

Finnish entry-level students' views of teacher knowledge and the characteristics of a good mathematics teacher

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This paper reports a study of the views held by Finnish students at the start of their university studies concerning their understanding of the knowledge and characteristics of a good mathematics teacher. A total of 97 students following a basic university course responded to a questionnaire. The results showed that a knowledge of teaching mathematics was more often used to describe the good mathematics teacher than a knowledge of mathematics. According to the students' views, mathematics teachers need to be able to take the level of understanding of individual students into account in their teaching. Good mathematics teachers were also considered to be skilled in explaining, simplifying and transforming mathematical contents for their students. A good mathematics teacher was often described by the respondents as a patient, clear, inspiring and consistent person. On the other hand, characteristics such as humorous, likeable, empathetic, or fair were seldom used in the students' responses to describe a good mathematics teacher. Those respondents who planned to become teachers demonstrated a more learner-centred concept of a good mathematics teacher than did those who were aiming at some other subject specialist profession than that of teaching.

Finnish teacher education and the national education system have attracted international interest primarily as a result of Finnish students' good results in PISA and TIMSS tests. It has been suggested that Finnish teachers themselves are among the most important factors contributing to this success (Simola, 2005; Välijärvi et al., 2007). In consequence, the present study aims to explore Finnish entry-level university students' views of the knowledge and characteristics of a good mathematics teacher.

Generally speaking, the ideal good teacher can be characterised in terms of teacher knowledge and personal characteristics (Arnon & Reichel, 2007). Traditionally, teachers' subject matter knowledge has

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been considered the cornerstone of teacher knowledge, especially in the case of subject teachers (Leinhardt & Smith, 1985; Ball, Lubiensk. & Mewborn, 2001). In addition, a teacher must demonstrate pedagogical knowledge that is connected with the subject to be taught. Shulman (1986) named this knowledge type as pedagogical content knowledge (PCK). This means that a mathematics teacher, for example, has to have mathematical knowledge for teaching in order to be able to teach (Hill & Ball, 2009). Teachers also need several other types of knowledge, such as a knowledge of general pedagogical principles, learners and learning, learning environments, and the aims and values of education (Shulman, 1987, 1986).

There has recently been a growing number of studies focusing on the impact of teacher personality on teaching and learning (see Cornelius-White, 2007; Lerkkanen et al., 2016). What has been termed the person-centred teaching approach emphasises the characteristics of an effective teacher, such as empathy, warmth, genuineness, being non-directive, and having the ability to encourage others to think (Cornelius-White, 2007). In brief, this kind of person-centred teaching has been shown to be positively associated with the development of students' academic skills at school (Lerkkanen et al., 2016).

The focus of the present study is on the nature of the views that Finnish entry-level university students have concerning the knowledge and characteristics of good mathematics teachers. In their lower and upper secondary school education, the students have experienced a considerable amount of contact with mathematics teaching and teachers. In consequence, it is interesting to investigate the nature of their views of teacher knowledge and the characteristics of a good mathematics teacher that they have formed in the course of their school years. Even if mathematics education research has covered many aspects of mathematics teachers and their teaching, relatively little is yet known about students' views of what, in their eyes, constitutes the actual knowledge and characteristics of good mathematics teachers. Hence, the research questions to be discussed in this article are:

1. According to Finnish entry-level university students, what does a mathematics teacher need to know in order to be a good mathematics teacher?
2. According to Finnish entry-level university students, what kind of characteristics does a mathematics teacher require in order to be a good mathematics teacher?

3. What kinds of differences exist between the ideas of students who have opted to become teachers and those who have opted to become subject specialists?

On the basis of the responses to the first two research questions, it has been possible to construct an image of the views held by Finnish entry-level university students of what it takes to be a good mathematics teacher. It is assumed that the image of a good mathematics teacher consists of views of the kinds of knowledge that are important for a good mathematics teacher and also of the characteristics of a good mathematics teacher.

The third research question probes the potential differences between the views of students who aim to become teachers, i.e., *future teachers*, and those who have opted for a subject specialist's profession, i.e., *future subject specialists*. It is possible that the future teachers have pondered the knowledge and characteristics of a good mathematics teacher more than have the future subject specialists. In consequence, the future teachers' views of a good mathematics teacher may be more diverse and contemporary than the views expressed by the future subject specialists.

