Opportunities to learn ambitious mathematics teaching from co-planning instruction

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This study explores ambitious teaching practices teachers have opportunities to learn when co-planning instruction as part of their professional development. An analytical framework associated with *Sociocultural discourse analysis* is applied to identify utterances (dialogue moves) in the co-planning sessions that are essential in helping the teachers to develop their reasoning together. The findings reveal that the participants work on the ambitious practices of predicting student responses, representing these responses and aiming towards the goal for the lesson when co-planning to introduce the distributive property of multiplication to their students. Dialogue moves in the reasoned dialogues such as expressing shared ideas and agreements, providing arguments and challenging each other's ideas are found to be essential for providing the teachers with opportunities to learn to predict student responses, to represent these responses and to aim towards the learning goal for the lesson.

Planning can be regarded as a way to ensure effective classroom instruction, as a core competence in itself and as a means for teachers' professional development (PD) (Kelly, 2009). Aiming at understanding mathematics teaching practices teachers have opportunities to learn while co-planning instruction, this study explores reasoned dialogues (see "Analytical approach") in collective planning (co-planning) in learning cycles of enactment and investigation in the research and PD project *Mastering ambitious mathematics teaching* (MAM). In these *learning cycles*, planning is regarded as the pre-active phase of teaching and as a means for PD.

Based on their research review, Munthe and Conway (2017) suggest that planning involves "a complex combination of knowledge, skills, understanding, values, attitudes, and desire" (p. 836). When exploring developments in the literature, they identify a shift from studies of what

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Fauskanger, J. & Bjuland, R. (2021). Opportunities to learn ambitious mathematics teaching from co-planning instruction. *Nordic Studies in Mathematics Education*, 26 (3-4), 53–70.

teachers do to studies of "how planning involves shared knowledge construction and professional learning" (p. 837). When the focus shifts from the individual to the group, questions of shared knowledge in decision making emerge. Based on a research review focusing on what in-service mathematics teachers learn from PD, Goldsmith et al. (2014) call for explorations of PD elements, such as co-planning and how teachers in such sessions might be given opportunities to learn mathematics teaching practices. This is the focus of attention in the present study addressing the following research questions:

1 Which ambitious mathematics teaching practices do teachers have opportunities to learn while participating in reasoned dialogue in co-planning sessions?

2 How do specific utterances of teachers' reasoned co-planning dialogues provide them with opportunities to learn ambitious mathematics teaching practices?

Many terms are used to describe approaches to teaching mathematics that centre on students' sense-making. In this article, the term *ambitious teaching* will be used. Ambitious teaching aims to help students "develop in-depth knowledge of subject matter, gain higher-order thinking skills, construct new knowledge and understanding, and effectively apply knowledge to real world situations" (Smylie & Wenzel, 2006, p. 7). Since ambitious teaching practices aim to develop all students' conceptual understanding, procedural knowledge, adaptive reasoning and engagement in mathematical problem solving (e.g. Ghousseini et al., 2015; Kazemi, 2017; Lampert et al., 2013), learning such practices is a complex and challenging endeavour. This complexity becomes visible when highlighting the following principles of ambitious teaching: formulating clear instructional goals, treating all students as sense-makers, engaging deeply with students' mathematical thinking and designing instruction so that all students have equitable access to learning (Ghousseini et al., 2015; Lampert et al., 2013). These principles involve knowing the students, developing positive relationships and being responsive to students in culturally appropriate ways (Ghousseini et al., 2015). Core ambitious practices are "identifiable components (fundamental to teaching and grounded in disciplinary goals) that teachers enact to support learning" and consist of "strategies, routines, and moves that can be unpacked and learned by teachers" (Grossman et al., 2018, p. 4). Building on Lampert et al. (2013), the ambitious practices worked on in MAM include launching problems, using mathematical representations, aiming towards a mathematical goal, facilitating student talk and eliciting and responding to students' mathematical ideas (Wæge & Fauskanger, 2020). Learning the principles and practices of ambitious teaching is an ultimate goal for PD and designing practice opportunities for teachers is important for supporting their learning of such teaching practices (Kavanagh et al., 2020). To support teachers in learning the practices identified as key to the principles of ambitious teaching, in MAM we gave them repeated opportunities to co-plan, rehearse and co-enact a set of intentionally selected instructional activities embedded in learning cycles.

