New mathematics teachers' learning when participating in induction on mathematics education

A case study of two lower secondary teachers

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Newly qualified mathematics teachers often face teaching problems during their initial teaching years. We report on a study in which we designed an induction programme in mathematics education to support new teachers in tackling these problems. Using a social practice perspective, we investigate what and how two new lower secondary teachers learn by participating in the programme. We show that, while one teacher exceeds our expectations for learning, the other learns very little. We also show that the two teachers' learning is reflexively related to school-local and broader contexts, which helps to explain the differences in their learning.

Studies show that the initial years of classroom teaching can be particularly demanding for newly qualified mathematics teachers (McGinnis et al., 2004; Richter et al., 2013; Potari & Georgiadou, 2009). This demanding situation is generally referred to as a "reality shock" (Veenman, 1984, p. 143), and, in mathematics education, it is often studied as a discrepancy between the mathematics teacher's teacher training programme and the school context. The new mathematics teacher's perception of school culture has been highlighted as the main limiting factor for the teacher's long-term and successful enactment of their teaching visions (McGinnis et al., 2004; Skott, 2013) in Denmark (Skott et al., 2011) and in other countries (Frykholm, 1999; McGinnis et al., 2004; Potari & Georgiadou, 2009).

The new mathematics teacher not only has to teach specific students specific content but also has to learn to teach within a particular workplace setting and with particular colleagues (Haggarty et al., 2011). Mastering this reality exposes new teachers to external pressures, feelings of

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uncertainty and disequilibrium (Potari & Georgiadou, 2009), and even the decision of whether to leave the profession.

It is only since the mid-1980s that the idea of providing support to teachers during their initial teaching years has been attracting attention politically and in research. This attention resulted primarily from alarmingly high attrition rates of teachers in Western countries (Ingersoll & Smith, 2004). In the US and some European countries, but not in Denmark, a variety of induction programmes for teachers have been implemented systematically (European Commission, 2010; Ingersoll & Strong, 2011). These programmes aim to help new teachers adjust to their school culture, support competent classroom practice, and, ultimately, retain teachers (Schwartz & Ticknor, 2018). Most of the programmes focus on pedagogical issues and only a limited number on subject teaching and students' learning of the subject.

In Denmark, the initial support offered to new mathematics teachers depends on the individual school and usually it does not relate to mathematics teaching or students' mathematics learning (EVA, 2011). New mathematics teachers often feel alone in tackling the problems they face in classrooms. The dual aim of our study is therefore to design an induction programme that focused specifically on mathematics education (hereafter ME-IP) and to investigate new mathematics teachers' learning when participating in it.

In this article, we first provide an outline of teacher training and induction initiatives in Denmark. We then present a literature study on teacher induction research with a focus on aspects relevant for the design of a ME-IP. Following this, we present our social practice theoretical approach to teacher learning, *Patterns of participation* (PoP) (Skott, 2017). After describing the ME-IP design and methodological approach, we present and discuss our results.

The Danish context

In Denmark, teacher training (to teach years 1–10) comprises four years of training at bachelor level at university colleges. Student teachers normally specialise in three school subjects. The mathematics programme has two further specialisations: primary (years 1–6) and lower secondary (years 4–10), each with a scope of 40 ECTS (UFM, 2015). Compared with other countries, including Norway and Sweden, this scope is limited, especially for the lower secondary level (Mullis et al., 2016). The specialisation programmes emphasise both the subject itself and educational issues, and they are formally described as "an interplay between mathematics subjects, mathematics competencies, mathematics education and the mathematics teachers' practices" (UFM, 2015). The student teachers are expected to work in ways that model the teaching-learning processes they will have to initiate once they've graduated, with a specific focus on engaging students in mathematics processes (i.e. competencies) such as reasoning or modelling.

In Denmark, the teacher attrition rate is relatively high, with 1 in 6 new teachers leaving the profession during their first year (AE, 2016). But, in Denmark, unlike in Norway and Sweden, there have been no initiatives to implement formalised induction programmes or mandatory initial support for teachers (European Commission, 2018; Olsen et al., 2020). Initial support is thus arbitrary and depends on the individual school. Only 50 % of new teachers are allocated local-school mentors. These mentors tend to focus on practical and pedagogical aspects, yet new teachers also claim to face problems related to the subjects they teach, such as subject-specific needs of individual students (EVA, 2011).

