Authority in students' peer collaboration in statistics: an empirical study based on inferentialism

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Students' peer collaboration efforts in mathematics and statistics is a topic that has increasingly gained attention in research. In any collaboration, authority relations play a role for how meaning is constituted: Whenever things are discussed and decisions are made, authority is involved in a sense that some arguments or persons may be more convincing and powerful than others. In this article, we investigate how authority changes dynamically in type and in distribution as groups of fifth grade students collaborate in data generation processes. We identify and categorize authority using an epistemological framework, which is based on the philosophical theory of inferentialism. The results show that the three different types of authority described in inferentialism are all identifiable in students' collaborative work. We also find and categorize further types of authority connected to the statistics group work, some of which are hardly addressed in previous research.

In recent decades, researchers in mathematics education have paid increased attention to collaborative activities and to the importance of communication in the joint process of meaning-making in mathmatics classrooms (Mercer, 1996; Resnick, Michael & O'Connor, 2010; Wells, 2007). Many researchers have concluded that group discussions can favorably affect students' learning (e.g. Hufferd-Ackles, Fuson & Sherin, 2004; Krummheuer, 2000; O'Connor & Michaels, 1996; Stein, Engle, Smith & Hughes, 2008). Consequently, several researchers have developed various frameworks to explore different aspects of collaboration (e.g. Engle, Langer-Osuna & McKinney de Royston, 2014; Krummheuer, 2015; Langer-Osuna, 2016; Ryve, 2006; Sfard & Kieran, 2001).

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One concept that has attracted interest on the subject of collaboration is authority (Amit & Fried, 2005; Engle et al., 2014; Langer-Osuna, 2011, 2016, 2017; Oyler, 1996; Oyler & Becker, 1997). Typically, having authority is taken to mean having the right to act in a certain way, being entitled, for instance, through a certain legitimized status or through the course of a discussion. As such, authority can be understood as an attribute of persons or as being established within social situations. Research on authority has shown, for instance, how the distribution of authority between teacher and student can influence relationships between students and, thus, also student collaboration (Amit & Fried, 2005). Therefore, Langer-Osuna concludes that the way authority is "constructed, organized, and distributed among students has implications for [...] mathematics learning" (2017, p. 237). Theoretical frameworks based on authority have been created to explain how influence plays out in collaboration (Engle et al., 2014). However, many studies on authority are theoretical, abstract, or general in nature (see Buzzelli & Johnston, 2001). Empirical studies have mainly explored interactions between teachers and students (Engle & Conant, 2002; Oyler & Becker, 1997), while many of the interactions in classrooms are between students. Existing studies, which deal with collaboration (e.g. Buzzelli & Johnston, 2001; Cohen & Lotan, 1995), do touch authority relations but rarely describe them expli-citly. Hence, the increased interest in collaboration has not been followed up by shifting the research focus from student-teacher interactions to student-student interactions and the investigation of authority in student collaboration is largely unexplored. This holds especially true for statistics education.

Our empirical study aims at investigating authority and authority relations in the context of student collaboration. We pursue the research question: *What types of authority can be identified in students' peer collaboration in data generation processes*? Through the identification and analysis of authority types, our research contributes to the body of research aiming at understanding the mechanisms of collaboration in the statistics classroom in order to make collaboration a stronger process for teaching and learning (see e.g. Ben-Zvi, 2007). We investigate what types of authority emerge in fifth graders' collaborative group work and, in doing so, build on the philosophical background theory of semantic inferentialism (Brandom, 1994, 2000), which has proven to be beneficial for investigating students' group work in statistics (e.g. Ben-Zvi, 2007; Schindler & Seidouvy, 2019). Specifically, inferentialism gives us an appropriate conceptualization for understanding different types of authority embedded in student collaboration.

Previous research on authority

Within the social sciences, there are many different understandings of what authority can be (e.g. Friedrich, 1958; Metz, 1978; Weber, 1947). In this article, we use Amit and Fried's (2005, p. 147) description as a starting point:

> [A] relation of authority exists when one person (or group of people) tends to obey, act on, or accept without question the statements or commands of another person (or group of people or any other entity capable of producing statements or commands).

We have identified three fundamental kinds of approaches within research on authority in mathematics and statistics education (table 1). First, there are approaches in which teacher-student collaboration is in the focus (1); second, we see approaches where authority is investigated predominantly in student-student collaboration (2); and third, there are approaches examining both teacher-student and student-student collaboration (3a and 3b) but where authority is considered in relation to other phenomena such as power, gender, or equity in general.

Table 1. *Perspectives on authority in mathematics and statistics education*

	Only authority	Mixed approach
Teacher-student	1	3a
Student-student	2	3b

The first kind of approach focuses on the interaction between the teacher and the students and particularly on how the teacher's authority influences and controls interactions in the classroom. Authority is typically understood as a personal attribute. Most research in this area is dominated by an *authoritarian* perspective, where the teacher is regarded as the controller of the learning environment in the classroom, the transmitter of knowledge, and the orchestrator of interactions (Oyler, 1996). The teacher can be understood as "*an* authority and knowledgeable, and also [...] *in* authority regarding classroom procedure" (Oyler, 1996, p. 149) (e.g. Skemp 1979; Wagner & Herbel-Eisenmann, 2014). Contrary to the authoritarian approach, Spring (1999) and Welker (1992) describe the *anti-authoritarian* approach as one that seeks to minimize the role of the teacher by combining a "soft" teacher authority with student contributions to the development of knowledge in classrooms (Oyler, 1996). One trait of such *shared authority* approaches (Oyler, 1996), which may help to differentiate them from authoritarian, is the emphasis on negotiating the norms of participation in the classroom. Oyler's (1996) study shows that providing students with multiple opportunities to speak and act as experts encourages them to become producers and not just consumers of knowledge in the classroom. Amid and Fried (2005) stress that shared authority is significant for collaboration. Shared authority may provide certain opportunities in the classroom: It has, for instance, shown to enable productive interactive cross-talk among peers when working collaboratively (Lemke, 1990). Despite such advantages, Langer-Osuna (2017) points out that shared authority in collaboration can also limit productive mathematical sense-making.

