Teachers' participation in practice based research – a methodological retrospect

HANNA PALMÉR AND JORRYT VAN BOMMEL

The focus of this article is methodological, on how teachers' participation in practice-based research coacts with research quality. Educational design research is an example of a practice-based research approach often used in mathematics education with the goal of developing both the theories and the practice of teaching and learning mathematics. In this article, one such educational design research study on problem solving in Swedish preschool class is used as an example of how teachers' participation in practice-based research can develop and of how different kinds of collaboration between researchers and teachers coact with research quality. One conclusion of the methodological meta-analysis is that there is a challenging tension between ensuring external validity of a study versus enabling internal validity and improvement of practice.

Over the past 40 years, research on mathematics education has increasingly used theories where meaning, thinking and reasoning are viewed as products of social activities. This direction has been labelled "a turn to social theories" (Lerman, 2000, p. 20). As part of this social turn, there are arguments for using theoretical perspectives that conceptualize learning and individuals' use of knowledge as an aspect of their participation in social practices (Borko, 2004; Peressini et al., 2004). This social turn has, among other things, led to researchers studying classrooms using approaches that aim at simultaneously investigating and developing mathematics teaching and learning (Cai, 2019). The context of this article is one such approach, a longitudinal Swedish educational design research study on problem solving in preschool class that aims to simultaneously investigate and develop the teaching and learning of problem solving. Educational design research is an approach developed to increase

Hanna Palmér, *Linnaeus University,*
Jorryt van Bommel, *Karlstad University*

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the impact of research on practice where basic research is integrated with applied research. Integrating basic research with applied research implies that the studies are conducted in realistic settings through recursive cycles of hypothesizing, testing, refining and rehypothesizing. The realistic settings are of importance as instructional problems are situated in contexts where solving instructional problems requires a detailed understanding of the contexts. Gaining such understanding includes building productive and sustainable partnerships between researchers and teachers as well as exploring the traditional professional roles of teachers and researchers (Cai, 2019).

The focus of the article is methodological, on how teachers' participation in practice-based research coacts with research quality. One educational design research study on problem solving in Swedish preschool class is used as an example of how teachers' participation in practice-based research can develop and of how different kinds of collaboration between researchers and teachers influence research quality. In the educational design research study used as example, two researchers and approximately 40 preschool class teachers have been collaborating over several years. Cai (2019) emphasizes that the costs and benefits of different pathways in research need to be evaluated as different collaborations between researchers and teachers will change the culture and professional expectations of both teachers and researchers. In this article, the development of the collaboration between researchers and teachers will be presented as three phases. In each phase, the researchers and the teachers have been collaborating for a period of time in designing, implementing, evaluating and redesigning several problem-solving lessons. In the first phase, all the problem-solving lessons and all the interviews with the children were conducted by the researchers. In the second phase, the teachers implemented the lessons and collected documentations from their students while interviews were conducted by the researchers. In the third phase the teachers continued to implement lessons and collect documentations and they also conducted interviews with their students. Thus, through the three phases the participation of the preschool class teachers' developed from their merely giving access to their students to their being responsible for gathering data. The focus of this article will be on the challenges that these different kinds of collaborations between researchers and teachers pose for research quality. More specifically, the following research question will be elaborated on:

> How do different kinds of collaboration between researchers and teachers coact with the quality of practice-based research on mathematics teaching and learning?

The article begins with a review of the role of the teacher in previous design research studies, followed by a presentation of the case to be focused on in this article. Thereafter follows a presentation of the theoretical framework that is used to evaluate the challenges that different kinds of collaboration between researchers and teachers pose for research quality. The article ends with a discussion and implications for further research.