It is also interesting to discover, if these groups demonstrate features of *teacher-centred teacher*, according to that a teacher is seen as a manipulator of learning situations and as a transformer of knowledge to students, or features of *learner-centred teacher*, which holds that a teacher has to be able to take the individual learning, motivation, and developmental needs of students into account in their teaching (Schuh, 2004).

Knowledge and characteristics of good teachers

Good teachers have been examined from two major viewpoints: the kind of knowledge that good teachers demonstrate and the nature of the characteristics demonstrated by good teachers. Both of these perspectives of good teachers will be discussed in this section.

When the various studies of views of what constitutes a good teacher are examined, it can be seen that a variety of adjectives have been used to describe the nature of an ideal teacher. If it is assumed that the 'goodness' of a teacher can be measured against the gains made in students' achievement, then good teachers are often termed effective teachers (e.g. Kane, McCaffrey, Miller & Staiger, 2013). If teachers are considered as teaching professionals who implement teaching based on their best knowledge and expertise, then those who are good are often referred to as skilful or good teachers (see Crosby, 2000). In the present article, the term good teacher is used because it best suits the prevailing ideology of Finnish teacher education: Finnish teachers are considered professional teachers

who teach on the basis of their research-based education and personal expertise (Westbury, Hansén, Kansanen & Björkvist, 2005).

Teacher knowledge

The knowledge demonstrated by teachers has been previously examined to gain an understanding of the kind of knowledge base that makes a good teacher. It has been argued, for example, that the quality of a teacher's knowledge has a strong influence on how the teacher is able to link and use his/her knowledge in both the preparation of lessons and also in teaching (Schoenfeld, 1992). Shulman introduced the term *Subject matter knowledge* to describe the extent and organization of a teacher's knowledge of the subject to be taught (e.g. Shulman, 1986). In addition to *Subject matter knowledge*, a teacher should also have general *Pedagogical knowledge* for effective teaching that includes a knowledge of the general variables of instruction, such as classroom management, pacing, and questioning strategies (Boz & Boz, 2008). When a teacher's Subject matter knowledge and Pedagogical knowledge combine, *Pedagogical content knowledge* (PCK) develops. For instance, a teacher's understanding of how certain topics, problems or issues are organized, represented and tailored to the various interest and abilities of learners is a part of PCK (Shulman, 1987, p. 8).

Shulman's conceptualization is generic, but there is a wide range of literature devoted in particular to teacher knowledge in mathematics. To develop an understanding of mathematics for teaching, one has to be able to think pedagogically about mathematics. Hill and Ball (2009) refer to this knowledge as *Mathematical knowledge for teaching*, a form of mathematical knowledge that teachers use when they teach mathematics (see also Stylianides & Ball, 2008). Research has shown that effective mathematics teachers have a rich and flexible knowledge of both mathematics and pedagogy: if the teacher is able to integrate mathematics and pedagogy and to apply the composite result in a particular learning context, excellent teaching can take place (Steele, 2005).

According to Goulding, Rowland and Barber (2002), many studies of teacher knowledge in mathematics are based on the belief that learning is an interactive process comprised of what the learner is taught and what the learner brings to the learning situation. This indicates that a knowledge of students' preconceptions and attitudes is also essential for a teacher endeavouring to promote students' learning. A teacher has to be cognizant of the common conceptions and misconceptions as well as the reasoning models that students have and use (Cobb, Yackel & Wood, 1992; Cobb & Steffe, 2011). This type of knowledge can be acquired from the

research literature related to students' conceptions and understanding. In addition, a knowledge of students' attitudes is important for any mathematics teacher since research has shown that such attitudes can be related to students' achievement in mathematics (Zan, Brown, Evans & Hannula, 2006; Ma & Xu, 2004). It is important that mathematics teachers are aware of potentially negative attitudes in order to be able to affect them.

It seems that there is no extensive literature on students' or teachers' views of teacher knowledge even if the concept of teacher knowledge has a strong theoretical background. Kaur (2004) has investigated in general terms Singaporean mathematics teachers' views of what constitutes a good mathematics teacher and found a number of the features of teacher knowledge. According to Kaur's (2004) results, good mathematics teachers are skilled in mathematics, know how students learn mathematics, and are able to arouse and sustain their students' interest. They also engage students by using a variety of teaching strategies and offer timely and meaningful feedback to the students and their parents. Asikainen, Pehkonen & Hirvonen (2013) examined the views related to the teacher knowledge of a good mathematics teacher held by mentor mathematics teachers (i.e., teachers at university practice schools) in Finland. All of the mentor mathematics teachers under study rated mastery of subject matter knowledge as the prerequisite of teacher knowledge. Furthermore, they emphasised the teacher's ability to transform mathematical knowledge so that it could be used for teaching purposes. They also stressed the importance of knowledge of how students learn mathematics, as did the teachers participating in Kaur's (2004) study.

