Practice-based research on mathematics teaching: A developmental turn?

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When I began my career in mathematics education research in the early 2000s, I was fascinated by a number of what were, at the time, recent and emerging studies of mathematics teaching. These studies noted an important shift away from mathematics teaching studied via a range of "proxies": level of prior mathematics qualifications of teachers or number of years of teaching experience, for instance. Instead, these emerging studies described themselves as "practice-based" studies of mathematics teaching: the focus was on understanding mathematics teaching *in situ*. While key studies were directed at a range of different goals, there were common threads in this body of work – particularly around understanding and theorizing teaching quality or effectiveness (e.g. Askew et al., 1997; Hill et al., 2008) and characterizing the nature of the professional knowledge base of mathematics teachers through studying their teaching (Ball & Bass, 2003).

In looking across the papers presented in this special issue of Nomad, all written some twenty years after my introduction to – what is still to me – a fascinating field of work in mathematics education, I am struck by the fact that while all the papers are, indeed, practice-based studies of mathematics teaching, they are all geared quite explicitly, albeit in different ways, to the *development* of mathematics teaching. It is this feature that leads to the question in the title of this paper. In this commentary paper, I ask whether we are in the midst of a "turn" in the foci of practice-based studies of mathematics teaching, away from the observational studies of mathematics teaching that sought to characterize and theorize classroom practices in relation to teaching quality or teachers' mathematical knowledge bases, towards an emphasis on practices related to mathematics teaching development? And if so, what are the currents at play in this turn?

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Now mathematics education research is somewhat awash with "turns" – we have had social turns (Lerman, 2000), linguistic turns (Radford, 2016) and sociopolitical turns (Gutierrez, 2013), but shifting trends in our attention remain important to understand. Shifts reflect changes in our priorities for what we want mathematics education research to try to achieve. All the papers in this issue are focused on studies directed at producing and understanding the development of mathematics teaching practices through collaborative working between teachers and researchers. In this paper, I comment on this shift in focus in two ways. Firstly, I offer a brief overview of the concerns that appeared to drive the earlier generation of practice-based studies of mathematics teaching. Then, through offering a commentary on the papers in this special issue, I consider the more recent currents in our field that may have contributed to my sense of a marked "developmental turn" in practice-based research in mathematics education. The paper concludes with some reflections on what this writing related to mathematics teaching development offers the field, and on the possible drivers for this shift in priorities within the field of practice-based research in mathematics teaching.

The 1990s and 2000s: earlier generation practice-based studies

In this section, I summarise the approaches and outcomes of the three seminal practice-based studies of the earlier generation of work mentioned above (Askew et al., 1997; Ball & Bass, 2003; Hill et al., 2008), and consider the currents that led to their common motivations to undertake practice-based research on mathematics teaching. The first of the three articles was Askew et al.'s (1997) *Effective teachers of numeracy* report. This study sought to identify constellations of factors related to the teaching of classes that showed high gains on numeracy assessments in comparison to other teachers. A key finding of this study was that high learning gains were frequently related to an orientation to mathematics teaching that the authors described as "connectionist" – involving seeing: mathematical ideas as connected, teaching and learning as connected, and learning and application as connected. The connectionist orientation was contrasted with two other orientations that were related to lower gains: transmission and discovery-based orientations.

The authors take pains in this report to emphasize that the orientations derived from their studies of mathematics teaching are "ideal types" – a composition of an emergent theory that sits at a level above the specific beliefs and practices of any particular teacher in their study:

> no one teacher did, or is ever likely to, fit exactly within the framework of beliefs of any one of the three orientations; many combined several characteristics of two or more orientations.
>
> However, it was clear that those teachers with a strongly connectionist orientation were more likely to have classes that made greater gains over the two terms than those classes of teachers with strongly discovery or transmission orientations. (p. 28)

In searching for factors related to effective and efficient student working on numeracy, a distilled theorization thus emerges – a theorization that has been widely cited in the mathematics teaching literature and more generally, in the research-based effective teaching literature.