Roles of teachers and researchers in educational design research

Educational design research has taken on different forms, names and criteria throughout the years. Development research, design experiments, formative experiments and design-based research are some examples of educational design research (Bakker, 2018; McKenney & Reeves, 2014). These different approaches and terms have emerged from specific historical and educational contexts and traditions and differ in more or less minor aspects, differences that, however, might be crucial to each specific approach. However, in general, one could say that the design of new educational materials is essential within each of these approaches (Bakker, 2018; McKenney & Reeves, 2014). Such educational materials can be specific tasks, worksheets, digital tools or extensive professional development programmes (Bakker, 2018). Furthermore, even though educational design research studies differ in both direction and size, they encompass the design and implementation of teaching as part of the research. The aim is to develop theories and new forms of instruction (Anderson & Shattuck, 2012), and to solve educational problems to help learners achieve specific goals (Bakker, 2018). Thus, the studies aim to bring value for both research and practice (Bakker, 2018; Prediger et al., 2015). To develop theories that inform and guide the practice of teaching and learning, educational design research studies are conducted in an iterative cyclic process of designing and testing interventions situated within an educational context. Each design cycle includes preparing for teaching, implementing the teaching, and finally, conducting a retrospective analysis of the teaching and learning (Cobb & Gravemeijer, 2008).

In such design cycles, collaboration between teachers and researchers is required (McKenney & Reeves, 2014). This collaboration can vary, and Cai and colleagues (2017a; 2017b; 2018) call for a reconceptualization of the role of teachers and researchers and suggest that this complementary expertise could be blended. "The interdependency is a necessary feature of the world we envision, a world in which research and practice in mathematics education are tightly intertwined, a world in which the boundaries between research and practice become blurred" (Cai et al., 2017a, p. 472).

One way to distinguish between the roles of teachers and researchers in design research is to look at their specific tasks and their joint work (Cai et al., 2018). Examples of joint work are defining the instructional problem, identifying the learning goals, applying available research and analysing data to revise hypotheses. Identification and formulation of educational problems can be done by teachers, with researchers providing ”useful alternative perspectives that shape the way problems of practice are perceived” (Cai et al., 2018, p. 515). The teachers' role in the collaboration is connected to their familiarity with their students and with the educational context, which enables them to anticipate the students' needs and to ”provide an insider's nuanced and reality-based perspective” (Cai et al., 2018, p. 518). The researcher, on the other hand, can synthesize relevant research and proved an outsider's perspective, most often a broad and holistic one. Because they can collaborate with several schools, researchers can also identify possible partnerships and networking opportunities (Cai et al., 2018).

Problem solving in preschool class

In this article, the focus is not empirical on problem solving but methodological on how different kinds of collaboration between researchers and teachers coact with the quality of research. However, as problem solving in preschool class is the context of the different kinds of collaboration to be meta-analysed, some background to this setting is given here.

Preschool class is one year of schooling that Swedish children attend the year before formal schooling begins. Therefore, when problem solving is introduced in preschool class, the children are not used to formal mathematics (nor to problem solving). The aim of the preschool class is to provide a smooth transition between preschool and primary school, which is why the teaching in preschool class ought to be a combination of preschool and primary school pedagogy, with play, curiosity and agency as important parts (National Agency for Education, 2014). In Sweden as well as in many other countries, problem solving is emphasized in the syllabus, aiming to educate children to become competent problem solvers. Integration of problem solving into the regular mathematics teaching is of importance, instead of teaching it as a separate topic after other skills have been taught (Cai, 2010). When working on problem solving, the solver needs to struggle with the mathematics at hand, which is significant in developing a deep understanding of mathematics (Hiebert & Grouws 2002). A problem-solving task, according to Lesh and Zawojewski (2007), is an activity with a clear goal and one that ”becomes a problem (or problematic) when the 'problem solver' [...] needs to develop a more productive way of thinking about the given situation” (p. 782).

A case of educational design research on problem solving

There are many different kinds of case studies; Stake (1995) distinguishes between intrinsic, instrumental and collective case studies. Intrinsic case studies focus on understanding one case specifically, while instrumental case studies focus on one case but with the aim of developing a general understanding beyond the case itself. In collective case studies, several individuals are studied within one case. According to Patton (2002), what constitutes a case, or unit of analysis, sometimes emerges during fieldwork. In the educational design research study in focus here, the initial unit of analysis was the teaching and learning of problem solving in preschool class, while the case of collaboration between researchers and teachers has emerged in parallel. Based on Stake's categories, the example in this article is a collective instrumental case on which a methodological meta-analysis will be presented by the use of qualitative quality criteria. In line with instrumental case studies, the aim is to develop a general understanding beyond the presented case itself, a general understanding of how different kinds of collaboration between researchers and teachers may coact with the quality of practice-based research.

