What characterises mathematical conversations in a Norwegian kindergarten?

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This paper presents a study of young children's mathematical conversations in a Norwegian kindergarten and provides examples to illustrate some of their characteristics. Using points made in previous research about conversations and mathematics, an analysis of an interaction involving a group of children who are putting a toy together exemplifies what can and what cannot be considered a mathematical conversation. For a conversation to be considered mathematical, it is suggested that it must include not only references to mathematics, but also specific structural elements in the conversation, as well as valuing participants' contributions and encouraging participants to reflect on the mathematical theme, thereby promoting learning.

In 2006, the Norwegian ministry of education developed for the first time a kindergarten curriculum that included mathematics, *the Norwegian framework plan for the content and task of kindergartens* (Kunnskapsdepartementet, 2006). Mathematics is included as a learning area in the Framework plan under the heading "Numbers, spaces and shapes". However, it was expected that these topics would be learnt in the same way as other knowledge and skills, particularly through engagement in conversations. In the Framework plan, the relationship between learning and conversations is described as follows:

> Learning takes place in everyday interactions with other people and with the community, and is closely related to play, care, and formation. Children can learn from everything they experience in all areas of life. Children's questions must be responded to in a challenging and investigative manner, to form the basis for an active and developmentally pedagogical environment at the kindergarten.

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> The children's own interests and questions should form the basis for learning processes and themes at kindergartens. The way in which staff respond to children's expressions in terms of body language, verbal language, feelings, and social relationships will affect how they learn. (Kunnskapsdepartementet, 2006, p. 29)

Thus, the Framework plan highlights the importance of communication in learning in kindergartens, indicates the ways children should learn and states that children learn when they participate in everyday activities in cooperation with adults. The Framework plan emphasises the way staff should respond to children's questions and expressions, because language and social relationships are considered to have an impact on children's learning and development. However, the lack of detail regarding what this kind of interaction would look like in kindergartens may cause teachers to struggle with what the distinctive features of a mathematical conversation are that would support children's learning.

The aim of this paper is to describe the content and structure of mathematical conversations in kindergartens. Knowledge of mathematical conversations, including how to use and initiate them, is likely to be beneficial to kindergarten teachers who want to promote mathematical learning.

Mathematics in kindergarten

In order to describe mathematical conversations, it is necessary to describe what mathematics in kindergarten might be. Most adults have some expectations about mathematics from their time in school. However, Nunes and Bryant (1996) describe mathematical activities and knowledge as involving much more than what is generally viewed as mathematics in the school curriculum. They stress that children need to be able to use mathematical knowledge to solve problems. Nunes and Bryant provide examples of how children can be successful in mathematical activities outside school but fail in similar activities in the classroom. Further, mathematical knowledge is something children already have experience with, at home and in kindergarten, before they start school (Nunes & Bryant, 1996). Therefore, rather than describing mathematics in kindergarten in terms of what occurs in school, it is more productive to consider it in relationship to the experiences that children are likely to have, either at home or in kindergartens.

Alan Bishop (1988a, b) developed an approach to defining mathematics that does not rely on school curricula. He describes six universal mathematical activities that can be found in all cultures: Counting (comparing and ordering discrete phenomena), Playing (devising and engaging in

games with rules), Designing (creating spatial objects), Measuring (comparing quantifiable qualities), Locating (engaging with the spatial environment) and Explaining (accounting for the existence of phenomena). Bishop's six activities have been used in considering young children's learning of mathematics in several earlier studies (see Johansson et al., 2012; MacMillan, 1998). This indicates that it is possible to describe mathematics in kindergarten using Bishop's (1988a, b) six activities, since it would make relevant children's experiences at home and at kindergarten.