Teacher characteristics

A number of studies focus on the effects of teacher personality on teaching and learning (see Cornelius-White, 2007; Lerkkanen et al., 2016). A *person-centred teaching approach* emphasises such teacher characteristics as empathy, warmth, genuineness, non-directiveness, and the ability to encourage others to think (Cornelius-White, 2007). It has shown that person-centred teaching is positively associated with the development of students' academic skills at school (Lerkkanen et al., 2016). Studies of teacher characteristics opened up when researchers and policy makers started to take more notice of the quality of teaching per se, (Graham & Heimerer, 1981). Initial studies of teacher characteristics were reported in the 1960s and 1970s, for example, Barr (1960), Ryan (1960) and Barr, Worcester, Abell et al. (1961) listed several characteristics typical of effective teachers, such as buoyancy, pleasantness, friendliness, emotional stability, and personal magnetism (see Check, 1986; Cruickshank & Haeefe, 2001).

This knowledge was considered to be useful, for instance, whenever a school management has to select the best teachers from amongst all of the qualified applicants, or it could be used to explain why certain teachers are more effective than others.

In the 1970s, the opinions of pupils and students regarding the nature of good teachers also started to interest researchers. Murray (1975), for instance, reported the results of a survey in which college students evaluated their teachers by means of a questionnaire that probed the students' conceptions of their teachers' characteristics and teaching. The results indicated that characteristics such as leadership, objectivity, extroversion, and lack of anxiety may explain two-thirds of the between-teacher variance in student ratings. Check's (1986) study focusing on college students showed that students valued teachers who were compassionate, able to relate to their students, communicate on the students' level, enthusiastic and knowable, and humorous. On the other hand, teachers who were unable to communicate and deliver the subject, who were boring and monotonous, and disorganized or deficient in knowledge were not respected by their students. Since 1980's, research on good teachers shifted on examining connections between certain teacher characteristics and efficient teaching (see Murray, Rushton & Paunonen, 1990; Allinder, 1994; Clotfelter, Ladd & Vigdor, 2007)

It has been observed that individual views of good teaching and good teachers start to develop early in childhood and youth: even primary school students are able to describe the characteristics of a good teacher (Murphy, Delli & Edwards, 2004; Beishuizen et al., 2001). Even if views and beliefs are sometimes said to be resistant to change, there is some evidence that views concerning the characteristics of a good teacher may develop in the process of education itself. Murphy et al. (2004) studied second graders' and pre- and in-service teachers' beliefs about the characteristics of a good teacher using a revised Tuchman Teacher Feedback Form. Caring, not boring, and polite were considered to be the important characteristics of a good teacher by all of the respondent groups. In addition, the pre-service and in-service teachers emphasised the characteristic of being patient more than did the second-graders. The second-graders, in turn, valued the quality of likeability more than did the pre- and in-service teachers. In contrast, being outspoken, ordinary, or strict were qualities that were regarded as less important for a good teacher. These results indicate that views of the nature of a good teacher may be changeable, rather than stable. Arnon and Reichel's (2007) findings support this conclusion. They compared the views of Israeli student teachers and newly qualified teachers concerning the image of the ideal teacher and also in relation to their own image as a teacher. Their results showed that

student teachers' images of themselves resembled their images of the ideal teacher, emphasizing personality, especially empathy and attentiveness, but understating teacher knowledge. The newly qualified teachers, for their part, tended not to see themselves as ideal teachers based on their own characteristics, but they emphasized teacher knowledge to roughly the same extent as personal characteristics.

More specifically, students' or teachers' views of the characteristics of a good mathematics teacher have yet to attract widespread research interest. However, Kaur (2004) has examined the views of a good mathematics teacher as expressed by Singaporean secondary school mathematics teachers. The teachers surveyed ranked the personal characteristics of teachers in the following order: patient, systematic/organised, helpful, hardworking, and enthusiastic. A good teacher was also considered to be caring, understanding, approachable, encouraging/motivating/inspiring, and firm/friendly, as ranked in order of prevalence. According to our understanding, however, these characteristics may suggest that teaching mathematics requires certain special characteristics, such as being systematic and hardworking, that may be unique to mathematics teachers. It has already been observed that there is a relationship between mathematics teachers' enthusiasm and the quality of teaching: more enthusiastic teachers may demonstrate higher quality instructional behaviour (Kunter et al., 2008).