The few national studies conducted in this area show that new mathematics teachers tend to adopt the traditional practices found in many classrooms within their first teaching years (Skott et al., 2011). In this context, traditional practices refer to a focus on mathematics products (e.g. a specific algorithm) and not on processes.

We were motivated to design a ME-IP in order to support new lower secondary mathematics teachers in tackling the problems they face in classrooms. To describe what inspired this design, we will now review the limited empirical research available on teacher induction programmes in mathematics education and supplement this review with general results and trends.

Induction and its role in teacher development

Our literature review shows that empirical research into induction tends to take place in the US, to focus on pedagogical issues, and to be based on quantitative data. Most research in this area investigates the effects of induction in relation to one of three perspectives: teacher characteristics, student achievements, or classroom practices (Ingersoll & Strong, 2011). In particular, by conducting large-scale surveys, studies examine the effects of induction on teacher characteristics (e.g. sense of self-efficacy, job satisfaction) (Richter et al., 2013) and whether these effects lead to teacher retention (Barnatt et al., 2017; Matsko et al., 2007). Such studies generally report positive effects (Ingersoll & Strong, 2011).

One criticism of this research is that no single teacher characteristic or workplace condition determines teachers' career decisions (Barnatt et al., 2017). Instead, Cochran-Smith et al. (2012) argue that we should examine "multiple variations of practice-coupled-with-career decisions" (p. 846). In a long-term case study, these authors explore relations between the quality of new teachers' teaching (strong, adequate, weak) and their career decisions (stayed at the same school, moved schools, left teaching). They deduce five configurations, one of which describes the role we wish our ME-IP to play: "going strong and staying on". Teachers that go strong and stay on are committed to the quality of their classroom teaching and to the success of all students, and they seek support within or outside their school when conditions are challenging. It is important that new teachers develop such commitments and support strategies to tackle classroom problems. For this, they need meaningful *Professional development* (PD) and appropriate opportunities for collaboration (Cochran-Smith et al., 2012).

However, it is less obvious what constitutes meaningful PD and appropriate collaboration opportunities for new mathematics teachers. Some studies argue that subject teaching is a more effective context for meaningful PD than general induction programmes (Luft et al., 2011; McGinnis et al., 2004; Santagata & Lee, 2019). Yet research into ME-IPs is very limited. We would like to highlight three results from two studies: Haggarty et al. (2011), and Schwartz and Ticknor (2018), which examine how a formalised ME-IP based on school mentoring in the UK and an innovative university-based ME-IP in the US, respectively, support new mathematics teachers.

The first result is that school contexts and those with power in these contexts (e.g. mentors) influence the nature and quality of new teachers' PD during the ME-IPs. In both studies, the schools expected the new teachers to accept their norms for teaching and PD, and the teachers' mentors focused on classroom management, considering it a prerequisite for teaching mathematics – to the extent that, once such issues were resolved, the mentors saw no need for further support (Haggarty et al., 2011). The new teachers were not encouraged to consider teaching through other lenses or to employ more creative teaching strategies (e.g. from teacher training).

The second result is that collaboration with colleagues tended to confine the new teachers' learning space. Some colleagues made the new teachers conform to teaching that contradicted their visions (Schwartz & Ticknor, 2018), and, together with the mentors, they often acted as a powerful filter that turned an otherwise collaborative setting into a restrictive setting (Haggarty et al., 2011).

The third result is that external support was crucial for the new teachers' PD; for example, providing opportunities to see "what's possible" (Schwartz & Ticknor, 2018, p. 404) in classrooms in order to enact the expected mathematics curriculum. The external mentors in this study taught at other municipal schools and had Master's degrees in elementary mathematics education.