Magne (2003) also provides an alternative definition of mathematics to that which applies in school. He emphasises three areas that are important for young children's learning of mathematics: 1) Problem solving and language, 2) Geometric understanding of space, shapes and measurements, and 3) Numbers or numeracy. These areas have some similarities with Bishop's activities. For example, explaining and playing are similar to Magne's problem solving and language. Bishop's three activities locating, measuring and designing share some elements of Magnes' second area, geometric understanding. The mathematical activity of counting is also closely connected to numbers or numeracy. Since these descriptions of mathematics in kindergarten, which are not based on what occurs in school, seem to highlight similar features, it would be useful to consider mathematics in kindergarten as those interactions that young children engage in which can be classified using Bishop's six activities.

Thus, the content in mathematical conversations in kindergarten needs to include references to one or more of Bishop's (1988a, b) mathematical activities: Counting, Playing, Designing, Measuring, Locating and Explaining.

Conversations, inquiry and orchestration

As well as defining the mathematical content, there is a need to define what constitutes a conversation in regard to mathematics in kindergartens. In everyday life, we have many conversations that reflect the great variety of ways people communicate with each other. Gjems (2009) stated that conversation is talk in which people exchange, for example, thoughts, feelings, knowledge and news. Conversations can differ with regard to topic, length, participants and structure. The structure of question–answer is a classic conversation type, but conversations can also involve other types of utterances. A conversation may include utterances such as open-ended statements and rephrasing of what the other person said. Säljö (2000) points out that conversation genres make what happens between people seem cohesive, logical and understandable. Having a conversation contributes to a greater degree of intersubjectivity, than just being in each other's presence (Gjems, 2009).

Conversations develop intersubjectivity and support learning. Children have an innate desire to learn and use their knowledge in conversations (Gjems, 2009). Vygotsky (2001) views language as a tool for communication between people and as a tool for thinking, thus highlighting its role as the social means of thinking. Vygotsky (1978) emphasizes that learning and development occur in the relationship between people, and thus within conversations. He describes development as the interaction between two zones, the zone of actual development and the zone of proximal development:

> what is in the zone of proximal development today will be the actual developmental level tomorrow – that is, what a child can do with assistance today she will be able to do by herself tomorrow.
>
> (Vygotsky, 1978, p. 87)

Vygotsky's zones provide tools for studying children's development and learning. The theory of the zone for proximal development indicates that children may need adults or another capable person to give directions and advice so that the child is guided to reflect on mathematical knowledge. Wood, Bruner and Ross (1976) label the process whereby a more knowledgeable person guides a less knowledgeable person as scaffolding.

Vygotsky stresses that culture affects the learning process. He writes: "human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those around them" (Vygotsky, 1978, p. 88). In a kindergarten setting, this would imply that the child's learning process is dependent on the learning environment that already exists there, and what the child is interested in will be dependent on what needs the child has, as well as on the stimulation provided by the available tools and artefacts. The child operates in an environment where artefacts already have a culturally determined meaning. Consequently, kindergarten teachers have to consider how artefacts are presented in the kindergarten setting. Vygotsky points out that:

> the most significant moment in the course of intellectual development, which gives birth to the purely human forms of practical and abstract intelligence, occurs when speech and practical activity, two previously completely independent lines of development, converge.
>
> (Vygotsky, 1978, p. 24)

The conversations that support children learning mathematics in kindergarten are likely to be the ones that occur within a child's zone of proximal development. I argue that, in order to move children into the zone of actual development, where children can work independently (Vygotsky, 1978), these conversations could include inquiry. Inquiry can be seen both as an instructional method and as a way of being –

a profound and basic attitude when encountering challenges and problems (Jaworski, 2005, 2007; Wells, 1999). The purpose of the inquiry is not primarily to develop a certain kind of knowledge, but rather to focus on how the individual is predisposed to and develops the ability to apply knowledge in future situations. Lindfors (1999) stresses that inquiry lives in the Zone of Proximal Development (Vygotsky, 1978). From research in schools, Lindfors (1999) identifies two forms of inquiry acts: information seeking and wondering. These acts are distinguished on the basis of their purpose. An information-seeking inquiry act is characterized by seeking to find closed, factual answers. In contrast, a wondering act opens up the conversation and is reflective in character. Wondering is playful and invites different approaches.