Method

Data collection

The data collection was based on a questionnaire that was implemented in conjunction with a study that probed student's views on mathematics (Viholainen, Asikainen & Hirvonen, 2014). The questionnaire consisted of the following open-ended questions (originally in Finnish) related to this study:

- What does a mathematics teacher need to know in order to be a good mathematics teacher? Specify the three most important issues.
- Specify three characteristics of a good mathematics teacher.

The questionnaire was distributed in paper-and-pencil form during a lecture given at the beginning of a course in basic mathematics. The course is the first in mathematics in the students' mathematics studies and is compulsory for everyone who wishes to complete at least the

basic studies in mathematics. Both of the questions had a five-line space for the students' responses. Completion of the questionnaire took approximately 10 minutes.

Separate questions concerning teacher knowledge and teacher characteristics were presented because our earlier experiences in this research area showed that in many cases the respondents focused on either the knowledge or the characteristics: if the respondent is requested to describe in free form the nature of a good teacher, the other topic will be neglected, and vice versa. In addition, to acquire useful contextual information, details were requested concerning the respondent's major, study programme, and the number of mathematics courses taken and grades obtained at upper secondary school.

Respondents' demographics

A total of 97 university students, 58 male and 39 female, responded to the questionnaire. According to the results of the questionnaire, 49% of the respondents were students within a teacher education programme, and in most cases their major was mathematics, physics, or chemistry. These respondents are referred to here as 'future teachers'. 51% of the respondents were not pursuing a teaching career but had opted for some other form of subject specialist's career, such as researcher, mathematician, physicist, or chemist, or they indicated uncertainty about their future careers. This respondent group is referred to in this study as 'future subject specialists'. It is, however, entirely possible that some of the respondents in both groups might later change their minds, and hence it must be emphasised that these were their views concerning their future careers when the questionnaire was administered. Table 1 shows the respondents' basic demographics.

Table 1. *Respondents' demographics*

Major	Study programme		Total
	Teacher education	Others	
Mathematics	19	20	39
Chemistry	6	15	21
Physics	8	9	17
Education	12	2	14
Other*	3	3	6
Total	48	49	97

Note. * Other is e.g. computer science, biology, forestry.

As table 1 shows, mathematics majors formed the largest respondent group. Roughly half of them were participating in a teacher education programme, but they also included chemistry, physics, and education majors. A majority of the chemistry and physics majors had not opted for teacher education. In contrast, most of the education majors were studying within the framework of a teacher education programme.

Data analysis

The questionnaire responses were all transcribed into a word processing program so that they could be investigated using a content analysis method (Mayring, 2000). The responses were coded and analysed to reveal relevant features of teacher knowledge and characteristics. To discover the respondents' views of teacher knowledge, attention was focused on locating phrases or statements describing what a good teacher needs to know. In addition, to discover the respondents' views of teacher characteristics, our analysis concentrated on finding descriptive adjectives in the text. Responses that broadly resembled each other were grouped together to form categories of respondent views. It was also our aim to understand what respondents really meant when they listed the particular characteristics of a good mathematics teacher, and hence the responses were read through several times so that a proper understanding of their content could be gained.

Because the third research question was to examine the differences between students in terms of those aiming at becoming teachers (future teachers) and those orienting towards some other subject specialist than teacher (future subject specialists), the responses were categorised on the basis of this information so that the ideas of these two groups could be compared. The main author of the present article performed the analysis. Once it had been completed, all of the authors discussed the categorisations, especially those of some of the more ambiguous responses, until consensus was reached.

Views of knowledge of a good mathematics teacher

University student's views of the knowledge important for good mathematics teachers formed three categories: Knowledge of teaching mathematics, Knowledge of mathematics, and Knowledge of interpersonal and communication skills. In what follows, we will attempt to discuss in greater detail what these respondents' views mean.