The second kind of approach puts emphasis on student-student interaction in various participation structures. Langer-Osuna (2016), for example, studied social (”directive”) and intellectual forms of authority in two students’ collaboration during problem-solving. The study indicates that interactions that position a student with the right to issue directives to peers (directive authority) can become linked to interactions that position the same student as a credible source of mathematical knowledge (intellectual authority).

In the third identified approach, authority is often examined through or together with frameworks that deal with other social relations (e.g. Buzzeli & Johnston, 2001; de Freitas et al., 2012; Engle et al., 2014). Buzzelli and Johnston (2001) combine authority, power, and morality to explore the issues of teacher authority in a third-grade classroom. Oyler and Becker (1997) introduce the concept of shared authority and shared vulnerability. The key idea in their framework is that students take risks in taking initiatives and open themselves to their peers’ challenges and arguments. Although students show authority in these situations, they also make space for vulnerability, which surfaces when they fail to provide evidence. Vulnerability can – in turn – hinder productive mathematical sense-making (see Langer-Osuna, 2017).

Looking beyond our categorization, we note that most of the research and analyses on authority are general in nature (Buzzelli & Johnston, 2001). Research on authority largely adheres to an individualistic, person-based conceptualization of authority. Compared to teacher-student interaction, student-student interaction has received relatively little attention in research on authority. Following Langer-Osuna (2016), we note a tendency to emphasize how authority – connected to individuals and individual attributes – regulates interactions more than how interactions regulate – or confer – authority for individuals.

Aiming to complement existing research, the perspective we chose for the present study builds on the following three assumptions.

Authority is dynamic

In line with Wertsch (1998), we do not consider authority as static but as dynamic and emerging in social situations, as something that is being constituted and negotiated in the interactions between people and through the resources that are available to them in situations at hand. We address authority in student-student interaction because we believe that, in collaboration with peers, students are given plenty of opportunities to experience and exercise authority as well as expose and develop their understandings.

Authority is sophisticated

We share Gore's view (1994, 1996) in seeking more details about specificities of authority in classroom collaboration, especially in student-student collaboration. Our hypothesis is that inferentialism offers certain opportunities to understand authority in student collaboration.

Authority is constituted through interaction

We base our approach on the theory of inferentialism (Brandom, 1994): we assume that authority, and how it is distributed and constituted, is tightly connected to communication and to the act of meaning making, where authority and interaction mutually influence one another.

Theoretical framework based on inferentialism

In this article, we approach the topic of authority by drawing on inferentialism, which provides both the epistemological and methodological foundations for our study, in line with Schindler et al. (2017). Inferentialism is a contemporary philosophical theory (Brandom, 1994, 2000). Using philosophical ideas as offered by, for instance, Kant, Hegel, and late Wittgenstein, inferentialism is a semantic theory (see Noorloos et al., 2017) that aims to explain concept use (Brandom, 2000). Inferentialism offers a sophisticated account of authority in communication. In particular, it treats authority as inherent in students' communication and allows for a detailed and fine-grained analysis of the role and impact of different kinds of authority in student communication. It provides a lens through which we can examine the role of different kinds of authority in students' saying and doing as well as its impact on students' ongoing discourse and their understanding of statistical concepts.

Language games and the Game of giving and asking for reasons

Following Wittgenstein in his late works, communication can be understood as language games, as linguistic practices following more or less implicit rules involving habits and conventions of certain communities. Language games are ruled by various factors, such as authority relationships, obligations, and responsibility relationships. Brandom (1994, 2000) argues that human communication can generally be understood as a game of giving and asking for reasons (GoGAR) involving bringing claims forward, undertaking claims, justifying claims, attributing claims to others, and acknowledging claims. In this game, humans negotiate meanings and are concept users. Mastering the game consists of the awareness that concepts are connected to one another as well as the awareness of antecedents and consequences for applying a concept. Brandom explains that this means ”being able to distinguish, what follows from the applicability of a concept, and what it follows from” (Brandom, 2000, p. 48).

Authority in the GoGAR

Brandom points out that ”[t]he game of giving and asking for reasons is an essentially social practice” (Brandom, 2000, p. 163). How to adequately present or challenge claims depends on the prevailing norms brought into play by the participants.

One way to understand how meaning is negotiated is through focusing on the emergence and the distribution of authority in the GoGAR. Authority is a mechanism in normative practices. Brandom (1994) explains that having authority means that one is entitled to having ones speak acts or other actions recognized in the GoGAR. This means, in terms of GoGAR, that claims are acknowledged or undertaken by others. Conversely, authority is in play whenever claims are undertaken and actions are carried out. Brandom (1994) describes three forms of authority. First, claims can have ”content-based authority” (Brandom, 1994, p. 175). In this case, when a speaker's claim is challenged, the speaker gives authority to the claim through entitling it with (an)other claim(s). Brandom (1994) explains that content-based authority is ”invoked by justifying the claim through assertion of other sentences from which the claim to be vindicated can appropriately be inferred” (p. 175).

