# Improving word problem performance in elementary school students by enriching word problems used in mathematics teaching

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The traditional practice of using word problems in classroom mathematics has been heavily criticized by educators and researchers. Students have an inclination to apply superficial strategies and exclude several important steps of the modeling process. In this study, a Word Problem Enrichment program (WPE) was developed to encourage teachers to use innovative self-created word problems to improve student mathematical modeling and problem solving skills. Participants included 170 fourth-, and sixth-graders and ten teachers from elementary schools in southwest Finland. The intervention effectiveness on student problem solving performance was investigated. The results suggested that the enriching word problems used in mathematics teaching is a promising method to improve student problem solving skills when solving non-routine and application word problems.

Mathematics has always been a major part of school curricula in most countries. It provides a variety of useful tools that help students to solve problems that they encounter in everyday life (Muller & Burkhardt, 2007). Mathematical tools are considered powerful, as long as one knows how to employ them across a range of suitable situations (Lave, 1992). However, selecting and using these tools appropriately appear to be challenging for many students (Muller & Burkhardt, 2007). Because of this, word problems have been included in mathematics education to offer practice for students in applying mathematical skills effectively in various problem situations confronted in everyday circumstances (Verschaffel, Greer & de Corte, 2000).

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A word problem is text which describes a situation with question(s) to be answered by performing mathematical operation(s) based on a provided set of descriptions (Verschaffel et al., 2000). Word problem solving refers to the whole process of engaging with a word problem in order to solve it. In this paper, three types of word problems are discussed (see table 2): routine, non-routine, and non-routine word problems requiring the use of realistic considerations (or also called application word problems). First, routine word problems are word problems that can be solved straightforwardly by a routine application such as the keyword approach (a strategy to solve word problems with the help of individual words, e.g. ”together” = addition). Students could solve these word problems by applying simple arithmetical operations using the numbers provided in the word problem.

In contrast, non-routine word problems cannot be solved by straightforward strategies. They require students to have a deep understanding of the context of the word problems in order to solve them correctly. By deep understanding we mean an ability to understand the situations described in word problem texts (Cummins, Kintsch, Reusser & Weimer, 1988). This ability can be achieved by reading word problem texts attentively and paying attention to all relevant aspects of the problem description. To gain understanding of the context of word problems is an important step in the modeling process before one can construct a proper situation model. Lastly, application word problems are similar to non-routine word problems. One additional requirement is the use of non-direct translation of the word problem texts on the basis of real-world knowledge and assumptions into the mathematical model. For example, the bus problem: ”450 soldiers must be bused to their training site. Each army bus can hold 36 soldiers. How many buses are needed?” Instead of the answer ”12.5 buses” which derives from a mathematical model translated directly from the problem’s statement (450 ÷ 36), students need to consider whether their answer is appropriate for the situation being modeled, and provide an alternate more suitable answer (13 buses).

Word problems have been used to teach and learn mathematical modeling and problem solving, as well as to connect the real-world to the mathematics classroom (Verschaffel et al., 2000). By real-world we mean the world outside mathematics that one experiences directly and indirectly (e.g., through the media, other people’s experience) in everyday life. It was assumed that word problems would foster realistic reasoning and mathematical modeling in students. Realistic reasoning is defined as a process of thinking derived from reasonable logic that is adopted to produce assertions and reach conclusions in problem solving (Verschaffel et al., 2000). However, throughout the past few decades, the traditional practice of using word problems in classroom mathematics has been heavily criticized by several educators and researchers. It has become evident that after being immersed in the traditional practice of word problems in school, students have an inclination to apply superficial and non-realistic strategies, and exclude several important steps of the modeling process (Cummins et al., 1988; De Corte, Greer & Verschaffel, 1996; Dewolf, Van Dooren, Ev Cimen & Verschaffel, 2014; Greer, 1992; Van Dooren et al., 2005; Verschaffel et al., 2000; Verschaffel, Greer, Van Dooren & Mukhopadhyay, 2009). Many students solve word problems by working instantly on the mathematical operations with given numbers without a deep understanding of the context of word problem and the proper use of realistic reasoning.

In this study, a *Word problem enrichment* program (WPE) was developed. The WPE included a professional development program that encouraged teachers to use innovative self-created word problems to improve students' mathematical modeling and problem solving skills. The purpose of this study was to investigate the effectiveness of the program on student problem solving performance.