Carlsen (2010) uses Kennewell's (2001) concept of orchestrating, to describe the process of inquiry developed by kindergarten teachers and a researcher to improve children's learning in mathematics. Orchestration includes the kindergarten teachers' planning and carrying out of mathematical actions and activities. In this way, it includes both the pre-session preparations, and what the teacher is thinking, saying and doing during the interactions with the children.

Carlsen, Erfjord and Hundeland's (2010) describe how a kindergarten teacher interacted with children in a measuring activity. They could find six different categories of questions that the kindergarten teachers used in their conversations with the children. The categories are: suggesting actions; requesting information in an open manner; asking for supporting information for the argument; inviting problem solving; re-phrasing; and concluding. The kindergarten teachers commonly use open questions. This is in contrast to Gjems' (2013) finding in which children were usually invited to answer closed questions in conversations about everyday activities in the kindergartens and seldom are invited to work together with the kindergarten teacher to develop understanding. This indicates that everyday conversations and mathematical conversations can be different.

Children's interactions with each other are also conversations. For example, Matre (2000) reveals that children are concerned about each other and each other´s thinking. When adults are not present, older children take a more active role in initiating playful and argumentative dialogues. The children argue with each other and try out hypotheses. Matre also shows that children provide examples and explanations, and establish connections.

The characteristics of a mathematical conversation include mathematical content and have a structure based on inquiry that generates a wondering attitude, assisted by a variety of questions.

Method

The data was collected during a three-week observation period at a kindergarten in the western part of Norway. The data consists of audio recordings of conversations and field notes taken when observing children and adults' play and conversations, both outside and inside the kindergarten. The observation was open and the participants knew that I was there to observe the everyday life in the kindergarten. My focus on mathematics was known to the teachers, but was not communicated to the children. Ethical issues are important in research about and with children. In my study, parental approval was obtained for the children's participation in the research and every child was anonymised.

In the audio recordings when children were playing, there was a lot of noise and sometimes it was difficult to hear exactly what was going on. Therefore, I made field notes to complement the recordings. I sat in the background a little bit away from the children when they were engaged in their activities. The conversation analysed in this paper was chosen because it exemplifies the characteristics identified in the previous discussion as essential to mathematical conversations.

The conversation is from an activity in which a group of children were building a Lego[1] airplane together with the help of an instruction booklet. The setup of the activity meant that the children needed to interact in order to succeed. In previous work with children using building blocks such as Lego, connections to mathematics have been noted (see for example Ferrara et al., 2011). Therefore, it seemed likely that, in building with Lego, the children would engage in a mathematical conversation.

Bishop's (1988a, b) six activities were chosen as a way of identifying the mathematics in the three excerpts from the conversation. Thus, alongside the transcripts of the extracts, there is a column indicating which of Bishop's six mathematical activities are present. In table 1, I describe how Bishop's six activities were operationalised in order for the different utterances to be categorised. Bishop's original research was not about mathematics in kindergartens, therefore it was important to consider what features were present for an utterance to be classified as involving one activity or another. By looking at the complete set of data, it was possible to identify links to all six of Bishop's mathematical activities. In the three excerpts presented in this paper, four of Bishop's activities were present: Explaining, Locating, Designing and Counting. These are the ones included in table 1.