Knowledge of teaching mathematics

A total of 89% of the future teachers and 76% of the future subject specialists referred to the teacher's knowledge of teaching mathematics when describing their views of the knowledge of a good mathematics teacher. A majority of the future teachers (53%) mentioned that a good mathematics teacher will take into account either the individual student or his/her level of understanding (see table 2). 12% of the future subject specialists expressed this notion. Many of these respondents referred to the individual needs of students based on their mathematical background, as the following extract demonstrates:

[S/he] takes the personal strengths and weaknesses of students into account. [S/he] uses step-by-step progress in teaching: A teacher does not let students calculate by themselves until end of class and then ask if anything had been unclear. [future subject specialist]

According to the respondents, a good teacher provides both talented and academically weak students with equal opportunities for learning mathematics. A good mathematics teacher is also able to step into his/her students' shoes in order to gain insight into their level of mathematical understanding.

Table 2. *Knowledge of teaching mathematics demonstrated by a good mathematics teacher, according to future teachers (n=48) and future subject specialists (n=49)*

Category of teacher knowledge	Future teachers (%)	Future subject specialists (%)
A. Teacher takes the level of understanding of individual students into account	53	12
B. Teacher simplifies, explains or transforms mathematical content for students	34	24
C. Teacher has good teaching skills	19	28
D. Teacher uses a variety of methods	13	4
E. Teacher uses examples	11	4
F. Teacher stimulates or motivates	4	10
G. Teacher illustrates	6	4
H. Teacher emphasises doing or inventing	2	4
I. Teacher connects mathematics with everyday life	4	2
J. Teacher is concise in teaching	-	6
K. Teacher answers correctly	2	-
L. Teacher advises	-	2

The skill to simplify, explain or transform mathematical knowledge was also often referred to by respondents as a type of knowledge relevant to a good mathematics teacher (see table 2). The future teachers brought up this idea more frequently than did the future subject specialists. The respondents often stated that a teacher has to be able to teach explicitly or thoroughly by employing several different explanations of the topic at hand. On the other hand, only a few of these respondents referred to the students' role in the teaching of mathematics, as the following extract shows:

[A teacher has] knowledge about how a student thinks. [S/he] is also able to explain complicated things clearly. [*future subject specialist*]

The respondents often assumed that if a teacher uses language that students understand or if s/he understands how students think, then clear and comprehensive mathematics teaching makes students learn mathematics, as in the case of the following future teacher:

[S/he] is able to explain and present things clearly and simply. [S/he] makes things easy to understand and assimilate. [*future teacher*]

Some respondents only expressed a view that a good teacher demonstrates good teaching skills, without specifying in greater detail. This kind of conception was more typical amongst the future subject specialists (28%) than amongst the future teachers (19%).

The respondents also mentioned several other features of a good mathematics teacher (table 2). Relatively few respondents referred to the use of a variety of methods, examples, stimulating or motivating teaching, or the use of illustrations (Categories D, E, F and G). Only a few respondents suggested that a good mathematics teacher would encourage her/his students to invent or do mathematics by themselves (Category H) or would connect mathematics with aspects of students' everyday life (Category I).

Knowledge of mathematics

A total of 75% of future teachers and 62% of future subject specialists referred to mathematical knowledge when they described a good mathematics teacher's knowledge (see table 3). Most respondents considered that a mathematics teacher must have an extensive knowledge of mathematics (Category A). According to the respondents, this means that a teacher has to master all of the topics in the curriculum and must also have a thorough understanding of the relevant mathematical topics rather than simply mechanical know-how, as the following excerpt shows:

[A good mathematics teacher] knows how to calculate and really understands what s/he is doing. S/he hasn't learnt the examples by heart. [future subject specialist]

Some of these respondents also emphasised that it is important to know the origin of mathematical formulas. It was also rather common among the respondents to have a somewhat obvious conception of the mathematical knowledge of a good teacher: a mathematics teacher has to know mathematics well, must know what s/he is teaching, or must understand what s/he is talking about (Category B):

[S/he] has a good knowledge of the topics to be taught. [future teacher]

Interestingly, one fifth of respondents expressed the conception that a basic knowledge of mathematics would be sufficient for a good mathematics teacher (Category C).

[A good mathematics teacher] has a good basic skills and knowledge of mathematics. [future teacher]

A few respondents also emphasized that the ability to apply his/her knowledge was important for a skilled mathematics teacher (Category D).

[S/he] possess theoretical knowledge and an ability to put the knowledge into practice. [future teacher]

[S/he] is capable to apply her/his knowledge instead of having mechanical skills. [future subject specialist]

According to our understanding, these examples may refer either to an ability to apply learned knowledge in practice or to using such knowledge in further problems requiring more than mechanical calculation skills alone.