Learning cycles for professional development

In their review of research on PD, Kazemi and Hubbard (2008) recommend that the design of and research on PD should include enactments of routine teaching activities, or what Grossman et al. (2018) refer to as approximations of practice. The school-based MAM project (e.g. Wæge & Fauskanger, 2020), explores learning cycles including specific instructional activities designed for learning ambitious mathematics teaching practices (Lampert et al., 2010). Quick images (figure 1) are recommended as an instructional activity providing teachers with opportunities to learn ambitious mathematics teaching principles and practices. Included in learning cycles, they are found to help teachers learn ambitious practices (e.g. Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2020).

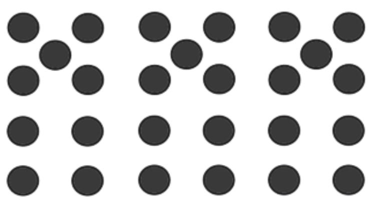

Figure 1. *An example of a quick image*

Quick images are suggested as a way of inviting students to learn about the commutative, associative and distributive properties of multiplication (Schumway, 2011). They are designed to help students to visualise numbers and form mental representations of a quantity by being invited to look at the image for a few seconds (i.e. a "quick" look and not enough time to count the dots one by one) and then explain how they organised and subitised quantities to find the total number of dots in the image.

MAM was designed to provide teachers with opportunities to learn ambitious practices by inviting the teachers into learning cycles. Each of the learning cycles includes the following six steps:

1 The teachers prepare for the cycle by reading given articles and by watching a video showing enactment of the cycle's instructional activity. Some teachers try out the activity with their own students.

2 One of the supervisors (teacher educators) leads a discussion/ analysis of the literature as well as the video clip.

3 Supported by a supervisor, the groups of teachers co-plan the given activity for given groups of students.

4 One of the teachers teaches the instructional activity in a rehearsal where the supervisor and the other teachers act as students (including teacher time-outs).

5 The same teacher enacts the activity together with a group of students. All participants can ask for teacher time-outs.

6 Each group of teachers analyses the enactment together with their supervisor, followed by a similar analysis with all the participating teachers and preparation for the next cycle's instructional activity.

The co-planning phase is explored in this study. The supervisor participates in the co-planning and is thus in a position to direct its focus on key practices of ambitious teaching and important mathematical ideas.

The present study will address the call for explorations of PD elements (Goldsmith et al., 2014) such as co-planning, and examinations of interactions in PD (e.g. Horn & Kane, 2015). Moreover, Fauskanger's and Bjuland's (2019) and Wæge's and Fauskanger's (2020) findings highlighted the need to further explore the learning cycles' potential for supporting the participants' collective learning. In this article, this is done by delving deeper into the teachers' interactions in co-planning sessions.

When using learning cycles in PD, learning is understood as it emerges in activities in each step of the cycle. Informed by sociocultural views on teacher learning, this study draws on Lave's (1991) description of learning, thinking and knowing as "relations among people engaged in activity in, with, and arising from the socially and culturally structured world" (p. 67). The distributed nature of teachers' reasoning and the importance of exploring the nature of PD interactions is of importance for research into dialogues in PD settings (Horn, 2005). Explorations of interactions in PD suggest that research would benefit from a clearer account of how conversations contribute to professional learning (e.g. Horn & Kane, 2015). The present study contributes to this body of work as it seeks to understand teachers' opportunities to learn through interactions in co-planning discussions with colleagues.

Opportunities to learn to teach the distributive property

Whereas learning of ambitious teaching practices is set as the goal for teachers in MAM, in the analysed co-planning sessions, the distributive property of multiplication, $a \times (b + c) = a \times b + a \times c$, was planned as a goal for the students' learning. This property is considered to be difficult to learn (Carpenter et al., 2005) and also to teach (Larsson et al., 2017).

