Mathematical modelling in textbook tasks and national examination in Norwegian upper secondary school

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Modelling competence is defined as the ability to carry through all steps of a mathematical modelling process, to solve a non-mathematical problem by mathematics. In this study, Norwegian textbook modelling tasks and tasks from the national exam are analyzed through the lens of a modelling cycle. The findings discussed as the *enacted* curriculum (textbook tasks) and *assessed* curriculum (exam tasks) are seen in relation to the *intended* curriculum. The results show different starting points of the modelling process in the intended curriculum and the tasks from textbooks and exam. The findings indicate different perspectives on mathematical modelling in the curriculum (modelling for developing modelling competence) and the textbook tasks and the national exam, where only parts of the modelling process are included.

The teaching and learning of mathematical modelling is an important research field all over the world (Schukajlow et al., 2018), but the presence of modelling activities in day-to-day teaching is still limited at many places (Frejd, 2012). Mathematical modelling is rather vaguely defined as a curriculum concept and comprises many different practices (Jablonka & Gellert, 2010; Kaiser & Sriraman, 2006). Brown and Stillman (2017) argue that mathematical modelling seems to be introduced diffusely so that the students eventually see modelling as a limited part of mathematics. According to Bracke and Geiger (2011), mathematical modelling should be integrated as long term experiences throughout the whole mathematics education. There are different perspectives on mathematical modelling concerning the aim of the activity, which will be exposed in the theory section.

In this study modelling tasks in textbooks and national exams in a Norwegian context are compared and seen in relation to the curriculum.

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Berget, I. K. L. (2022). Mathematical modelling in textbook tasks and national examination in Norwegian upper secondary school. *Nordic Studies in Mathematics Education*, 27 (1), 51–70.

A modelling cycle is used to analyze tasks. The modelling tasks from the textbooks give insight into the classroom activity concerning mathematical modelling. The tasks from the national examination give an idea of what students are supposed to learn about mathematical modelling. The coherence between curriculum, classroom activities and what is assessed about mathematical modelling, will be discussed as the *intended* curriculum, *enacted* curriculum and *assessed* curriculum as used in Porter (2006). The contribution to the literature through this study is the use of a modelling cycle to analyze modelling tasks and insight into the tasks given in the teaching and assessing of mathematical modelling in Norwegian upper secondary schools.

Theory

Different perspectives on mathematical modelling in education have been expressed, and there have been various attempts of classifying them (see e.g. Galbraith, 2012; Kaiser & Sriraman, 2006; Stillman et al., 2016). Blomhøj and Ärlebäck (2018) divide modelling as a means for 1) learning mathematics and 2) developing modelling competence. This can be seen in relation to Julie's (2002) perspectives *modelling-as-vehicle* when the aim of the modelling activity is to learn specific mathematical issues, and *modelling-as-content*, when the aim is the modelling in itself. A third perspective, *modelling-as-critic*, is when the aim is to critically assess given models (Barbosa, 2006), which is included in the modelling competence (Niss & Jensen, 2002). The two perspectives correspond to Niss and Blum's (2020) two overarching reasons for including mathematical modelling in mathematics teaching and learning, 1) modelling for the sake of mathematics and 2) mathematics for the sake of modelling. The two reasons are not contradictory, but they "give rise to different consequences in terms of priorities and activities" (Niss & Blum, 2020, p. 28). As Niss and colleagues (2007) expressed, in any application of mathematics a mathematical model is involved. "[A] mathematical model is a deliberately simplified and formalized image of some part of the real world" (Blum, 2015, p. 77). Instead of the term "real world", "extra-mathematical world" is sometimes used, or "the rest of the world" as formulated by Pollak (1979). "The rest of the world" includes nature, culture, society or everyday life. Among the purposes of models are not only describing and explaining (descriptive models) but also predicting and even creating parts of the real world (normative models) (Blum, 2015). Stillman (2015, p. 792) expresses *applications of mathematics* to have "the direction (mathematics → reality) in focus. In *mathematical modelling* the reverse direction (reality → mathematics) is the focus". Several modelling cycles have been developed to show the modelling process (see e.g. Borromeo Ferri, 2018; Blum, 2015). In many cycles, real life is separated from mathematics to emphasise modelling as a connection of mathematics to the rest of the world. I will now present the modelling cycle by Blum and Leiß (2007), which is "widely accepted in the modelling debate" (Hankeln et al., 2019, p. 144). This cycle is from a cognitive perspective and takes the students' perspective while solving a modelling task (Borromeo Ferri, 2018). It is about analyzing and understanding the cognitive procedures that take place in modelling problems (Greefrath & Vorhölter, 2016). Therefore, I find this cycle relevant for this study, which focuses on students' solving processes of modelling tasks. The cycle is modified placing mathematics as a part of life, instead of a separated part from "the rest of the world", to clearer communicate mathematics as a human activity. The steps are the same as in the cycle from Blum and Leiβ (2007).