The longitudinal educational design research study on problem solving in preschool class that serves as the case in this article has been ongoing for several years. The study was initiated in 2014 and is still ongoing (e.g. van Bommel & Palmér, 2018; Palmér & van Bommel, 2020). The aim of the educational design research study has been to investigate the potential in teaching mathematics through problem solving with the six-year-olds. The teaching and learning of problem solving within the study is not the focus of this article. Instead, it is the development of the collaboration between the researchers and the teachers during the years of intervention. The development of this collaboration can be described as three phases, all including several iterative design cycles. This implies that in each phase, the researchers and the teachers have been collaborating for a period of time in designing, implementing, evaluating and redesigning several problem-solving lessons. In all three phases, the participating teachers have been given verbal and written information about the study and have consented to participate in line with the ethical guidelines provided by the Swedish Research Council (2017).

Phase one

When the educational design research study was initiated in 2014, there were few studies on problem solving in mathematics involving young students. Thus, the first step of the intervention became to explore how to teach problem solving to young children who may not know how to read and write. The researchers selected and contacted schools where they themselves implemented problem-solving lessons in preschool classes. Neither the students nor the teachers in these preschool classes had previously been working with problem solving. The purpose of this phase in the educational design research was partly to develop knowledge of how to design and implement problem solving in mathematics with young students, and partly to develop knowledge of how young students perceive working with problem solving in mathematics. Approximately 145 students from eight Swedish preschool classes at three different schools were involved in this phase of the study. In each of these preschool classes, five to eight problem-solving lessons on different content (for example, probability and three-dimensional geometry) were implemented. After the series of lessons, interviews were conducted with the students to investigate how they perceived working with problem solving in mathematics. In this first phase, all the problem-solving lessons and all the interviews with the children were conducted by the researchers. The preschool class teachers were observers during the lessons as they were the ones familiar with the students. The researchers collected and analysed documentations from the students, and afterwards, they presented results from the analysis to the teachers.

Phase two

In the second phase, the results from phase one (Palmér & van Bommel, 2018a; van Bommel & Palmér, 2016; 2018) led to a professional development programme on problem solving for preschool class teachers. For one year, the researchers and preschool class teachers (approximately 35) from one municipality met once a month. These teachers included some of the participants from phase one. As preschool class students continue to first grade after one year, none of the students in phase two had participated in phase one. Within the professional development programme, the researchers and the teachers met eight times. The first time, problem solving was discussed and the teachers read articles on the subject. At the following meetings, the problem-solving lessons that had been developed in the first phase were presented and jointly discussed. One problem-solving lesson was focused on at each meeting. Between the meetings, the teachers implemented the problem-solving lesson that had been focused on in their own preschool classes. In this implementation, the teachers were told to adapt the lessons to the conditions of their classrooms and to their students. In total, six lessons were implemented by the teachers. The teachers wrote reflections before and after each problem-solving lesson. In these reflections they wrote about how they adapted the lessons and how the lessons turned out. They also wrote reflections on how the lessons could be improved as well as on the continued design. In connection with the implementation of the lessons, the teachers collected the documentations from their students. These documentations were written solutions of different kinds (see figure 1 for examples).

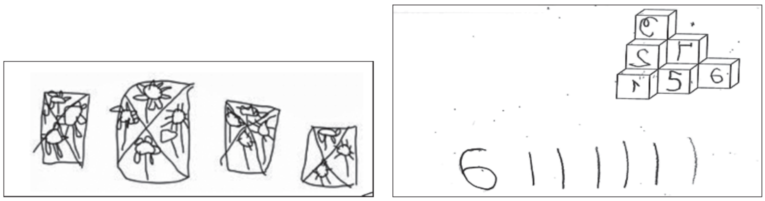

Figure 1. *Example of students' documentations*

Teachers' reflections and students' documentations were jointly discussed at the meetings and handed to the researchers. Between the meetings, the researchers conducted an analysis of the teachers' reflections and the students' documentations, and these analyses were presented to the preschool class teachers at the next meeting.