The second kind of authority is ”person-based authority” (Brandom, 1994, p. 175). In this case, the speaker does not provide content-related reasons herself but attributes authority to the claim of another person. In other words, the speaker reasserts the claim of another person. In turn, entitlement to the claim is ”inherited” from the original speaker who uttered the claim in the first place. The responsibility is passed to the original speaker.

The third kind of authority is ”observational authority” (Brandom, 1994, p. 226). Brandom explains that ”[t]he noninferential undertaking of a commitment by a reliable reporter can inferentially authorize another to undertake a commitment with that content” (p. 216). In this case, a claim gains authority because somebody is taken or treated as a reliable reporter of some empirical matter.

In sum, ”[a]uthority is not found in nature” (Brandom, 1994, p. 51) but gained in the GoGAR through providing evidence based on content, observation, or personal attributes for one's claims and actions.

Method

Setting and participants

The data used in this article consists of video-recorded group works sessions in a fifth grade in a primary school in Sweden. The class was composed of 18 students, eight girls and ten boys.

The statistical topic of the study is *data generation*: the process of data collection involving clarifying the significance of the phenomenon under investigation, delineating relevant aspects of the phenomenon that should be measured, and considering how they should be measured (Cobb & McClain, 2004). Naturally, in this process of discussing and conducting the data collection process – which inherently belongs to statistics (education) – students in lower grades do not necessarily talk about concepts such as variability, distribution, mean value, or median explicitly and consciously – concepts that they never explicitly discussed in statistics education before and whose ”names” or terms the students have probably never heard of.

Following Vergnaud (1982, 1996), learning can be understood in terms of conceptual fields, where conceptual field is understood as ”an informal and heterogeneous set of problems, situations, concepts, relationships, structures, contents, and operations of thought, connected to one another and likely to be interwoven during the process of acquisition” (Vergnaud, 1982, p. 39f). In the case of data generation in our study, the conceptual field involves many concepts, such as variability, distribution, mean, median, measurement, length, and many more – all interconnected through inferential relations between them. Whereas concepts such as variability and distribution may still be under emergence for students in lower grades like in this very study, they may draw on other involved concepts from prior experiences, such as their concepts of measurement, length, fairness, etc.

For the experiment, two groups were allocated in the classroom and the other three groups were each assigned to a study room (smaller than a classroom). The groups were given a task by their teacher, which was designed by the first author of this article (based on Vogel, 2009). In this task, students were to experiment with paper frog models of different sizes. Different-sized frogs had different colors. The frogs could be pushed down with a finger and then released to make a small jump. The task was set in a narrative where the students were called in as experts to assist a company in finding "the frog that would sell the best". This was explained to mean the frog that jumps furthest. In the instructions, the students were asked to collaboratively test and record five jumps for each of the three frogs and then to come up with a recommendation for the company. The students had measuring tapes, the frogs, and a prepared worksheets where they could record their measurements.

Data collection

This article focuses on two groups' collaborative work (group A: two boys and two girls; group B: three girls and one boy) both working in separate study rooms. For each group, a single camera was placed on a tripod that pointed downwards onto each group, capturing their interactions, gestures, body movements, and shared materials. The groups' written notes were collected as well. All sections of the video recordings that addressed the data generation were transcribed.

Data analysis

The data analysis focused on how the students came to an agreement on which number to note for every measurement of a jump. Agreement is here taken to be indicated through the students finally noting a specific number (length of the frog jump) on the designated paper and no student objecting to this number. The analysis was conducted in three steps. First, drawing on Brandom (1994, 2000), authority in students' discussion was identified through identifying claims that were acknowledged or undertaken by others and had an effect on the subsequent collaborative work. Once such instances of authority were identified, we looked for the speech acts and actions leading to the authoritative claim. Second, we categorized each instance in which authority was apparent according to the three types described above: *content-based*, *person-based*, or *observational authority*. We call this type of classification our *deductive analysis step*. Third, since Brandom's authority framework does not deal with the consequences certain claims might have in a statistical learning situation in particular, we once again looked at all instances of authority and applied an *inductive analysis*, which aimed at characterizing and examining authority specific to statistics, specifically, to data generation.

Results

The data that our study draws on is comprised of approximately 12 minutes of video for group A and 14 minutes for group B. Below, we first present an overview of the kinds of authority observed in the group work. This is followed by illustrative instances of the inferentialist authority types and further authority types as found in our inductive analysis.

Overview of authority types in group work

In figure 1, the dominant categories of authority in the decision making for every jump are presented. It was not unusual for an episode to contain different types of authority. However, it was always possible to assign one predominant type. The figure shows the chronological order of dominant authority types connected to each jump.

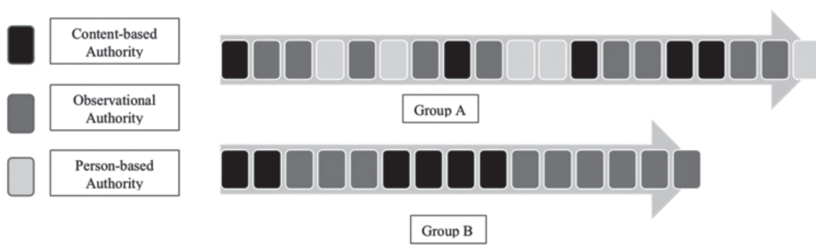

Figure 1. *Inferentialist authority types per jump*

In group A, despite instructions to count every jump, the group disregarded four jumps and redid the jumps, which explains why we have analyzed 19 jumps instead of 15. Group B conducted exactly 15 jumps, as instructed. The sequence of authority types are different for the two groups. For group A, the pattern is rather irregular and exhibits several person-based authority claims. As we will explore later, some of the jump measuring sessions of group A include interesting content-based discussions resulting in content-based authority. But reviewing the sequence as a whole reveals that the content-based discussions do not generate a shared understanding in how to carry out jumps and measure them, that means how to generate data. This view is strengthened when compared with the discussions in group B. Here, instances of content-based authority are followed by sequences of observational authority; specifically, the four consecutive content-based authority instances are followed by five observational authority instances. In this case, the group sequentially generated an increased agreement on how to measure, which resulted in more efficient measuring; thus, the students gained observational authority and did not need to discuss the way to measure each jump any longer. Below we further explain our analysis as well as the characteristics of the different inferentialist authority types.