These three results show that it is important to carefully select and train mentors in relation to specific aspects of mathematics teaching and student learning. Comparing different kinds of mentoring in a large-scale study in Germany, Richter et al. (2013) further conclude that *constructivist-oriented mentoring* (i.e. when mentors initiate inquiry stances towards teaching) supports new mathematics teachers more appropriately than *transmissive-oriented mentoring* (i.e. when mentors convey their own teaching ideas and focus on technical skills). Their conclusion is based on teachers' self-reported experiences of (among other things) self-efficacy and teaching enthusiasm.

To summarise, in order to design a ME-IP that supports new teachers in "Going strong and staying on", we build on three ideas inspired by the above studies. The first idea is that, to be meaningful, PD should view new teachers' classrooms as sites for teacher learning and as opportunities to address the problems these teachers face. The second idea is that the new teachers and their mentors should collaborate on developing an inquiry teaching stance (constructivist-oriented mentoring), and the third idea is that new teachers should be offered the opportunity to collaborate with external experts in mathematics education.

A social practice theoretical approach

For our purpose, two results from Goldsmith et al.'s (2014) review of mathematics teachers' learning are important. The first result is that most research focuses on teachers' beliefs and their dispositions to act upon them. The second result is that research so far has had little to say about how teachers learn.

In contrast to the first result, a small number of studies focus on the mutual transformative relationship between teachers and their social worlds and, in particular, how it influences new teachers' initial teaching experiences and PD. Such studies (e.g. Barnatt et al., 2017; Cochran-Smith et al., 2012; Losano et al., 2018; Skott, 2017) take what Russ et al. (2016) call a situated and socio-cultural perspective on learning to teach. They highlight new teachers' experiences from contexts such as teacher training, classroom practice, and collaboration with colleagues and parents as influential for what and how new teachers learn.

We adopt a situated perspective, called *Patterns of participation* (PoP) (Skott, 2019), that focuses on how school-local and broader social enterprises are reflexively related to and co-determiners of new teachers' learning. A PoP analysis enables us to interpret and explain teachers' learning in context (C.K. Skott et al., 2021), which is promising given the above call for insight into how teachers learn.

PoP is inspired by social practice theory and draws on the notions of practice and *Figured worlds* (FWs). A practice is defined as ”doing in a social and historical context that gives structure and meaning to what we do” (Wenger, 1998, p. 47), while FWs are defined as ”socially and culturally constructed realm[s] of interpretation, in which particular characters and actors are recognised, significance is assigned to certain acts, and particular outcomes are valued over others” (Holland et al., 1998, p. 52).

Unlike other studies of new teachers based on social practice theory (Barnatt et al., 2017; Cochran-Smith et al., 2012; Losano et al., 2018; Ma & Singer-Gabella, 2011), PoP focuses on changes in how a teacher participates in social interactions over time (e.g. classroom interactions) rather than how a teacher moves from peripheral participation towards more comprehensive participation in a specific, pre-established community or FW.

In PoP, a teacher's participation in social interactions is understood as influenced by their interpretation of the immediate situation and their simultaneous meaning-making in which they continuously interprets ”others' actions symbolically, including their actual or possible reactions to one's own behaviour” (Skott, 2019, p. 472). The teacher thus approaches themself with the attitude others approach them with. This ”other” could be a colleague or a parent, but it could also be a social group or community, in which case the teacher approaches themself with the attitude of generalised others. PoP interprets practices and FWs as possible generalised others. While a teacher ”relates at any point in time to a multitude of practices and FWs” (Skott, 2017, p. 139), there are ”patterns in the ways in which [the teacher] participates in these practices and contributes to their continuous reconstitution and renegotiation” (Skott et al., 2011, p. 32). We use *constellation* to denote the practices and FWs that play the most prominent roles for a teacher's pattern of participation in specific situations.

Our research aim is to investigate whether participating in a ME-IP helps new mathematics teachers learn to tackle their classroom problems. Using PoP to conceptualise teacher learning as changes in a teacher's patterns of participation in classroom and collegial interactions, our research question is:

> What and how do new lower secondary mathematics teachers learn by participating in a ME-IP in terms of shifts among their practices and FWs?