Phase three

In the third phase, the preschool class teachers from phases one and two continued working with the problem-solving lessons from phase one but now without meetings and instructions from the researchers. As preschool class is only one year of education, there are new students in the classes each year, so the same lessons could be used each year. Occasionally, the researchers asked the teachers to implement new problem-solving lessons in their classes and to collect students' documentations (as in phase two) from these lessons. The lesson the preschool class teachers were to use with their students was sent to them by mail. The teachers' participation was voluntary; those who volunteered were given verbal and written information on how to conduct the specific lesson, and after the lesson, students' documentations were mailed to the researchers. Also, the teachers wrote reflections on the lessons. At times, the preschool class teachers were asked to conduct interviews with their students, in which students' reflections on specific problem-solving lessons were addressed. These interviews followed a questionnaire developed by the researchers, and the teachers wrote down the students' answers. The questionnaire included both open-ended and multiple-choice questions. The interview data together with the students' documentations and the teachers' reflections were analysed by the researchers.

Framework: quality in research

In this article, we will use quality criteria developed by Kilpatrick (1993) to analyse how different kinds of collaboration between researchers and teachers coact with the quality of practice-based research on mathematics teaching and learning (this selection of criteria was in turn based on Sierpinska, 1993). In 1993, when writing his criteria, Kilpatrick himself addressed the question of whether they were really up to date. He then argued that even though there are different approaches to research, these criteria can help to evaluate scientific quality as they are quite general. Thus, the criteria can be used to evaluate the scientific quality of studies conducted by different kinds of research designs, recently applied or referred to by other researchers as well (see for instance Grundén (2017), Niss (2018) or Scheiner (2019) for an application and prompt of the framework). Below, we first present the eight criteria by Kilpatrick (1993), and then we identify which of these will be used in the results and explain why these have been chosen.

The first criterion is *relevance*, that is, to whom and for what purpose is the research useful. As most research in mathematics education is conducted to raise the quality of mathematics education, the relevance and usefulness for teachers are of great importance. However, mathematics education research has typically been of less value for teachers than for researchers, and researchers have often failed to involve teachers as participants in research. The second criterion is *validity*. Besides internal and external validity, Kilpatrick emphasizes that no study is valid in itself but that validity refers to the conclusions that are drawn from the study. When making generalizations, the question "What is this a case of?" is of importance as are questions about how well the methods used made it possible to investigate what was intended. The third criterion is *objectivity*. Even though absolute objectivity is unattainable, this criterion stresses that researchers ought to identify the biases they bring into research. Furthermore, researchers ought to provide as much evidence as possible concerning how the biases they bring into research may have coacted with their findings.

The fourth criterion is *originality*, which does not mean that the research should lack connection to prior research but that it ought to provide new insights. The situation of the research as such may be well known, and a study may even be a replication, as long as old things are viewed in a new light. The fifth criterion, *rigour and precision*, has, according to Kilpatrick, both positive (exactitude, accuracy, precision) and negative (rigidity, inflexibility, no variation, inability to respond to the moment) meanings. Kilpatrick broadens the perspective of rigour and precision from precision of measurement to precision of meaning, where a study may be rigorous not because it follows a strict design but because it shows sensitivity to the meanings of the study participants as well as to interpretations others may have.

The sixth criterion, *predictability*, has traditionally been understood as a cause-effect relation, and such predictions are, according to Kilpatrick, not possible in classrooms as these are not closed systems. However, even though human behaviour is too complex to be predicted in detail, researchers may strive to find patterns of regularity when it comes to people and situations. Making generalizations then becomes a question of presenting tendencies and patterns that to different degrees make it possible to anticipate what will happen in a given situation. The seventh criterion is *reproducibility*, which implies that a research report ought to include details about both the generation and analysis of data, making it possible for others to reproduce the study. The eighth and last criterion, *relatedness*, implies that research in mathematics education ought to be related to both mathematics and education. This criterion does not imply, however, that research in mathematics education should not borrow ideas from other fields as the research field is indeed interdisciplinary.

In the results, a methodological meta-analysis of the three previously presented phases of the collaboration between researchers and teachers in the educational design research study on problem solving in preschool class will be presented based on six of these eight criteria. The two criteria that will not be focused on are originality and relatedness, as these are not influenced by the changes in collaboration, which are the focus of the articla. However, we will come back to these two criteria in the discussion, but in relation to the whole study.