Illustrations of inferentialist authority types

Observational authority

In the first jump session in group A, Grace and Harold (all names are pseudonyms) start by straightening the measuring tape, with one person holding each end. Felicia asks to execute the first jump, and Eric gives explicit instructions to Felicia on how to position the frog at the starting point: the nose of the frog is to be placed on the zero of the starting point. Felicia acknowledges the instruction by following the suggested rule. The transcript starts when the frog lands.

Grace:	20
Eric:	No, wait
Felicia:	It's the one that is the closest to the frog's nose
Grace:	19, 20 [...] what does it say here?
Harold:	No it's [...]
Eric:	19 [...] 19
Harold:	29
Eric:	19, no, 19
Harold:	Yes, 19

Grace's "20" is challenged by her peers and the group returns to the spot of the jump to look for evidence to support their challenge. Each member of the group takes part in this new evaluation including Grace, leading to a unanimous agreement on the measurement, 19 cm. Both measurements are observational reports and emerge from observations in similar circumstances and directions. There is no appeal to logical rules, inferences or personal reasons in the new evaluation. The first report is rejected for the second solely on the basis of observation. In sum, both the challenged claim and the proposed claims are observational: the authority basis for the reported value (19 cm) is observational.

Person-based authority

Jump 6 by group A serves as an illustration of person-based authority. At this point, the group has already disregarded two jumps. The sixth jump is executed by Felicia. The frog lands backwards. She is not satisfied with the length and tries to remove the frog. Eric then reacts saying, "No, no, no," preventing Felicia from removing the frog. The group then continues; they measure the jump and report it. Eric's action in this context can be characterized as person-based authority. There were no claims of explanations as to why to let the jump count, nor any references to empirical observables. The group followed Eric's initiative to count the jump on the basis of his reaction alone. We will come back to the case of Eric later to explain how his general behavior might have granted him the potential for person-based authority.

Content-based authority

An example from Group B illustrates content-based authority. The jump is executed and measured as 40. However, a disagreement emerges about the units of measurement: 40 mm or 40 cm?

Alice:	Yes, this is centimeters
Bob:	No, this is not centimeters
Alice:	Yes, this is millimeters [pointing at the measuring-tape and comparing cm-marks and mm-marks]. If this is centimeters, what is this then? What is this then, if it is not centimeters?
Bob:	This is 4 centimeters [taking the measuring tape and showing what is actually 40 centimeters]
Alice:	No [...]
Jane:	This cannot be 4 centimeters
Bob:	No, it is not. This is like half a meter
Jane:	Exactly
Bob:	Ok, so it will be 40 centimeters
Jane:	Yes, it is 40 centimeters

First, Alice challenges Bob by showing a contradiction in Bob's reasoning. Her statement using the logical construction, "If ... then ..." helps Bob to discern the consequences of his commitment. At this stage, Bob still does not seem to be convinced despite the fact he cannot reject Alice's challenge or vindicate his own claim. Next, Jane challenges the incompatibility in Bob's claim. Indirectly, Jane is asking for more evidence. In his attempt to provide more evidence to support his "4 centimeters" claims, Bob then compares it to half a meter. At this moment, he realizes the incompatibility in his reasoning. The group agrees on 40 cm and records it. Content-related, inferential, and logical reasoning are used; the reason that pushes Bob to revise his commitment have content-based authority.

Further authority types in the data generation process

Our inductive analysis resulted in five categories. While some categories emerge from a single example, others build on several, sometimes consecutive episodes.

Authority of socially negotiated content (acceptable data)

Consider the following transcript from group A:

Eric: 32.6
Felicia: No 30 [...]
Eric: Yes
Felicia: 7
Eric: No, 32.6
Felicia: Oh okay [*they are all about to start writing*], but it was 7
Harold: 32.7
Eric: Was it 7?
Felicia: Yes!
Gael: Point 7 [the group then recorded 32.7 as the value of the jump]

Each of the four students makes at least one claim about the length of the jump. Our interest is focused on why 32.7 eventually gains authority. There are no content-related reasons. And despite the disagreement, there is no attempt to verify the reported length. Instead, Eric asks if 7 is correct and Felicia says yes. Gael then acknowledges this claim verbally and writes it down. Since Felicia does not make an additional observation, the claim is acknowledged by person-based authority. Our interpretation is that social considerations are behind the decision to record 32.7. One possible interpretation is that the group is more interested in writing down a number to get the job done than in actually recording a correct jump length. Possibly, issues concerning turn taking, or not pressuring individuals by questioning their claims are also at play. The full analysis of group A's work shows that it is in fact symptomatic that they do not develop a shared understanding of how to carry out the measurements; something that is done effectively in group B, as pointed out earlier. However, even in group B we identified one case of social negotiation. Christy and Bob report 14 and 16 respectively. This is a large discrepancy and it is unreasonable that both were results of serious efforts to measure. But instead of a careful re-check, it seems that the group agrees on 15 as a compromise.