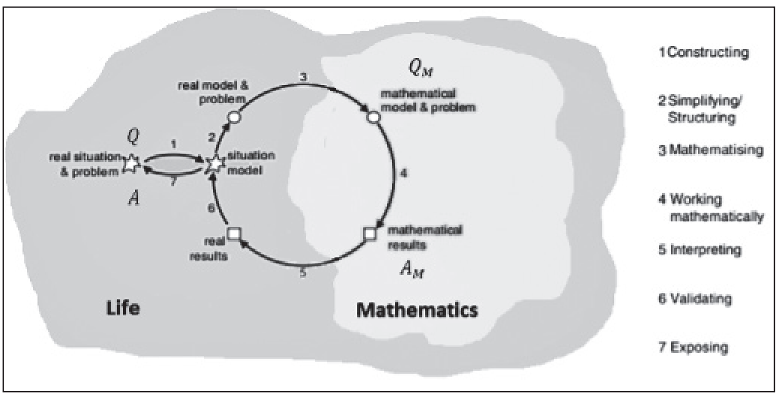

Figure 1. *Modelling cycle, modified from Blum and Leiß (2007)*

The seven steps in the cycle are shown to the right in figure 1. It is pointed out that students don't follow the modelling cycle step by step when solving modelling tasks, but are going back and forth between the steps (Borromeo Ferri, 2010). Each step will now be briefly explained, based on Czocher (2016) and Blum and Ferri (2009). 1) *Constructing* is to form an idea about what the problem is asking for. 2) *Simplifying/Structuring* is to identify critical components of the problem situation, and assumptions must be made. 3) *Mathematising* is to transform the "real model & problem" into a "mathematical model & problem", to express it in mathematical terms. 4) *Working mathematically* is to do calculations, solve equations, curve fitting, graphing and so on. 5) *Interpreting* is to re-contextualize the mathematical result. 6) *Validating* is verifying results against the situation model. 7) *Exposing* is to explain the assumptions that were made, and how the model and the answer to the original problem consider this.

Modelling as a means for learning mathematics

Within *Realistic mathematics education* (RME), models are seen as representations to visualize problem situations (van den Heuvel-Panhuizen, 2003), and mathematical activity is seen as mathematising in both a horizontal and a vertical direction. The horizontal mathematising leads, according to Freudenthal (1991, p. 41) "from real life to the world of symbols", and in the vertical mathematising you "stay in the world of symbols". In RME models are used as a tool to understand mathematics and is therefore classified as *modelling-as-vehicle* (Galbraith, 2012) or modelling as a means for learning mathematics. Here, the direction of the process could be from one mathematical content to another, for example from algebra to geometry, and not necessarily from real life to mathematics.

Another perspective within *modelling as a means for learning mathematics* is to apply specific mathematical content in a given practical context. Modelling as curve fitting is for example classified by Galbraith (2012) as *modelling-as-vehicle*. This is criticized by Borromeo Ferri (2018). She claims that when all the needed data is given in a context, together with the method to solve it, the task can be recognized as a pseudo-realistic problem, and not modelling. Højgaard (2009) proposes that invitations to mathematization too often is replaced by pseudo-realistic oriented tasks, which do not challenge students. In real life, the assumptions are not already made, and simplification must be done to solve problems. But if the aim is to learn mathematics, and not the use of mathematics in daily life, this can still be seen as mathematical modelling. Hankeln (2020) compares students' modelling processes in Germany and France, and points out that the French community of mathematics didactics is grounded on the fundamental idea that, according to Trouche (2016, p. 242), "each teaching and learning analysis starts from the mathematical content of what is to be learnt". This idea supports the understanding of modelling where the mathematical content is in focus, and modelling is seen as a means for learning this specific mathematical content. In German traditions, "mathematical content is more of a tool for solving an authentic problem than the ultimate aim of an exercise" (Hankeln, 2020, p. 214). This leads us to the other perspective of modelling.