The new national curriculum in Norway highlights the importance of the arithmetic properties of multiplication. For example, in year three, one goal is for the students to "use commutative, associative and distributive properties to investigate and describe strategies for multiplication" (Utdanningsdirektoratet, 2019, our translation). Distributivity is important because it underpins mental calculation strategies, as well as algorithms for multi-digit multiplication where the factors are partitioned (e.g. Izsák, 2004). Using distributivity in this way is suggested as a method for demonstrating a high level of understanding (Ambrose et al., 2003).

Case studies conducted by Larsson et al. (2017) provide detailed examples of how students' conceptualisations of multiplication as equal groups and repeated addition support their understanding of distributivity. These researchers found that although the students showed a robust conceptualisation of multiplication as equal groups and this "supported their utilisation of distributivity to multi-digits but constrained their utilisation of commutativity" (Larsson et al., 2017, p. 1), they failed to connect calculations to multiplication models. Transforming an equal-groups model of multiplication into a rectangular array – where distributivity can be demonstrated by partitioning the rows and the columns into partial products represented by smaller rectangles – is suggested as a useful instructional approach (Carpenter et al., 2005). According to Larsson et al. (2017), such rectangular arrays are found to support students' understanding of distributivity (Barmby et al., 2009; Izsák, 2004). Using quick images as instructional activities might provide such support for students' learning (Schumway, 2011) but also for teachers' learning to teach (e.g. Wæge & Fauskanger, 2020). Based on findings that show it might take several years to develop multiplicative understanding (e.g. Verschaffel et al., 2007), it is important to study ambitious practices for teaching the multiplicative properties teachers are given opportunities to learn through participating in PD (Larsson et al., 2017).

Design, data material and participants

In MAM, the heads at each primary school in a city in Norway selected teachers which could serve as future mentors for their colleagues, resulting in the participation of 30 mathematics teachers from 10 schools. The teachers were divided into four groups, and all teachers in three of these groups agreed to participate in the research. Two groups (14 teachers) were randomly chosen to be part of the research reported on here. They teach years five to seven (i.e. the students are 11 to 13 years). The supervisors were teacher educators working at a university and the researchers were from two different universities.

The participants took part in nine full learning cycles over the course of two years. All steps in each cycle were videotaped. The analysed data material for this study comprises video recordings from co-planning sessions in both groups in two cycles where it was decided to use quick images (figure 1) as an instructional activity.

Analytical approach

We build on the recommendation from Littleton and Mercer (2013) to explore interactions among teachers for the potential for them ”to engage in reasoned dialogue” (Littleton & Mercer, 2013, p. 112), that is the potential to develop their reasoning together (Warwick et al., 2016). Reasoned dialogue is put forward as interactions where opportunities for teachers to learn are witnessed (Littleton & Mercer, 2013). A key part of this kind of dialogue is that ”everyone engages critically but constructively with each other's ideas”, treating everyone's ideas ”as worthy of consideration” (Littleton & Mercer, 2013, p. 16). People question each other's ideas in the interest of achieving a joint goal, ”they ask each other what they think, they all participate and they appear to reach consensual decisions” (Littleton & Mercer, 2013, p. 19). When analysing teachers' interaction in post-lesson discussions, Warwick et al. (2016) found that the most significant aspect in their analysis was the power of particular utterances identified as dialogue moves, denoting ”questioning (including negotiating meaning), building on each other's ideas, coming to some agreement, providing evidence or reasoning and challenging” (Warwick et al., 2016, p. 566). These moves allowed for a cumulative building of ideas, leading towards agreements on pedagogic development, and thus opportunities to learn for the teachers.