A methodological meta-analysis of the three phases

As previously described, the collaboration between researchers and teachers can be divided into three phases, where each phase includes several design cycles. Below, a methodological meta-analysis of each of the three phases is presented based on six of the eight criteria presented above. As the criteria of *originality* and *relatedness* are not affected by the changes between the phases, these two will instead be considered in the discussion.

Phase one

As mentioned, in the first phase, the researchers selected schools where they implemented problem-solving lessons in preschool classes themselves, and the teachers were observers. In this phase, the study was possibly of some *relevance* for the teachers as they agreed to have the researchers conduct the problem-solving lessons in their classrooms. As the teachers were observers of the lessons, it was possible for them to learn from the researchers and to pay attention to the actions of their students. However, in this phase, the relevance was probably greater for the researchers, who wanted to investigate, implement and evaluate a novel approach for early mathematics education. As the researchers conducted the lessons in all preschool classes, the lessons became more or less similar, increasing the external *validity*. However, the internal *validity* became lower as a mathematics lesson taught by a researcher is not like the ordinary mathematics lesson in these classrooms. While the researchers knew the content and the goal of the intervention well, they did not know the students. Thus, phase one became a case of showing that problem solving as a starting point for early mathematics was both possible and reasonable, at least if an enthusiastic researcher conducted the lessons (Palmér & van Bommel, 2018a). During this phase, only the researchers' biases needed to be considered regarding *objectivity* as the researchers both conducted and analysed the lessons. As the starting point of the project was to investigate whether problem solving was possible with young children, the researchers had high expectations and wanted the students to succeed, which could influence the results. To minimize biases, the results were presented at several seminars and conferences and were reviewed in journals. As the researchers were in the classroom and met the students, the criterion of *rigour and precision*, as in being sensitive to the meanings of the study participants in the analysis, was well met. Being in the classroom conducting the lessons also met the criterion of *reproducibility* as it was possible to provide all details of generating and analysing data. Finally, regarding *predictability*, tendencies and patterns could be identified because several classes were involved, making it possible to anticipate what would happen in a given situation.

Phase two

As mentioned, in the second phase, approximately 35 preschool class teachers from one municipality participated in a professional development programme focused on problem solving. In this phase, it was the preschool class teachers who implemented the problem-solving lessons in their classes, and while doing this they collected documentation (written solutions) from their students. As such, the teachers became part of the research process. It is reasonable to assume that the study had more *relevance* for the teachers in this phase as it influenced their everyday work in their classrooms. At the joint meetings, the teachers often emphasized, in both oral and written reflections, how participation in the study affected their teaching and their professional development in a positive way. The relevance was still high for the researchers as they could now implement and study their novel idea on mathematics education with the participation of a larger population. Furthermore, the researchers could study the necessary conditions for regular preschool class teachers to conduct the lessons. As the teachers now conducted the lessons in their preschool classes, the external *validity* became lower while the internal *validity* increased. This phase of the study became a case of showing that problem solving as a starting point for early mathematics is both possible and plausible also when the lessons are taught by the regular class teachers. In this phase, both researchers' and teachers' biases needed to be considered regarding *objectivity* as the teachers conducted the lessons and the researchers analysed the students' documentations. In these analyses, the researchers could not know exactly how the lessons had been conducted as they had not been present. Thus, *rigour and precision* became lower; it is hard to interpret meanings based only on documentations, and thus sensitivity to the participants' meanings decreased. Furthermore, the researchers not being in the classrooms conducting the lessons decreased *reproducibility* as details regarding the generating of data were harder to provide. Finally, regarding *predictability*, tendencies and patterns were more readily identified, as more teachers and students became involved, making it more possible to anticipate what would happen in a given (natural) situation. The extended role of teachers as implementers of problem-solving lessons in their classes and collectors of documentations was a prerequisite for being able to scale up the study this way and thus increasing predictability.