## Theoretical and empirical background

It has been widely believed that learning mathematics in a meaningful context could enhance transfer between school mathematics and a wide variety of contexts outside of school (Boaler, 1993). Experience with word problems could create a meaningful bridge for connecting classroom mathematics to real-life applications. However, several studies have indicated a linkage problem between mathematics and the real-world (e.g., Greer, 1993; Verschaffel, De Corte & Lasure, 1994). Students develop their own problem solving behavior which clearly shows that they do not see the importance of context in learning word problems. Students use superficial comprehension strategies and do not develop an adequate understanding of the situations described in word problem texts. They try to start immediately on the mathematical operations with the given numbers without basing the mathematical model on a proper situational model (Reusser & Stebler, 1997; Van Dooren, De Bock, Vleugels & Verschaffel, 2010). Moreover, even if students use deeper comprehension strategies they have difficulties in making proper use of realistic thinking. Evidence to support this claim is presented in two related studies conducted with 100 lower-secondary students in Northern Ireland (Greer, 1993) and 75 upper-primary students in Belgium (Verschaffel et al., 1994). In these studies, students were asked to solve two types of word problems: routine and application word problems. The findings of these two studies (Greer, 1993; Verschaffel et al., 1994) demonstrate that students have a very strong tendency to exclude real-world knowledge and realistic considerations when solving application word problems. This issue was brought to the forefront when replication of these two studies in various countries consistently reported evidence of elementary school students not making use of realistic mathematical modeling in their solutions (for an overview, see Verschaffel et al., 2000; Verschaffel et al., 2009; Verschaffel, Van Dooren, Greer & Mukhopadhyay, 2010).

Characteristics of word problems included in traditional textbooks and the way in which word problems are conceived and handled by teachers in regular mathematics lessons are two of the main reasons why students use superficial comprehension strategies, do not construct adequate situation models, and neglect common sense knowledge and realistic considerations (Verschaffel et al., 1999; Verschaffel et al., 2000).

### *Characteristics of word problems in traditional textbooks*

Traditional word problems do not provide opportunities for students to practice their problem solving and realistic reasoning skills. Most word problems usually include a simple goal and only the numbers required to solve them (Cognition and Technology Group at Vanderbilt, 1992b). Moreover, traditional word problems often request a precise numerical response which leaves little to no room for realistic considerations to be integrated into the solution process (Freudenthal, 1991). Though the situation has changed over time, the current situation of word problems in school mathematics textbooks remains unresolved. A comparative analysis of Greek mathematics textbooks by Gkoris and colleagues (2013) revealed that neither old nor new mathematics textbooks for grade 5 provided teachers or students with varying word problems that included more application word problems. Around 90 percent of word problems in old and new textbooks can be solved by direct translation of the problem texts into the mathematical operation without the need for any realistic considerations. Similar to Greek textbooks, in 5th grade Finnish mathematics textbooks, 94 percent of word problems are word problems that include a simple goal and always have only one correct answer (Joutsenlahti & Vainionpää, 2008) indicating the lack of application word problems.

Several researchers argued that the stereotyped nature of word problems in traditional textbooks encourage students to use superficial solving strategies such as the keyword approach without constructing an adequate model of the situation described in the word problem (De Corte & Verschaffel, 1986; Verschaffel et al., 2000). Traditional word problems have been described as too simple or straightforward, and solved easily by using a superficial strategy (Schoenfeld, 1991; Wyndhamn & Säljö, 1997). Gravemeijer (1997) pointed out that students developed their problem solving behavior according to the given word problems. When given mainly routine word problems, students developed a habit of skipping several important steps of the modeling process. The students read the problems superficially to determine what calculation to perform without any critical reflection of whether the calculation fits with the context of the word problem (Van Dooren et al., 2010). This superficial solving behavior in elementary school students strongly suggests a lack of heuristic, metacognitive, and affective aspects of mathematical competence (Vauras, Kinnunen & Rauhanummi, 1999; Verschaffel et al., 1999). Many students do not apply useful heuristic strategies and meta-cognitive skills (e.g., drawing of the problem situation, decomposing the problem into parts, or checking the answer) in their solution processes (Kajamies, Vaurus & Kinnunen, 2010; Lester, Garofalo & Kroll, 1989).