We see co-planning as a context for reasoned dialogue (Littleton & Mercer, 2013) and explore the specific utterances of the co-planning dialogues that are found to be essential for helping teachers collectively move their pedagogy forward. Following Littleton and Mercer (2013) who analysed teachers' dialogues, where pedagogical intentions were evidenced, analysing co-planning of mathematics teaching makes it possible to explore pedagogical intentions. In co-planning dialogues, chains of utterances illustrating the teachers' collaborative efforts to participate in building on each other's initiatives can be identified. In line with

Warwick et al. (2016), an analytical framework associated with *Sociocultural discourse analysis* with five dialogue moves (table 1), has been used to identify reasoned dialogues within co-planning in MAM. We are aware that critical voices could be raised in order to ask for more specific criteria for interactions to be reasoned dialogues or not. Following Warwick et al. (2016), we think it is important to emphasise that all these five dialogue moves are considered to have the potential to take the dialogue further into a reasoned dialogue and thus, collaborative learning experience and opportunities for teachers' learning (Warwick et al., 2016). However, a dialogue can, to some extent, express different levels of reasoned dialogue. In our coding process, we have particularly seen that dialogue moves, building on and expressing shared ideas [DM3], moves providing reasoning and arguments [DM4] and moves denoting the issue of challenge [DM5] have been essential for a high level of reasoned dialogue, giving the teachers' particularly opportunities to learn ambitious mathematics teaching from co-planning instruction. In the second part of the result section, this is illustrated by two representative excerpts from the empirical material, focusing on high and low levels of reasoned dialogue.

We approached the data in three steps: 1) Identification of episodes, 2) Coding the episodes according to the ambitious teaching practices in focus in each episode and 3) Analysing representative episodes by using an analytical framework with five dialogue moves (table 1). We started out by dividing the four co-planning sessions into episodes according to different thematic foci in the teachers' dialogues. We identified 51 episodes in total in the four co-planning sessions analysed. One episode represents part of a co-planning session where the teachers have a reasoned dialogue. Another episode begins when there is a clear shift in the focus of the dialogue as indicated by an utterance (e.g. a question or a statement). The episodes in one of the co-planning sessions were first identified by the two researchers individually before coming together and agreeing on them, and here there was then total agreement. This was followed by the process of coding the episodes in this session according to the ambitious teaching practices in focus in each episode. In MAM, these were launching problems, using mathematical representations, aiming towards a mathematical goal, facilitating student talk and eliciting and responding to students' mathematical ideas (see Wæge & Fauskanger (2020) for more information about codes for ambitious practices). There was also total agreement on this coding that was first undertaken individually by both researchers. For this reason, the rest of the data material was coded by one researcher and checked by the other.

With the overview of ambitious teaching practices discussed in the episodes as the point of departure (see results section), we watched the videos together to analyse representative episodes using an analytical framework developed by Warwick et al. (2016, p. 567), with the following five dialogue moves, (table 1): [DM1] – ”Requesting information, opinion or clarification”, [DM2] – ”Making positive and supportive contributions”, [DM3] – ”Expressing shared ideas and agreements”, [DM4] – ”Providing evidence or reasoning”, and [DM5] – ”Challenging ideas or re-focusing talk”.

In our study, an utterance is conceived as a participant’s verbalisation, including non-verbal actions (e.g. pointing at or writing mathematics symbols in a quick image) as long as the interlocutor has the floor. Following this, each utterance in the transcribed episodes was coded as [DM1] to [DM5]. Utterances identified as not codable according to these five categories were coded as ”other moves” [OM]. Frequent examples of such moves are comments from students or comments given to students by the MAM participants (e.g. ”Is that a correct interpretation of what you said, [student’s name]?”). The participating teachers are named T1–T6 and the supervisor is named S. The utterances (table 1) illustrate examples of the dialogue moves.

Table 1. *Examples of dialogue moves*

Utterance	Dialogue moves [DM]
18. T4: Are we thinking that the goal is to arrive at, whatever it’s called? I mean the distributive law or property?	[DM1]: Asking for clarification as to whether the distributive property should be the goal for the lesson.
13. S: Yes [points at the third example] as an example, we have a very good link here in relation to the distributive property.	[DM2]: Supportive and points to the third representation. [DM3]: Building on and expressing a shared idea and agreement by connecting the third representation to distributive properties.
19. T5: Or perhaps more that you see that 3 times 5 plus 3 times 4 is equal to 3 times [parenthesis 5 plus 4].	[DM4]: Providing evidence or reasoning by connecting the first and third representations.
25. T3: Yes, that they [the students] can explain what it is.	[DM5]: Elaboration on (19) making the suggestion to challenge the students to explain that these two mathematical expressions have equal value.