Modelling as means for developing modelling competence

Within the perspective *modelling-as-content*, where the aim is to use mathematics to deal with the world around us, the development of modelling competence is essential. Blum and Ferri (2009, p. 47) define modelling competence as "the ability to construct models by carrying out those various steps appropriately as well as to analyze or compare given models". The perspective *modelling-as-critic* is also included in the modelling competence, to compare given models. Blomhøj and Højgaard Jensen (2003) divides modelling competency into several sub-competencies, such as the steps in the modelling cycle in figure 1. They emphasize the importance of both working with full-scale mathematical modelling process (holistic tasks), where all steps of the modelling cycle are needed, and focusing on parts of the modelling process in different tasks (atomistic tasks). If the aim is for the student to evolve modelling competence to be used in everyday life, experience with holistic tasks is preferable (Hankeln et al., 2019). The sub-competence *mathematising* (step 3 in the modelling cycle), is often experienced as cognitively demanding and frustrating for students and should be given priority (Blomhøj & Højgaard Jensen, 2003). Blum (2015) points out three different steps of the modelling cycle to be challenging. That is step 1, 2 and 6; to *understand* the problem, to make assumptions to *simplify*, and to *validate*. One can therefore argue that these sub-competencies should be in focus in atomistic tasks given to students.

Frejd (2011) studied mathematical modelling in the Swedish national course tests. He found that only fragments of the modelling process were tested in the national course tests. The connection to real-life situations was lacking in most of the tasks, and a given correct answer was often the only thing assessed. The mathematical models were already given in the tasks, and there was no need to *mathematise* to solve the tasks. Hankeln et al. (2019) developed a test instrument to assess student's sub-competencies of mathematical modelling. They argue that in a test situation it is expedient to atomistic tasks that only tests one or a few of the steps in the modelling cycle. If the students get stuck in the step of simplifying in a holistic task in a test situation, they might not reach the point of interpreting a mathematic result. Then they are not able to show other parts of their modelling competence. In the PISA framework it is pointed out that especially in the context of an assessment it is not necessary to engage in every stage of the modelling cycle. "Significant parts of the mathematical modelling cycle have been undertaken by others, and the end-user carries out some of the steps" (OECD, 2018, p. 11). Urhan and Dost (2018) analyzed a coursebook made for the new curriculum in Turkey, based on model-eliciting principles developed by Lesh and colleagues (2003). The least frequently found criteria was the self-evaluation principle, whether students must assess their model and result. In light of the seven-step modelling cycle, this corresponds to step 6, validating.

Reasons for modelling activities in mathematics education

"Real-world problem-solving expertise is a foremost educational goal that continues to be reinforced internationally, at least officially" (Galbraith, 2012, p. 4). Mathematical modelling competence is still a part of the conceptual basis for the PISA study, included in the term mathematical literacy (OECD, 2018). The teaching of mathematical modelling has moved forward in mathematics classrooms worldwide (Burkhardt, 2006), and one reason is to see the use of mathematics, and connect school mathematics and life outside of school (Niss, 1993). Blum (2015) presents groups of justifications for the inclusion of application and modelling in curricula and everyday teaching. I will now present two groups of justifications from Blum (2015). *The pragmatic justification* is that you need mathematical modelling competence to understand and master real-world situations. Suitable applications and modelling examples must be explicitly treated. This justification can be connected to modelling as a means for developing modelling competence. *The psychological justification* is that real-world examples may contribute to raise students' interest in mathematics, motivating or structuring mathematical content, to better understand it and to retain it longer. This justification can be connected to modelling as a means for learning mathematics. The aim is to better understand mathematics. Vos (2018) discusses authenticity in mathematics education and gives an example of studies where students find mathematics more useless because of given real-world contexts. She argues that authentic situations should imply an authentic question, questions that people in that context would ask. It is therefore important to evaluate the use of real-world contexts. The two groups of justification could be seen as reasons to learn modelling and show that both perspectives on mathematical modelling are relevant.

Mathematical modelling in the curriculum

Porter (2006) uses the term curriculum assessment when evaluating differences and similarities in the *intended*, *enacted*, *assessed* curriculum. The *intended* curriculum is what is stated for what students must know and be able to do by some specified point in time. The *enacted* curriculum concerns what happens in the classrooms. In the Nordic countries, the use of textbooks seems to be even more intense than in other parts of the world (Grevholm, 2017). This implicates that evaluating textbook tasks as the *enacted* curriculum is relevant. The *assessed* curriculum concerns student achievement tests (e.g., national examinations as in this study). It is suitable to use this framework to evaluate the results from the analysis of textbook tasks (*enacted* curriculum) and exam tasks (*assessed* curriculum) and relate it to the *intended* curriculum. This study can be seen as curriculum assessment concerning mathematical modelling.