Table 3. *Mathematical knowledge of a good mathematics teacher, according to the future teachers (n=48) and future subject specialists (n=49)*

Category of teacher's mathematical knowledge	Future teachers (%)	Future subject specialists (%)
A. Extensive knowledge of mathematics	26	22
B. Mathematical knowledge generally	26	20
C. Basic knowledge of mathematics	17	18
D. Application skills	6	2
Total	75	62

There were no notable differences in this category between the views of the future teachers and those of the future subject specialist. Generally speaking, however, the future teachers tended to be more verbose in the questionnaire.

Knowledge of interpersonal and communication skills

The third main category of teacher knowledge formed from the respondents' answers was Knowledge of interpersonal and communication skills. We acknowledge that this is not usually discussed as a part of teacher knowledge, but because the categorisation is based on the respondents' answers, and the respondents discussed it as something that a good mathematics teachers have, Interpersonal and communication skills forms a separate category. This knowledge was emphasised more by the future teachers than by the future subject specialists. 30% of the future teachers referred to interpersonal skills, but only 14% of the future subject specialists mentioned the kind of knowledge as belonging to this category (see table 4).

Table 4. *Knowledge of interpersonal and communication skills of a good mathematics teacher, according to future teachers (n=48) and future subject specialists (n=49)*

Category of teacher's mathematical knowledge	Future teachers (%)	Future subject specialists (%)
A. People/Social skills	19	6
B. An ability to empathize	11	4
C. An ability as a public performer	-	4

Further, the most commonly mentioned knowledge type related to interpersonal and communication was people skills or social skills (category A). Altogether, 19% of the future teachers and 6% of the future subject specialists addressed this skill. The following excerpt illustrates this issue.

[S/he] gets along with pupils of certain age. For instance, discipline at lower secondary school. [future teacher]

Some of the participants also suggested that a good mathematics teacher needs to be able to empathize, to consider a problem from the perspective of a student, and to converse with a student to discover the level of his/her understanding. This category was named as An ability to empathize.

[S/he] listens a student and is able to put oneself in students' shoes. [future teacher]

A few future subject specialists also emphasized public performance (category C) as an ability stating very shortly as follows:

- | | |
|--------------------------------|-----------------------------|
| Ability as a public performer. | [future subject specialist] |
| Appearance. | [future subject specialist] |

Views of characteristics of a good mathematics teacher

When the respondents were asked to specify the three most important characteristics of a mathematics teacher, a total of 27 different characteristics were mentioned. Altogether, the data consisted of 214 mentions, which means students expressed an average of 2.2 characteristics. The characteristics that were categorized are presented in table 5.

The characteristics most frequently mentioned were the ability to be patient and the ability to be clear. Half of the future subject specialists and 28 % of the future teachers considered that a good mathematics teacher should be patient (see table 5). The following quotations show typical responses.

- [A good mathematics teacher] is patient. [S/he] doesn't rush forward but gives time for the student. [future subject specialist]

Some respondents argued that a mathematics teacher has to be patient because learning mathematics is not as easy for every student. They also reasoned that a patient mathematics teacher leaves enough room for student learning. If necessary, a patient mathematics teacher also explains the topic to be learned several times, if a student does not understand immediately. Some respondents also mentioned that a patient mathematics teacher does not become frustrated or lose his/her temper if a student does not grasp the topic to be learned.

- [A good mathematics teacher] is patient (doesn't get frustrated). [future teacher]

We interpret this generally in terms of a patient mathematics teacher taking the individual student into account in his/her teaching.

Some 38 % of the future teachers mentioned that a teacher should be clear. This characteristic was almost as common amongst the future subject specialists (30 % of the respondents). According to respondents, by using a clear and simplifying teaching style, a good mathematics teacher facilitates students' understanding.

- [A good mathematics teacher] has a clear manner of explaining and a clear order in issues to be learned. [future teacher]

Table 5. *Characteristics of a good mathematics teacher according to future teachers (n=48) and future subject specialists (n=49)*

Teacher characteristic *	Future teachers (%)	Future subject specialists (%)
Patient	28	50
Clear	38	30
Inspiring	21	20
Consistent	23	16
Enthusiastic	17	8
Encouraging	15	4
Helpful	11	8
Calm	13	4
Relaxed	9	8
Humorous	9	4
Authoritarian	2	8
Likeable	2	8
Empathetic	4	6
Creative	6	4

Note. * Characteristics mentioned only once or twice are not presented in the table: *adaptive, approachable, good-looking, fair, exemplary, open-minded, person of interest, sprightly, smart, has clear handwriting, spontaneous, demanding and thorough.*

A mathematics teacher can also be clear in the sense of discussing the most salient aspects of the topic to be learned. Some respondents also suggested that a good mathematics teacher uses clear speech or thinking.