ME-IP design

In addition to induction research, our ME-IP design is also inspired by practice-based research, particularly Cai et al.'s (2017a, 2017b) emphasis on the problems faced by teachers, and Morris and Hiebert's (2011) concept of an *instructional product*. An instructional product is inspired by how professionals outside and inside education (e.g. Japanese lesson study groups) build useful and changeable products to solve problems and to improve their practice. The authors identify three important features to support a work culture in creating this product: that all members: 1) share the same problem to which the product offers solutions, 2) collect data to test small product changes, 3) contribute to improving the product so that it profits from multiple sources of expertise and is owned by all members. In line with Morris and Hiebert (2011), the instructional product for which we are aiming in our ME-IP is a lesson plan, and parts of the design (phase 2 and 3) are thus inspired by the three collaborative processes of lesson study: planning a lesson, observing the lesson, and sharing reflections (Murata, 2011). Our ME-IP consists of four phases:

1 *Selection of a shared problem*: In discussions with the new teachers, we (the authors) selected a problem that all the new teachers faced during their initial classroom teaching.

2 *Mentor training*: We provided mentors with guidance on constructivist-oriented mentoring (Richter et al., 2013) and mathematical and didactic aspects of the shared problem.

3 *School-based collaboration*: We facilitated two rounds of lesson study-like collaboration between pairs of new teachers and their mentor, where each pair twice planned a lesson to tackle the shared problem, the mentor observed the new teacher teach the lesson, and they reflected on this lesson together. We participated in the first round.

4 *Broader collaboration*: We collaborated with all the participants to create a lesson plan (i.e. the instructional product) to tackle the shared problem. To improve this plan, each new teacher took turns to teach a lesson based on the plan while their mentors and we observed. During the three reflection sessions, all participants suggested improvements to the plan and for the next new teacher. Finally, all participants met to create a final lesson plan. Two of us participated in these activities, while the third observed.

Methodological approach

Context of the study

Initially, five new teachers participated in the ME-IP in 2019. However, two of them left before completing it. The three new teachers completing the ME-IP were all females and in their twenties. They graduated as lower secondary mathematics teachers from prestigious Danish university colleges in 2018 and started working at different schools in the Copenhagen area immediately after. Two of them, Amy and Laura, expressed strong commitments to the shared problem and high levels of frustration regarding the lack of support they received from, among others, their mentors (i.e. experienced mathematics teachers). We therefore selected Amy and Laura as our two cases in this multiple case study (Cohen et al., 2011).

Generation of data

We collected data in three of the phases listed above, omitting phase 2 due to our focus on new teachers. In phase 1, we conducted a semi-structured interview with all five new teachers (2 hours), which focused on the problems they faced during their first six months as teachers. They agreed on inquiry teaching as a common pressing problem. To gain a deeper insight into their difficulties, we conducted classroom observations (2 x 45 min) and follow-up interviews (2 x 1 hour) with, among others, Amy and Laura.

We collected data from both rounds in phase 3: classroom observations (4 x 90 min) and shared reflections (4 x 1 hour) with Amy and Laura. In phase 4, we collected data from: the joint planning (90 min), classroom observations of (2 x 1 hour) and shared reflections with (2 x 1 hour) Amy and Laura, and the joint final session at the university college (2 hours).

We video-recorded all classroom observations and audio-recorded the other activities. All data, apart from observations, were transcribed.

Analysis of data

We conducted a two-step analysis. In the first step, we analysed data from phase 1 to empirically infer the practices and FWs that played the most prominent roles in how Amy and Laura dealt with inquiry teaching during their initial classroom teaching. On this basis, we constructed an initial constellation of prominent practices and FWs for each of them.

In the second step, we analysed data from phases 3 and 4 to similarly infer the practices and FWs that played a prominent role in how they participated in the ME-IP in relation to inquiry teaching. We constructed a second constellation for each of them. By comparing their two pairs of constellations, we both derived, what each of them had learned, and gained an insight into how they learned or what hindered this learning.