In what follows, representative excerpts from the reasoned dialogues are presented and discussed to illustrate our findings.

Results

In line with Wæge and Fauskanger (2020), who focused on teachers’ learning in rehearsals, we suggest that teachers have opportunities to learn the ambitious mathematics teaching practices that are reasoned about in the dialogues in the co-planning sessions. An important starting point for our analyses was that the teachers and their supervisor only had opportunities to learn what was worked on and talked about in the co-planning sessions. The aim of the first part of this results section is thus to present the ambitious mathematics teaching practices identified in teachers' reasoned dialogues while co-planning instruction. When analysing all episodes of the co-planning sessions, the ambitious teaching practices that were most common in the teachers' reasoned co-planning dialogues were: predicting student responses, representing predicted student ideas in the quick image and aiming towards a mathematical goal for the lessons. This is in line with a recent study within the MAM project conducted by Fauskanger and Bjuland (2019) who explored ambitious teaching practices the participants had opportunities to learn through a single learning cycle. From our analyses, we identified that predicting student responses for finding the number of dots in the quick image (figure 2) was worked on the most in the co-planning sessions when quick images were used as an instructional activity. Prediction was often followed by reasoned dialogues about how to elicit students' thinking and how to represent the students' ideas in the quick image and/or mathematically.

The aim of the second part of this section is to present the results of the exploration of the specific utterances (dialogue moves) in teachers' reasoned co-planning dialogues that might provide them with opportunities to learn ambitious mathematics teaching practices.

Predicting student responses represented in the quick image

In the following, we will focus on one of the analysed co-planning sessions. The reasoned dialogue below illustrates ambitious mathematics teaching practices the teachers have the opportunities to learn while co-planning instruction. Before this part of the co-planning session (episode 3), the number 15 has already been circled in the quick image (figure 2). After a reasoned dialogue about several other predicted student responses, the dialogue returns to 15, this time parallel to representing the response on the board (figure 2):

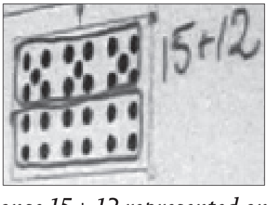

Figure 2. *The predicted response 15 + 12 represented on the board*

139 S: So, they just know that it's 15, right [writes 15 next to the quick image].

140 Many: Yes.

[...]

145 S: Yes, but if we look at exactly this picture [points to the quick image where 15 is marked]. If we, the students who see 15 here.

146 T2: But they quickly see 12 on all, yeah.

147 S: So, you think that [they see] 15 plus 12?

148 T2: Yes, they might do that.

149 S: Yes [writes + 12 on the board next to 15 on the board (figure 2)].

150 Many: Yes.

151 T4: Then I'm thinking a bit in relation to if you take 15, 15 [and ask the students] "how do you see 15 here?"

152 S: Yes [at the same time frames the three fours in figure 2].

The supervisor starts by writing 15 on the board next to the circled 15 dots (139) and writes + 12 next to 15 (149) after having clarified that T2 predicts that some students will see the three fours as 12 (146–148). The supervisor illustrates this representation by circling the three fours as one 12 (152). The reasoned dialogue in episode 3 continues by deciding to ask the students how they saw the number 12 in the quick image (151–152). This dialogue gives a glimpse of how the supervisor and two of the teachers T2 and T4 make important inputs, illustrating how the co-planning enabled a learning situation for the teachers in which they were making sense together of predicting student responses and representing students' ideas. Such reasoned dialogues were prominent throughout the co-planning sessions. Figure 3 summarises all predicted student responses presented on the board towards the end of this particular co-planning session and shows how these predictions were represented in the quick image as well as in mathematical notation.

The teachers and their supervisor were negotiating how to represent student ideas as accurately as possible in the quick image whilst simultaneously paying attention to the mathematical correctness of the representation. Thus, they were trying to make sense of these two practices simultaneously and also to see them in relation to each other.