The two missing categories are Playing and Measuring. That does not mean that they were not present, but that I cannot identify them in the utterances. Bishop (1988b) defines Playing as: "Devising, and engaging in,

Table 1. O*perationalising Bishop's six activities*

Bishop's activities	Bishop's (1988a, b) definition	Identifying features	Examples in the data
Explaining	"Finding ways to account for the existence of phenomena be they religious, animistic or scientific." (1988b, p. 183). "It focusses attention on the actual abstractions and formalisations themselves, [...] explaining is concerned with answering the complex question of "Why?" (1988a, p. 48). "classifying is a universal activity, the classifications obtained are not. The diversity of languages brings a diversity of classifications." (1988a, p. 48).	Giving reasons for something. Classifying something based on an attribute.	Explanations example: "We should open this afterwards." Classification example: "It is hair." Classification example: "All that belongs to humans is over here."
Locating	"Exploring one's spatial environment and conceptualising and symbolising that environment, with models, diagrams, drawings, words or other means." (1988b, p. 182).	Use of prepositions. References to the position of people or objects.	Example: "in", "by", "under". "Which way should it be?"
Designing	"Creating a shape or design for an object or for any part of one's spatial environment. It may involve making the object, as a 'mental template', or symbolising it in some conventionalised way." (1988b, p. 183).	References to a plan or mental visualisation of what is being built. Could include instructions on how to build the airplane.	Example: "Do you see that the green one should be up against the grey?"
Counting	"The use of a systematic way to compare and order discrete phenomena. It may involve tallying, or using objects or string to record, or special number words or names." (1988b, p. 182).	Number terms or aspects to do with calculating how many of something are needed.	Example: "Here are two more."

games and pastimes, with more or less formalised rules that all players must abide by" (p. 183), and Measuring as: "ordering, using objects or tokens as measuring devices with associated units or 'measure-words' ". (p. 182–183). One can argue that playing is connected to the building activity and measuring may have been more obvious if the activity was filmed. Moreover, many single utterances could belong to more than one category. Therefore, each utterance was considered in relationship to preceding utterances, including questions, to facilitate its categorisation as primarily involving one activity.

Conversations related to Lego constructions

The teacher planned for groups of children to build Lego figures which some children had done previously and which the teacher knew they enjoyed. Four children, Ina, Chi, Dan and Jil, all around five years old, were to build a Lego airplane together. Inside the Lego box, there was an instruction booklet and several bags of Lego pieces. The teacher assigned Ina the task of managing the group work, since she had built a similar airplane before. The teacher wanted to observe how the children managed on their own. Three excerpts from the children's interaction are analysed.

Excerpt 1

This excerpt starts with the children opening the bags with Lego pieces.

		[Bishop's activities]
Dan:	Here is the plastic bag. Are we going to throw it in the bin?	Explaining
Ina:	Is there anyone who has seen the head?	
Ina:	We should open this afterwards. It is hair. All that belongs to humans is over here.	Explaining
Jil:	Are we good at building Lego? [She looks at the teacher, who smiles back.]	
Dan:	It should be there.	Explaining Locating
T.:	Do you have the instruction book Ina? Here is the bag with the number 1 and here is one with the number 2. What number is that? [Asks the group] ... one. Then we must open this.	Counting Explaining
Chi:	[Trying to open a bag]	
Dan:	I can do it.	
Chi:	I will do it.	

There was a lot going on simultaneously in this sequence. The organisation of the children in a group contributed to them conversing. If they had to build by themselves, there would not have been the opportunity to have a conversation. Intersubjectivity was present in the way they asked questions of each other: "Are we going to throw it in the bin?" and gave one another instructions: "All that belongs to humans is over here". The children were at the beginning of the task and there was an emerging collaboration among the group members. Ina had been told that she was the manager of the building project and had started to sort the Lego pieces and find out what pieces belonged together, such as everything connected to human figures. In this process, she was classifying the pieces

according to certain attributes. As mentioned earlier, Bishop (1988a, b) considered classification to be a form of Explanation. Therefore, Ina's utterance about classifying was labelled as Explaining.

Throughout the conversation, Jil placed herself a little to the side. She did not take any verbal initiative to join the building. Her only question was directed at the teacher: "Are we good at building Lego?", and was not related to the actual construction process. Her utterance highlighted her need to gain support for her feelings and to be seen as part of the group by the teacher. This, therefore, makes it a part of the conversation, but not a part of a mathematical conversation because the content did not include mathematics.