The third and fourth most common mathematics teacher characteristics were Inspiring and Consistent. One fifth of the respondents in both of the groups considered that a good mathematics teacher is inspiring. According to the respondents, a good mathematics teacher is able to stimulate students' interest in mathematics. This can be done, for example, by using various teaching approaches in the form of exercises that make mathematics an interesting subject to study.

[A good mathematics teacher] has a personality that gets students become enthusiastic in mathematics. [future teacher]

Some respondents also thought that good mathematics teachers have a positive attitude towards teaching and learning mathematics.

Almost one quarter of future teachers used the adjective Consistent to describe a good mathematics teacher, but amongst the future

subject specialists it was less frequent, with only one out of eight respondents referring to this characteristic. According to the respondents, a consistent mathematics teacher teaches mathematics systematically.

[A good mathematics teacher] doesn't jump from one thing to another. [future subject specialist]

Some respondents also argued that consistency is an important characteristic for a mathematics teacher, since this kind of teacher is able to teach mathematics in a way that is "easy to follow".

The characteristics of Enthusiastic, Helpful, Encouraging, Calm, and Relaxed were less frequent among the respondents than the characteristics mentioned previously. In addition, all of them were more commonly suggested by future teachers than by future subject specialists. According to the respondents, good mathematics teachers love to learn new things and share them with their students.

[A good mathematics teacher] wants to learn new things and share this knowledge with students. [future teacher]

A good mathematics teacher is also interested in, or even passionate about, mathematics and its teaching. A teacher's personal interest in mathematics was seen by some respondents as a prerequisite for the passion required to help students succeed in mathematics.

A good mathematics teacher was considered to be Helpful. According to the respondents, this means that a teacher either offers help when students need it or s/he is pleased to help. A helpful mathematics teacher helps students to overcome problems and succeed in learning mathematics.

[A good mathematics teacher] helps and guides if necessary. [future teacher]

Good mathematics teachers also encourage and support students in studying mathematics. They encourage their students to try, fail, and succeed.

According to the respondents, a good mathematics teacher is Calm in the sense that s/he does not hurry in teaching, and that mathematics teaching is unhurried. We interpreted calm as a separate characteristic from patient. In addition, the respondents explained the characteristic of patient as more connected with versatility than the characteristic of calm.

Some respondents also stated that a good mathematics teacher is Relaxed. This characteristic was described in terms of "an easy-going atmosphere" or "lets students keep their own habits".

Some respondents mentioned that good mathematics teachers are Humorous or that they have to be Authoritarian or demonstrate leadership

qualities. Likeable, Empathetic and Creative were other infrequently mentioned characteristics that were not explained by the respondents.

Discussion

In the present study, attention was paid to Finnish university students' views of the knowledge and characteristics required by a good mathematics teacher. The respondents were students participating in a basic mathematics course who intended to become teachers (future teachers), researchers, or other types of subject specialists (future subject specialists). Based on our results, we conclude that the students' conceptions of a good mathematics teacher include views related to both teacher-centred and learner-centred mathematics teachers. The view of a good mathematics teacher of future teachers seems to involve more learner-centred characteristics than does the view of future subject specialists.

Knowledge of teaching mathematics was the most common category of teacher knowledge in both of the respondent groups. This is interesting, because it has been observed in earlier studies (e.g. Asikainen et al., 2013) that practicing mathematics teachers prioritise knowledge of mathematics. Approximately half of the future teachers introduced the basic idea of learner-centred thinking, which holds that mathematics teachers have to be able to take the individual learning, motivation, and developmental needs of students into account in their teaching (Schuh, 2004). However, only 12% of the future subject specialists expressed this conception in the questionnaire, and hence it can be concluded that they were less familiar with this idea than were the future teachers.

Roughly every third respondent also mentioned that a good mathematics teacher has to have the ability to simplify, explain, and transform mathematical content for their students. The respondents argued that a mathematics teacher has to have the tools that will enable them to chop mathematical content into sufficiently small pieces that are easier for their students to understand. This idea, in turn, often projected an image of teacher-centred teaching where the teacher transmits information and manipulates learning situations in order to obtain the desired outcomes framed by the general characteristics of learners (Schuh, 2004). This view does not emphasise students as individuals but rather as groups of learners. This kind of view, where the transfer of knowledge is salient for a teacher, has also been acknowledged by Beishuizen et al. (2001).