Many word problems in textbooks do not represent the characteristic of problems that usually occur in real life. Problem is a situation that challenges one intellectually who is not in immediate possession of the direct procedures or methods to answer the questions (Blum & Niss, 1991). By problem solving we mean an entire process of dealing with a problem for which the solution method is not known in advance (National Council of Teachers of Mathematics, 2000). However, it has been found that the sequence of word problems presented in traditional textbooks (e.g., providing students with whole pages of the same type of word problems and examples illustrating how to solve that particular type of word problems) is already clear to students (Pongsakdi et al., in press; Stigler, Fuson, Ham & Kim, 1986). By presenting word problems in this manner, students merely practice their computation skills by recalling facts and imitating a solution procedure illustrated in the textbooks without using conceptual understanding and proper mathematically founded reasoning (Boesen, Lithner & Palm, 2010; Jonsson, Norqvist, Liljekvist & Lithner, 2014; Lithner, 2008; Porter, 1989). According to these criticisms, it can be concluded that, in pedagogical practices, word problems are often trivialized and do not fully serve their purpose in connecting classroom mathematics with real-world mathematics.

### *Teacher conception and use of word problems*

Teacher actions in mathematics classrooms have been found to be influenced by their beliefs about mathematics and teaching mathematics (De Corte et al., 1996). It was also found that the way in which students are likely to view learning mathematics is influenced by teachers' actions in the classroom (Carter & Norwood, 1997). Chapman (2006) presents empirical evidence to support these findings where the classroom culture of teaching mathematical word problems by 14 elementary, and high school teachers was observed. Chapman (2006) adopted a cognitive functioning theory from Bruner (1986) to create a distinction between a *paradigmatic-oriented* and *narrative-oriented* approach to word problems. The paradigmatic approach is based on conceptualization which concentrates on the mathematical model or structures that are universal and context-free. For instance, the teacher suggested students see how the mathematics structure could be independent of the context. In contrast, the narrative approach deals with the social context of the word problem which focuses on context-sensitive and particular explications (e.g., situations, storyline). The teacher, who held this narrative-oriented perspective, provided opportunities for students to describe situations of the problem in which students saw themselves, or real-world experiences, and used that in handling with the problem (Chapman, 2006).

According to Bruner (1986), these two modes of cognitive functioning cannot be treated separately, as "Efforts to reduce one mode to other or to ignore one at the expense of the other inevitably fail to capture the rich diversity of thought" (p.11). However, overall findings of Chapman's study (2006) suggested that the paradigmatic approach was more dominant and was combined with the narrative approach differently among the teachers. More recently, Depaepe and colleagues (2010) conducted a study to investigate how two upper elementary school teachers treat word problems in their actual pedagogical practice. The results revealed that the word problem lessons provided by the two teachers were more characterized by a paradigmatic than a narrative approach. These suggested that the narrative approach used in the classroom should be more emphasized.

### *Previous research on improving modeling and reasoning skills*

In response to these discussions on the impact of traditional textbooks and pedagogical practice towards word problems, several researchers have designed experimental programs aimed at enhancing students' mathematical modeling and reasoning skills (Blum, Galbraith, Henn & Niss, 2007). Verschaffel and De Corte (1997) conducted an experiment aimed at improving realistic mathematical modeling by using application word problems. The results of Verschaffel and De Corte's (1997) study indicated that it is possible to enhance realistic mathematical modeling by integrating more application and less routine word problems into mathematics classrooms.

A study in the U.S. (CTGV, 1992a) used new information technologies called "The Jasper Series" to promote problem posing, problem solving, reasoning, and effective communication in students. The Jasper Series was developed based on the idea of anchored instruction which used real-world situations including challenges to provoke thoughtful engagement that helps in the development of critical thinking and realistic reasoning skills. Together in small groups, students explore the word problem, search for extra information required to solve the word problem, discuss possible options, and develop solutions. The findings indicated that the program has a positive impact on problem solving skills and planning word problems.