In the Norwegian context, modelling has already been a part of the intended curriculum for decades (Berget & Bolstad, 2019). From 1994, modelling, experimentation and exploration was one of nine aims in upper secondary school mathematics (Ministry of Church Education and Research, 1994), and is also mentioned in the curriculum years before. In the intended curriculum valid in the period 2006–2020/2021, the following explanation is given, concerning the purpose of mathematics as a school subject: "Solid competence in mathematics involves using problem-solving techniques and modelling to analyze and transform a problem into mathematic form, solve the problem and evaluate the validity of the solution" (Ministry of Education and Research, 2013, p. 1). Here, the steps in the modelling cycle in figure 1 can be recognized: "To analyze a problem", to "transform a problem into mathematic form", to "solve the problem" and to "evaluate the validity". In the specific curriculum, for the upper secondary course named 2P, where this study is placed, modelling is one of four main subject areas (in addition to "functions in practice", "numbers and algebra in practice" and "statistics"). Mathematical modelling is explained as "a fundamental process in the subject, where the starting point is something that actually exists. This is described in mathematical terms through a formulated model, and the results are discussed in light of the original situation" (Ministry of Education and Research, 2013, p. 3). Here, the processes of modelling are also prominent in the explanation of modelling, like in the definitions of modelling competence. In the curriculum from 2020/2021 *modelling and application* is expressed as a part of the whole mathematics curriculum at all school levels, 1–13, as one of six "core elements". In this study, the aim is to analyse tasks from textbooks and national examination, as a way of getting insight into the teaching of mathematical modelling. The research question is formulated as follows:

> Which steps of the modelling cycle are needed to solve textbook modelling tasks and tasks from national examinations?

Further, the findings will be discussed identifying differences and similarities in the *intended*, *enacted* and *assessed* curriculum. The process of the analysis will now be explained before the results will be presented and discussed.

Method

This study is a content analysis of mathematics textbook tasks and national examination tasks. It is placed in practical mathematics in upper secondary school in Norway. The tasks from the textbooks and national exam correspond to the curriculum valid in the period 2006–2021. The students are 16 and 17 years old, in their second year of upper secondary school, in their 12th school year. Textbook tasks from the three greatest textbook publishers are analysed; Cappelen Damm (Oldervoll et al., 2014), Gyldendal (Øgrim et al., 2013) and Aschehoug (Heir et al., 2014). In Cappelen Damm, the topics ”modelling” and ”functions in practice” are mixed in two chapters, and I have therefore analyzed all tasks from both two chapters (257 tasks). In Gyldendal and Aschehoug, I have analyzed all the tasks from the modelling chapters in the books (149 and 108 tasks). Overall, 514 tasks from four chapters in three different textbooks. A 5-hour written national examination in two parts is arranged. In the first part (2h), only paper and pen are allowed. In the second part (3h), it is expected to use a computer (included GeoGebra and spreadsheet). The sample for this study is the tasks from the written national examination from the years 2014–2018. I have analyzed each of the 112 tasks from the 10 exams, and not only modelling tasks as in the sample from the textbooks.

The analysis is based on a modelling cycle which could include different perspectives on mathematical modelling (see e.g. Nortvedt, 2013; Niss, 2015) comparing mathematical competencies and PISA framework). For each of the 514 tasks from the textbooks and the 152 examination tasks, I identified the different steps of the modelling cycle in figure 1 in the theory section. I will now present each step with remarks for when the steps could be argued to be used and give an example task that involves the given step.

Step 1, *constructing*, is relevant if the problem is formulated without defining parameters or presenting numbers to use. ”Do the students need to make assumptions and decide and identify critical components themselves?” is a relevant question to ask to identify this step. Step 1 is connected to step 2, *simplifying/structuring*, and is identified by the same question in this analysis. An example of a task where the first two steps are relevant is ”How should I travel to school?”, where the students must identify parameters and make assumptions (this task was not found in the textbooks or the exams). If the task is presented in a daily language and without mentioning how to solve it, step 3, *mathematising* (horizontal mathematising) is relevant. An example task is ”investigate if there is a connection between arm strength and the number of pushups a person can do”. The parameters are identified, but the students must express them mathematically and collect data to make a mathematical model. If the task is formulated in a mathematical language and there is given a way of solving it, the mathematising is done by the authors of the task. In the task in figure 2 mathematical terms are used, and a method of solving the tasks is given. The textbook authors have already mathematised this task. Step 4, to *work mathematically* (vertical mathematising), is relevant if the students need to calculate by hand or by digital tools, to ”do” mathematics, like ”use the first and last numbers given in the table to make a linear model”. To ”stay in the words of symbols” (Freudenthal, 1991, p. 41). The next step, 5, is to *interpret* the mathematical results back to the context. This is relevant if there is a question that is formulated in everyday language, where a mathematical result is used to answer it. The question ”what time of the day was the temperature 0° C?”, assumes step 5. The process of operationalizing the step of *validating*, step 6, was not straightforward. Most of the tasks have detailed questions (See figure 2 and figure 3 for examples). Then it is sufficient to look for a question that asks for validation. If the task is a more open question (which there were a few examples of in the textbooks, see figure 4), there is no specific question for validation. But if there is a given right answer to textbook tasks, there is no need to validate the answer by students, because they only need to check their answer. Then step 6 is not needed to solve the task. If the task is open and no answers are given, the student must validate. There were no open questions in the exam tasks. Step 7, *exposing*, is relevant if the task is formulated as an open task where students must make their assumptions and decisions about how to solve it, and is therefore connected to step 1 and 2. If it is enough to find a given right answer to solve the task, there is no need to expose further what is done. Using a spreadsheet, I marked ”1” if the step was relevant to use in the given task. For the textbook tasks, I also marked if the tasks had one correct given answer in the own section in the back of the textbook.