The children were enthusiastic about participating, which was evident in Dan and Chi's disagreement about who should open the bag. They were engaged as they tried to get the pieces out of the bag.

Excerpt 2

In this segment, the teacher was more involved and Dan was the centre of the interaction.

		[Bishop's activities]
Chi:	What do we do now?	
Dan:	Now we need two green.	Counting Explaining
T.:	Where do you think this piece should be, Ina? [Ina is sorting pieces and does not answer.]	Locating
T.:	Now we need to find pieces the same as this [picks up a Lego piece and shows it to them]. We need five. How many do we have Dan?	Designing Counting
Dan:	One, we need one, two, three, four. [Counts the pieces in the instruction manual]	Counting
Dan:	Here are two more.	Counting
T.:	This ...	
Dan:	The tyre is hard to stick on.	Explaining
T.:	Wait a minute.	
Dan:	Now there should be two white on each side and a blue brick.	Counting Explaining Locating
T.:	Which way should it go? .... It should be like all the others. Dan, press slightly on the piece in front, there. There you go.	Locating Explaining
Dan:	There must be some kind of piece on top here.	Designing Locating

T.:	Do you see that the green one should be up against the grey?	Locating Designing
Dan:	The grey should be in the middle. First this one and then the other one on the other side, there. Here we go.	Explaining Locating Counting
T.:	Great.	
Dan:	I have put it on.	Locating

The conversation among the children gave them the opportunity to inquire into and use their mathematical knowledge. As Gjems (2009) indicated, sharing knowledge is an important characteristic of a conversation because the participants become aware of their own and other's knowledge. For instance, when Chi asked the question: "What do we do now?" Dan answered: "Now we need two green."

Through his answers in this segment, Dan showed that he was engaging in the mathematical activities Counting, Explaining and Locating. When the teacher asked about the number of pieces they had, Dan counted the pieces and figured out how many they needed also by counting. He answered: "One, we need one, two, three, four". By responding in this manner, he showed that he understood the question and that he had the relevant knowledge, connected to the mathematical activity, Counting, to answer it. Dan also showed a connection to Bishop's activity Explaining when he described what he was building and why. For example, "the tyre is hard to stick on" was an instance of Explaining as it provides the reason for why it took so long to put the two pieces together. In the utterance, "The grey should be in the middle", the term "middle", was a spatial reference and indicated that he was engaging in Locating.

It is important for the learning process for children to be aware of their own knowledge. For example, Vygotsky (2001) considers that verbalising inner speech is crucial for learning. In this exchange, opportunities for reflection were supported by the group interaction and conversation structure. For example when Dan made the statement "there must be some kind of piece on top here", he appeared to be wondering about possible solutions to his building problem. He expressed his uncertainty, but showed that he had some understanding about what piece should be on top. Here he was engaging in Explaining and displaying some knowledge about Locating. With his utterance, he showed the rest of the group that he understood the next step but he also allowed them to provide suggestions about what should be done. The teacher also made him aware of the picture in the instruction booklet by asking, "Do you see that the green one should be up against the grey?" The teacher provided Dan with an opportunity to learn how to solve the problem, by consulting the picture

in the instruction booklet. In the process, he learnt how to interpret the booklet, which was also an aspect of learning about the mathematical activity Locating. This conversation, therefore, can be considered mathematical. As well as discussing the mathematical activities, it also seemed to provide an opportunity for further learning, which can be considered an important characteristic of mathematical conversations.

However, all of the participants are not involved in the building process. Jil, for example, did not participate actively, which Gjems (2009) suggested would restrict her learning. However, this conversation may have provided Jil with opportunities for passive learning. Wells (1999, p. 84) emphasised that:

> experience is not what happens to a person, but the meanings that are constructed in the course of participation in the successions of events that make up his or her life trajectory, as these events are construed in terms of the individual's existing model of the world.