In summary, we characterize this type of relationship as socially negotiated authority. Conflict and disagreement about data value are solved by social negotiations rather than by collaborative efforts to clarify the nature of the disagreement and its relationship to the material or logical inferences related to the data generation.

Authority of logical inferences

We now refer back to the transcript concerning content-based authority above, where reasoning about the relationship between 4 cm and half a meter made Bob change his claim to 40 cm. Inferentialism does not in itself differentiate according to content, but for the purpose of mathematics and statistics education – and data generation in particular –, it is interesting to identify content-based authority for inferences made from mathematical and statistical matters, as is the case in the 4/40 cm discussion. Another example of logical inferences, where measurement units were discussed, comes from group A. Here, centimeter and millimeter are also discussed but in relation to the meaning of these two concepts in terms of their relevance and possible impact on the context of the task. These two mathematical concepts are inferentially related to their roles in decision making in data generation. In group A, the meaning of centimeter and millimeter (and even meter) are connected to the length, while in group B, these two concepts are connected to the possible impact of ignoring millimeter and rounding up or down to centimeters in the decision-making. However, in both cases, the logical inferences are related to the content of mathematics and statistics. Identifying and promoting content-based authority of this type is significant in relation to promoting reasoning about reasonable data (Garfield & Ben-Zvi, 2008), mathematical and statistical reasons and concepts; this comes in the form of either formal statistical knowledge or students' informal statistical reasoning (e.g. Bakker & Derry, 2011; Makar, Bakker & Ben-Zvi, 2011).

Authority of variability

Another interesting case of authority stems from the observation that all reported lengths for group B are whole numbers. This is in fact a result of a decision the group makes early on and it has the effect that following measurements require less accuracy. The authority base that legitimized whole numbers is the acceptance of variability within a centimeter as inherent to the jump lengths. This connects to the substantiated research on students' difficulties to deal with variability

(Garfield & Ben-Zvi, 2008). In our study, variability appears at two levels. The first level of variability is the variability due to the measurement process. The students in group B grant authority to the *variability within centimeters* to handle variation connected to the measurement process. The second level of variability is the *variability of the reported jump lengths*. The second level of variability is not in the scope of this study, however, note that the discussions in both groups reveals that students eventually deal with the second level of variability.

As reported above, group A – in contrast to group B – takes the opposite decision and decides to record jump lengths in millimeters. While these decisions do not relate to any particular instance of authority, the data seems to indicate that it affects the distribution of authority types. In group B, there is a shared understanding of how to conduct the jumps. Therefore, observational authority is easily exercised in this group. In group A, it is likely that students understand that it is difficult to substantiate a claim about how to interpret a jump and the respective measurement. This makes it more difficult to exercise observational authority. Any observational claim could, in fact, be challenged since, practically, it is hard to decide exactly between millimeters. This, we believe opens up for more person-based arguments in group A.

Authority of motion and location in space

This category is comprised of instances where authority is gained from motion and location in physical space during experimentation. Brandom (1994) acknowledges that being in the right circumstances is a main factor in gaining observational authority. In particular for group B, our analysis identifies several such instances. Further, our analysis suggests that, on a more general level, movement and location have the potential to grant access to critical features in the experiment. In group work, this type of authority requires group members to be aware and recognize what specific moves and locations boost individual persuasiveness in both experimentation and discussion with peers.

For example, Eric in group A positions himself at the starting point, close to the point where the frog will be launched. Right after each launch, he moves towards the spot and takes a new position near the frog's landing spot. When the jump is complete, he tries to read the measurement quickly before the others. His access to the landing spot of the jump and the measuring tape create circumstances for Eric's claims to gain authority. This happens in several instances.

Authority of procedural expertise

In group A, early in the experimentation, Eric influences the group work by exhibiting his skills and abilities. He explains to his peers how to hold the measuring tape, from which point to execute the jump, and how to spot the landing of the frog. The expert status gained actively is used when Eric becomes the person who legitimizes and monitors the experimentation. Eric puts himself in this position by demonstrating to peers his understanding of the physicality of the phenomenon under investigation and his mastering of the tools connected to the measurement of the jumps. Hence, through exercising procedural expertise, Eric gains a potential for person-based authority, which he also uses several times. In contrast, group B operates on turn taking. While some students in group B also exercise particular procedures effectively, there is not the same asymmetry as in group A.

In general, we see that experimentation requires certain practical skills. Students can have status as experts by, for instance, leading, mastering the tools allocated to the experimentation, and following and implementing instructions. In fact, in collaborative group work the expert status is often associated with authority (Amid & Fried, 2005; Inglis & Mejia-Ramos, 2008; Langer-Osuna, 2017). We understand this kind of authority as authority of procedural expertise.

Discussion

Following the research question – What types of authority can be identified in students' peer collaboration in data generation processes? – we have investigated authority in students' peer collaboration efforts in the context of statistics lessons that incorporate data generation. Our metho-dological approach builds on the semantic theory of inferentialism, which has been used in recent studies in mathematics and statistics education (Bakker & Derry, 2011; Bakker et al., 2017; Schindler et al., 2017). However, authority has been addressed only to a small extent in research using inferentialism (see Schindler & Seidouvy, 2019), and research using inferentialism for investigating collaboration is rare in general (see Seidouvy et al., 2018, Seidouvy & Schindler, 2019, and Schindler & Joklitschke, 2016, as examples). We see that inferentialism attributes a strong role to authority in communication and, hence, in students' meaning making. For any claim or action addressing content, students can ask for reasons or clarifications. Authority is a mechanism that regulates how and when questions are presented. Though students can always ask for other, underlying reasons ("chains" of why-questions), authority determines when and how it is appropriate to stop questioning. In data generation, the production of each data point is the act of making assumptions as well as – especially in social situations – claims and actions. Through investigating what speech acts or other actions precede such claims, we extracted information on different authority types. Therefore, this paper makes the methodological contribution that inferentialism can be operationalized to identify and categorize authority in student collaboration.