To infer practices and FWs, we coded transcriptions of our data without using pre-defined codes inspired by grounded theory (Charmaz, 2014). As examples of initial codes, we initially coded Amy's utterance "I don't always sum up the lesson with students ... as many zoom out ... though in theory they would benefit from talking about what we have been doing" (phase 1) as "experiencing a dilemma between student reactions and teacher training". We then gathered our initial codes into focused codes, such as "steering by student reactions", which we assembled into practices and FWs – in this case relationing, which denotes Amy's ways of establishing relationships to her students inside and outside the classroom. In similar ways, we identified the following practices and FWs for Amy and Laura: *teacher training* (TT world) (their teacher training with the characteristics described above), *school world* (their school's culture and teaching approach), and *own schooling* (characters, acts and values they have themselves experienced as students). Although these practices and FWs share general characteristics, Amy and Laura seem to derive different meaning from them as they participate in classroom interactions and collegial collaborations.

Our participation in the ME-IP

As teacher educators, we were attentive to our participant role. However, we had no prior knowledge of the participants that could influence our own nor their participation. We strived to work with them as colleagues involved in solving a shared problem and to create an atmosphere of mutual respect and equality. Based on their feedback, we assessed that we succeeded in creating such a collaborative environment, and also a working environment based on Morris and Hiebert's (2011) features mentioned above (i.e. participants sharing the problem, providing data, and contributing to improve its solution).

Our participation should also be considered as the new teachers' opportunities to collaborate with external experts. We highlight two ways in which we have particularly contributed to this. First, we decided to challenge their approach to inquiry teaching prior to phase 4, especially their emphasis on extensive out-of-class investigations. We proposed two mathematical inquiries and they selected the *sum problem*: Which natural numbers can be written as a sum of consecutive natural numbers?

Second, we adapted Artigue and Blomhøj's (2013) definition of inquiry teaching, in which "students are invited to work in ways similar to how mathematicians work" (p. 797) by asking questions, making observations and formulating hypotheses. We added two purposes to this. The first purpose is to motivate students and to give them responsibility for themselves and each other. The second purpose is to expand the mathematical objectives to focus not only on products but also on processes in such a way that students learn about products and simultaneously learn to engage in processes (Skott & Skott, 2019). We did not discuss the definition with the new teachers, but we used it to guide our participation.

Results

Amy – becoming a teacher

Amy first meets inquiry teaching during her teacher training and a character emerges for her "I really want to be an investigative teacher ... who promotes dialogues about mathematics". Amy stresses her relationship with students as her main motivation for teaching "I really love to immerse myself in what happens [socially in classrooms]". She considers good relations as a prerequisite for student learning. When Amy graduates, her orientations towards *relationing* seem to play a role in her recontextualisation of inquiry teaching, in which "we do not necessarily need the right result" but students' many ways to approach a solution are important.

When Amy as a new teacher engages her students in inquiry activities, she fails to relate their various approaches to the content. The students get confused, and Amy describes the activities as "a waste of time". She seeks advice from her colleagues, also on more pedagogical aspects, but she receives only a few advices that do not help her. She feels left on her own and experiences a school culture that prioritises the independence of its teachers. Amy then turns to her students. They prefer their familiar product-oriented tasks, and Amy gradually gives up on her inquiry character and resorts to the practices that dominate the *school world* and that she knows from her *own schooling*. Amy recounts how she learns to constantly adjust her teaching decisions-in-the-moment to fit the students' moods and energy levels, often at the expense of opportunities to teach the content more deeply. *Relationing* thus becomes dominant for Amy's participation in classroom interactions.

After half a year, *relationing*, *school world* and the product-oriented practices constitute Amy's initial constellation. Feeling confident in this teaching approach, Amy completely rejects *TT world* and accuses inquiry teaching of being ”a non-evidence based, random paradigm”. She feels acknowledged as a competent teacher by her students, and she ignores her potential doubts about her approach by ”not taking things too seriously like my colleagues” (*school world*).

Amy's learning

Based on data from phase 4, it seems that Amy's initial constellation remains largely unchanged by her participation in the ME-IP, with only two minor shifts towards the *TT world*.