Therefore, by making meaning of what the other children were doing, Jil may have also learned something. It is just that an analysis of the children's conversation does not reveal this.

The use of questions in the conversation varied from problem-solving invitations like "Which way should it be?" to suggesting actions such as "What is it a picture of?" and concluding questions like "Do you see that the green one should be up against the grey?" Through the use of questions, the teacher structured and thus orchestrated the conversation. Through her questions, she directed the children's attention to specific parts of the building process.

This excerpt was also structured by the task of building the airplane. Of the four children, Dan in particular showed his knowledge in building Lego, which may have contributed to supporting the development of the skills of the whole group. From Vygotsky's theory of zones and Bruner's concept of scaffolding (Wood, Bruner & Ross, 1976), Dan appeared to be more skilled and, through scaffolding, together with the teacher, could have supported the others to do more than they could do by themselves. The other children could observe Dan and participate in the building process following his example. This is discussed after the next extract.

However, at times Dan was also confronted by the teacher, as, for example, when she challenged him to respond to the question; "Do you see that the green one should be up against the grey?" From his response, it is possible to see this question as being in Dan's zone of proximal development – "The grey should be in the middle. First this one and then the other one on the other side, there". The question has forced Dan to elaborate on what he knows about how the bricks should be placed.

Gjems (2009) emphasised that it is important for children to be challenged so they could develop their knowledge. As Dan's knowledge was challenged, these interactions increased all the children's possibilities to benefit from the building process (Wells, 1999). Thus, this illustrates the importance of incorporating challenges in mathematical conversations in kindergarten.

In the third excerpt, the teacher actively constructed a conversation in which the children were encouraged to work with the building task. As well as asking questions, the teacher praised and supported the children's utterances and actions with statements such as "there you go", "here we go" and "great". Encouraging the children in their building was likely to enhance the possibilities of learning.

Excerpt 3

		[Bishop's activities]
T.:	How will you build it? [Points to the instruction booklet.]	
Dan:	We need tyres.	Explaining
Chi:	We can look in the booklet. [Turns page] There were many red squares.	Explaining
Jil:	[picks up a lot of things]	
T.:	What are you thinking about Chi?	
Chi:	We need to have white and blue.	Explaining
T.:	What is this a picture of? [Points to the instruction booklet.]	
Chi:	The steering wheel.	Explaining
T.:	Where should it be?	
Cni:	Right behind.	Locating
Jil:	[Moving Lego pieces]	
Ina:	Jil stop just taking ... [uses a high voice]	
Ina:	Do you need a grey brick? You need a chair in between the steering wheel.	Explaining Locating
Dan:	I have a white.	Explaining
Ina:	We need many white pieces.	Counting
Chi:	We have reached 10. [She shouts it out to another group. "10" refers to a stage in the booklet.]	Counting
X:	We have reached 9 ... [Says someone from another group].	Counting
Dan:	We need more of these pieces.	Counting

In this excerpt, the construction project had progressed with the children participating in different ways. The teacher asked the question "Where should it be?" in order to challenge the children to wonder. It was an open question, but by pointing to the instruction booklet, there was a clear suggestion of what an appropriate action could be. The instruction booklet gave the children an opportunity to compare the plane they were building with the pictures in the booklet.

The teacher's questions were an essential part of the conversation structure, in that the participants could both take the initiative and be invited to participate. Taking the initiative is connected both to posing a question and to responding to one. The use of open questions, and the wondering attitude they promoted, was important in the development of communities of inquiry, and in creating communities that cultivated mathematical conversations (Carlsen, et al., 2010). Therefore, the structure of a mathematical conversation requires the inclusion of more wondering questions than may be found in an everyday conversation.