Phase three

Finally, in the third phase, the preschool class teachers continued working with the problem-solving lessons without instructions from the researchers, and on occasion the preschool class teachers who wanted to, implemented new problem-solving tasks developed by the researchers. Some of the preschool class teachers participated on every occasion and others participated occasionally. Thus, because the preschool class teachers could choose for themselves whether to participate, *relevance* for the teachers can be considered high. The relevance was still high for the researchers as they could expand the study with new problem-solving tasks and new interviews without the study being too time consuming for them. As in phase two, the teachers were conducting the lessons, which is why the external *validity* was low but the internal *validity* high. One could argue that the external validity became even lower than in the second phase, as in phase three the teachers tried out new lessons that had not been taught and analysed by the researchers beforehand. As such, there was no explicit model for them to follow. However, the teachers became more autonomous in deciding how to conduct the lessons, which became evident in their written reflections. This phase of the study became a case of showing that problem solving as a starting point for early mathematics is both possible and plausible even when the teachers are given considerable freedom in how they conduct the lessons. As in phase two, both researchers' and teachers' biases needed to be considered regarding *objectivity* as the teachers conducted the lessons and the researchers analysed the students' documentations. Now that the teachers had more autonomy in conducting the lessons, even more attention needed to be paid to their biases, which were sometimes apparent in their written reflections. In the analyses of students' documentation, the researchers could not know exactly how the lessons had been conducted as they had not been present. *Rigour and precision* became low; it is hard to interpret meanings based only on documentations, and thus sensitivity to the meanings of the participants decreased. Also, in this phase, the interviews were conducted by the teachers, and when analysing the students' answers the researchers had to assume that the questions were asked as they were written in the questionnaire. Furthermore, the researchers could not know whether and to what extent the teachers may have clarified any of the questions for the students. However, the teachers are probably more sensitive to the students than the researchers would be, and thus were able to give each student the best conditions for answering the questions. Similar to phase two, since the researchers did not conduct the lessons and were not in the classroom, *reproducibility* decreased as details regarding the generation of data were harder to provide. Finally, regarding *predictability*, tendencies and patterns regarding children's work with problem-solving tasks could be easily identified as many teachers and students became involved, making it more possible to anticipate what would happen in a given (natural) situation. Such tendencies and patterns were identified regarding children's understanding of the mathematical content (Palmér & van Bommel, 2018a; van Bommel & Palmér, 2016; 2018) and regarding the views children expressed on problem solving (Palmér & van Bommel, 2018b; 2020). Since the internal validity was high, the predictability was also high.

Summary

The methodological meta-analysis above shows that when the teachers were given an expanded role in the collaboration, the *internal validity* and the *predictability* increased. Regarding *relevance*, the study always had relevance for the researchers, but probably its relevance has increased for the teachers as they have been given an expanded role. Regarding *external validity*, *rigour and precision*, and *reproducibility*, these all decreased whilst the teachers were given an expanded role in the collaboration. Finally, *objectivity* became more complex as both teachers' and researchers' biases needed to be considered.

Discussion

The aim in educational design research is to develop theories that inform and guide the practice of teaching and learning. These theories are developed in an iterative cyclic process of designing and testing interventions situated within an educational context (Cobb & Gravemeijer, 2008). In such design cycles, collaboration between teachers and researchers is required (McKenney & Reeves, 2014), and different collaborations between researchers and teachers are of relevance to evaluate as different collaborations change the culture and professional expectations of both teachers and researchers (Cai, 2019). In this article, we have offered some insights into methodological constraints of such a development, for instance regarding the tension between ensuring external validity of a study versus enabling internal validity and improvement of practice. One threat to external validity is that teachers' notes might become influenced by what the they think will support the study most, and thus skewed data may be obtained. Also, teachers might provide students with clarifications of interview questions, clarifications not known by the researchers when analysing the data. However, concerning ethical issues on research with young children (Swedish Research Council, 2017), teachers are likely to be sensitive to the students and thus are able to give each student the best conditions for answering questions in interviews or on questionnaires.

Two of Kilpatrick's (1993) criteria have not been used in the results. One criterion not focused on is *originality*, which, according to Kilpatrick (1993), does not mean that the research should lack connection to prior research but that the research ought to provide new insights. When the study presented here was initiated, there were few published studies on problem solving with young students. This lack of knowledge initiated the design of the study in the first phase, with the goal of providing new insights. Before implementing problem solving in several preschool classes in phase two, we needed to investigate whether such an implementation was at all possible and plausible. Working in close collaboration with teachers in phases two and three is in line with decreasing the disconnection between research and practice, enabling research to have an actual impact on practice (for more details, see Cai, 2019). One issue often ignored in research is the detail in information needed for teachers to implement new forms or types of instructions (Cai, 2019). In the study presented here, phase three would not have been possible had the teachers not first participated in phase two. The instructions and collegial evaluations of the problem-solving lessons were crucial for the design of the third phase. Developing the study from phase one to phase three without phase two would have implied a different design with a different outcome.