When working on the sum problem during the joint lesson planning, Amy focuses on its potential to motivate students – ”I think they'll think it's fun” (*school world/relationing*) – and not on its learning potentials (*TT world*). When asked (by us) to solve the problem, Amy focuses on ”finding a system ... there are many ways to find the solution” which is in line with her re-contextualisation of inquiry teaching. It appears that she quickly and mentally plans the lesson in her usual way by drawing on her initial constellation. She acts with self-confidence, and her mentor often seeks her affirmation, not the other way around.

When teaching the lesson in year 7, Amy uses the sum problem as her focal point, but she engages in product-oriented teaching practices. We highlight two aspects of this. First, Amy focuses on the products of the students' inquiries and does not prompt the students to explain, validate or argue for their systems (as stressed in the lesson plan). For instance, when Amy asks about which systems they discovered, a student replies ”The odd numbers, because one of them is almost half of it and then we just have to add”. However, Amy ignores the student's attempt to justify her system and only focuses on the product aspects ”The odd numbers. Can we do it with 87?”. Second, as just exemplified, Amy's communication with students is characterised by an initiation-response-evaluation approach (Mehan, 1979), even when asking the plan's open questions.

In the reflection session, Amy holds on to her initial constellation. As such, she insists that the two goals of the sum problem are to find products (*own schooling*) and to ”solve this problem together”, which relates to the pedagogical purpose of inquiry teaching (*school world*). When confronted with student observations (like the one above), Amy realises that reasoning was not part of the lesson. She legitimises this by referring to *school world*, ”Usually, as a teacher, you do not spend time working with reasoning”, and remains confident in her approach. Drawing on her initial constellation, she dismisses the idea of spending more time on the problem (as proposed by the other new teachers), as ”students would then have to do more of the same. Doing bigger and bigger calculations would just tire them out". However, when repeatedly pushed to interpret the students' actions in terms of opportunities for their processual learning, she finally asks, "It's fun to do such an inquiry, but ... what competences do the students gain by doing it?". We interpret this as her first minor shift to *TT world* as, by asking this question, she seems to render her previous entertainment reasons insufficient.

When Laura, in the final session, stresses the importance of students justifying their systems, Amy replies, "I forgot this so much in my first class", and she recounts how she taught the lesson differently in another class, "I asked, 'Why can we do it?' and then suddenly it became more difficult for them as they had to explain it". By requiring explanations, Amy seems to contribute differently to the classroom interactions, which indicates her second minor shift towards *TT world*.

Importantly, Amy's mentor, Rau, is the only mentor who teaches the lesson, and Amy and Rau observe each other's lessons. Rau himself strives to teach in a more inquiry-based manner, and he is unable to offer Amy meaningful support on how to do this. He is inspired by her ways of managing the classroom and, rather than mentee and mentor, they act like colleagues.

Laura – becoming a teacher

Laura describes her path to teacher training as coincidental. She disliked mathematics as a student until she got a "fun" teacher in year 8 and a "structured" teacher in year 9 "the combination of the two made me become good at maths but also made me really enjoy it". Both these teaching characters represent traditional teaching as they focus on memorising products, but they differ in their pedagogical approaches. The *fun* teacher is playful (e.g. exclaiming "Oh no, a negative sign outside the parentheses!"), while the *structured* teacher covers the content systematically ("we must focus on exams"). We interpret both as important characters for Laura's way of becoming a teacher.

Inquiry teaching, which Laura first encountered during her teacher training, fits well with the *fun* character from Laura's *own schooling*. She uses opportunities for fun by focusing on out-of-class activities, by finding mathematics "in things that normally aren't maths", and by not following "the textbook from A to B".

As a new teacher, Laura draws on the *fun* character and *TT world*, distancing herself from the product-oriented parts of her *own schooling*. However, she is surprised to find that students oppose her "I have many good ideas on paper. But when I enter the classroom ... they turn out to be really bad". She describes a harsh classroom atmosphere of "let's see who we can kick out fastest", and she leaves the job after two months. However, she returns soon after, supported by her colleagues. This marks a shift in her constellation of practices and FWs. She responds to student resistance by turning to the *structured* character and *own schooling* and by following the textbook more systematically. She also uses her colleagues' approach to classroom management (e.g. using hand signs for specific behaviour), thus orienting herself towards *school world*. She distances herself from *TT world*, criticising it for being "unstructured" and claiming that, contrary to the textbook, it does not provide tools to structure students' work.