The children engaged in four of Bishop's (1988a, b) mathematical activities as they built the plane. For instance, the children sorted the Lego pieces into different categories like tyres and steering wheels, and according to colour and length. Bishop (1988a, b) considered classifying to be a form of Explaining because a specific set of attributes was used in the sorting process. The children also used Counting, to keep track of where they were in the building process. Utterances like, "we have reached 10" or "first this one, then ..." are connected to the ordinal rather than the cardinal number understanding of Counting. References to spatial relationships in utterances like "right behind", and shapes such as "wheel" relate to Bishop's activities Locating and Designing, respectfully. Designing is the ongoing activity. They are building the airplane together and while they are doing so they comment on the actual design and how to build it in accordance with the instructions.

Although it was useful for the children to have an adult who gave directions and advice, the children also used each other as resources. Dan's earlier building, observed by the other children, could have contributed to them being more active in this final extract. By sharing this experience, the children are in the process of what Wells (1999) calls knowledge building. The participants are experiencing the building process together and acting upon this to build knowledge.

Characteristics of mathematical conversations

The aim of this article was to describe the content and structure of mathematical conversations in kindergarten. From the analysis of the

extracts and references to the literature, I would suggest that mathematical conversations in kindergarten include five features 1) being mathematical, 2) structuring, 3) requiring reflection, 4) involving interaction among participants and 5) aiming at further learning. The components *being mathematical*, *reflecting* and *aiming at further learning* are related to the content in the conversations; while *structuring* and *interaction among participants* are related to the structure of mathematical conversations.

Being mathematical

This refers to the fact that the conversation was related to one or more of Bishop's six activities. In these three excerpts, the children appeared to be engaged in four of Bishops (1988a, b) activities.

Many of the conversations illustrated how several mathematical areas were present simultaneously. Lego-building is concerned with Bishop's Designing and Locating activities. This is illustrated in Dan's statement: "there must be some kind of piece on top here"; and Ina's comment: "you need a chair in between the steering wheel." The children also engaged in the mathematical activity Explaining a lot, such as when Chi told the others, "we need to have white and blue". Finally, Counting was evident when Dan proclaimed: "One, we need one, two, three, four".

The content of the mathematical conversation in kindergarten must include reference to at least one of Bishop's six mathematical activities. Although mathematical conversations need to be about mathematics, this characteristic is not sufficient in itself.

Structuring

The structuring of the conversation in and around the activity influences the conversation. Structure is the aspect of the conversation that keeps it on track. A conversation without a clear structure will easily drift towards other topics. A mathematical conversation should be structured so that the mathematics is not incidental but central, with the participants being invited to inquire further into this feature of the activity. By posing a variety of questions in a way that involve the participants in an inquiry, kindergarten teachers can orchestrate conversations so that they stay focussed on the mathematical aspects of the situation.

Motivation and encouragement are also part of structuring the conversation. Often this is achieved through the teacher's orchestration. For example, when the teacher praises Dan in excerpt 2.

The Lego conversation was structured around the building of the airplane. In general, a conversation is always structured around the topic under discussion and the activity one is engaged in. The structure of the

topic shapes the way the children can act and participate in the conversation. The conversation among the participants developed around the Lego construction: First, there was an organisation phase and then there was a building phase. In each of these phases, certain utterances were more relevant than others. For instance in the first excerpt, Jil asked, "are we good at building Lego?". However, no one responded orally, perhaps because what she said did not fit other participants' expectations about what was needed in the organising phase. By only responding to her utterance with a smile, the teacher did not legitimise this utterance as being a part of the mathematical conversation.

A common tool for structuring conversations is the use of questions. In the Lego construction context, the questions posed by the teacher and the children determined the course of the conversation and, as was already noted, kept it focused on mathematics.

Requiring reflection

In mathematical conversations, the participants reflect on mathematics. This may be related to the types of questions being asked in the conversation. In addition, other utterances such as those used when engaging in inquiry can foster participants' reflection. For example, wondering acts (Lindfors, 1999) can contribute to reflecting. Reflecting on mathematical knowledge opens up possibilities to express and develop mathematics in elaborate ways.