To strengthen the reliability of the analysis, a group of seven master students analyzed different parts of the textbook tasks that I had already analyzed. 91 % of the posed questions were given the same answer yes/no by the student, as I did in my analysis. The differences between the two task situations must be kept in mind when comparing the results. The written examination is limited by a given time and number of tasks. The purpose is to assess, and the tasks are not mainly for learning, as for the textbook tasks. The assessment for the exams is standardized, and the tasks have a given right answer. On the other side, the examination is an assessment of what is seen as important to learn and can therefore be seen in relation to the textbook tasks.

Results

Three tasks will be presented to expose the results: One common type of textbook tasks, and one exam task typical for all the exams. Then I will present an example task from the textbooks which can involve all the steps of the modelling cycle (there were only a few examples of such tasks). The tasks are given on the left-hand side in figures 2–4, and the steps to the right.

The table shows driving length and tire tread depth for 7 summer tires.

Driving length in 1000 km	14	17	24	35	37	38	39
Tire tread depth in mm	5.7	6.5	4.0	1.9	2.7	1.9	2.3

a) Make a linear model for the tread depth $f(x)$ in mm as a function of driving length x in 1000 km
b) Use the model to calculate what the tread depth can be for new tires
c) The minimum legal tread depth for summer tires is 1,6 mm. How long can you legally drive before changing tires, according to this model?
d) What is the domain of the model, for legal car drivers?
e) Recommended tread depth is 3.0 mm. How many per cent shorter than legal driving length is the driving length if you follow this advice?

1) NO
2) NO
3) NO
4) YES = 1
5) YES = 1
6) NO
7) NO

Figure 2. *Analysis of textbook task 3.59 (Heir et al., 2013, p. 112) (transl. by author)*

In this task, a dataset is presented, and it is asked for a linear model. The problem is already mathematised, and it is given how to solve it. Previous tasks and examples in the textbook pointed to the use of "two-variable regression analysis" in GeoGebra to make the linear model. To make the model, the students only need to work mathematically on the given numbers. There is a given right model in the answer section of the textbook. None of the questions asks for validation of the model. Therefore, if the students get the right answer to question a), they do not need to validate and argue why their model is reasonable. The next questions are formulated in everyday language, and the answers to the questions must be translated from the mathematical model. There is no need to expose the model because no assumptions are made by the student. Several examples of this kind were also found in the exams. Only step 4 and 5 are needed to solve this task, as for 41 % of the textbook tasks and 39 % of the exam tasks (see table 1). The same steps are needed in the following exam task, given in figure 3.

In this task, the model is given. Someone has already *constructed*, *simplified* and *mathematised* the problem. Mathematics must be used to solve the task, and the answers must be interpreted to the practical situation. The task has given correct answers, and it is not needed to argue for the given model. In the exam task in figure 3, the mathematical model is given. In the textbook task in figure 2, the numbers and the way of solving it is given. The model can be found after straightforward use

The function V, given $V(x) = 0.064x^4 - 2.41x^3 + 28.4x^2 - 105x + 39, \quad 0 \le x \le 18$ shows the sea level $V(x)$ centimetres above or under mean sea level in Tromsø one day, x hours after midnight. a) Use a graphing tool to graph V b) Show that the sea level is approximately 40 cm under mean sea level one hour after midnight and approximately 31 cm above mean sea level 12 hours after midnight. c) Find the difference between the highest and lowest sea level in the period from midnight and to 06.00 pm. d) Find the instantaneous rate of change to the function V at 07.00 am. Interpret the result in the practical context.	1) NO 2) NO 3) NO 4) YES = 1 5) YES = 1 6) NO 7) NO

Figure 3. *Example of exam task, Exercise 1, part 2, spring 2019 (transl. by author)*

of GeoGebra and does not provoke more of the steps in the modelling cycle. Even if the two tasks in figure 2 and figure 3 seem different, the same two steps in the modelling cycle are needed to solve them. Only a few examples were found in the textbooks of the following kind of task, without several sub-questions.