Thus, after half a year, Laura's initial constellation comprises the product-oriented practices from *school world* and *own schooling*, and the *structured* character. Although it has become easier for her to teach, she is unhappy. Asked to describe a typical day, she replies "long". Shortly after, she moves to a nearby private school.

Laura's learning

We present three shifts in Laura's initial constellation. The first shift is a re-contextualisation of inquiry teaching, and it takes place at her first school in phase 3. Laura, her mentor and one of the authors discuss how to expand a closed task after observing two students working on it in an inquiry-based way. Laura realises that she "got the image of inquiry teaching wrong. I though it should be huge, wild ... that we had to explore things without using real maths". Laura re-contextualises inquiry teaching to also include mathematical investigations, and she subsequently draws on this.

The second shift takes place when Laura succeeds in connecting the unstructured process-oriented practices from her *TT world* with her *structured* character in phase 4. During the joint lesson planning, Laura struggles to connect them "it's difficult to introduce the problem so I don't give them too much but still give them something". She keeps asking for suggestions on how to structure the students' work. When teaching the lesson in year 6, Laura is able to connect them by using two suggestions from the mentors: to introduce the problem as a claim to be disproved (e.g. "I claim that there are 6 numbers between 1 and 30, that cannot be written as a sum of consecutive numbers. Am I right?") and to visually represent the students' investigations by using a chalk-drawn number board, where students cross out numbers as they find their sum(s). She gathers all the students around the board to listen to and discuss their findings using the board as a mean to structure their work, and she provokes their reasoning by asking "why", "Why can all uneven numbers [be written as sums of consecutive numbers]?". She thus moves away from her *own schooling* and *school world*.

The third shift is a further expansion of her ways of contributing to inquiry teaching. In the final session, Laura describes how she has encouraged specific students to share and explain their strategies in the next lesson, "I noticed students who had made good systems, so I asked the ones I wanted to hear from if they had a system [...] and to explain it to the others".

Hence, Laura fundamentally changes her ways of participating in classroom interactions. From drawing initially on the product-oriented practices and the *structured* character from *school world* and *own schooling*, Laura learns to structure inquiry teaching by transforming and connecting process-oriented practices (*TT world*) with her *fun* and *structured* characters (*own schooling*). At her new school, she positions herself with confidence as an inquiry mathematics teacher aiming to inspire colleagues "this is my personal school project: to show that inquiry teaching is manageable" (final session).

Discussion

Previous research into teacher induction tends to be quantitative and focus on pedagogical issues. Only a few qualitative studies have investigated induction programmes that focus on mathematics education. Within these, school contexts and those that involve expert figures, such as mentors, are shown to influence the quality of new mathematics teachers' PD. Mentors tend to focus on classroom management, considering it a prerequisite for mathematics teaching. Collaboration with colleagues is also shown to confine the new teacher's learning space.

Our study fills a gap in induction research and takes as a starting point that pedagogical issues such as classroom management are tied to the subject content. Our contribution is to understand how it may work in induction when we apply a content perspective and thus challenge the views that pedagogical issues should be dealt with separately from teaching mathematics or that they should be seen as a prerequisite for mathematics teaching.

It is important to ask whether our ME-IP succeeded in supporting the new teachers in tackling the problems they faced: in one case, it succeeded beyond expectation, and, in the other case, it made little difference. Amy hardly changed her pattern of participation in inquiry teaching but kept to her initial constellation dominated by traditional classroom practices, while Laura fundamentally changed her participation with major shifts between her initial and final constellations.

Importantly, aspects of what Laura learnt can be said to be pedagogical but tied to the content, such as how to introduce an activity so that it is accessible for all students. Laura further seemed to transform her overall ways of interacting with students and colleagues beyond the context of the ME-IP, which will help her tackle future classroom problems. The two teachers thus differ markedly in terms of *what* they learnt by participating in the ME-IP.