Opportunities to learn ambitious teaching practices

Littleton and Mercer (2013) emphasise the importance of allowing participants in PD to interact and develop their reasoning together, giving them the opportunity to engage in a reasoned dialogue by questioning each other's ideas in the interest of achieving a joint goal. Participation

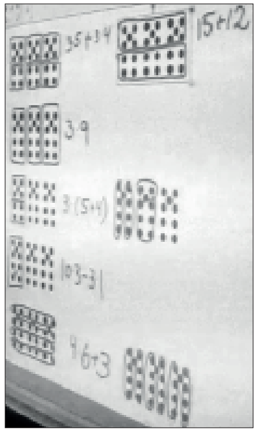

Figure 3. *All predicted responses presented on the board*

in such dialogues might provide teachers with opportunities to learn (Warwick et al., 2016), in our case, teachers' opportunities to learn ambitious mathematics teaching practices from co-planning dialogues. The analysis below will particularly focus on the teachers' opportunities to learn ambitious teaching practices by exploring the specific utterances (dialogue moves, table 1) of the co-planning dialogues which are found to be essential for helping teachers collectively move their pedagogy forward.

Aiming towards goals from predicted student responses

In the continuation of the co-planning of this particular group (episode 6), we identify chains of utterances, illustrating how the ambitious teaching practices are discussed in a reasoned dialogue in the participants' co-planning when agreeing to focus on the distributive property of multiplication as the learning goal for the lesson.

This extract (table 2) illustrates how the teachers and their supervisor participate in a reasoned dialogue, discussing the distributive property of multiplication as a possible learning goal for the lesson. Some of the dialogue moves seem to be particularly important for the cumulative building of ideas (13, 19, 25). The supervisor uses the predicted student response (see figure 3), building on and expressing the shared idea to focus on the relation between the first and the third representation (13, DM3). T5 builds on this initiative by providing evidence to see that these two mathematical expressions have equal value, expressing the connections in a mathematical language (19, DM4). T3 suggests that they could challenge the students to explain that these two mathematical expressions have equal value (25, DM5). These three dialogue moves seem to

Table 2. *The distributive property – a possible goal for the students' learning*

Utterances	Dialogue moves
13. S: Yes [points at the third predicted student response] as an example, we have a very good link here in relation to the distributive property.	[DM2]: Supportive and points to the third representation. [DM3]: Building on and expressing a shared idea and agreement by connecting the third representation to distributive properties.
14. Several: Mm [agree].	[DM2]: Several teachers make a supportive contribution.
15. S. What about you [addresses the group including teachers 4, 5 and 6].	[DM1]: Requesting information, inviting the other small group (T4, T5 and T6) to make a suggestion for a possible goal.
16. T4: We thought, yes, when thinking if we should have focused on that.	[DM3]: Responding by starting to express shared ideas and agreements in this small group.
17. S: Yes.	[DM2]: Making a supportive contribution.
18. T4: Are we thinking that the goal is to arrive at, whatever it's called? I mean the distributive law or property?	[DM1]: Asking for clarification as to whether the distributive property should be the learning goal for the lesson.
19. T5: Or perhaps more that you see that 3 times 5 plus 3 times 4 equals 3 times [parenthesis 5 plus 4] [$3 \times 5 + 3 \times 4 = 3 \times (5 + 4)$].	[DM4]: Providing evidence or reasoning by connecting the first and the third representations, expressing the connection in mathematical language.
20. S: Yes.	[DM2]: Making a supportive contribution.
25. T3: Yes, that they [the students] can explain what it is.	[DM5]: Elaboration on (19) making the suggestion to challenge the students to explain that these two mathematical expressions have equal value.

be of particular importance for agreeing on which student strategies to use while aiming towards the goal for the lesson. Through this reasoned dialogue (Littleton & Mercer, 2013) the teachers' have opportunities to learn what goal they might aim for in a lesson when using a quick image as instructional activity including that quick images could be used to reach the learning goal (the distributive property) as well as how to state a learning goal in relation to predicted student responses. We consider this as an example of a high-level reasoned dialogue. The analysis has revealed that the participants engage critically but constructively with each other's ideas, illustrated particularly by the dialogue moves [DM3], [DM4] and [DM5].