The second common response category of teacher knowledge of a good mathematics teacher was *Knowledge of mathematics*. This result is in line with the common view that a teacher has to know what s/he is teaching (Shulman, 1987). However, almost one fifth of the respondents

expressed the idea that basic mathematical knowledge is adequate for a mathematics teacher. It seems that some of these respondents think that it is enough if a teacher knows the mathematics to be taught at a general level but her/his knowledge of teaching mathematics has to be good. This rather naïve view contrasts with the results obtained by Ball, Hill and Bass (2005). The study suggests that a deep understanding of mathematical concepts may be essential for teachers so that they can develop a wide variety of teaching strategies. According to our understanding, this may mean that moderate mathematical understanding may very well restrict a teacher's development of teaching skills.

The third category of teacher knowledge formed on the basis of the data that we obtained was *Knowledge of Interpersonal and communication skills*. Even if this type of knowledge is not typically included in models of teacher knowledge (e.g. Shulman, 1986; Hill & Ball, 2009), it is claimed that interaction is one of the most important factors in teaching because it is related to order in the classroom, and hence it is a critical factor for achieving the aims of teaching and also of wellbeing in the teacher's work (Friedman, 2000). This knowledge type was mentioned by 30 % of the future teachers, but by only 14 % of the future subject specialists. On the other hand, many of the teacher characteristics mentioned by the respondents are related to Interpersonal and communication skills, such as *patient* and *clear*. Patience has been reported as an important feature of a good teacher, according to pre-service and in-service teachers (Murphy et al., 2004) and secondary mathematics teachers (Kaur, 2004). In this study, the respondents described patience as a student-centred characteristic (Schuh, 2004). Interpersonal and communication skills are undoubtedly characteristics of a good mathematics teacher that should also be discussed further in mathematics teacher education.

These characteristics, together with the fourth common characteristic in both the groups, *consistency*, describe the ability of a teacher to transfer knowledge to students and can therefore be interpreted as indication of a teacher-centred teaching view (Schuh, 2004). This kind of view has been previously observed with primary school students (Beishuizen et al., 2001).

In contrast, the characteristics of *inspiring* and *encouraging* as aspects of a teacher's ability to motivate and inspire students to learn mathematics were not as common as those previously mentioned in our study, even if they can be assumed to portray a learner-centred teacher who understands the importance of student interest, engagement, and motivation in the learning of mathematics (Singh, Granville & Dika, 2002). Furthermore, the respondents seldom used characteristics such as humorous, likeable, empathetic, or equal to describe a good

mathematics teacher, even if these characteristics have often been reported in other studies in descriptions of good teachers (see Check, 1986; Kaur, 2004; Murphy et al., 2004).

It seems that many of the respondents in our study, especially the future subject specialists, regarded good mathematics teachers as experts of mathematics teaching and mathematics, but not as student-centred, empathetic, friendly or humorous persons. It is possible that the more traditional, teacher-centred teacher view of the future subject specialists in this study may have a closer connection to their orientation towards a research profession than to teaching per se. It has been observed that students who think they have the right qualities for working as a teacher do indeed enter teacher education programs (Löfström, Poom-Valickis, Hannula & Mathews, 2010). If a student's view of a mathematics teacher is teacher-centred, then they may not regard their own characteristics as suitable for that profession. Students may also ask themselves: "Is my maths teacher the kind of person that I am or that I want to become?" At the moment, becoming a mathematics teacher is not a popular career choice among young people in the Nordic countries. Since every mathematics teacher is an important role model for young people pondering their career paths, this issue needs to be better addressed both in the literature on teacher research and in mathematics teacher training. Teacher training programmes should offer opportunities for students to explore their own views concerning good teachers in order to make them visible. This kind of method is often used at the beginning of teacher education (Minor et al., 2002). The results presented in this article may provide a fruitful base for teacher educators in mathematics to use to identify and discuss the views and beliefs of their own students.

The next stage in this research project will be to design a survey instrument for a large-scale study of students' views of the knowledge and characteristics of a good mathematics teacher. It would also be interesting to explore students' views of good mathematics teachers in different countries to discover potential similarities and discrepancies. This kind of study may also explain why becoming a mathematics teacher is not a popular career option amongst young people and help in finding possible solutions to this problem.

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