A simple game for two persons is "first to 50". Throw a dice as many times as you like and add the numbers of pips from all the tosses. You can stop whenever you want. Then the points of the round will be added to the total sum, and it is the other player's turn. But if you get 1, you will lose both your rounds sum and your turn. The game is over when one of the players reaches 50 points. Find out which sum to stop at, before the chance to lose your rounds points is too big.	1) YES = 1 2) YES = 1 3) YES = 1 4) YES = 1 5) YES = 1 6) YES = 1 7) YES = 1

Figure 4. *Example of a textbook task (Øgrim et al., 2013, p. 188)*

This task is formulated in everyday language, and it is not given how the students should solve the task. They must decide and identify critical components themselves and mathematise. Further, they must calculate, interpret and validate the answer. There is not one given correct answer. Since the students have to make assumptions while solving the task, it is also necessary to explain how the assumptions are influencing the answer, step 7, to expose.

The results from the analysis of the textbook tasks and tasks from the national examination are presented in figure 5 and table 2. In figure 5 we can see a bar chart of the percentage use of the seven different steps in the modelling cycle needed to solve textbook tasks and exam tasks. Step 4, *working mathematically*, is needed for 95 % of the textbook tasks and 93 % of the tasks from the national examination. The next step, to *interpret* is asked for in over 50 % of the tasks, both in the textbooks and exam tasks. This means that over 50 % of the tasks are formulated in a context. There is a difference between the tasks in the textbooks and the examination when it comes to step 6, *validate*, which is needed in 22 % of the

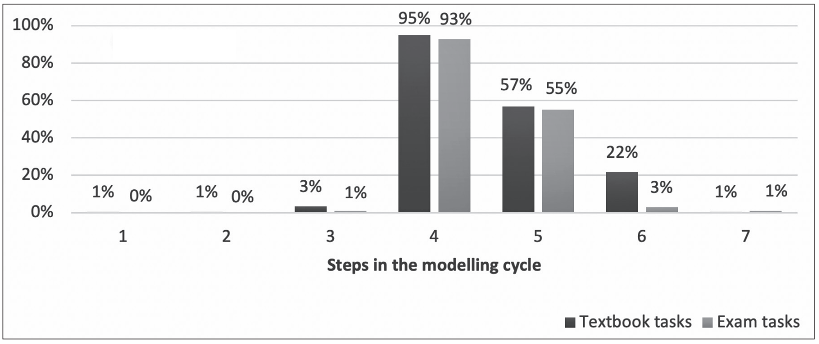

Figure 5. *Result from the analysis. Percentage of the task requiring each of the steps*

textbook tasks, and only 3 % of the tasks from the national exam. This will be discussed under the next heading.

Most of the tasks' starting point is to the right in the modelling cycle in figure 1, in mathematics, to work mathematically. In over 50 % of the tasks, *interpretation* of the mathematical results is done as well, moving to the left in the modelling cycle. In the direction Stillman (2015) points out to be in focus in the *application of mathematics*. Since the tasks are given in a practical mathematics subject, the mathematics is expected to be connected to contextual situations.

Step 3, to mathematize are only required in 3 % of the textbook tasks and 1 % of the tasks from the exam. If this step is the characterizing step of a modelling task, the direction from the real world to mathematics, as Stillman (2015) points out, would be expected to be found in modelling tasks in the perspective of modelling as evolving modelling competence. Steps 1, 2 and 7 are only needed in three textbook tasks. Two tasks in one textbook and one task in another textbook. In the last textbook, there are no tasks including step 1, 2 and 7. Step 3 is only needed in 17 of the 514 textbook tasks. As we can see in table 1, this is in line with the national examination. Here, step and 1 and 2 are not needed at all to solve the 152 exam tasks, and step 3, to *mathematise*, is only needed in 1 of the 152 tasks. Even if many of the tasks are formulated in a context, in which the mathematical answer is interpreted, most of the tasks are already mathematised by the authors of the textbooks and exam task. The tasks are formulated using mathematical language, and numbers are given, as in the task in figure 2. The focus is mainly on *working mathematically* and *interpreting* the mathematical results.