A challenge, if student responses are not connected to the learning goal

We will now illustrate a learning situation in which the teachers are considering if possible predicted student suggestions are not connected to the learning goal of the distributive property of multiplication.

Table 3. *When student responses are not connected to the learning goal*

Utterances	Dialogue moves
47. T6: But I think it's difficult to know how to present this [for the students].	[DM5]: Challenging the participants to consider how the mathematical goal for the lesson can be presented to the students (see utterance 49).
48. T4: Yes.	[DM2]: Making a positive contribution.
49. T6: What do they [the students] suggest of ideas, in a way? Because that has much to say.	[DM5]: Continuation and elaboration on (49), challenging the participants to consider if anticipated student responses are not focused on the distributive property.
50. Several: Yes.	[DM2]: Several participants make a supportive contribution.
51. T6: What patterns they see, but also what we talked about if they don't find [any pattern] in a way, then we have to introduce it for them, the pattern they find.	[DM3]: Building on (47, 49), and also expressing a shared idea if students do not come up with patterns that are related to the distributive property, the teachers can introduce examples that help them to focus on the property.

In the previous extract, we observed that both the supervisor and the three teachers T3, T4 and T5 were engaged with each other's ideas in a high-level reasoned dialogue. Here, it is another teacher (T6) who follows up and supports the participants' collective opportunities to learn the relations between the three ambitious practices when he expresses that it is difficult to focus on the learning goal if the students do not have any ideas, or if their responses are not connected to the distributive property. In this reasoned dialogue, one teacher, T6 challenges the other participants to consider this difficulty (47), (49). Several teachers (48), (50) make supportive moves without bringing in important dialogue moves such as [DM3], [DM4] and [DM5]. To some extent, we can say that this reasoned dialogue has a lower level than the previous one since there do not seem to be the same contributions from the other teachers to engage critically but constructively with each other's ideas. On the other hand, the teachers' supportive moves invite T6 to continue his predictions on this issue (51). This illustrates that it is not an easy task to give precise criteria for a talk to be characterised as a high-level reasoned dialogue.

In episode 7 the supervisor follows up T6's initiative and challenges them to consider what to do with the representations on the board (see figure 3) that cannot easily be connected to the distributive property of multiplication (i.e. $15 + 12$, $4 \times 6 + 3$ and $10 \times 3 - 3 \times 1$). T6 suggests a reorganisation of the different representations on the board so that it is easier for students to see the link between representations (i.e. $3 \times 5 + 3 \times 4$, 3×9 and $3 \times (5 + 4)$) and the distributive property of multiplication ($3 \times 9 = 3 \times (5 + 4) = 3 \times 5 + 3 \times 4$).

To sum up, the dialogue moves expressed in the two extracts from reasoned dialogues in this particular co-planning session seem to provide the teachers opportunities to learn the three ambitious teaching practices (predicting student responses, representing student ideas, aiming towards a mathematical learning goal) in their co-planning. We have seen how some dialogue moves have expressed shared ideas [DM3], introduced challenging ideas [DM5] and initiated a reasoned dialogue in which the teachers have reflected on and provided arguments [DM4] for introducing the distributive property of multiplication. This is in line with Warwick et al. (2016), who claim that the issue of challenge is very important as a dialogue move since it seems to move the dialogue positively forward towards development of teacher learning. These dialogue moves ([DM3], [DM4], [DM5]) have been initiated both by the supervisor and several teachers (T3, T4, T5, T6). We have also identified how dialogue moves like asking for clarification [DM1] and making supportive contributions [DM2] have been important triggers for the three other moves which are particularly essential for a high-level reasoned dialogue.