In order to be called mathematical conversations, verbal exchanges must engage the participants in mathematical reflection. For example, in the conversation about Lego building, the teacher asked, "How will you build it?" With this question, the teacher tried to promote reflection among the children by indirectly referring to what they did in previous tasks by pointing at the instruction booklet. There was no direct instruction but the teacher wanted through her gesture to remind the children of what they had done previously. After reflecting on the question and listening to the other children's responses, Dan stated, "We need more of these pieces". He realised that he did not have enough Lego pieces and needed some more. Another example was the teacher's question: "what are you thinking about Chi?" and Chi's answer: "we need to have white and blue." In this way, the teacher told Chi that she had observed her and that she looked like she had been thinking about something. In addition, Chi's answer showed that she had found the answer to her problem. Reflections about the mathematical content, such as Chi is engaged in in this excerpt, show how the mathematical activity Explaining is essential for reflecting in a mathematical conversation.

Involving interaction among participants

Learning is a social practice and for a conversation to be mathematical there has to be interaction among the participants in such a way that all are actively engaging with mathematical ideas. Knowledge is constructed and reconstructed in conversations between people who do things together and talk together (Gjems, 2009). In the Lego conversation, there were interactions between Dan and the teacher. In addition, there is a short exchange between Ina and Dan in the last excerpt: "Do you need a grey brick?", this illustrated how they negotiate which pieces they needed. By interacting, the participants were involved in the conversation. This is a characteristic of all conversations, but since the participants were engaged in a common activity like building the Lego-airplane, it was a property of a mathematical conversation.

Aiming at further learning

For a conversation in kindergarten to be mathematical, it has to include opportunities for further learning. According to Vygotsky (2001), the child's intellectual growth depends on the child mastering the social means of thinking, which is language. Conversations are a useful tool for development in the children's *zone of proximal development* (Vygotsky, 1978) and can provide scaffolding (Wood, et al., 1976) for the participants' development. Through conversations, there is the potential of accomplishing more together than individually. From the excerpts, it is possible to see that the participants had an ongoing conversation in which they exchanged and perhaps created new knowledge. Dan, in particular, was very active and it seemed that he built up the other group members' knowledge. For example, in his conversation with the teacher in the second excerpt the teacher asked: "Now we need to find pieces the same as this [takes up a Lego piece and shows it around]. We need five. How many do we have Dan?" and Dan answered, "one, we need one, two, three, four [counts the pieces in the instruction manual]." For the children who were not yet counting fluently, Dan's counting could have increased their understanding of how to solve problems with quantities.

The teacher's questions arose spontaneously and helped structure the conversations in a way that included both reflection on current activity and supporting children to develop further learning about building the plane. By being present and answering questions, the child made use of the teacher's questions to find the solution. From the way the teacher posed the question, Dan may have learnt to work out what to do when similar problems occur at a later time.

Conclusion

Based on the literature and the analysis of the extracts, I have proposed five characteristics for mathematical conversations. For a conversation to be considered mathematical, it must involve the participants in discussing mathematical knowledge and be structured so that participants reflect on what they had done, as well as possible future actions. It is also important to consider how the conversation is structured; how participants' contributions are valued in the on-going interactions, how the participants reflect on the mathematical theme and how the conversation promotes further learning. Thus, a kindergarten teacher can inspire conversations that are mathematically enriching. Such conversations can be carefully planned or arise more or less spontaneously. A topic for future research would be to consider the teachers' role in mathematical conversations and how they can orchestrate these conversations. A dynamic environment for mathematical conversation can be achieved when children participate in a variety of conversations in a community of inquiry.

In this paper I have described the content and structure of mathematical conversations in kindergartens. The kindergarten teacher can initiate and promote mathematical learning by focusing on the five features of these conversations; these are related to content like *being mathematical*, *reflecting* and *aimed at further learning*; and structure like *structuring* and *interaction*.

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Notes

1 Lego is the brand name of plastic blocks that come in construction kits.

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