The three phases presented and meta-analysed in this article are not to be considered as the answer to how all collaboration between researchers and teachers ought to be conducted in educational design research. The three phases are *one* example of how collaboration can develop and how the development coacts with the quality of the research. As has been shown, the three different phases have different strengths and weaknesses and they depend on each other. Sometimes we will find a need to conduct design cycles in line with phase one, for example, when implementing content with which researchers and teachers have had limited prior experience. However, if we aim for educational design research studies to have an impact on practice, to decrease the disconnection between research and practice (Cai, 2019), and to bring value for both research and practice (Bakker, 2018; Prediger et al., 2015), teachers need to be given a prominent role in the studies. As the internal validity is of great importance to the reproducibility of the results of a study, an increase of internal validity, at the cost of a decrease of the external validity, might be worth it.

The other criterion not focused on in the results is *relatedness*, which implies that research in mathematics education ought to be related to both mathematics and education. The study per se aimed to provide new insights into the teaching of problem solving in mathematics with young students. In other articles, the study's implications for the teaching and learning of mathematics have been described (Palmér & van Bommel, 2018a; 2020; van Bommel & Palmér, 2016, 2018). Thus, the study per se relates clearly to both mathematics and education. However, the three phases described in this study might seem to be disconnected from the mathematical content and be regarded as merely a methodological result. Even though mathematics is the content of the educational design research used as an example in this article, the collaboration between researchers and teachers is not related specifically to the mathematical content of the intervention. Yet, the content of problem solving clearly frames the design of each of the three phases described. Different designs of collaborations between teachers and researchers in studies are of importance for enabling research to have an impact on practice and decrease the disconnection between research and practice (Cai, 2019). The methodological meta-analysis presented in this article is therefore of importance for the community of mathematics education as it discusses *how* collaboration between teachers and researchers may coact with the results of studies, including the relevance of the studies, and thus influence both mathematics education and research on mathematics education.

References

Anderson, T. & Shattuck, J. (2012). Design-based research: a decade of progress in education research? *Educational Researcher*, 41 (1), 16–25. doi: 10.3102/0013189X11428813

Bakker, A. (2018). *Design research in education: a practical guide for early career researchers*. Routledge.

Bommel, J. van & Palmér, H. (2016). Young children exploring probability: with focus on their documentations. *Nordic Studies in Mathematics Education*, 21 (4), 95–114.

Bommel, J. van & Palmér, H. (2018). Enhancing young children's understanding of a combinatorial task by using a duo of digital and physical artefacts. *Early Years*, 1–14. doi: 10.1080/09575146.2018.1501553

Borko, H. (2004). Professional development and teacher learning: mapping the terrain. *Educational Researcher*, 33 (8), 3–15. doi: 10.3102/0013189X033008003

Cai, J. (2010). Commentary on problem solving heuristics, affect, and discrete mathematics: a representational discussion. In B. Sriraman & L. English (Eds.), *Theories of mathematics education: seeking new frontiers* (pp. 251–258). Springer. doi: 10.1007/978-3-642-00742-2_25

Cai, J. (2019). Research pathways that connect research and practice. *Journal for Research in Mathematics Education*, 50 (1), 2–10. doi: 10.5951/jresematheduc.50.1.0002

Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V. & Hiebert, J. (2017a). A future vision of mathematics education research: blurring the boundaries of research and practice to address teachers' problems. *Journal for Research in Mathematics Education*, 48 (5), 466–473. doi: 10.5951/jresematheduc.48.5.0466

Cai, J., Morris, A., Hwang, S., Hohensee, C., Robison, V. & Hiebert, J. (2017b). Improving the impact of educational research. *Journal for Research in Mathematics Education*, 48 (1), 2–6. doi: 10.5951/ jresematheduc.48.1.0002

Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V. & Hiebert, J. (2018). Reconceptualizing the roles of researchers and teachers to bring research closer to teaching. *Journal for Research in Mathematics Education*, 49 (5), 514–520. doi: 10.5951/jresematheduc.49.5.0514

Cobb, P. & Gravemeijer, K. (2008). Experimenting to support and understand learning processes. In A. E. Kelly, R. A. Lesh & J. Y. Baek (Eds.), *Handbook of design research methods in education: innovations in science, technology, engineering, and mathematics learning and teaching* (pp. 68–95). Routledge.