Our study also shows that new teachers' actions and meaning-making are dependent on contexts and reflexively related to school-local and broader social enterprises. For Amy, *relationing* had become so dominant that, in the ME-IP, it was difficult for her to establish a content-based profile. She had already highlighted the importance of having a productive relationship with students during her teacher training, but, during her initial teaching, it became *the* way Amy survived as a teacher. The students' mood, expectations, and energy levels became her dominant orientations. For her, inquiry teaching primarily served the pedagogical purpose of motivating students. Our analysis shows that this orientation significantly limited her in developing inquiry teaching practices related to the content. In similar ways, our analysis shows that Laura's learning is reflexively dependent on contextual aspects related to both the local context (*school world*) and beyond (e.g. the *fun* and *structured* characters from her *own schooling*). These analyses provide insight into *how* Amy and Laura learn to teach.

Hence, our study contributes an exploration of *how* new mathematics teachers learn, as called for by Goldsmith et al. (2014). Contrary to most research on mathematics teachers' learning, which focuses on the teachers' beliefs, our study focused on the dialectical relationship between new teachers and their social worlds, and it offered an contextual interpretation of the teachers' learning that seems particularly relevant considering the complex situations new teachers find themselves in.

In line with induction research (e.g. Ingersoll & Strong, 2011), the two teachers in our study experienced being left alone in succeeding or failing within the confines of their classrooms, and, despite their strong teaching visions, like many other new mathematics teachers, they soon taught in a traditional way (e.g. Haggarty et al., 2011; Richter et al., 2013). Amy and Laura would most likely have continued in this way without the ME-IP. Our study thus confirms that the initial years of teaching seem to act as an indicator for long-term teaching effectiveness, making induction an essential part of new teachers' PD (Schwartz & Ticknor, 2018).

Our study further testifies that mentors, even when experienced mathematics teachers, tend to focus on practical and pedagogical issues (Haggarty et al., 2011; Schwartz & Ticknor, 2018). However, although unaccustomed with inquiry teaching, our mentors gradually engaged in constructivist-oriented mentoring, and they each established a more equal than hieratical relationship with the new teachers. We claim that these two aspects are vital in establishing a collaboration in which both mentors and the new teachers can learn. The ME-IP played a significant role in this regard, especially the mentor training in phase 2 and the apprenticeship-like mentoring in phase 3, in which we (as teacher educators) modelled in practice the kind of mentoring for which we were aiming. This part of the ME-IP seems particularly important when compared with other studies on ME-IP, such as Haggarty et al. (2011) and Schwartz and Ticknor (2018).

These considerations indicate that teacher induction, besides supporting new teachers, could be used to provide PD to other mathematics teachers at the school, as also suggested by Haggarty et al. (2011).

Cochran-Smith et al. (2012) argue that meaningful PD and appropriate collaboration opportunities are important for new teachers to ”go strong and stay on”. Our study suggests that meaningful PD is not achieved by easing new teachers' entry into teaching by providing practical and emotional support or by dealing with (expected) deficits in their practices, like the mentors in Haggarty et al. (2011). Instead of viewing the new teacher's classroom as a place for social reproduction (Losano et al., 2018), our study suggests using it as a site for teacher learning and as an opportunity to address new teachers' problems, as this gives rise to important discussions of the complexities of mathematics teaching and student learning. Our study further suggests that opportunities arise when teachers and their mentors collaborate to create an instructional product in a lesson study-fashion, since both parties are challenged to find ways to promote good practices. We conclude that the ME-IP opened a necessary (but not sufficient) forum to discuss and collaborate on teaching complexities and that this forum offered the new teachers meaningful PD and appropriate collaboration opportunities.

Conclusion

Our study contributes to practice-based research by exploring what and how new mathematics teachers learn when participating in an induction programme focused on mathematics education. The study shows the complexity of how new teachers learn to tackle problems in their classrooms, especially when they are not supported by their mentors, or their schools, and when their teacher training is limited. The aim of our ME-IP – that new teachers should ”go strong and stay on” – was high, yet one teacher almost reached it by ”going strong but moving on”. Our study indicates that, if new teachers wish to learn to tackle classroom problems without compromising teaching quality, it is essential to provide them with support connected to mathematics education during their initial teaching years.

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