The other two mathematics teaching practice-based studies introduced earlier are related. Firstly, a series of studies located within the *Mathematics teaching and learning to teach* (MTLT) and the *Learning mathematics for teaching* (LMT) projects, led by Deborah Ball, traced back from studies of the work of mathematics teaching to understand the mathematical demands of teaching, and hence the mathematical knowledge required for effective teaching. Findings from the MTLT project led to the internationally well-known "Mathematical knowledge for teaching" (MKT) model, with its breakdown of teachers' mathematical knowledge into subject matter knowledge and pedagogical content knowledge, and then into sub-categories across both of these major categories (Ball et al., 2008). The follow-up LMT study led to the development of items across each of these sub-categories that allowed for mathematical knowledge for teaching to be measured in broader surveys (Learning mathematics for teaching project, 2011).

A further set of studies led by Heather Hill focused on testing the relationship between Ball's MKT measures and the mathematical quality of instruction (MQI). For this work, a literature-based model of MQI was constructed, described by Hill et al. (2008, p. 431) in the following terms:

> By "mathematical quality of instruction" we mean a composite of several dimensions that characterize the rigor and richness of the mathematics of the lesson, including the presence or absence of mathematical errors, mathematical explanation and justification, mathematical representation, and related observables.

Of interest in relation to my purposes across these two sets of studies is the development of further theorizations – the MKT model and the MQI model. The MKT and MQI models have been widely taken up by researchers in a number of countries and contexts, with the items

developed within the LMT project used in several studies to extend understandings of teachers' mathematical knowledge, and to reflect on the possibilities of success of policies aimed at improving learning outcomes (e.g. Hill, 2007).

A range of common dissatisfactions feature commonly across the motivations for these three seminal studies. Among these, the following aspects featured prominently in the literature.

- That teaching practice, on the one hand, had tended to be studied in generic, rather than discipline-specific ways, while – on the other hand – a substantial body of critique had identified the prevalence of highly procedural teaching in mathematics and, in consequence, highly imitative modes of learning
- Widespread evidence of gaps in teachers' mathematical content knowledge, and of largely negative memories of mathematics teaching and learning in teachers' own histories, with these features marked particularly in primary mathematics teaching
- That mathematical qualification nature and level indicators offered poor predictive validity for mathematics teaching quality, leading to a much more direct focus on mathematics teaching practice.

Nearly two decades on, the interest for me was in trying to pull apart how our dissatisfactions have shifted, in order to understand why the practice-based studies that feature in this issue (and more broadly) are so much more directly oriented towards studying mathematics teaching practice from the perspective of development. It is this question that drives my overview of the papers in this issue, their approaches and outcomes, and then a commentary on shifts in the patterns of motivation that may feature within the change in direction.

Considering the special issue papers

All of the papers in this issue involve teacher-researcher collaborations, and interestingly, concerns with the research-practice or the theory-practice nexus itself are recurringly stated as among the key motivations guiding the collaborative focus on teaching development in several of the papers. Säfström et al. problematize explicitly in their opening sentence, offering a range of citations that stress that the concern pre-exists their specific context:

> Educational research has long been criticised for its weak relation to classroom practice (Burkhardt & Schoenfeld, 2003; Coburn & Penuel, 2016), regarding lack of attention to practical key teaching issues, lack of development of empirically-tested and easily-implemented tools and processes, and lack of collaboration between teachers and researchers (Burkhardt & Schoenfeld, 2003; Stylianides & Stylianides, 2013).

The critique of the lack of "practical" input is of particular interest to me given what I have said about the earlier studies' offer of categorizations that emerged from analyses of practice, and yet stood "above" practice in their distillations of orientations. The papers in this issue are oriented in rather different ways to these earlier papers. All of them make use of theory: often, it provides the lens for considering differences in a range of aspects of teaching practices: teacher planning, teacher attention to problem-solving, teacher reflections on practice, etc. Some of the papers go forward from the particulars of teacher change into key aspects that featured within the supporting of teaching development. The commentary below focuses on the nature of theory use and theory development in each paper, noting also, the key findings that are presented. The papers focused on pre-school teaching practices are engaged with first, followed by the primary mathematics teaching focused papers and then the secondary and cross-phase papers.