Although these programs proved to have positive effects on students, this does not guarantee successful implementation in a large-scale setting. Literature from various fields of study has revealed a gap between research and practice (Vanderlinde & Van Braak, 2010). It is argued that studies conducted in controlled settings miss the range of "messy variables" that might occur in real-life (DeAngelis, 2010). In pedagogical practice, the challenges of integrating research findings into practice could include difficulties in implementation and limitation of resources. Concerned about these issues, we tried to find effective and widely applicable pedagogical methods, which could improve mathematical modeling and problem solving skills. Inspired by the works of Verschaffel and De Corte (1997) and CTGV (1992a), we developed a *Word problem enrichment* program (WPE). The idea of the WPE is to provide examples of non-routine and application word problems that resemble those presented in Verschaffel and De Corte (1997) and CTGV (1992a) for the teachers with additional guidelines on how they can create innovative word problems themselves or together with students. The teachers were encouraged to use more non-routine and application word problems with meaningful context that related to real-world in mathematics classrooms. The WPE included only the professional development of teachers and no previously planned procedures which experimental group teachers should apply in detail in their teaching. Thus the effects of the program were dependent on the way teachers in the experimental group applied the new ideas and skills (e.g. skills to produce themselves pedagogically meaningful word problems) provided by the WPE in their own teaching. However, we assume the WPE would have a positive impact on students' problem solving skills.

The purpose of this study was to investigate whether the WPE, which was aimed at facilitating teachers to develop their own word problems and implement these innovative word problems in their teaching, would influence student achievement in solving word problems. In this study, we attempted to answer these two research questions:

1. Does the *Word problem enrichment program* improve student problem solving skills on non-routine word problems when compared with traditional mathematics teaching?
2. Does the *Word problem enrichment program* enhance student problem solving performance on an application word problem when compared with traditional mathematics teaching?

## Method

### *Participants and overall design*

This study used a quasi-experimental pre-test post-test design. A total of ten teachers and 170 students from fourth- and sixth-grade participated in this study. Even though the students were drawn from different elementary schools located in socioeconomically varied areas in southwest Finland, the households were predominantly middle-class.

The experimental group consisted of 5 teachers and 98 students, and the control group comprised of 5 teachers and 72 students (see table 1). The experimental group teachers ($n$ = 5) were those who volunteered to participate in the professional development program. The other teachers ($n$ = 5) were not offered to join the program, but they volunteered to participate in the study as a control group, and they followed traditional practice of word problems in mathematics lessons.

Table 1. *Number of participants per grade and experimental condition*

| | Condition | | | |
|---|---|---|---|---|
| | WPE | | Traditional | |
| Grade | Student | Teacher | Student | Teacher |
| 4 | 62 | 3 | 49 | 3 |
| 6 | 36 | 2 | 23 | 2 |
| Total | 98 | 5 | 72 | 5 |

### *Professional development program*

The experimental group teachers entered into a professional development program which was organized over three afternoon seminars each lasting around three hours. Each seminar was arranged once a month between January and March 2013. The professional development facilitators consisted of an expert who worked for the Centre of teacher training in mathematics (Turun Matikkamaa) and a researcher from University of Turku.

In the first seminar, several issues related to the use of word problems in traditional mathematics lessons, such as beliefs about word problems and the stereotyped nature of school word problems, were actively discussed. The teachers were shown the empirical evidence concerning the impact of traditional textbooks and pedagogical practice on students' problem solving, and realistic reasoning skills (Verschaffel et al., 2000). Moreover, it was emphasized to the teachers why it is important to implement a narrative-oriented approach in their teaching and how that can be achieved.

In the second seminar, the main purpose was to emphasize that many word problems in regular textbooks were too simple, and inhibited a genuine disposition towards mathematical modeling in students. Several examples of non-routine and application word problems were introduced to the teachers with additional guidelines specifying how they could create innovative word problems themselves or together with students. When solving these word problems, the teachers were advised to instruct their students to apply two steps of heuristic strategies based on Verschaffel et al.'s (1999) study; a) build a mental representation and b) decide how to solve the problem.

In the last seminar, teachers were guided on how to create non-routine and application word problems that are interesting for their students and also related to real-world situations. Real-world situations refer to the situations that one experiences directly and indirectly in everyday life. However, the situations that one perceives as "real" might be differed, due to their previous experience and cultural background (Pongsakdi, Brezovszky, Hannula-Sormunen & Lehtinen, 2013). Moreover, for elementary school students, the fantasy world of fairy tales could be also considered to be realistic, since they are real in the student's mind (Depaepe, De Corte & Verschaffel, 2009). Therefore, the concept of real-world situations that was emphasized in the seminar are not limited to their possible occurrence in the real-world, but rather situations that students can conceive.