Our first investigation focused on distinguishing three fundamental types of authority presented in inferentialism in two student groups. As a first discussion point based on observations, it is worth noting that a portion of authority was content-based: Students discussed how to measure – and evaluated the variability in the measurement – based on their conceptual knowledge. This knowledge involved in data generation does not touch only a single mathematical concept, but involves a whole conceptual field (Vergnaud, 1996) of interconnected concepts, which all play a role for conducting and understanding the data generation process. For instance, the concepts of measurement, units of measurement, and natural numbers were involved in our study; and also an intuitive concept of variability. This supports the view of data generation as more than just simple data collection (Cobb & McClain, 2004). It also strengthens the importance of data generation in statistical inquiry because data generation appears to offer opportunities to reason about data (Garfield & Ben-Zvi, 2008), to view data in different ways (Konold, Higgins, Russell & Khalil, 2015); and to experience variability, in important statistical concept. Researchers such as Cobb (1992) argue that variability is not best understood by lecture, but should – instead – be experienced in intuitive ways. In this sense, the statistical activity, in particular the data generation process addressed in this article, provides opportunities for conceptual development in statistics: Conceptual development which connects mathematical concepts (e.g. natural numbers) with everyday concepts (e.g. fairness) and statistical concepts (e.g. variability).

Our study indicates that observational authority emerges from discussions that are regulated by content-based authority. In group B, two series of content-based authority are followed by two series of observational authority (figure 1). It is during the series of content-based authority that the conditions to grant observational authority are adjusted, refined, or expanded. This finding is in line with what can be derived from the theory of inferentialism. For anyone to gain observational authority, certain actions or competences related to measuring need to be acknowledged by the collaborators (Brandom, 1994). Through making such actions or competences explicit, the collaborators can more easily gain observational authority. The two groups in our study worked quite differently, and it turned out that both the distribution of authority types as well as how the authority types were distributed sequentially differed substantially, which we analyze in more detail elsewhere (Seidouvy et al., 2018).

This difference, we claim, could be explored and used for didactical purposes. In our case, both groups had the same task, but it would be possible to adjust the formulation of the task to increase or decrease the ease with which observational authority could be gained. For a teacher, establishing access to observational authority would essentially mean that the group can produce more measurements more easily (if the details of data generation are not an explicit learning goal). On the other hand, setting up a situation where the content-dependent essentials of data generation and measurement are not regulated in the task makes it likely that different ideas about data generation are discussed, which may be advantageous for conceptual learning purposes. In this case, students need to explicitly discuss measurement and related concepts; which also provides the opportunity to discuss variability as described above.

Our second analysis consisted of taking another look at the instances of authority to see if we could find other aspects of authority with educational relevance. This revealed five complementary types: *authority of socially negotiated content (acceptable data)*, *authority of logical inferences*, *authority of variability*, *authority of motion and location in space* and *authority of procedural expertise*. Below we briefly discuss how we see these types of authority as related to authority types previously discussed in the literature.

First, when looking at *authority of socially negotiated content*, it is worth noting for educational purposes that even in a well-set-up collaborative task, social considerations can be the basis of authority just as easily as content-based considerations, meaning that content-related issues might not get discussed. This case is exemplified in group B where the jump length is legitimized through compromise rather than content-based discussions. Our findings connect to Langer-Osuna (2011, 2016), who found that social forms of authority (directive authority) can be connected to intellectual authority. Langer-Osuna found that a female group leader's directive authority is more frequently challenged compare to a male's. She concluded that challenging directives authority affect intellectual authority.

The *authority of motions and location in space* is linked to the concept of intellectual authority discussed by Langer-Osuna (2011, 2016). Engle et al. (2014), observed that, during a group discussion, access to the conversational floor may affect intellectual authority. This view is supported also in a more recent study which shows that students with good mathematical ideas may have trouble ”gain[ing] the conversational floor or hav[ing] [...] their ideas attended to” (Langer-Osuna, 2017, p. 240). We think that gaining access to the conversational floor is a variation of authority of motions and location in space.

Authority of logical inference focuses on students' understandings of mathematical and statistical concepts in experimentation. Authority of logical inference is similar to expert authority with mathematical knowledge (Amit & Fried, 2005) and intellectual authority (Engle, 2012) in the sense that the students' claims related to data generation are valuated as credible by peers.

Variability authority exposes aspects of the measurement that are relevant to students. In group A, the students reflect on the possible impact of the variation within a centimeter on deciding "the best frog". In group B, the focus was on having a shared understanding on how to measure the jumps. In Cobb and McClain's (2004) perspective, group A reached the goal of data generation process which is to reflect and connect actions in how data is produced to the conclusion to be drawn: The students intuitively reflect on the variability, which guides their data generation process.

Finally, *authority of procedural expertise* sheds light on what types of procedural abilities and skills are influential and persuasive in experimentation. It is a kind of expert authority (Amit & Fried, 2005) with procedural skills as the authority base.

Yet, our method obviously also has limitations. In particular, at times, drawing conclusions about the type of authority in play requires us to make hypotheses about different social norms in play. As described when discussing authority of socially negotiated content, for example, it is *some* sort of social norm that makes the student refrain from challenging the reported data and require a re-measurement. It would however require much more information than we have to draw solid conclusions about how these norms look like in detail. Inferentialism depicts authority as something constituted by the act of communicating, but not only. Some aspects of authority obviously also relate to norms that are established and rather stable, but not necessarily easy to infer for a researcher who only gets a snapshot of students' communication.