The intervention study at the heart of Björklund and Ekdahl's paper is framed in key tenets of variation theory (Marton & Booth, 1997). Variation ideas frame the design of the tasks that the pre-service teacher uses in her mathematics teaching, while also framing the expansions in the focal teacher's ways of teaching and her ways of experiencing and reflecting on her own teaching of early number. With data drawn from observation of video clips of teaching and from a reflective interview with the teacher, the authors highlight the connections between changes in teaching acts over time and shifts in the teacher's ways of experiencing mathematics. I found the micro-level descriptions of this teacher's gradual appropriation of variation theory into her own work with artifacts and activities particularly interesting. Initially, the teacher describes trying to remember the order of working with examples that was discussed in the larger group discussions in these terms:

> I had to write down exactly how to ask, otherwise I mixed it all up. There were quite many questions to ask. Ten-seven, ten-six, ten-five. And then some discussion which I could not remember. I was so focused on the discussion. If I was to ask ten-six or was it ten-three, I had to write it all down, maybe my memory is bad.

The authors go on to analyse an increasing ownership of the principles and purposes of the task and activities, seen in small changes in how the teacher poses and discusses problems, framed within an orientation to connected number relations rather to isolated additive results. The collective development of activities in the pre-service teacher project group, with focus on better understanding children's inputs is also noted as important for maintaining commitment to the project over time.

From the frameworks offered in the earlier generation of practice-based studies, it may be possible to claim that these small changes can be analysed in terms of increasing attention to connections or better mathematical and pedagogical content knowledge. However, the approach to analysis in Björklund and Ekdahl's paper remains much more grounded in the minutiae of the data, and being responsive to the particular teaching development needs of the intervention participants. This flags a distinction that I find interesting – between remaining with micro-level changes in the analysis with an eye on "local" development, or transcending the local to create meta-level theory from patterns that emerge from the local. Björklund and Ekdahl's focus is on the former; the earlier studies' focus was on the latter. This is an idea that I return to later in the discussion.

Palmér and Van Bommel's paper is a reflection on the multi-year trajectory of a practice-based design research study working with teachers on the development of problem-solving approaches in pre-school settings. The authors describe changes in the researcher-teacher collaboration model, and in particular, teachers' increasing ownership and leadership of problem-solving lessons and the associated student interviews over time. Shifts in collaboration were linked with shifts in markers of research quality. Specifically, they note that while internal validity and predictability markers tended to increase with the aforementioned shifts, external validity, rigour, precision and reproducibility markers tended to decrease as larger parts of the rollout and data collection moved into the hands of participating teachers.

The effects of changes over time in the researcher-teacher collaboration model are therefore at the centre of this paper, with the authors noting that such collaborations are critical to research that aims to translate into educational development. A central part of this translation is the development of tools or materials linked to the goals of the design study (Bakker, 2018). In Palmér and Van Bommel's paper, it is clear that the longitudinal research has involved the development of problem solving lessons and an interview model for gaining insight into how children experienced the problem-solving lessons. This has parallels with the

design of early number and additive relations tasks and activities in Björklund and Ekdahl's paper.

Fauskanger and Bjuland's developmental attention is on providing teachers with professional development (PD) opportunities to focus on collective lesson planning activities across several learning cycles. The authors look at what are described as teachers' "reasoned dialogues" in the PD sessions. They analyse these within a sociocultural discourse analysis drawn from the work of Warwick et al. (2016) on dialogic processes and discourse frames. The authors note the emergence of ambitious teaching practices related to anticipating student responses and representing these responses. The PD discussions coupled with tasks based on structured images of quantities are used to provide a basis for attention to learning about the distributive law, and comprise the core tools for the professional learning cycles in this study.

In Eriksson et al.'s paper, the goal is to support teachers to work, from a Davydovian base in learning activity, to develop students' collective theoretical thinking in the context of algebraic reasoning. Jointly planned research lessons using problem situations and learning models drawn from Davydov's curriculum were iteratively adapted through trialing, documenting and jointly reflecting with a Swedish grade 7 class. Drawing from classroom practice data analysis, the authors conclude with an identification of "didactical tools and strategies" related to tasks, their design and associated elements that appeared to be important for supporting theoretical working on algebraic expressions: contradictions, playful formats for student working and openings for collective reflection.