Concluding discussion

Based on their research review on planning, Munthe and Conway (2017) call for studies of how co-planning could support shared knowledge construction. Moreover, explorations of what teachers learn from PD elements such as co-planning (Goldsmith et al., 2014) and explorations of how conversations might support teacher learning through involvement in problems of practice are also called for (Horn, 2005, 2010; Horn & Kane, 2015). To satisfy these calls, this study was designed to better understand which ambitious mathematics teaching practices teachers might have opportunities to learn through interactions with colleagues in co-planning sessions in learning cycles and how teachers' reasoned co-planning dialogues might provide them with opportunities to learn such practices. When analysing the co-planning dialogues to identify what was worked on, our findings revealed that the following three main ambitious practices were present: predicting student responses, representing these responses in the quick image and using mathematical notation and the distributive property of multiplication as the goal for the lesson. By using an analytical framework developed to understand how involvement in reasoned dialogues might support teachers' opportunities to learn (Warwick et al., 2016), we found that the teachers were provided with opportunities to learn how to use predicted student responses when aiming to determine and satisfy the goal for the lesson. Meeting the call from Larsson et al. (2017) to study ambitious practices for teaching multiplicative properties, our study indicates that through participation in learning cycles, teachers are provided with opportunities to learn what goal they might aim for in a lesson when using a quick image as instructional activity, how to state a learning goal in relation to predicted student responses as well as why aiming towards the particular mathematical goal of distributive property of multiplication is important.

When exploring learning cycles, rehearsals are recommended as a pedagogy that approximates the work of ambitious teaching (e.g. Kavanagh et al., 2020; Lampert et al., 2013; Wæge & Fauskanger, 2020). In line with Wæge and Fauskanger (2020) but focusing on co-planning instead of rehearsals, we suggest that working on the practices of ambitious teaching in co-planning sessions have the potential to help the teachers develop a shared understanding which can enable adaptive teaching. Using the analytical framework from Warwick et al. (2016), we have been able to explore the reasoned dialogue among the participants. We have also raised the issue in order to ask for more specific criteria for a talk to be a high-level reasoned dialogue. Our findings suggest that particularly dialogue moves, building on and expressing shared ideas [DM3], moves providing reasoning and arguments [DM4] and moves denoting the issue of challenge [DM5] have been essential. On the other hand, the analysis has also illustrated that it is not an easy task to give precise criteria for a talk to be characterised as a high-level reasoned dialogue. The dialogue moves are closely linked together. The participants requested clarification, they made supportive contributions, built on and expressed shared agreement, provided reasoning and challenged each other to discuss the distributive property as the learning goal for the lesson. As our analysis of teachers' co-planning dialogues indicates, the teachers' opportunities to learn are related to their engagement and participation in a reasoned dialogue in which they have discussed how to introduce the learning goal from predicted student responses represented in a quick image. This illustrates the potential the use of quick images as instructional activities has for supporting teacher learning (Schumway, 2011; Wæge & Fauskanger, 2020).

The reasoned dialogue from the participants' co-planning has revealed that the teachers discussed how to invite the students to learn the distributive property of multiplication when they co-plan instruction using quick images as an instructional activity. One reason for this might be that quick images invite teachers to discuss models complementing equal groups (Larsson et al., 2017). Using distributivity in this way is suggested as a method for demonstrating a high level of understanding (Ambrose et al., 2003). Co-planning can thus provide contexts for learning ambitious practices for teaching what the new Norwegian curriculum expects students to learn (Utdanningsdirektoratet, 2019).

All in all, the present study has identified opportunities to learn ambitious mathematics teaching practices in two groups of teachers and their supervisors' co-planning sessions. Our study has thus revealed that the use of co-planning as a context in teachers' PD (Kelly, 2009) has the potential to help the teachers to develop a shared understanding that can enable adaptive teaching.

One limitation of the analytical approach used in this study is that we do not foreground individual teachers' learning trajectories. One implication for future research will therefore be to focus on individual teachers' opportunities to learn from co-planning over time. Since dialogue moves relate to participants' learning (e.g. Warwick et al., 2016), more research is also needed to compare and contrast which dialogue moves are used by participants in different contexts and which of these moves build a reasoned dialogue that can help the teachers to collectively move their pedagogy forward. Lastly, knowing more about how co-planning settings can be developed into reasoned dialogue, in which teachers are provided opportunities to learn about ambitious mathematics teaching practices will be interesting to explore in future research.

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