The teachers were suggested to use various sources available on the Internet to create stories of the word problems. Ideally, the word problems are similar to those word problems presented in Verschaffel and De Corte (1997) and CTGV (1992a) that provided opportunities for students to use their imagination and real-life applications to think of possible solutions, and discuss their thoughts in small groups to develop solutions. An example of a non-routine word problem with real-world situation developed by the teacher and her students is provided in appendix A.

Unlike typical intervention studies that had systematic instruction on how the experimental group teachers should implement the new method in the classroom, in this study, we were concerned about ecological validity, and tried to maintain natural settings as much as possible. Therefore, after the program, researchers did not interfere at all in the classroom teaching, but let teachers naturally apply the new ideas and skills that they learned from the professional development program in their teaching by themselves. This follows the general idea of teacher autonomy in Finnish comprehensive schools, that the teachers had the freedom to design their own teaching.

### *Measures*

Student problem solving performance was assessed with a word problem test, containing five word problems: 1 routine, 3 non-routine, and 1 application (see table 2). A *routine word problem* was an adaptation from a typical routine word problem which is often used in textbooks. This routine word problem was used as a warm up word problem. *Non-routine word problems* were constructed in such a way that they could not be solved by straightforward strategies. For example, avoid using keywords in the word problems and provide meaningful data in the written form instead of numbers. An *application word problem* was adapted from an original word problem mentioned in Depaepe and colleagues' (2009) study which demands the use of realistic considerations. However, this application word problem is slightly different from the bus problem used in Verschaffel et al.'s (1994) study, since the use of realistic considerations is integrated in the problem. To clarify, in order to solve this problem correctly, it requires students to understand the situation of the word problem that there are 4 children who participate in the party instead

Table 2. *Example of word problems used in pre-test (translated to English)*

| Type | Word problems |
|---|---|
| Routine | Pekka has 7 adventure books. Pirkko has 6 adventure books more. How many adventure books does Pirkko have?<br>(correct answer: 13) |
| Non-routine | There was a bowl full of chocolate pieces on a desk. Liisa took 2 pieces of chocolate every day. After two weeks all the chocolate pieces were gone. How many chocolate pieces were there at the beginning?<br>(correct answer: 28) |
| Application | Paula is preparing some food and drinks for her birthday party. She buys two packets of chips (1 packet costs 2.50 euros), a big packet of mixed candies (1 packet costs 3.60 euros), and 4 bottles of lemonade (1 bottle costs 1.25 euros). Three friends come to the party. How much does the snacks and drinks cost for each participant?<br>(correct answer: 3.4) |

of 3 children (see table 2), but for the bus problem, students need to consider whether their original answer is appropriate to the situation being modeled (e.g., answer 5 buses instead of 4.5 buses).

A parallel version of the word problem pre-test was developed for the post-test. The problems were structurally identical at different measurement points with the same given values, but the problem contexts differed. The number of word problems included in the test was quite small. However, in this study, it is important for us to understand how students solved the problems when there was no time pressure and overwhelming number of word problems.

## *Procedure*

The pre-test was administered to students by their teachers at the beginning of the professional development program (the beginning of the spring semester of 2013). The parallel test was given to students two months after the professional development program (the end of the spring semester of 2013) in the post-test. Students were instructed to describe how they solved each word problem as well as explain how they understood the word problem either by writing short descriptions or using visual representations (e.g., drawing picture, chart). Students had around 35 minutes to do the test.

## *Analysis*

### Scoring systems

Two types of scoring systems were used to analyze different types of word problems. For routine and non-routine word problems, 1 point was given for each correct answer and 0 for an incorrect answer, or no response. For an application word problem, it appeared that students had difficulties to do calculation, especially multiplication and division with decimal numbers. Many students showed that they understood how to solve the application word problem (e.g., writing the mathematical model, explaining the situation by drawing pictures), but they could not complete the calculation, or made calculation errors. In this study, we emphasized on how students understand the context of word problems and whether they are able to create a mathematical model derived from a proper situational model, therefore, although students made calculation errors, 3 points were given if students could provide a completed correct mathematical modeling (either by writing a short description or drawing pictures) that included the use of realistic considerations (see figure 1). Two points were given to students who provided a completed mathematical modeling without the use of realistic considerations (see figure 2), 1 point was given when students provided incomplete mathematical modeling (partly solve the word problem), and 0 for an irrelevant answer (the answer that is not at all related to the word problem), or no response. The inter-rater agreement between two independent coders scoring this problem is very high (κ= .89).