To conclude: Inferentialism provides a good theoretical basis for examining authority as a dynamic and sophisticated phenomenon that is constituted through interaction. Our methodological operationalization of inferentialism theory allowed us to identify the types of authority known from Brandom's (1994) work as well as additional types. Our study hence contributes to existing research in several ways. It addresses a need in mathematics and statistics education research by looking closely at authority in student-student interactions in collaborative work. It shows that collaborative data generation can be a process where content-related arguments also get discussed, and it offers insights into how different types of authority can influence a discussion. In teacher-guided situations, the teacher has certain opportunities to distribute authority and to, in turn, influence which aspects are discussed, even in relatively student-driven discussions (Oyler, 1996). In peer collaboration, this is harder to control (Oyler & Becker, 1997). Yet, the types of authority in play influence the claims that are taken as true and assumed by the students. Therefore, the authority used influences the content that is in play and that is accessible for learning. It is impossible to completely steer autho-rity relationships through, for example, the task design. But it is our hope that knowledge about how authority operates can help researchers and teachers to develop teaching activities in more deliberate ways. Finally, we hope that our research can be a springboard for further research on authority, which is an important and inherent part of all social learning situations.

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References

Amit, M. & Fried, M. N. (2005). Authority and authority relations in mathematics education: a view from an 8th grade classroom. *Educational Studies in Mathematics*, 58 (2), 145–168.

Bakker, A., Ben-Zvi, D. & Makar, K. (2017). An inferentialist perspective on the coordination of actions and reasons involved in making a statistical inference. *Mathematics Education Research Journal*, 29 (4), 455–470.

Bakker, A. & Derry, J. (2011). Lessons from inferentialism for statistics education. *Mathematical Thinking and Learning*, 13 (1-2), 5–26.

Ben-Zvi, D. (2007). Using wiki to promote collaborative learning in statistics education. *Technology Innovations in Statistics Education*, 1 (1), 1–18.

Brandom, R. B. (2000). *Articulating reasons. An introduction to inferentialism*. Cambridge: Harvard University Press.

Brandom, R. B. (1994). *Making it explicit: reasoning, representing, and discursive commitment*. Cambridge: Harvard University Press.

Buzzelli, C. & Johnston, B. (2001). Authority, power, and morality in classroom discourse. *Teaching and teacher education*, 17 (8), 873–884.

Cobb, P. & McClain, K. (2004). Principles of instructional design for supporting the development of students' statistical reasoning. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 375–395). Dordrecht: Kluwer.

Cobb, G. W. (1992). Teaching statistics. In L. A. Steen (Ed.), *Heeding the call for change: suggestions for curricular action* (pp. 3–43). Washington: Mathematical Association of America.

Cohen, E. G. & Lotan, R. A. (1995). Producing equal-status interaction in the heterogeneous classroom. *American educational research journal*, 32 (1), 99–120.

Engle, R. A., Langer-Osuna, J. M. & McKinney de Royston, M. (2014). Toward a model of influence in persuasive discussions: negotiating quality, authority, privilege, and access within a student-led argument. *Journal of the Learning Sciences*, 23 (2), 245–268.

Engle, R. A. (2012). The resurgence of research into transfer: an introduction to the final articles of the transfer strand. *Journal of the Learning Sciences*, 21 (3), 347–352.

Engle, R. A. & Conant, F. R. (2002). Guiding principles for fostering productive disciplinary engagement: explaining an emergent argument in a community of learners' classroom. *Cognition and Instruction*, 20 (4), 399–483.

Freitas, E. de, Wagner, D., Esmonde, I., Knipping, C., Lunney Borden, L. & Reid, D. (2012). Discursive authority and sociocultural positioning in the mathematics classroom: new directions for teacher professional development. *Canadian Journal of Science, Mathematics and Technology Education*, 12 (2), 137–159.

Friedrich, C. J. (Ed.). (1958). *Nomos I: authority*. Cambridge: Harvard University Press.

Garfield, J. & Ben-Zvi, D. (2008). *Developing students' statistical reasoning: connecting research and teaching practice*. New York: Springer.

Gore, J. M. (1996, November). *Understanding power relations in pedagogy*. Paper presented at the joint meeting of the Australian Association for Research in Education and the Educational Research Association, Singapore.

Gore, J. M. (1994, April). *Power and pedagogy: an empirical investigation of four sites*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans.

Hufferd-Ackles, K., Fuson, K. C. & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for research in mathematics education*, 35 (2), 81–116.

Inglis, M. & Mejia-Ramos, J. P. (2008). How persuaded are you? A typology of responses. *Research in Mathematics Education*, 10 (2), 119–133.

Konold, C., Higgins, T., Russell, S. J. & Khalil, K. (2015). Data seen through different lenses. *Educational Studies in Mathematics*, 88 (3), 305–325.

Krummheuer, G. (2015). Methods for reconstructing processes of argumentation and participation in primary mathematics classroom interaction. In A. Bikner-Ahsbahs, C. Knipping & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education. Examples of methodology and methods* (pp. 51–74). Dordrecht: Springer.

Krummheuer, G. (2000). Mathematics learning in narrative classroom cultures: studies of argumentation in primary mathematics education. *For the learning of mathematics*, 20 (1), 22–32.

Langer-Osuna, J. M. (2017). Authority, identity, and collaborative mathematics. *Journal for Research in Mathematics Education*, 48 (3), 237–247.