Mårtensson and Ekdahl describe a learning study involving 30 pre-service primary teachers in a 5-week intervention module where groups of students created, taught, reflected and adapted tasks for use. The authors' focus is on bridging the widely cited theory-practice divide in education. Their design research learning study introduces their pre-service teachers to key tenets of variation theory, with small groups then supported to work on the initial design, and teaching of lesson tasks based on these tenets, followed by reflection and redesign of these tasks. Analysing initial and redesigned tasks in the teachers' submitted reports, the authors detail a categorization of key ways in which tasks were redesigned using variation theory: expanding tasks, making tasks more explicit, making tasks less explicit, bringing metaphors and representations to the foreground, and creating new tasks. Using illustrative examples of tasks in each of these categories, the authors note the broader point that this project model provides evidence of theory being used in the development of pre-service teachers' pedagogic practice.

Moving beyond the realms of pre-service teacher education, Krog Skott, Laursen Falkenberg and Redder Honoré offer an analysis of the learning of two newly qualified teachers in the design research context of a mathematics-specific induction programme aiming to support the retention of teachers in the profession. The induction programme model, as in some of the earlier papers in this issue, rests on the collective development, trialing, and reflecting on, a lesson plan focused on a shared problem and goals of constructivist, process-oriented mathema-tics teaching. Collaboration in their model included mentor teachers for pairs of newly qualified teachers in each of the participating schools.

Drawing on Jeppe Skott's (2017) writing on "patterns of participation" as a social practice framing for considering teacher learning, the authors illustrate two contrastive stories of how contextual features of new teachers' histories and social worlds interact to afford/constrain, in different ways, their possibilities for taking on board more open process-oriented practices. In this paper, as in Mårtensson and Ekdahl's paper, there are exemplifications of the actual playing out of teaching that point to some less than desirable outcomes. One of the two teachers in focus in this study retains a pedagogy dominated by traditional instruction. There are echoes here of Mårtensson and Ekdahl's illustration of instances of the "mechanical" incorporation of variation theory, where much of the potential for a sensitive and nuanced relational view of the teaching-learning nexus is negated. I return to this point in the discussion that follows this section.

Säfström et al.'s paper is also focused on teacher-researcher collaborations (TRCs) for the development of mathematics teaching. Specifically, the authors use two notions that have been proposed in earlier research as critical for successful TRC functioning – symmetry (commitment and attention to the needs and goals of teacher and researcher communities) and complementarity (utilisation of the different areas of expertise of the two constituent communities) – to study processes of exploration, design and evaluation within a collaborative design research study set up to promote problem-solving approaches with groups of primary and secondary mathematics teachers. The author group studied the interactions between participants across a number of the broader TRC group and smaller sub-group meetings, aimed at producing problem-solving tasks and a teacher guide to support other teachers in teaching for mathematical reasoning. Refining tasks following iterative trialing in classrooms featured as a central part of the work of the design research in this study as well. In this paper too, as in the Krog Skott et al. and Mårtensson and Ekdahl papers, there is attention to the contingent ways in which a range of factors, in particular, the time available in Säfström et al.'s

paper, play out, feeding into different and dynamic patterns of mutual learning from the TRC and different trajectories of symmetry and complementarity.

Commonalities and motivations for focus on practice development-based research

I have already noted that all the papers in this issue share a focus, not just on practice-based research but on practice development-based research. However, a number of other features also permeate these papers. A longitudinal design research orientation is evident across all the studies, as is teacher-researcher collaboration. Attention to mathematics teaching practice is evident in the artifacts of practice that are at the centre of the studies – lesson plans, classroom tasks, supporting students' problem-solving and reasoning, and mathematical representations among these. Further, adapted lesson study-type formats – involving groups of teachers working with university teacher educators and/or researchers on iterative trialing of these artifacts, followed by collective reflection and adaptation – also feature commonly. Additionally, as pointed out above, several papers discuss the features that appear to have contributed to deviations from desired outcomes as well as exemplifying the successes.

I find the commonalities interesting, and not least because they appear to draw on shifting emphases in the broader international mathematics education literature base rather than shifts that are localized to the Nordic countries' research base. In this section, I consider the emphases that this set of papers draws from, and how these emphases differ from the influences on the earlier generation of practice-based research on mathematics teaching.