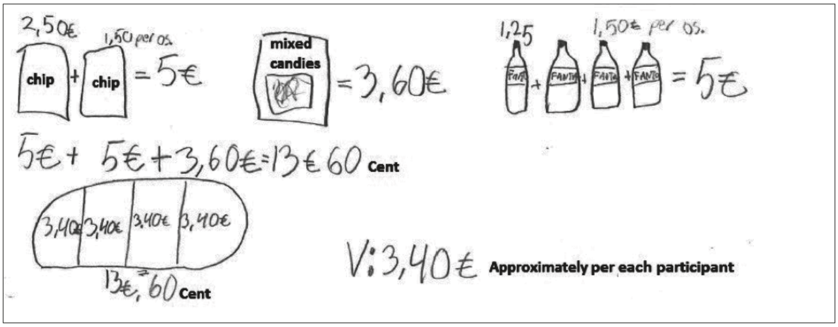


*Figure 1. Example of responses given 3 points (translated to english)*

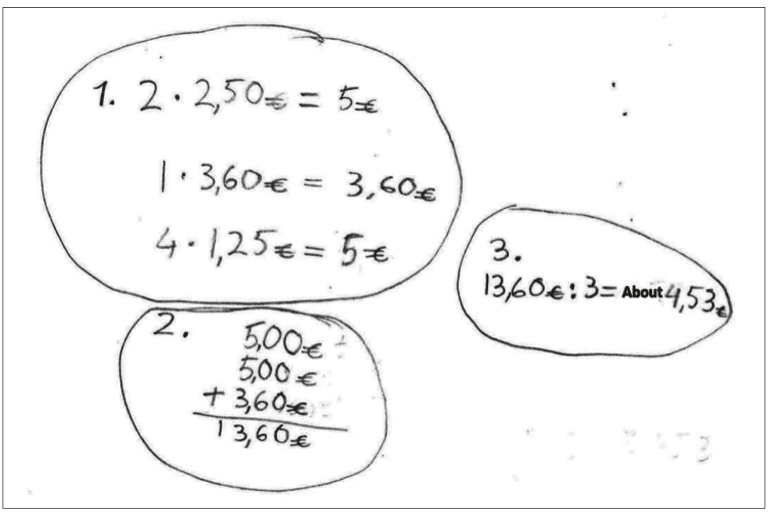


Figure 2. *Example of responses given 2 points (translated to english).*

The routine word problem appeared to be too easy (a ceiling effect occurred with the pre-test and post-test). To examine the WPE's effectiveness, the analyses were divided into two parts. The first part examined the impact of the WPE on student problem solving skills with non-routine word problems. The sum score of non-routine items was used in this part of the analysis. Cronbach's alphas for the pre-test and the post-test were .63 and .65 respectively, which are considered sufficient (Hair et al., 2006). The second part investigated the intervention effectiveness on student performance on an application word problem. Three students were excluded from the analyses as they did not attend the post-test.

**Variance component analyses**

Concerning the fact that this data is from separate classes, there might be a need for multilevel approach. However, the sample size is too small to use multilevel perspectives (Kreft, 1996). Thus, a variance component analyses of the post-test scores (pre-test scores as covariates) was conducted to compute intraclass correlation coefficients (ICC) to find out how much of the variation is explained by the class. The results showed that the ICC level of the non-routine word problems and an application word problem is 0.22 and 0.08 respectively which is lower than the cut-off level proposed by Kreft (1996). These results suggested that the nested nature of the data does not result in misleading significant test value, and the data can be analyzed using traditional methods (such as ANOVA) exclusively at the individual level (Kreft, 1996; Sagan, 2013).

## Results

### *WPE's impact on student performance with non-routine word problems*

To investigate the impact of the WPE on student problem solving performance with non-routine word problems, the sum score of non-routine items was used in the analysis. First, an independent-sample t-test was conducted to investigate student problem solving performance on the non-routine word problem pretest. Results revealed that there was no significant difference between students in the experimental group ($M$ = 1.72, $SD$ = 1.07) and the control group ($M$ = 1.49, $SD$ = 1.10) on non-routine word problem pretest scores; $t(168) = 1.49$, $p > .05$.