Langer-Osuna, J. M. (2016). The social construction of authority among peers and its implications for collaborative mathematics problem solving. *Mathematical Thinking and Learning*, 18 (2), 107–124.

Langer-Osuna, J. M. (2011). How Brianna became bossy and Kofi came out smart: understanding the trajectories of identity and engagement for two group leaders in a project-based mathematics classroom. *Canadian Journal of Science, Mathematics and Technology Education*, 11 (3), 207–225.

Lemke, J. L. (1990). *Talking science: language, learning, and values*. Norwood: Ablex.

Makar, K., Bakker, A. & Ben-Zvi, D. (2011). The reasoning behind informal statistical inference. *Mathematical Thinking and Learning*, 13 (1-2), 152–173.

Mercer, N. (1996). The quality of talk in children's collaborative activity in the classroom. *Learning and instruction*, 6 (4), 359–377.

Metz, M. H. (1978). Clashes in the classroom: the importance of norms for authority. *Education and Urban Society*, 11 (1), 13–47.

Noorloos, R., Taylor, S. D., Bakker, A. & Derry, J. (2017). Inferentialism as an alternative to socioconstructivism in mathematics education. *Mathematics Education Research Journal*, 29 (4), 437–453.

O'Connor, M. C. & Michaels, S. (1996). Shifting participant frameworks: orchestrating thinking practices in group discussion. In D. Hicks (Ed.), *Discourse, learning, and schooling*, (pp. 63–103). 63–103. New York: Cambridge University Press.

Oyler, C. (1996). Sharing authority: student initiations during teacher-led read-alouds of information books. *Teaching and Teacher Education*, 12 (2), 149-160.

Oyler, C. & Becker, J. (1997). Teaching beyond the progressive-traditional dichotomy: sharing authority and sharing vulnerability. *Curriculum Inquiry*, 27 (4), 453–467.

Resnick, L. B., Michaels, S. & O'Connor, C. (2010). How (well structured) talk builds the mind. In D. Preiss & R. Sternberg (Eds.), *Innovations in educational psychology: perspectives on learning, teaching and human development* (pp. 163–194). New York: Springer.

Ryve, A. (2006). *Approaching mathematical discourse: two analytical frameworks and their relation to problem solving interactions* (PhD thesis). Departement of Mathematics and Physics, Mälardalen University.

Schindler, M., Hußmann, S., Nilsson, P. & Bakker, A. (2017). Sixth-grade students' reasoning on the order relation of integers as influenced by prior experience: an inferentialist analysis. *Mathematics Education Research Journal*, 29 (4), 471–492.

Schindler, M. & Joklitschke, J. (2016). Designing tasks for mathematically talented students. In K. Krainer & N. Vondrová (Eds.), *Proceedings of CERME 9* (pp. 1066–1072). Retrieved from https://hal.archives-ouvertes.fr/hal-01287313/document

Schindler M. & Seidouvy A. (2019) Informal inferential reasoning and the social: understanding students' informal inferences through an inferentialist epistemology. In G. Burrill & D. Ben-Zvi (Eds.), *Topics and trends in current statistics education research* (pp. 153–171). Cham: Springer.

Seidouvy, A., Helenius, O. & Schindler, M. (2018). Data generation in statistics – both procedural and conceptual. An inferentialist analysis. In J. Häggström, Y. Liljekvist, J. Bergman Ärlebäck, M. Fahlgren & O. Olande (Eds.), *Perspectives on professional development of mathematics teachers. Proceedings of MADIF 11* (pp. 191–200). Gothenburg: SMDF.

Seidouvy, A. & Schindler, M. (2019). An inferentialist account of students' collaboration in mathematics education. *Mathematics Education Research Journal*, 1–21. doi: 10.1007/s13394-019-00267-0

Sfard, A. & Kieran, C. (2001). Cognition as communication: rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity*, 8 (1), 42–76.

Skemp, R. R. (1979). *Intelligence, learning, and action: a foundation for theory and practice in education*. Chinchester: Wiley.

Spring, J. H. (1999). *Wheels in the head: educational philosophies of authority, freedom, and culture from Socrates to human rights*. Mahwah: McGraw-Hill College.

Stein, M. K., Engle, R. A., Smith, M. S. & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: five practices for helping teachers move beyond show and tell. *Mathematical thinking and learning*, 10 (4), 313–340.

Vergnaud, G. (1996). The theory of conceptual fields. In L. P. Steffe & P. Nesher (Eds.), *Theories of mathematical learning* (pp. 219–239). Mahwah: Lawrence Erlbaum.

Vergnaud, G. (1982). A classification of cognitive tasks and operations of thought involved in addition and subtraction problems. In T. P. Carpenter, J. Moser & T. A. Romberg (Eds.), *Addition and subtraction: a cognitive perspective* (pp. 39–59). Hillsdale: Lawrence Erlbaum

Vogel, M. (2009). Experimentieren mit Papierfröschen. *Praxis der Mathematik in der Schule*, 51 (26), 22–30.

Wagner, D. & Herbel-Eisenmann, B. (2014). Mathematics teachers' representations of authority. *Journal of Mathematics Teacher Education*, 17 (3), 201–225.

Weber M. (1947) The types of authority and imperative co-ordination. In A. M. Parsons & T. Parsons (Eds.), *The theory of social and economic organization* (pp. 324–423). New York: The Free Press.

Welker, R. (1992). *The teacher as expert: a theoretical and historical examination*. Albany: SUNY Press.

Wells, G. (2007). Semiotic mediation, dialogue and the construction of knowledge. *Human Development*, 50 (5), 244–274.

Wertsch, J. V. (1998). *Mind as action*. Oxford University Press.

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