When it comes to holistic modelling tasks (Blomhøj & Højgaard Jensen, 2003), only 1 % of the textbook tasks falls into this category (see

Table 1. *Results from the analysis, comparing textbook tasks and examination tasks*

	Textbook tasks	Exam tasks
All the steps 1 to 7	1 %	0 %
Step 3, 4, 5 and 6	1 %	0 %
Step 4, 5 and 6	18 %	3 %
Step 4 and 5	41 %	39 %
Only step 4	32 %	53 %
One, two or three of the steps 3, 4, 5, 6	8 %	5 %

table 2), where the students have to proceed through a complete modelling cycle to solve the problem (a total of 3 tasks, 2 from one textbook and 1 from another). There are not given any holistic modelling tasks in the national exams. But as pointed out both in the PISA framework (OECD, 2018) and by Hankeln et al. (2019), atomistic tasks are more suitable in a written examination. The three holistic tasks included in the textbooks are placed in the most difficult path of the textbook tasks. Some students may not solve any textbook modelling task where all the steps of the modelling cycle are needed. These tasks are the only tasks where step 1, 2 and 7 are needed. There is no need to expose the model (step 7) when the assumptions are already made by the author of the task, and the correct answer is given.

In the atomistic tasks, steps 4–6 are emphasized. Most of the tasks are already *simplified*, *structured* and *mathematised*. The correct answer was already given in 94 % of the textbook modelling tasks. A typical task presents a given set of numbers, and for example, the question "Use ICT and find an exponential function that fits the given data" (Oldervoll et al., 2014, p. 146). Often a similar example is given on the previous pages before the tasks, giving a method for solving them, and in this case, using regression analysis in GeoGebra.

The result from the analysis eliminates the understanding of modelling which Blomhøj and Ärlebäck (2018) classify as *developing modelling competence* both in the textbook tasks and exam tasks. More holistic tasks would then be expected, and atomistic tasks focusing on all the different steps of the modelling cycle. In both the textbook tasks and the tasks at the national exam it is emphasized to *work mathematically* and to *interpret* the mathematical answers to the context. This also excludes the RME's use of modelling, where the formation of the model assumes a situation to be mathematised. The result will now be discussed considering the *intended*, *enacted* and *assessed* curriculum.

Discussion

In the intended curriculum in the Norwegian context, modelling is expressed by the modelling process. The steps are not explicitly pronounced as sub-competencies or sub-processes of modelling competence. Nevertheless, one can recognize the presence of all steps, as explained in the theory section. This leads in the direction of the perspective *modelling as developing modelling competence*. Concrete mathematical content is not mentioned in the descriptions of mathematical modelling. The starting point of the modelling process is in real life, which is essential in the perspective of developing modelling competence. Through the PISA framework, real life-mathematics is implemented in the Norwegian intended curriculum from 2006 (Nortvedt, 2013), not only for primary and secondary school but also for upper secondary school. But the teaching traditions in upper secondary school Norway may be similar to the French, as described in the theory section, where the activity usually originates from given mathematical content, rather than a real-life problem. This can be acknowledged by the results from the analysis, at least it shows that mathematics is the starting point in almost every given textbook modelling task and tasks from the national examination. Especially in the upper secondary school in Norway, it has been a focus on theoretical mathematics without a practical context before this subject of Practical mathematics was introduced in 2006. The *intended*, *enacted* and *assessed* curriculum seems to disagree on the starting point of a modelling process, whether it is mathematical content or ”something that actually exist” (Ministry of Education and Research, 2013, p. 3).

In the *intended* curriculum, it is made clear that ”[m]odelling provides an overarching perspective on the subject” (Ministry of Education and Research, 2013). Nevertheless, it is placed as one of four main subject areas. In the *enacted* curriculum, represented by the textbooks, modelling is one of four chapters in the books and connected to the subject area functions. Modelling may therefore be seen as a separate part of mathematics, as Brown and Stillman (2017) indicates in their study, instead of ”an approach to life and a way of thinking”. In the *enacted* curriculum there is a lack of holistic modelling tasks and atomistic tasks where the students must identify mathematics themselves. Therefore, if the students only solve textbook tasks, they have few opportunities to develop good modelling skills. It is possible that students only dealing with textbook tasks, never proceed through a whole modelling process in a holistic modelling task. To work with such tasks is preferable for connecting school mathematics to everyday life because it is similar to everyday situations (Hankeln et al., 2019). As mentioned in the theory section, students should focus on specific sub-competencies to evolve modelling competence. Step 3 (Blomhøj & Højgaard Jensen, 2003) and step 1, 2 and

6 (Blum, 2015) are seen as challenging steps. Based on the result of this study, these are the steps that are least needed to solve textbook modelling tasks. This also points at different aims for modelling than evolving modelling competence. The focus in the *enacted* and *assessed* curriculum is rather to work mathematically, and the perspective on modelling is difficult to identify because of a lack of modelling tasks.