Grundén, H. (2017). Diversity in meanings as an issue in research interviews. In A. Chronaki (Ed.), *Mathematics education and life at times of crises: proceedings of MES 9 conference* (Vol. 2, pp. 503–512). University of Thessaly.

Hiebert, J. & Grouws, D. A. (2002). The effects of classroom mathematics teaching on students learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). NCTM & Information Age.

Kilpatrick, J. (1993). Beyond face value: assessing research in mathematics education. In G. Nissen & M. Blomhøj (Eds.), *Criteria for scientific quality and relevance in the didactics of mathematics* (pp. 15–34). Danish Research Council for the Humanities.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Greenwood Publishing.

Lesh, R. & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763–799). NCTM & Information Age.

McKenney, S. & Reeves, T. (2014). Educational design research. In J. M. Spector, M. D. Merrill, J. Elen & M. J. Bishop (Eds.), *Handbook of research on educational communications technology* (pp. 131–140). Springer. doi: 10.1007/978-1-4614-3185-5_11

National Agency for Education (2014). *Preschool class: assignment, content and quality*. Swedish National Agency for Education.

Niss, M. A. (2018). The very multi-faceted nature of mathematics education research. In E. Bergqvist, M. Österholm, C. Granberg & L. Sumpter (Eds.), *Proceedings of PME 42* (Vol. 1, pp. 35–50). Umeå University.

Patton, M. Q. (2002). *Qualitative research & evaluation methods*. Sage.

Palmér, H. & Bommel, J. van (2018a). Problem solving in early mathematics teaching: a way to promote creativity? *Creative Education*, 9, 1775–1793. doi: 10.4236/ce.2018.912129

Palmér, H. & Bommel, J. van (2018b). Young students' feelings towards problem solving tasks: What does "success" imply? In B. Rott, G. Törner, J. Peters-Dasdemir, A. Möller & Safrudiannur (Eds.), *Views and beliefs in mathematics education* (pp. 69–78). Springer. doi: 10.1007/978-3-030-01273-1

Palmér, H. & Bommel, J. van (2020). Young students posing problem-solving tasks: What does posing a similar task imply to students? *ZDM*, 52 (4), 743–752. doi: 10.1007/s11858-020-01129-x
Peressini, D., Borko, H., Romagnano, L., Knuth, E. & Willis, C. A. (2004). Conceptual framework for learning to teach secondary mathematics: a situative perspective. *Educational Studies in Mathematics*, 56 (1), 67–96. doi: 10.1023/B:EDUC.0000028398.80108.87
Prediger, S., Gravemeijer, K. & Confrey, J. (2015). Design research with a focus on learning processes: an overview on achievements and challenges. *ZDM*, 47 (6), 877–891. doi: 10.1007/s11858-015-0722-3
Scheiner T. (2019). If we want to get ahead, we should transcend dualisms and foster paradigm pluralism. In G. Kaiser & N. Presmeg (Eds.), *Compendium for early career researchers in mathematics education*. Springer. doi: 10.1007/978-3-030-15636-7_27
Sierpinska, A. (1993). Criteria for scientific quality and relevance in the didactics of mathematics. In G. Nissen & M. Blomhøj (Eds.), *Criteria for scientific quality and relevance in the didactics of mathematics* (pp. 35–74). Danish Research Council for the Humanities.
Stake, R. E. (1995). *The art of case study research*. Sage.
Swedish Research Council (2017). *Good custom in research*. Vetenskapsrådet.

Hanna Palmér

Hanna Palmér is Professor of Mathematics Education at Linnaeus University. Her main research interests are mathematics teaching and learning among preschoolers as well as problem solving in early mathematics education.

hanna.palmer@lnu.se

Jorryt van Bommel

Jorryt van Bommel is an associated professor at Karlstad University in Mathematics Education as well as at Høgskolen i Innlandet, Norway. Her research interests are related to problem solving in early year mathematics and professionalisation of mathematics teachers.

jorryt.vanbommel@kau.se