Table 3. *Mean (and Standard deviations) of student problem solving performance in non-routine word problems for the experimental and control group*

| Group | Time | |
|---|---|---|
| | Pre | Post |
| Experimental ($n$ =97) | 1.72 (1.07) | 2.02 (0.98) |
| Control ($n$ = 70) | 1.49 (1.10) | 1.29 (1.14) |

Then, repeated-measures ANOVA was conducted, and the results revealed that there was a significant interaction between time and group, $F(1,165) = 13.10$, $p < .001$, with a moderate effect size ($\eta_p^2 = .07$). Although the mean score in problem solving performance of students in the control group in the post-test ($M = 1.29$, $SD = 1.14$) was lower than in the pretest ($M = 1.49$, $SD = 1.10$), the decrease in mean score is not significant; $t(69) = 1.78$, $p > .05$. The result suggested that the WPE has a positive effect on students' problem solving skills on non-routine word problems.

### *WPE's impact on student performance on an application word problem*

To examine the impact of the WPE on student performance when solving an application word problem, the score of the application item was used in this analysis. Independent-sample t-test showed that there was a significant difference on application word problem pretest between the experimental ($M = 1.69$, $SD = 0.98$) and control groups ($M = 1.33$, $SD = 1.05$); $t(168) = 2.37$, $p < .05$.

Table 4. *Mean (and standard deviations) of student problem solving performance in an application word problem for the experimental and control group*

| Group | Time | |
|---|---|---|
| | Pre | Post |
| Experimental (*n* =97) | 1.69 (0.98) | 1.97 (0.97) |
| Control (*n* = 70) | 1.33 (1.05) | 1.30 (1.04) |

Repeated-measures ANOVA was conducted, and the result revealed that there was a significant interaction between time and group, $F(1,165) = 4.01$, $p < .05$, with a small effect size ($\eta_p^2 = .02$). It suggested that the WPE has a positive impact on students' problem solving skills on the application word problem.

## Discussion

Previous studies have shown that it is possible to improve student realistic mathematical modeling and problem solving skills by using more non-routine and application word problems with real-world situations (CTGV, 1992a; Verschaffel & De Corte, 1997). However, these studies were conducted in controlled settings that miss several chaotic variables (DeAngelis, 2010), such as difficulties in implementation and limitation of resources. In the present study, we tried to simplify the ideas of research findings and transfer this knowledge to the teachers through a professional development program which makes it feasible (in terms of resources) to integrate this program in a large-scale setting in the future (e.g., pre-service and in-service teacher curriculum). The aim of this study was to evaluate the effect of the Word Problem Enrichment program (WPE) on student problem solving performance on non-routine and application word problems. The results suggested that the WPE, aimed at facilitating teachers to enrich word problems used in mathematics teaching, has a positive impact on student problem solving skills. This was not only with non-routine problems (research question 1), but also an application word problem (research question 2).

Although WPE has a positive impact on student problem solving skills, the limitation of the quasi-experimental design used in this study must be considered. The experimental group teachers were those who volunteered to participate in the professional development program. This may imply that the level of teacher interest might be different between the two groups. However, the control group teachers were not offered participation in the professional development program, they volunteered to participate in the study because of their own interest in student problem solving performance. This suggested that they had some level of interest in how current teaching practices used regularly in traditional classroom teaching affected their students' problem solving performance.

Furthermore, if experimental group teachers would have been more engaged in developing their teaching than the control group, it should be reflected in student problem solving performance on both non-routine and application word problems in the pre-test. However, in the pre-test, there was no significant difference between students in the experimental group and the control group on non-routine word problem. There was only slightly better performance on an application word problem. For future studies, it could be meaningful to investigate how teacher interests in developing word problems mediate the effect of the professional development program. An additional limitation is that the results of student realistic mathematical modeling are based on one item. Moreover, the difficulties of students to handle decimal numbers included in the item made it difficult to draw a clear conclusion on the student mathematical modeling. To clarify these issues, future studies should partial out the effects of a different number of categories on problem solving and use a larger set of application word problems.

Effect sizes indicated a moderate intervention effect on student performance with non-routine word problems, and a small intervention effect on student performance with an application word problem. The smaller effect size might be due to the level of difficulty of this item.