In the international terrain, several countries continue to face problems in terms of expanding access to high quality mathematics education and closing the attainment gaps between high and low attaining students in mathematics. This goal remains marked in the face of evidence from large scale international comparative studies such as TIMSS that show associations between reductions in the attainment gap and overall increases in performance in mathematics and science (Broer et al., 2019). But what the international comparative literature has also pointed to is the need for attention to both the quality of mathematics teaching and to the systemic supports in place to develop this quality. Influential studies such as Liping Ma's (1999) comparison of Chinese and United States' primary teachers' mathematical knowledge delineated differences in teachers' discourses around key elementary ideas, while Stigler

and Hiebert (1999) drew attention to differences in teaching methods. They also noted contrasts in the systemic supports for practice-based professional development, and emphasized teaching as the key lever for improving learning outcomes. Importantly, they highlighted systems for supporting teaching development as critical for achieving improvements in the quality of teaching.

Given that mathematics teaching is an element that we might reasonably see as within the sphere of influence of teachers and the teacher education system (unlike more structural aspects such as socio-economic conditions), the focus on teaching was (and remains) highly understandable. However, in spite of mathematics teaching practices having been in the limelight of mathematics educational research for an extended period of time, the research base indicates some disappointments with the scale and the pace of change, and concerns in particular relating to the limited evidence of broad based change.

Lesh and Zawojewski (2007), reviewing the existing research on mathematical problem-solving note, for example "the lack of impact and cumulativeness" (p. 763) of this body of work for classroom practice. More general critiques of the limited impact of mathematics education research on pedagogic practice are among the papers commonly cited in the studies reported in this issue (e.g. Korthagen, 2010; Coburn & Penuel, 2016). Alongside, and drawing from these critiques, there has been an upsurge of interest in design research, constituted explicitly with the goal to connect theory and practice (Bakker, 2018). Cobb and Jackson (2015), writing in support of attention to design-based research, highlight though, the extent of the gap between existing practices and aspirations for pedagogy thus,

> disseminating the products of classroom design studies will often involve supporting large numbers of teachers not merely to extend or elaborate their current instructional practices, but to reorganize those practices. The teacher learning involved is substantial.
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> (p. 1028)

While closing the gap between empirical realities and research-based aspirations for teaching and learning is the initial goal of design-based research, the quote above also alludes to the scales required of this enterprise. Thus, as well as closing the gap there is also an urgent need to consider avenues for scaling up promising results from smaller scale studies (Maass et al., 2019). Arguably, in this latter area, an even larger shortfall exists. This is in spite of models that have drawn attention to how scaling up can be effected to go beyond the confines of initial direct collaboration between particular groups of researchers and teachers (Borko, 2004).

Palmér and Van Bommel's paper in this issue speak specifically to considerations related to this kind of scaling up, and attention to scaling is implied in Säfström et al.'s broader intervention design. Interesting too, is the increasing take up of adapted lesson study-type models, where groups of teachers are brought together to focus on some aspect or other related to their own perceived concerns about their teaching practice and student learning. This take up too is supported by a burgeoning international literature base (Huang et al., 2019; Seino & Foster, 2020), with adapted, culturally-sensitive models found that allow for a focus on practice in conditions that are far-removed from the time allocations, conditions and expectations for professional development described in Stigler and Hiebert's (1999) early descriptions of lesson study in Japanese settings. Building capacity within mathematics teacher education contexts for the expansion of support for mathematics teaching practice development is a further area that is receiving increased visibility (e.g. Krainer & Llinares, 2010), and is likely to be critical in the quest for scaling up.

Maass et al. (2019) note the need to be both "optimistic and cautious" about initiatives seeking to improve mathematics at scale. The papers in this issue appear, to me, to have worn both of these hats with some care. Perhaps, underlying the move to development-oriented studies of mathematics teaching practices, there is greater awareness of the scale of this work, and of the challenges posed by the contingent aspects of context and conditions. The trends highlighted already, coupled with these context and conditions findings, suggests greater caution, and perhaps some world-weary scepticism, with what distilled theory can offer. Korthagen and Kessels (1999), referring to pre-service teacher learning, have long stressed the need for smaller and more tailored, practically relevant theory in the following terms:

> More often, however, they need knowledge that is situation-specific and related to the context in which they meet a problem or develop a need or concern, knowledge that brings their already existing, subjective perception of personally relevant classroom situations one step further. [...] We could also call it "theory with a small t". (p. 7)