One of the reasons for implementing modelling in school mathematics is that mathematical modelling is useful in everyday life – pragmatic justification (Blum, 2015). However, if students only deal with textbook tasks where the problem is already *structured*, *simplified*, and *mathematised*, they do not experience the whole process of using mathematics in a real-life situation. This could lead to a lack of connection to real life in the *enacted* curriculum. This justification for mathematical modelling in school mathematics could therefore disappear. The psychological justification (Blum, 2015) is that real-world examples may contribute to raising students' interest in mathematics. For the task in figure 2, one could ask if the use of authentic context is preferable, concerning questions raised by Vos (2007). Who would collect the data and ask the questions given? Will a car owner collect the data in figure 2? How and why? If the task is not authentic, one can question the psychological justification. But on the other hand, if the students are given only theoretical tasks, they do not relate mathematics to everyday life (Boaler, 2001). If the psychological justification is not valid, and neither the pragmatic justification, one can question the reasons for working on such tasks. The tasks represented by the ones in figure 2 and figure 3 can be questioned to be modelling tasks at all.

The reasons for divergence between perspectives on mathematical modelling in the *intended* curriculum, the *enacted* and the *assessed* curriculum could be diverse. The administration of the assessment of national examination can conduct certain types of tasks. There is a difference between the *enacted* and the *assessed* curriculum when it comes to the frequency of the need of step 6, to *validate*. It is debated if traditional paper and pencil tests can be used to assess practical competencies connected to authentic tasks (Vos, 2007). Holistic tasks are comprehensive and time-consuming, and the use of such tasks in an exam situation could be problematic. The modelling tasks should be different in the *enacted* and *assessed* curriculum, because of the limitations in the situation of written examination. But on another hand, it is possible to assess the different sub-competencies by a standardized test, such as developed by Hankeln and colleagues (2019). They show that all the steps of the modelling cycle could be a part of the assessed curriculum in atomistic tasks. The written exam tasks could be seen as guidelines for what is important to learn (Momsen et al., 2013). The *assessed* curriculum is therefore influencing the *enacted* curriculum. The purpose of textbook tasks and exam tasks may be different, but in this study, tasks from previous exams are found in the textbooks and are to a great extent similar.

The exam tasks in this study lack the perspective *modelling-as-critic* and crucial steps of the modelling process, as mathematising. In the typical exam task presented in figure 2, it could be relevant to ask critical questions like: Where does the model come from? Why is it expressed as a polynomial of grade 4? Are there reasons to believe that it is a good model before midnight as well? But such questions are not found in the exam tasks. In the type of task as in figure 2, GeoGebra and regression analysis should be used. Neither here critical questions are asked, and the task can be solved by following a given procedure. Such straightforward tasks where all the needed information is given is not modelling, according to Borromeo Ferri (2018). Modelling is cognitively demanding and is not a ”spectator-sport” where you don’t need to get involved in the problem (Blum, 2015). It is not sufficient to solve tasks including a model and a context by finding a given correct answer, to develop modelling competence. To mathematise, the step Stillman (2015) points out as the most important when modelling, is not emphasized in the enacted or assessed curriculum. One can therefore question if modelling is included in the *enacted* and *assessed* curriculum, or if only application of mathematics is implemented in this subject of Practical mathematics.

Conclusion and final remarks

Using a modelling cycle to analyze tasks can give insight into how the different modelling sub-competencies is emphasized in the textbooks and the national exam. The results from this study show an absence of holistic modelling tasks and focus on given mathematical content in a context. One can question if the contexts are authentic. When modelling is implemented into school mathematics, something that is generated outside of school is transposed into school, out of its natural context. Is it challenging for textbook authors to bring a natural context for modelling activities inside of school? How could students evolve modelling competence if they don’t involve in all the sub-competences of modelling, but only receive already mathematised tasks where the data is given? The students might be given different tasks by the teachers, not included in the textbook, to provoke the development of modelling competence.

Now a new curriculum is being implemented in the Norwegian school system. Here, *modelling and application* of mathematics is expressed as one of the *core elements* of the whole mathematics curriculum at all school levels, grade 1–13. The purpose for working with mathematical modelling are presented as to learn how mathematics is used and to use mathematics in daily life, work-life and society (The Norwegian Directorate for Education and Training, 2019). It includes both *modelling as means for developing modelling competence*, and the perspective *modelling as means for learning mathematics* (Berget & Bolstad, 2019). This study shows a divergence between the former intended curriculum and the textbook and exam tasks. If the aim is for students to see the use of mathematics and be able to master real-world situations using mathematics, modelling should also be implemented with all its sub-competencies in both the *enacted* and *assessed* curriculum, allowing students to evolve modelling competence.

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