The application word problem appeared to be the most difficult item among other items included in the test, since it required students not only to understand a complex situation of the problem, but also to use realistic considerations in mathematical modeling. This finding is in agreement with the previous study demonstrating that the tendency of students to exclude realistic considerations is deeply entrenched (Yoshida, Verschaffel & De Corte, 1997).Thus it may be difficult to see dramatic change especially within short periods of time. For the future research, qualitative methods (e.g., video record, interview) could be used for better understanding of how teachers implemented the new methods in the classroom.

Our research project tried to close the gap between research and practice. To bring theories into practice, researches worked collaboratively with an experienced teacher (an expert who worked for the Centre of teacher training in math) to design and develop the professional development program for teachers. In each seminar, we provided opportunities for teachers to share their experiences and ideas about the new methods: what the teachers thought might be useful or challenging if they would implement these methods in their teaching. By having an experienced teacher as one of facilitators, it was possible for us to convince the experimental group teachers how they could implement these methods. A successful change in the classroom culture of word problems requires a major change in two key elements: 1) the types of word problems used in mathematics lessons (CTGV, 1992a; Verschaffel & De Corte, 1997) and 2) teachers' beliefs (Verschaffel et al., 1999). Kajamies and colleagues (2010) explained that it is not enough for effective change in the classroom culture if only more application problems are included in mathematics lessons, without taking classroom teachers into account. Successful change in the classroom culture also requires a major change in teacher beliefs (Verschaffel et al., 1999). Otherwise, more realistic mathematical modeling in word problem solving will not be emphasized by the teachers (Verschaffel, De Corte & Borghart, 1997). In the WPE, we not only promoted the use of more variable application problems in the classroom, but we also emphasized to teachers how the traditional practice of word problems in classroom mathematics impacts students, and why the current practice needs to be changed. Practically, we tried to convince teachers to change their beliefs about word problems. Although evidence from this study strongly suggested that the problem solving performance of students in an experimental group (WPE) with non-routine and application problems have been improved, direct investigations on the effects of the WPE on teachers' classroom practices and beliefs about word problems should also be examined.

Despite the limitations, the results of this study corroborate the view that the Word problem enrichment program (WPE) is a feasible method to enhance word problem performance in elementary school students, and is a much needed addition to current mathematics textbooks and teaching.

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# Appendix A

## *An example of non-routine word problem with real-world situations*

Volkan, Niklas and Juho decided to go together to see a movie. Mom gave Volkan 10 euros and he took 15 euros from his own savings. All moms agreed that all boys had to come back home before 20:00. The boys started their trip at 11:00. Each boy had 25 euros with them when they left home. The boys took a bus to the town center. Niklas and Juho paid for their trip with bus cards and Volkan paid 1.20 euros for his ticket.

After twenty minutes, the boys arrived and bought bags of candy. Niklas' candy bag cost 5 euros and Volkan's candy bag was the same price as Niklas'. Juho's candy bag cost one half less than Volkan's bag.

After fifteen minutes, the boys went to get movie tickets. One 3d-movie ticket cost 12 euros. Volkan used a free-ticket that he got from his mom. It took 10 minutes to buy tickets. Because there was still 2 hours before the movie started, the boys went to buy hamburgers. Everyone bought a hamburger meal, which cost 5 euros. After eating the burgers, the boys went to spend time at the city center until the movie started. The movie lasted for two hours.

After the movie ended, the boys decided to take a bus to a shopping mall. They waited ten minutes for the bus. The bus tickets were paid similarly as earlier. Traveling to the mall took 15 minutes. Inside the mall, there was a toyshop, which had pedal car testing on-going. The boys drove 1h 35min with the pedal cars. After that they went to another shop to test game consoles.

After playing on play-station and x-box for 25 minutes, Volkan decided to buy everybody Berlin doughnuts. Six doughnuts cost 10 euros, but the doughnut shop was closing so they got the doughnuts for half price. Juho and Volkan went to buy 2 packs of football cards. One pack of cards costs 2 euros. Niklas bought a 2.50 euros lemonade bottle from a shop. This took 20 minutes. Then the boys finally went back to the bus station. Unfortunately the bus had just left. So, the boys sat down on a bench to eat the doughnuts. The next bus took 50 minutes.

When the bus arrived, Niklas had to pay for Volkan's trip with his bus card because Volkan's money had run out. The ride back took 15 minutes. The walk home from the bus stop took five minutes.

1) How much money did the boys spend on their trip to the movies? How much money did they have left?

2) How many hours did the boys spend on their trip and did they manage to get back home on time or early?

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