Small theories create narratives that may be less grand, less all-encompassing than theories with larger t's, but they are geared towards the "practical theory" needed to be responsive enough to the ground to support changes in mathematics teaching practices. Additionally, and of interest, the design research/lesson study model of multiple iterations, with its production of carefully considered lesson plans offers avenues for, and incorporates some of, the conditions needed for scaling beyond the immediately local. A recent commentary on a group of papers

analyzing the quality of instruction of a set of three lessons using different frameworks (an MQI-based analysis among these) is interesting in this regard. Praetorius and Charalambous (2018) note that in spite of seeking overlaps between frameworks through this approach, they found limited convergence on how quality was defined and studied. Their reflection on this lack of synchronicity across frameworks, citing Gitomer (2009) in support, is that this lack is a function of teaching quality occurring in "an exceedingly noisy environment" (Gitomer, 2009, p. 229). The direction of the papers in this issue perhaps suggests, instead, local attunements to the "noise" of particular settings alongside attunement to quality as a process rather than as a destination.

Concluding comment: a time for "slow" research

It may be the case that, as a field, we are more acutely aware than we were twenty years ago of the gaps between the research enterprise in mathematics education and practice in mathematics classrooms. There is an increasing urgency for mathematics teaching development – given what we know about mathematics as a "gatekeeper" subject that features within the reproduction of social inequality. The global COVID-19 pandemic has highlighted the exacerbations of these inequalities of access around the world, and the fragilities of our own education systems. The "pause" provides, perhaps, a time to reflect on our work as academics in mathematics education and why we do the work we do. For me, this time has crystallized a sense that it is research-based mathematics teaching development that is at the heart of my motivations for this work. Theory development may happen, but it is geared towards an improved understanding of local change over time. The work is much more located in the minutiae of the particular – in studying small, actual changes rather than grand categories of change. This collection of papers reflects these motivations. The projects within this issue all involve researcher teams working with teachers in a developmental orientation in multi-year programmes of development activity iterations and data gathering. This kind of work tends to reflect the terms of what has been described as "slow research" (Lindquist, 2012) or "slow science" (Stengers, 2018). The work of teaching development, as an extensive body of work shows, is labour-intensive, personalized, drawn out, contingent and uncertain. Describing exploratory qualitative work with developmental orientations in the field of writing studies, Lindquist reflects that their research is: "emergent, abductive, unpredictable, time- and equipment-intensive, and entirely inefficient" (p. 651). Burkhardt and Schoenfeld (2003) noted, nearly twenty years ago now, that the constitution of research in the

academy tended to work against valuing teacher development activity. The pressures to publish extensively and quickly have expanded across this time in the knowledge economy that characterizes much of the academy. Finding spaces and resources for longitudinal developmental work in mathematics education research is not easy. And yet, it is this kind of work, more than any other, that is likely to lend relevance to why mathematics education departments should exist in universities. Stengers' description emphasizes that the adjective "slow" is not related simply to the timeframes of more applied ways of working. Rather, it involves scientists going into situations with knowledge of "pure theory" without being circumscribed by its contours:

> They would acknowledge that when what they have achieved leaves its native environment – the network of research laboratories – and intervenes in different social and natural environments, it may well be leaving behind its specific reliability. And they would recognise that restoring reliability means weaving new relations proper to each new environment, which entails welcoming new objections – no longer just the objections of colleagues, but those of other collectives concerned by aspects of the environment that the scientists themselves were not concerned with. (Location 1640 of 2483)

The theories developed in the careful (and often longitudinal) studies of twenty years ago have been highly influential – between them, the three studies mentioned in this paper have over 6000 citations. I have found them hugely useful in thinking about my own work in mathematics teacher development. But however useful, they are not sufficient for the work of mathematics teaching development. Instead, as the papers in this issue demonstrate, a much broader range of competences, of attention to differences in what is viewed as important, and concern for the contingencies and contexts on the ground, come into play in the actual work of supporting teaching development. The more "local" orientations of these papers may well mean that they do not go on to garner the breadth of international attention in the research field as the observation-based development of theory papers did two decades ago. Just as with the older observation-based studies, researchers are likely to need to adapt methodologies and artifacts if they are to take the intervention models into other settings. Nevertheless, in reading all of them, I learned much that I found relevant to my work in the South African teacher development context – micro stories of what was involved in small changes in ownership of more skilled mathematics teaching, artifacts for use in this work, and ways in which our research methodologies may need to be attuned differently if we are to seriously engage in and support this kind of work.

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