y. & Francis, D. C. (2018). Investigating the relationships among elementary teachers' perceptions of the use of students' thinking, their professional noticing skills, and their teaching practices. *The Journal of Mathematical Behavior*, 51, 118–128. doi: 10.1016/j.jmathb.2017.11.007

Leinhardt, G. & Steele, M. D. (2010). Seeing the complexity of standing to the side: instructional dialogues. *Cognition and Instruction*, 23 (1), 87–163. doi: 10.1207/s1532690xci2301_4

McDonald, M., Kazemi, E. & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education: a call for a common language and collective activity. *Journal of Teacher Education*, 64 (5), 378–386.

Munter, C. (2014). Developing visions of high-quality mathematics instruction. *Journal for Research in Mathematics Education*, 45 (5), 584–635. doi: 10.5951/jresematheduc.45.5.0584

Richards, J. & Robertson, A. D. (2015). A review of the research on responsive teaching in science and mathematics. In A. D. Robertson, R. E. Sherr & D. Hammer (Eds.), *Responsive teaching in science and mathematics* (pp. 36–55). Routledge.

Stein, M. K., Engle, R. A., Smith, M. S. & Hughes E. K. (2008). Orchestrating productive mathematics discussions: five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, 10 (4), 313–340. doi: 10.1080/10986060802229675

Stockero, S. L., Leatham, K. R., Ochieng, M. A., Van Zoest, L. R. & Peterson, B. E. (2020). Teachers' orientations toward using student mathematical thinking as a resource during whole-class discussion. *Journal of Mathematics Teacher Education*, 23, 237–267. doi: 10.1007/s10857-018-09421-0

Wenger, E. (1998). *Communities of practice. Learning, meaning, and identity.* Cambridge University Press.

Wæge, K. (2007). *Elevenes motivasjon for å lære matematikk og undersøkende matematikkundervisning* [Students' motivation for learning mathematics and inquiry based mathematics teaching] [doctoral thesis]. NTNU, Norwegian University of Science and Technology.

Wæge, K. & Fauskanger, J. (2022). In-service teachers' opportunities to learn ambitious mathematics teaching in rehearsals and co-enactments. *Scandinavian Journal of Educational Research*. Advance online publication.

Wæge, K. & Fauskanger, J. (2021). Teacher time out in rehearsals: in-service teachers learning ambitious mathematics teaching practices. *Journal of Mathematics Teacher Education*, 24, 563–586. doi: 10.1007/s10857-020-09474-0

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